Studies of Heat Transfer for Water-Diesel Two-Phase System in a Spiral Heat Exchanger

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In the present study, the main objective is to evolve a correlation to predict liquid-liquid two-phase heat transfer coefficients in a spiral plate heat exchanger. Experimental studies were conducted in a spiral plate heat exchanger using the liquid-liquid two-phase system of water-diesel in different volume fractions and flow rates as the cold fluid. Experiments were conducted by varying the volumetric flow rate and temperature, keeping the volumetric flow rate of hot fluid constant. The two-phase heat transfer coefficients were correlated with Reynolds number, Prandtl number and volume fraction in the form

$$Nu = a (Re)^b (Pr)^c \left(\frac{\phi}{\phi^2}\right)^d.$$  

The data obtained from fresh experiments were compared with the predictions of the obtained correlation. The predicted coefficients showed a spread of ±12% in the laminar range, indicating the potential use for practical applications.

Key words: Spiral plate heat exchanger, Reynolds number, Nusselt number, Single-phase and two-phase heat transfer coefficients, volumetric flow rate

Introduction

Heat exchangers are ubiquitous to energy conversion and utilization. They involve heat exchange between two fluids separated by a solid, and encompass a wide range of flow configurations. Systems involving multiphase fluid flow occur widely in nature and in industry. Liquid-liquid two-phase flows occur in many engineering applications, particularly in processing of petroleum, metal processing, petrochemical and power generation industries. A good understanding of the rates of momentum and heat transfer for liquid-liquid multiphase flow systems is a must for the optimal design of heat exchangers for such systems. This involves the understanding of flow instabilities, effects of curvature and their effects in possible enhancement of heat transfer. There is little published literature on such issues in the case of liquid-liquid two-phase systems. The first step in gaining such understanding is the development of transfer coefficient correlations using pure phase thermophysical properties and system parameters like flow geometries and flow velocities.

The first detailed study in two-phase flow in pipes was carried out by Lockhart and Martinelli and proposed a correlation for isothermal two-phase systems. The recent advances in the modeling of two-phase flow and heat transfer has been reviewed and a general thermal design method for two-phase heat exchangers based on local two-phase flow patterns and the flow structure has been studied. Oil-water two-phase flows in microchannels of 793 and 667 μm hydraulic diameters made of quartz and glass were investigated and by injecting one fluid at a constant flow rate and the second at variable flow rate, different flow patterns were identified and mapped and the corresponding two-phase pressure drops were measured. Measurements of the pressure drops were interpreted using the homogeneous and Lockhart-Martinelli models developed for two-phase flows in pipes. The results show similarity to both liquid-liquid flow in pipes and gas-liquid flow in microchannels. The correlation of pressure gradients was studied for the stratified laminar-turbulent pipeline flow of two immiscible liquids and identified parameters $\phi$ (ratio of the two-phase pressure gradient to the pressure gradient for one of the phases flowing alone) and $\chi$ (ratio of the pressure gradients which would exist if each phase flowed alone) that were useful in correlating data for the two immiscible liquids in closed conduits when at least one phase is in turbulent flow. The use of liquid-liquid slug flow in a capillary micrometer was studied for identifying heat and mass transfer in liquid-liquid reactions and the effect of...
various operating conditions on the flow regimes, slug size, interfacial area and pressure drop were investigated. Experiments were carried out using different Y-junction mixing elements with various downstream capillaries. The flow of immiscible fluids in a PMMA microchannel was studied. Dyed de-ionized water and kerosene were selected as the test fluids. Flow patterns were observed by using a CCD camera and identified by examining the video images.

The pressure drop, flow pattern and local water-volume fraction measurements of oil-water flow in horizontal and slightly inclined pipes were studied and observed, and the flow regimes categorized into eight different flow patterns, in accordance with the results presented by numerous literature. The local water-volume fraction measurements were compared for different mixture velocities, inlet water cuts and pipe inclinations. The frictional pressure drop was presented as a function of inlet water cuts for different mixture velocities for horizontal flows. Oil-water two-phase flow experiments in an inclined steel pipe were studied and the steady-state data on the flow patterns, two-phase pressure gradient and holdup were obtained over the entire range of flow rates for the various pipe inclinations. The characterization of flow patterns and identification of their boundaries was achieved via observation of recorded movies and by analysis of the relative deviation from the homogeneous behavior. The effect of upward and downward pipe inclinations on the flow patterns, hold up and pressure gradient during two-liquid phase flows for mixtures velocities between 0.7 and 2.5 m s⁻¹ and phase fractions between 10 % and 90 % were experimentally investigated and performed in a 38 mm ID stainless steel test pipe with water and oil as test fluids. High-speed video recording and local impedance and conductivity probes were used to precisely identify the different flow patterns. The experiments of two-phase liquid flows at +5⁰ inclination from the horizontal for mixture velocities between 0.7 and 2.5 m s⁻¹ and input oil fractions between 10 % and 90 % were studied. Dual continuous flow prevailed, while the two-phase pressure gradient was found to be lower than the single-phase oil or water. Compared to horizontal flow, water holdup was higher and frictional pressure gradient was lower.

However, while detailed studies of flow behavior of two-phase liquid-liquid mixtures have just begun, there is little reported literature on heat transfer involving such mixtures. The heat transfer studies in a spiral plate heat exchanger for water-palm oil two-phase system was experimentally studied and the two-phase heat transfer coefficients were correlated with Reynolds number in the form

\[ h = a \text{Re}^m. \]

Correlations were developed between the two-phase multiplier and the Lockhart-Martelli parameter. The two-phase system of nitrobenzene – water in spiral plate heat exchanger has been experimentally studied. The two-phase heat transfer coefficients were correlated with experimentally measured Reynolds number, Prandtl number and composition, while the developed correlations were validated against new experimental data.

In this study, hot water was used as the hot fluid, and immiscible two-phase mixtures of water with diesel in different ratios and flow rates were used as the cold fluid in a spiral plate heat exchanger. Experimental runs with single-phase fluids on the cold side were also carried out. The heat transfer coefficients on the cold side were correlated with Reynolds number, in addition to the development of a new correlation for Nusselt number.

### Experimental

#### Experimental setup

The experimental setup consisted of spiral plate heat exchanger, rotameters, pumps, control valves, tanks and thermometer as shown in Fig. 1. The specifications of heat exchanger are given in Table 1. Spacing between two channels in the exchanger was simply twice the plate thickness given in Table 1.

<table>
<thead>
<tr>
<th>Table 1 – Dimensions of the spiral plate heat exchanger</th>
</tr>
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<tbody>
<tr>
<td>Exchanger details</td>
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<tr>
<td>Flow length, m</td>
</tr>
<tr>
<td>Heat transfer area, m²</td>
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<tr>
<td>Plate spacing, m</td>
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<td>Plate width, m</td>
</tr>
<tr>
<td>Plate thickness, m</td>
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<td>Material of construction</td>
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</table>

The heating fluid used was water, heated in a stainless steel vessel by direct steam injection and agitated to maintain a uniform temperature. The hot-water inlet pipe was connected at the center core of the heat exchanger and the outlet pipe was taken from the periphery of the heat exchanger. The cold fluid was stored in a different stainless steel tank. Weighed quantities of diesel and demineralized water were charged into this tank to obtain the experimental range of volume fractions. Agitation in the tank was maintained by bubbling air. The
cold fluid inlet pipe was connected to the periphery of the heat exchanger and the outlet was taken from the center of the heat exchanger. Two centrifugal pumps were used for the circulation of the hot and cold fluid streams. Flow rates were measured by passing the fluids through rotameters. Thermometers were used to measure the temperatures of the inlet and outlet streams of the hot and cold fluids.

**Experimental procedure**

The inlet hot-water flow rate was kept constant for each cold-side two-fluid composition and the inlet cold two-phase fluid flow rate was varied using a control valve. Control valves \( C_1 \) and \( C_2 \) were used to vary the flow rates of hot water (for varying cold fluid composition) and cold fluid respectively. The pathway of heat exchange between hot water and cold fluid is shown in Fig. 2. Thermometers \( t_1 \) and \( t_2 \) were used to measure the inlet temperature of hot and cold fluids and \( t_3 \) and \( t_4 \) were used to measure the outlet temperature of hot and cold fluids, respectively. For different cold two-phase flow rate, all four temperatures were recorded after steady state was reached. Experimental runs with pure liquids in the cold side were also carried out.

**Calculation methodology**

The heat load was calculated using the expression

\[
Q = M_bC_p(\Delta T)_b.
\]

(1)

This leads to the estimation of overall heat transfer coefficient from the relation

\[
U = \frac{Q}{FA(\Delta T)_in}.
\]

(2)

Here, \( F \) denotes the temperature correction factor that accounts for differences arising from non-countercurrent operation. Our calculations show this factor to be very close to unity. Hence it is dropped in all further equations. The hot fluid side heat transfer coefficient \( h_b \) was calculated using an appropriate correlation for spiral plate heat exchangers in Perry *et al.*

\[
\text{Nu} = 0.0315(\text{Re})^{0.8}(\text{Pr})^{0.25}\left(\frac{\mu}{\mu_w}\right)^{0.17}
\]

(3)

where, \( \text{Nu} \) is the Nusselt number given by
In eq. 3, the term \((\mu/\mu_w)^{0.17}\) is the Sieder-Tate correction factor to account for the evaluation of the Reynolds and Prandtl numbers at bulk temperature rather than at local temperatures. This effect is significant only when the temperature differences are large. For the conditions in our experiments, they are negligible and hence are not used.

The cold fluid side (two-phase side) heat transfer coefficient \((h_{2\phi})\) was estimated from the overall heat transfer coefficient using

\[
\frac{1}{U} = \frac{1}{h_b} + \frac{B}{k_w} + \frac{1}{h_{2\phi}}. \tag{5}
\]

The corresponding Nusselt number of the two-phase cold fluid from eq. (5) (obtained from experiments, denoted as \(Nu_{\text{experimental}}\)) was calculated using

\[
Nu_{\text{experimental}} = (h_{2\phi})(d_e/k)_{\text{cold side}}. \tag{6}
\]

In developing predictive correlations for the cold-side two-phase heat transfer coefficient, the effect of composition has to be accounted for. Using weighted-average two-phase properties, dimensional analysis indicates that there are four dimensionless groups in the system. They are identified to be Nusselt number (\(Nu\)), Reynolds number (\(Re\)), Prandtl number (\(Pr\)), and the volume fraction \((\psi)\). Therefore, the general form of the correlations developed was

\[
Nu_{\text{predicted}} = a(Re)^b(Pr)^c(\psi)^d, \tag{7}
\]

where \(a\), \(b\), \(c\), and \(d\) are constants obtained from linear regression using Excel. Thermophysical properties of the pure substances were calculated based on the correlations given in Yaws.\(^{14}\)

The heat transfer coefficient so estimated is an average property and does not separate out influences of curvature, fluctuations in composition and other parameters.

### Results and discussion

#### Single-phase results

The experimental results of single-phase studies are presented in the form of a plot between Reynolds Number and \(h_{1\phi}\) in Fig. 3. The relation between \(Re\) and \(h_{1\phi}\) was correlated along with two-phase data (taking \(\psi = 1\)) by regression analysis. The correlation in eq. (7) reduces to:

\[
Nu = a(Re)^b(Pr)^c, \tag{8}
\]

with the constants as shown in Table 2. For pure water, eq. (3) was used to predict \(h_{1\phi}\).

#### Two-phase results

Two-phase studies were carried out with different volume fractions of diesel in water (20 %, 40 %, 60 % and 80 %). A sample set of experimental runs is presented in Table 3. Fig. 4 represents the two-phase heat transfer coefficients, \(h_{2\phi}\), as a function of Reynolds number, \(Re\), for various compositions. For two-phase systems, Reynolds number is based on the weighted-average thermophysical properties of the fluids at the respective mean bulk temperatures. The obtained data are fitted by regression to the correlation given in eq. (7) and the values of \(a\), \(b\), \(c\) and \(d\) are given in Table 2. Regression analysis yielded an \(R^2\) value.
of 0.981, small standard errors, and indicated a normal distribution of residuals and random nature of the residuals with respect to the fitted value. The result of statistical analysis for water-diesel is given in Table 4, and presented in Fig. 5.

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<th>Th₁/ K</th>
<th>Th₂/ K</th>
<th>Re of hot water</th>
<th>h₁/ W m⁻² K⁻¹</th>
<th>Q/W</th>
<th>Cold fluid flow rate/ (m³ s⁻¹) · 10⁴</th>
<th>Tc₁/ K</th>
<th>Tc₂/ K</th>
<th>Re of cold fluid</th>
<th>U/ W m⁻² K⁻¹</th>
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![Normal Probability Plot](image1.png)

![Residual Versus Fits](image2.png)

![Histogram](image3.png)

![Residual Versus Order](image4.png)

**Fig. 5** – Residual plots for water-diesel system
The developed correlation was tested against new experimental data on the same systems. The calculated values of $h_{2b}$ based on these constants agreed with the experimental data within an error of ±12%. The ranges of Reynolds number and Prandtl number for which the correlations are valid are given in Table 5.

Fig. 6 shows the comparison of the Nusselt numbers obtained from the experiments conducted with those calculated from the developed correlations. The predicted Nusselt numbers are within ±12% of the experimental values. Observed discrepancies between the measured data and calculated results may be due to the uncertainties of the correlation, as well as the averaging of properties. Further studies are needed to develop better correlations for two-phase convective transfer of polar systems, aromatics and highly viscous liquids. In addition, these experimental and predictive data may be used to develop sophisticated models and experiments examining the flow and temperature profiles in the exchanger.

**Table 4 – Statistical analysis of regression in water-diesel system**

| S.No | Coefficients | P-value | $R^2$ | S | n | $\sigma$
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<td>2</td>
<td>b</td>
<td>$1.50 \cdot 10^{-25}$</td>
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<td>–</td>
<td>–</td>
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</tr>
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<td>3</td>
<td>c</td>
<td>$1.99 \cdot 10^{-7}$</td>
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<td>–</td>
<td>–</td>
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<tr>
<td>4</td>
<td>d</td>
<td>0.142177</td>
<td>–</td>
<td>–</td>
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(S: standard error; n: number of observations; $\sigma$: standard deviation)

**Table 5 – Ranges of Re and Pr**

<table>
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<th>System</th>
<th>Re</th>
<th>Pr</th>
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<tbody>
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<td>Water</td>
<td>826 &lt; Re &lt; 2135</td>
<td>4.68 &lt; Pr &lt; 5.64</td>
</tr>
<tr>
<td>Water – diesel</td>
<td>658 &lt; Re &lt; 2098</td>
<td>4.65 &lt; Pr &lt; 6.54</td>
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</table>

**Conclusions**

Preliminary studies were initiated on heat transfer in liquid-liquid two-phase mixtures of water-diesel in the cold side of a spiral plate heat exchanger. Based on the inlet and outlet temperatures and flow rates of the hot and cold streams, heat transfer coefficients were estimated and new correlations for liquid-liquid two-phase heat transfer were developed, as an additional function of composition of the mixture. As expected, the heat transfer coefficients were poorer in mixtures with more organic fluid. Regression analysis was performed and it is seen that the correlations faithfully reproduce the variations observed in the experiments. The developed correlation predicts the experimental heat transfer behavior within ±12%. Experimental determination of two-phase heat transfer coefficients and development of predictive correlations are the first step towards initiating detailed studies on various aspects of liquid-liquid two-phase momentum and heat transfer, leading to better design capabilities for compact heat exchangers handling liquid-liquid two-phase mixtures.

**ACKNOWLEDGEMENT**

The authors acknowledge the contributions of Dr. Sriram Devanathan, Professor, Department of Chemical Engineering and Materials Science, Amrita Vishwa Vidyapeetham on the statistical analysis of the regression of heat transfer data.

**List of symbols**

$A$ – area of heat transfer, m$^2$
$C_p$ – specific heat, J kg$^{-1}$ K$^{-1}$
$d_e$ – equivalent diameter of the flow channel, m
$F$ – temperature correction factor
$h_h$ – heat transfer coefficient, W m$^{-2}$ K$^{-1}$
$h_{1h}$ – heat transfer coefficient, W m$^{-2}$ K$^{-1}$
$h_{2b}$ – heat transfer coefficient, W m$^{-2}$ K$^{-1}$
$k_w$ – thermal conductivity, W m$^{-1}$ K$^{-1}$
$k_s$ – thermal conductivity of the wall, W m$^{-1}$ K$^{-1}$
$M_f$ – mass flow rate, kg s$^{-1}$
$Nu$ – Nusselt number
$Pr$ – Prandtl number
$Q$ – heat transferred, W
$Re$ – Reynolds number
$B$ – wall thickness of the spiral plate, m
$U$ – overall heat transfer coefficient, W m$^{-2}$ K$^{-1}$
$v$ – velocity of fluid flow, m s$^{-1}$
$\psi$ – volume fraction of the organic component in the liquid-liquid two-phase mixture

**Fig. 6 – Comparison of Nu$_{\text{experimental}}$ with Nu$_{\text{predicted}}$ for water-diesel system**
Greek symbols

\((\Delta T)_h\) – temperature drop of hot fluid, K
\((\Delta T)_{hm}\) – logarithmic mean temperature difference between hot and cold fluid, K
\(\mu\) – viscosity of fluid, kg m\(^{-1}\) s\(^{-1}\)
\(\mu_w\) – viscosity of fluid at wall temperature, kg m\(^{-1}\) s\(^{-1}\)
\(\rho\) – density of fluid, kg m\(^{-3}\)
\(\phi\) – phase of liquid based on single and two-phase

Subscripts

\(h\) – hot fluid side
\(1\phi\) – single phase fluid
\(2\phi\) – two-phase fluid

References