# NEW GEOMETRY AND TECHNOLOGY OF FACE-GEAR FORMING WITH CIRCLE LINE OF TEETH ON CNC MILLING MACHINE

Different types of geometric models of face-gear with circle line of teeth have been shown in the paper. Generation of a new geometrical of a face-gear is performed on CNC milling machine. The basic direction of the development geometrical of a face-gear and technology is in the search of new trends and methods focused on improving the quality of products, shortening the production cycles, their mechanizations, automation and implementation of a high-precision technology.

*Key words:* face-gear, circular line, CNC milling-machine

**Nova geometrija i tehnologija oblikovanja hipoidnih zupčanika na CNC glodalici.** Različiti tipovi geometrijskih modela hipoidnih zupčanika su prikazani u radu. Izrada novog geometrijskog oblika zupčanika je provedena na CNC glodalici. Temeljni smjer razvoja geometrijskih oblika zupčanika i tehnologije izrade je u traženju novih trendova i metoda za unaprjeđenje kvalitete proizvoda, skraćenja trajanja proizvodnog ciklusa, njegova mehanizacija, automatizacija iimplementacija visokoprecizne tehnologije.

*Ključne riječi:* zupčanik, kružna linija, CNC glodalica

## INTRODUCTION

Invention of face worm gear drives with conical and cylindrical worms by Saari [1, 2] was a substantial contribution. Initially the design of the invented gear drives was based on application of worms provided by axial profiles as straight lines [3]. The generation of face worm gear drives of all types of existing design is based on application of a hob for generation of the face-gear. The disadvantage of such method of generation is the low precision of a hob used as a generating tool especially in the case of small dimensions of hob [3].

The generations of a face worm gear drives of all types of existing is based on application of a hob for generation of the face worm gear with conical and cylindrical worms. Saari [1, 2] and next researchers had proposed methods based on application a worm hob for manufacturing a face worm gear with conical or cylindrical worms. Litvin and coworkers [3] had proposed a tilted head-cuter for forming of face worm gear drives with conical and cylindrical worms. In work 4-10 and 11 presented developed a new technique of cutting a face worm gear on a CNC machining. For generation face-gear used the 4-axis vertical CNC milling-machine incorporate a rotary table and a NC spindle. The new process uses general purpose machine like vertical machining centre. The generations are performed by a tilted tool edge with straight line profiles of blades. The

process is giving better results with use of newly developed technique to generate a face-gear. Due to addition two rotational axes in 5-axis machining enables cutting face-gear and taken high surface quality.

For numerically controlled universal milling-machine a face-gear can be shaped with different front lines of teeth. Known methods for forming face-gear are based on kinematics of conventional machine tools. While notching the teeth, workgroups of machine performs movements at a constant speed, the tracks are rectilinear or rotary (NC rotary tables, spindle tool). One way to cut face-gear is the use of single blade tool in the form of a universal sintered carbide insert.

This method may be used to shape the toothing of a straight and involute line of teeth [4-10, 11]. The new geometry and technology proposed in this article is based on application of single blade tool and CNC milling-machine with special program of control.

## MODELING FACE – GEAR WITH CIRCLE LINE OF TEETH

Figure 1 illustrates schematically the generation of the face-gear.

In the following geometric models of shaping circle line, assumptions are that:
– a tooth line is shaped with single blade tool,
– a incision tooth line is rigidly linked to the machined teeth crown,
– a beginning of the system of coordinates is located at the intersection of the axis of symmetry shaped toothing,

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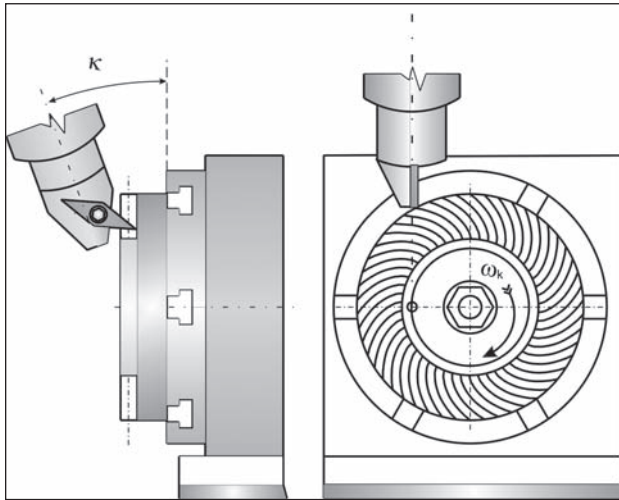A. Stoić, University of applied sciencies, Slavonski Brod, Croatia

**Figure 1** Scheme the generation of the face-gear

– the location of the curve, part of which is a tooth lines, is set in relation to the theoretical rolling circle,
– a trace location of the tool is described by the blade cutting edge is so located in relation to the shaped surface to have a common normal with the shaped line of the tooth.

Here are models of geometrical shape of the tooth which is a circle, with different radii of curvature of the teeth line.

In the simplest model of the toothing with circular line, the axis of symmetry of the circle, which is part of the line tooth, lies at the intersection of the theoretical rolling circle with a $R_b$ radius and the axis of system coordinates associated with the shaped toothing (Z). The geometric model is shown in the Figure 2.

From Figure 2 can be determined the coordinates of points in the ring-shaped toothing:

$$\left. \begin{array}{l} x_1 = \rho \cdot \cos(\Delta\psi) \\ z_1 = R_b + \rho \cdot \sin(\Delta\psi) \end{array} \right\}, \quad (1)$$

as well as
$$\left. \begin{array}{l} x_2 = R_v \cdot \sin(\psi_0 + \Delta\psi) \\ z_2 = x_2 \cdot ctg(\psi_0 + \Delta\psi) \end{array} \right\}, \quad (2)$$

where:
$\rho$ – radius of a circle of being the tooth line,
$R_b$ – theoretical rolling circle.

From the described model in Figure 1 it is possible also to set coordinates of points in polar coordinates:

$$\left. \begin{array}{l} \psi_0 = \arcsin\left(\dfrac{x_1}{R_v}\right) \\ R_v = \sqrt{x_1^2 + z_1^2} \end{array} \right\}. \quad (3)$$

Placing described relations to equations 2 with equations 3 we receive:

$$\left. \begin{array}{l} x_2 = \sqrt{x_1^2 + z_1^2} \cdot \sin\left[\arcsin\left(\dfrac{x_1}{\sqrt{x_1^2 + z_1^2}}\right) + \Delta\psi\right] \\ z_2 = x_2 \cdot ctg\left(\arcsin\left(\dfrac{x_1}{\sqrt{x_1^2 + z_1^2}}\right) + \Delta\psi\right) \end{array} \right\} \quad (4)$$

and after substituting formula 1 to equations 4 we receive (5).

Equations 5 describe the track of the tools in the forming process of toothing, the tooth-line as part of a circle with a $\rho$ radius. Variable in the division plane is an increase in the $\Delta\psi$ angle of rotation of toothing.

$$\left. \begin{array}{l} x_2 = \sqrt{[\rho \cdot \cos(\Delta\psi)]^2 + [R_b + \rho \cdot \sin(\Delta\psi)]^2} \times \\ \times \sin\left[\arcsin\left(\dfrac{\rho\cos[\Delta\psi]}{\sqrt{(\rho \cdot \cos(\Delta\psi))^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}}\right) + \Delta\psi\right] \\ z_2 = x_2 \cdot ctg\left[\arcsin\left(\dfrac{\rho \cdot \cos(\Delta\psi)}{\sqrt{(\rho \cdot \cos(\Delta\psi))^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}}\right) + \Delta\psi\right] \end{array} \right\} \quad (5)$$

Another solution toothing with a tooth line as a part of a circle is a circle line of tooth shift so that it does not lie on the Z-axis of face-gear. In the case of the circle line of tooth with the axis of symmetry moved in the direction of positive X-axis, in the toothing tooth lines can be obtained with less radius of curvature. Geometric model of such a solution is shown in Figure 3.



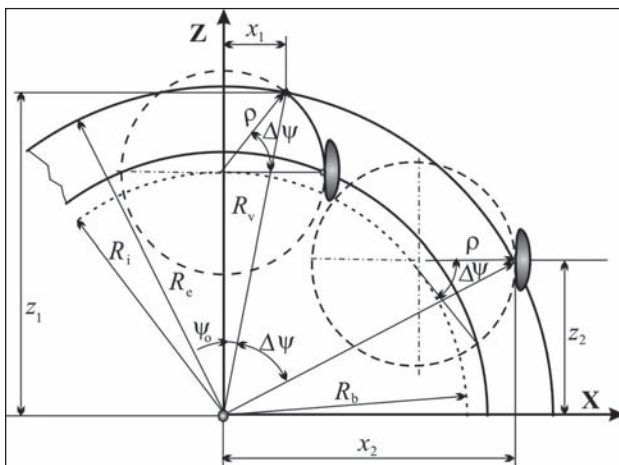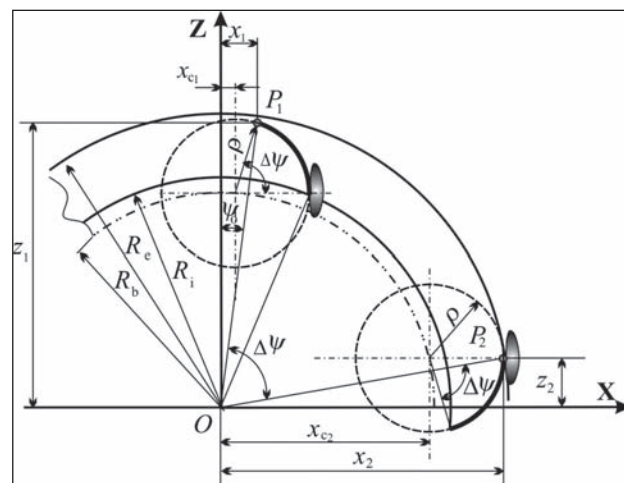**Figure 2** Geometric model of forming face-gear with circular line with symmetric axis of circle teeth line places on the axis of face-gear



**Figure 3** The geometric model of forming face-gear with circular line, the circle line of tooth with the axis of symmetry moved in the direction of positive X-axis values

From the Figure 3 can be determined the coordinates of points in the ring-shaped toothing:

$$\left. \begin{array}{l} x_1 = \rho \cdot \cos(\Delta\psi) + x_{c1} \\ z_1 = R_b + \rho \cdot \sin(\Delta\psi) \end{array} \right\}. \qquad (6)$$

From Figure 3 it is possible to set coordinates of points like in case of the description of the model shown in Figure 2 (equations 2, 3, 4), and after placing relation 6 to equations 4 relations circumscribing the line of the tooth are (7).

$$\left. \begin{array}{l} x_2 = \sqrt{[\rho \cdot \cos(\Delta\psi) + x_{c1}]^2 + [R_b + \rho \cdot \sin(\Delta\psi)]^2} \times \\ \times \sin\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) - x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) + x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \\ z_2 = x_2 \cdot ctg\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) + x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) + x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \end{array} \right\} (7)$$

With alternative of moving the arrangement of the circle of the line of teeth of toothing is reallocating his middle is so that it is in negative values of the X-axis. Such moving the circle of the line of the tooth will allow the tooth to obtain a larger radius of curvature of the tooth line. Geometric model of such a solution is shown in Figure 4.

Relations result from Figure:

$$\left. \begin{array}{l} x_1 = \rho \cdot \cos(\Delta\psi) + x_{c1} \\ z_1 = R_b + \rho \cdot \sin(\Delta\psi) \end{array} \right\}. \qquad (8)$$

Taking equation 2 and 3 into consideration, it is also possible to derive on the basis of model 4 and after substituting the equation to relation 4 describing the tooth line becomes (9).

$$\left. \begin{array}{l} x_2 = \sqrt{[\rho \cdot \cos(\Delta\psi) - x_{c1}]^2 + [R_b + \rho \cdot \sin(\Delta\psi)]^2} \times \\ \times \sin\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) - x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) - x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \\ z_2 = x_2 \cdot ctg\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) - x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) - x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \end{array} \right\} (9)$$
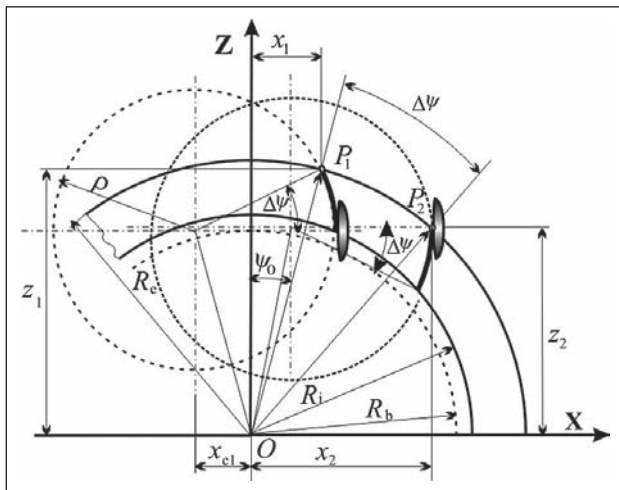


**Figure 4** The geometric model of forming face-gear with circular line, the circle line of tooth with the axis of symmetry moved in the direction of negative X-axis values

Including models shown in Figures 2, 3 and 4 and equations of the teeth line described with relations 5, 7 and 9 it is possible to represent equations describing synchronizations of steered pivots of the machine tool generalized with relation (10).

$$\left. \begin{array}{l} x_2 = \sqrt{[\rho \cdot \cos(\Delta\psi) \pm x_{c1}]^2 + [R_b + \rho \cdot \sin(\Delta\psi)]^2} \times \\ \times \sin\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) \pm x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) \pm x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \\ z_2 = x_2 \cdot ctg\left[ \arcsin\left( \dfrac{\rho \cdot \cos(\Delta\psi) \pm x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) \pm x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}} \right) + \Delta\psi \right] \end{array} \right\} (10)$$

The $x_{c1}$ value determines the location of the center of a circle line of teeth and the size of radius of curvature of the teeth line. In the case of $x_{c1} = 0$ circle line of tooth lies on the Z-axis of toothing. Addition of $x_{c1}$ in the equation 10 reduces and subtracting of $x_{c1}$ increases the radius of curvature of the circle.

## THE ALGORITHM OF STEERING AND EXPERIMENTAL EXAMINATIONS OF THE SYNCHRONIZATION OF STEERED AXIS OF THE MACHINE TOOL

In order of conducting attempts to synchronize the axis of the machine tool enabling forming face-gear with circle line of teeth control algorithm was developed as shown in Figure 5. This algorithm served to develop parameterized control program for the machine tool notching the teeth of toothing.
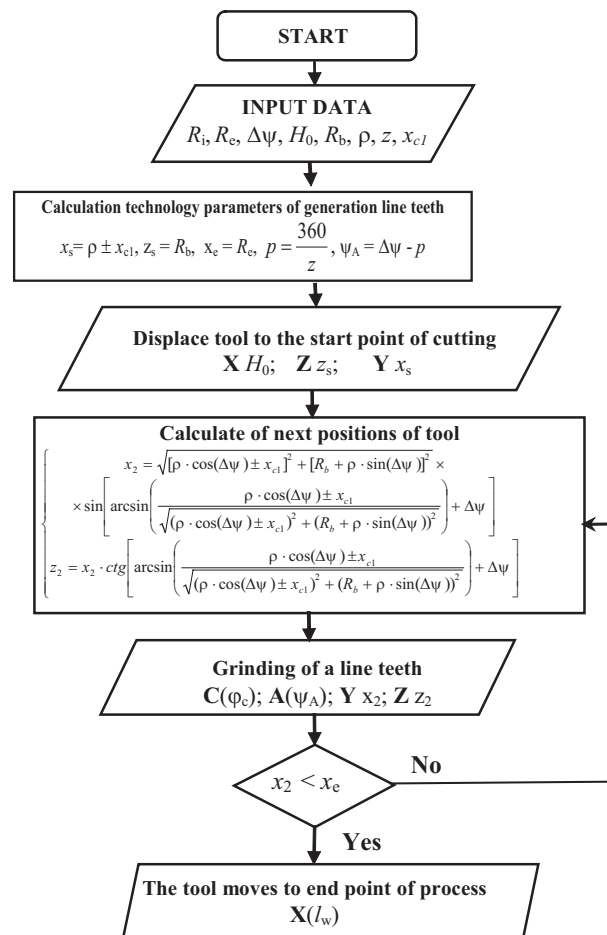
Attempts to shape face-gear were conducted on the milling machine FYN - 50ND type, equipped with numerically controlled rotary table (Figure 6a). The milling machine is holding the control system of the TNC 407 type of the Heidenhain.

The Heidenhain 407 controller enables simultaneous interpolation in three axes (linear or circular in three dimensional space). Steering of processing of the outline is held with digital speed control. Servo systems in each axis servo are position regulated type, controlled by deviation signals. Feed the axes X, Y, Z and A are carried out by four independent pulse-controlled AC motors. The drive of the spindle is equipped with a system for continuous variable speed transmission. In the axis of the spindle of the milling machine a rotational-pulse sensor was fastened, which signals are transmitted to the control system of the machine tool what allows to control spindle as a rotational axis (C). An example of cutting teeth line in the face-gear is shown at Figure 6b.

## CONCLUSIONS

Conducted examinations of the generation of the face-gear according to the relation described with formula 10 confirmed the possibility of shaping the circle line teeth on the CNC milling-machine. Despite of heavy-load of processor with complex calculations control system do not cause temporary detention of controlled machine tools units. The surveys are the basis

START

INPUT DATA
$R_i, R_e, \Delta\psi, H_0, R_b, \rho, z, x_{c1}$

Calculation technology parameters of generation line teeth

$x_s = \rho \pm x_{c1}, z_s = R_b, x_e = R_e, p = \dfrac{360}{z}, \psi_A = \Delta\psi - p$

Displace tool to the start point of cutting
**X** $H_0$;   **Z** $z_s$;   **Y** $x_s$

Calculate of next positions of tool

$$x_2 = \sqrt{[\rho \cdot \cos(\Delta\psi) \pm x_{c1}]^2 + [R_b + \rho \cdot \sin(\Delta\psi)]^2} \times$$

$$\times \sin\left[\arcsin\left(\frac{\rho \cdot \cos(\Delta\psi) \pm x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) \pm x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}}\right) + \Delta\psi\right]$$

$$z_2 = x_2 \cdot ctg\left[\arcsin\left(\frac{\rho \cdot \cos(\Delta\psi) \pm x_{c1}}{\sqrt{(\rho \cdot \cos(\Delta\psi) \pm x_{c1})^2 + (R_b + \rho \cdot \sin(\Delta\psi))^2}}\right) + \Delta\psi\right]$$

Grinding of a line teeth
**C**$(\varphi_c)$; **A**$(\psi_A)$; **Y** $x_2$; **Z** $z_2$

$x_2 < x_e$ — **No**

**Yes**

The tool moves to end point of process
**X**$(l_w)$

**Figure 5** Algorithm of steering the functioning of the machine tool during forming tooth line



a)



b)

**Figure 6** View: a) investigations stand for cutting a face-gear, b) example cutting teeth line in the face-gear

for further work on applying a face-gear with circle line of teeth in clutch connection.
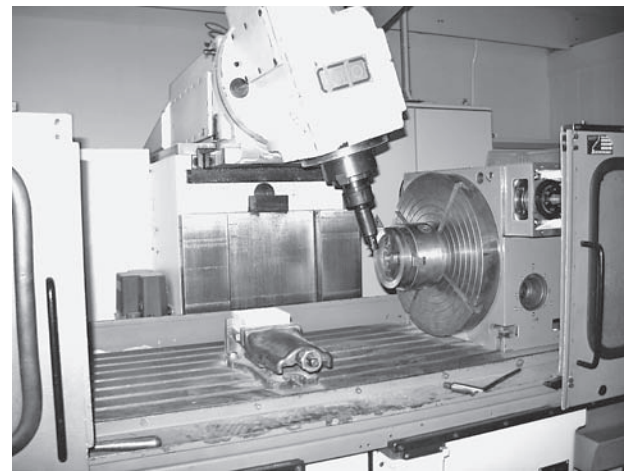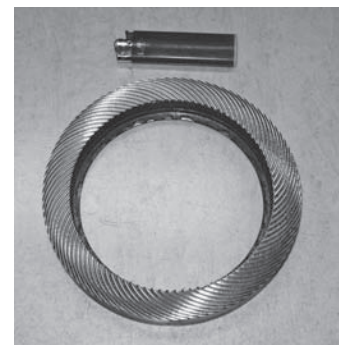
## Acknowledgement

## Legend of symbols

$R_i$ – inner radius of the face-gear
$R_b$ – theoretical rolling circle
$R_e$ – outer radius of the face-gear
X,Y,Z – coordinate system connected to machine-tool
$x,z$ – coordinate system rigidly connected to face-gear
$z$ – number of teeth of the face-gear
$\rho$ – radius of generation a circle tooth line of face-gear
$\varphi_C$ – angle of rotation of the grinding tool
$\psi_A$ – angle of rotation of the face-gear in the process of generation
$\Delta\psi$ – additional rotational motion of the face-gear during grinding tool motion
$P_1, P_2$ – points of contact on surface of tooth line and grinding tool trace
$x_{c1}$ – parameter of relative location of point of the center of the circle tooth line

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