1 Introduction

Squat is defined as the increase of the draught of a vessel due to its movement in shallow water [1]. One can calculate ship squat in shallow water by a number of methods such as analytical method [2], numerical and experimental methods [3] and [4]. Due to the existence of complicated three-dimensional flow around the ship in shallow water, experimental methods are the most viable option and the most accurate method.

Kreitner [5] was the first who calculated the squat of a given vessel by fundamentals of fluid mechanics. Havelock [6] obtained the squat of a boat with an elliptic hull form by analytical approach. Constantine [7] obtained some results for the squat by one dimensional hydraulics theory. Naghdi and Rubin [8] calculated the squat for a certain ship by two-dimensional hydraulics method and verified their results by experimental means. Barras [1] introduced experimental formulas for real vessels that now have applications for determining the squat in shallow and narrow canals. Tuck [9, 10] obtained another experimental formula for calculating the squat that has limited application and is not suitable for all ships and velocities. Due to the squat effect in shallow water, the resistance of a ship increases [11]. This phenomenon may be measured by testing a model and determining the extra force needed for towing the model in shallow water.

In very shallow water or canal, nobody cares about the resistance increase but about colliding of the ship floor and the seabed.
is highly concerned. This article deals with such a condition and discusses avoidance of ship grounding due to squat in shallow water by experimental methods.

In this research the authors introduced a new formula for the squat of commercial vessels for block coefficient, $C_B$, ranging from 0.6 to 0.8, in shallow water.

2 Fundamentals of model testing for ship squat

For the assessment of the ship behaviour by model test, one should establish geometrical, kinematical and dynamical similarities between the ship and its model. If all the ship dimensions are scaled down by a certain values and all angles of the model are kept the same as the ship, then the geometrical similarity is achieved as:

$$\lambda = \frac{L_s}{L_m}, \quad \lambda^2 = \frac{A_s}{A_m}, \quad \lambda^3 = \frac{V_s}{V_m} \quad (1)$$

where:

- $\lambda$: Scale
- $L$: Length
- $A$: Area
- $V$: Displacement
- $s$: Subscript referring to the ship
- $m$: Subscript referring to the model

For kinematic and dynamic similarities, the Froude, Strouhal, Webber and Reynolds numbers should be the same for the ship and its model:

$$Re = \frac{Vl}{\nu}, \quad We = \frac{V^2pl}{\sigma_s}, \quad Sh = \frac{aoV}{V}, \quad Fn_s = \frac{V}{\sqrt{gh}} \quad (2)$$

where:

- $Fn_s$: Froude Number based on water depth
- $We$: Weber Number
- $Sh$: Strouhal Number
- $Re$: Reynolds Number
- $V$: Ship speed
- $l$: Ship length
- $v$: Kinematic viscosity
- $h$: Water depth
- $\rho$: Water density
- $\sigma_s$: Surface tension
- $\omega$: Vortex shedding frequency

In order to reduce the risk of large errors due to inequality of Webber and Reynolds numbers, the size of the models should be large enough.

The Froude law of similarity is satisfied for a model in shallow water, which means that the Froude number of the ship and its model are the same. This condition is used for determining the speed of the model,

$$Fn_m = Fn_s = \frac{V}{\sqrt{gh_m}} = \frac{V}{\sqrt{gh_s}} \quad (3)$$

where, subscript “m” refers to the model and “s” refers to the ship.

In reality, some errors occur when the model test results are extrapolated for the ship, which is the so called scale effect. This is due to the inequality of the other non-dimensional numbers of the model and the ship.

3 Manufacturing the model and equipping the laboratory

Several tests were planned for the experimental analysis of the squat effect of vessels in shallow water. These tests were carried out at the marine laboratory of the Sharif University of Technology. To perform the tests, some models were built, a shallow water tank was prepared and measurement and data recording tools were provided.

Two models with the series 60 hull form and block coefficient of $C_B=0.7$ and $C_B=0.75$ and with the scale of 1:40 and 1:70 were precisely manufactured. The main particulars of the models are presented in Table 1. The models which were built of fiberglass are shown in Figures 1 and 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Length [m]</th>
<th>Beam [m]</th>
<th>$C_B$</th>
<th>Model Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>0.31</td>
<td>0.7</td>
<td>commercial</td>
</tr>
<tr>
<td>2</td>
<td>2.38</td>
<td>0.323</td>
<td>0.75</td>
<td>commercial</td>
</tr>
</tbody>
</table>

Figure 1 Model with block coefficient of 0.7
Figure 2 Model with block coefficient of 0.75

The water lines, aft perpendicular, fore perpendicular and the location of the theoretical frame are marked on the models. The water lines on the model are also marked by different colour.

In order to facilitate a shallow water condition with an exact gap between the model floor and the tank bottom, an adjustable false floor is installed in the tank. The movable false bottom enables us to adjust the gap by accuracy of 0.5 millimetres.

The squat is measured by recording the images of the vertical position of the model at amidship by a digital camera. For this purpose, a measurement indicator at the tank window is installed. These indicators are calibrated by 0.5 mm and installed at two places. When the model passes the window it is in steady state condition and its draught at amidship in comparison with the
static draught indicates its squat. Figure 3 shows the measurement indicator.

![Measurement indicator](image)

**4 Method of testing**

The existing measurement system of the laboratory is only suitable for recording the resistance of the model. However, for carrying out other tests such as a shallow water test one should adopt another method. The tests were carried out at four different dimensionless water depths such as \( \frac{h}{T} = 1.05, \ 1.10, \ 1.15 \) and \( \frac{h}{T} = 1.20 \), where \( h \) is the depth of the shallow water and \( T \) is the draught of the model. The tests were initially performed at a speed of 0.2 meters per seconds and then continued by the steps of 0.2 meters per seconds for low speeds and 0.1 meters per seconds for high speeds. The test parameters for model with \( C_B = 0.7 \) and \( C_B = 0.75 \) are shown in Tables 2 and 3 respectively. Following the advice of the International Towing Tank Conference, ITTC, each test was carried out three times and was repeated if the error was considerable.

**5 Test results**

**5.1 Introducing dimensionless parameters**

Because of the relationship between the squat and the block coefficient of the vessel (Varyani [12]), the tests were carried out with two models \( C_B = 0.7 \) and \( C_B = 0.75 \) and the results were recorded. After recording the first set of the results and plotting them, they seemed meaningless and no interpretation could be extracted from them. By careful considerations, the authors generalized their results by introducing new dimensionless parameters. These parameters are \( S^* \), dimensionless squat, and \( \delta \), dimensionless gap between the seabed and the model floor. The above dimensionless parameters in conjunction with hydraulics Froude number \( (F_{nh}) \) were used for further analysis. The parameters as defined above are as follow:

\[
\delta = \frac{h - T}{h} , \quad F_{nh} = \frac{V}{\sqrt{gh}} , \quad S^* = \frac{S}{h}
\]

where \( h \) is the depth of water, \( V \) is the speed of the model, \( g \) is the gravity acceleration, \( T \) is the draught, and \( S \) is the squat.

The extreme squat may result in colliding of the ship with the sea bed. The speed of a model, at which, the model collides with the tank bottom is named crucial speed. The crucial speed of the model is also studied in this research.

**5.2 Calibrating the tests and the instruments**

In order to validate the accuracy of the tests and limit the errors associated with the equipment and operators, the results obtained in this study are compared with the result of other researches. In Figures 4 and 5, the results of the present study are compared with the result of Olivieri [3] and Kriebel [13], which is reproduced by the aid of the above-mentioned non-dimensional parameters. However, the Olivieri and Kriebel results refer to the ship with a block coefficient of 0.506 and 0.6 respectively, while this study analyzes the squat for the block coefficients of 0.7 and 0.75. The general conclusion is that the recorded data of this study follow the same trend as the other two. On this basis, it is concluded that this test result is quite reliable.
5.3 The test results

The tests were carried out for two models at four different depths. The dimensionless squat versus the Froude number (see (4)) for different cases of $C_B=0.7$ and $C_B=0.75$ are shown in Figures 6 and 7. Due to interaction between bottom of the tank and the model floor, the increase in speed of the model leads to more squat.

As speed of the model increases the squat also increases unless the floor of the model collides with the tank bottom (crucial speed). The magnitude of the crucial speed increases when $\delta$ increases too.

Figures 8, 9, 10 and 11 show the comparison of the dimensionless squat for different $\delta$. The interesting observation is that the squat for a larger $C_B$ is greater than for a smaller $C_B$. The difference between the dimensionless squat for $C_B=0.75$ and $C_B=0.7$ is relatively large. This difference increases more rapidly by increasing the Froude number.

Figure 12 shows the crucial speed versus the dimensionless gap $\delta$. According to the results of the model experiment, with the increase of $\delta$, the crucial speed increases too. General form of this curve is more or less similar to s-curve. It means that for a small $\delta$ the rate of the crucial speed increase is lower, while for the medium $\delta$ the crucial speed increase rate is highly increasing. For a large $\delta$ the rate is getting a moderate form as in the case of a small $\delta$. 
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Figure 9 Demonstration of the dimensionless squat for $C_B=0.75$ and $C_B=0.7$ at $\delta = 0.091$

Slika 9 Prikaz bezdimenzionalne vrijednosti slege za $C_B=0.75$ i $C_B=0.7$ pri $\delta = 0.091$

Figure 10 Demonstration of the dimensionless squat for $C_B=0.75$ and $C_B=0.7$ at $\delta = 0.13$

Slika 10 Prikaz bezdimenzionalne vrijednosti slege za $C_B=0.75$ i $C_B=0.7$ pri $\delta = 0.13$

Figure 11 Demonstration of the dimensionless squat for $C_B=0.75$ and $C_B=0.7$ for $\delta = 0.1667$

Slika 11 Prikaz bezdimenzionalne vrijednosti slege za $C_B=0.75$ i $C_B=0.7$ za $\delta = 0.1667$

Figure 12 Demonstration of result of crucial Froude Number for $C_B=0.75$ and $C_B=0.7$

Slika 12 Prikaz rezultata pri ključnom Fr broju za $C_B=0.75$ i $C_B=0.7$

6. Introducing an empirical formula by regression method

By analyzing the results obtained, one can state that the squat of the vessel is dependent on the water depth, speed of the model, and the block coefficient. Representing the non-dimensional squat, $S^*$, by non-dimensional parameters such as $\delta$, $F_{nh}$, and $C_B$, enables us to find an appropriate empirical formula. A general form of the empirical formula that defines the effect of each non-dimensional parameter on the squat independently is given in (5).

$$S^* = f_1(\delta), f_2(F_{nh}), f_3(C_B)$$

(5)

With the aid of (5) and regression analysis of the test results, including the results of Olivieri [3] and Kriebel [13], one may obtain more general equation for the squat:

$$S^* = 1.501383C_B(1-\delta)F_{nh}^2$$

(6)

6.1 Verifying the empirical formula

To verify the accuracy of the proposed empirical formula, the squat for different depths were calculated by (6) and compared with the test results.

Furthermore, the results of the test cases are shown in Figures 8, 9, 10 and 11 while the results of (6) are shown in Figures 13, 14, 15, 16 and 17. It is clear that (6) approximates very well the squat of commercial vessels with different speed, block coefficients and shallow water depth.
Table 4 shows the relative error of the empirical formula in comparison with the test results for a certain case that is $\delta = 0.13$ and $C_\delta = 0.75$. The Root Mean Square of the error, RMS, is given by (7):

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (S^* - S_{Ex}^*)^2}$$

where:
- $S^*$: non-dimensional squat by (6)
- $S_{Ex}^*$: non-dimensional squat by model experiment
- $n$: number of data.

For the above case, the RMS based (7) is equal to 0.009483709. Comparison shows a very good agreement between the (6) and the test results. For other cases, the relative error is almost the same.

### Table 4 Calculation of Relative Error of (6) by (7) at $C_\delta = 0.75$ and $\delta = 0.13$

<table>
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<tr>
<th>Product</th>
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### 7 Conclusions

To ensure the safe passage of commercial vessels in territorial waters, one should determine the crucial speed of a ship in shallow water. In this research, this has been accomplished by the model test and the following results and conclusions have been obtained:

- An empirical formula, i.e. (6), is introduced to calculate squat of commercial vessel and it has been validated.
- The dimensionless squat $S^*$, dimensionless gap $\delta$ and the crucial ship speed are introduced.
The dimensionless squat is a linear function of the ship block coefficient.
- The dimensionless squat is a linear function of the dimensionless gap.
- The dimensionless squat is proportional to the square of Froude number.

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References