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Pricing Options on Commodity Futures: The Role of Weather and Storage

Abstract: Options on agricultural futures are popular financial instruments used for agricultural price risk management and to speculate on future price movements. Poor performance of Black's classical option pricing model has stimulated many researchers to introduce pricing models that are more consistent with observed option premiums. However, most models are motivated solely from the standpoint of the time series properties of futures prices and need for improvements in forecasting and hedging performance. In this paper I propose a novel arbitrage pricing model motivated from the economic theory of optimal storage and consistent with implications of plant physiology on the importance of weather stress. I introduce a pricing model for options on futures based on a generalized lambda distribution (GLD) that allows greater flexibility in higher moments of the expected terminal distribution of futures price. I use times and sales data for corn futures and options for the period 1995-2009 to estimate the implied skewness parameter separately for each trading day. An economic explanation is then presented for inter-year variations in implied skewness based on the theory of storage. After controlling for changes in planned acreage, I find a statistically significant negative relationship between ending stocks-to-use and implied skewness, as predicted by the theory of storage. Furthermore, intra-year dynamics of implied skewness reflect the fact that uncertainty in corn supply is resolved between late June and early October, i.e., during corn growth phases that encompass corn silking and grain maturity. Impacts of storage and weather on the distribution of terminal futures price jointly explain upward-sloping implied volatility curves.

Keywords: arbitrage pricing model, options on futures, generalized lambda distribution, theory of storage, skewness
JEL classification: G13, Q11, Q14

Određivanje cijena opcija na *futures* ugovore za poljoprivredne proizvode: utjecaj vremenskih uvjeta i zaliha

Sažetak: Opcije na *futures* ugovore za poljoprivredne proizvode su često korišteni financijski instrumenti u upravljanju cjenovnim rizikom u poljoprivredi i pri špekuliranju o smjeru kretanja *futures* cijena. Slabosti klasičnog Blackovog modela potakle su mnoge istraživače da predlože nove modele za određivanje cijena opcijskih ugovora koji su više u skladu s opaženim premijama opcijskih ugovora. Međutim, takvi modeli su gotovo bez iznimke osmišljeni isključivo na temelju značajki vremenskih nizova cijena terminskih ugovora i potrebe za poboljšanjem predviđanja i *hedginga*. Ovaj rad predlaže novi arbitražni model za određivanje cijena opcija polazeći od ekonomske teorije o optimalnom upravljanju zalihama i vodeći računa o implikacijama fiziologije bilja na važnost vremenskih uvjeta pri rastu. Predstavljen je model za određivanje cijena opcija na terminske ugovore zasnovan na generaliziranoj lambda distribuciji (GLD) koja omogućava veću fleksibilnost u višim momentima očekivane krajnje distribucije cijena terminskih ugovora. Podaci o svim zabilježenim transakcijama opcijskim i terminskim ugovorima za kukuruz u razdoblju 1995.-2009. korišteni su pri procjeni impliciranih parametara asimetrije, posebno za svaki radni dan. Kontrolirajući za utjecaj promjena u planiranoj površini za sadnju, nalazim statistički značajan negativan utjecaj odnosa zaliha i potražnje na implicirani parametar asimetrije, u skladu s hipotezom koja slijedi iz teorije o zalihama. Nadalje, dinamika promjena parametra asimetrije unutar iste godine reflektira činjenicu da se neizvjesnost o konačnoj veličini žetve razrješava od lipnja do listopada, tj. u vremenskom razdoblju koje obuhvaća reproduktivnu fazu rasta kukuruza i fazu dozrijevanja zrna. Utjecaj zaliha i vremenskih uvjeta objašnjava pojavu pozitivnog nagiba krivulja implicirane volatilnosti.

Ključne riječi: arbitražni model cijena, opcije na *futures* ugovore, generalizirana lambda distribucija, teorija zaliha, asimetrija
JEL klasifikacija: G13, Q11, Q14

1 Introduction*

Options written on commodity futures have been investigated from several aspects in the commodity economics literature. For example, Lence, Sakong and Hayes (1994), Vercammen (1995), Lien and Wong (2002), and Adam-Müller and Panaretou (2009) considered the role of options in optimal hedging. Use of options in agricultural policy was examined by Gardner (1977), Glauber and Miranda (1989), and Buschena and McNew (2008). The effects of news on options prices have been investigated by Fortenbery and Sumner (1993), Isengildina-Massa et al. (2008), and Thomsen, McKenzie and Power (2009). The informational content of options prices has been looked into by Fackler and King (1990), Sherrick, Garcia and Tirupattur (1996), and Egelkraut, Garcia and Sherrick (2007). Some of the most interesting work done in this area considers modifications to the standard Black-Scholes formula that accounts for non-normality (skewness, leptokurtosis) of price innovations, heteroskedasticity, and specifics of commodity spot prices (i.e., mean-reversion). Examples include Kang and Brorsen (1995) and Ji and Brorsen (2009).

In this article I revisit the well-known fact that the classical Black's (1976a) model is inconsistent with observed option premiums. Previous studies like Fackler and King (1990) and Sherrick, Garcia and Tirupattur (1996) address this puzzle by identifying properties of futures prices that deviate from the assumptions of Black's model, i.e., leptokurtic and skewed distributions of the logarithm of terminal futures prices and stochastic volatility. A common feature of past studies is the grounding of their arguments in the time-series properties of stochastic processes for futures prices and the distributional properties of terminal futures prices. In other words, their arguments are primarily statistical. In contrast to previous studies, I offer an economic explanation for the observed statistical characteristics. In this paper I analyze in detail options on corn futures. The focus is on presenting an alternative pricing model that is not motivated by improving the forecasts of options premiums compared to Black's or other models, but by linking option pricing models with the economics of supply for annually harvested storable agricultural commodities. In particular, I demonstrate the effect of storability and crop physiology (i.e., susceptibility to weather stress) on higher moments of the futures price distribution. Only by understanding these fundamental economic forces can we truly explain why classical option pricing models work so poorly for commodity futures.

The article is organized as follows. In the second section I examine in detail the implications of Black's classical option pricing model on the shape and dynamics of the futures price distribution. I follow by presenting the rational expectations competitive equilibrium model with storage, and testable hypothesis on conditional new crop price distributions that follow from it. In addition to storage, I present the agronomical

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research on the impact of weather on corn yields. I then develop a novel arbitrage pricing model for options on commodity futures based on the generalized lambda distribution (GLD) which I propose to use in calibrating skewness of new crop futures price to match observed option premiums. The third section describes the econometric model. In the fourth section I summarize the data used in the econometric analysis. Finally, I describe the estimation procedure and present results of statistical inference, followed by a set of conclusions and directions for further research.

2 Theory

2.1 Foundations of Arbitrage Pricing Theory for Options on Futures

Black (1976) was the first to offer an arbitrage pricing model for options on futures contracts. Despite numerous extensions and modifications proposed in the literature, and the inability of the model to explain observed option premiums, traders still use this model in practice, and widely used information systems for traders (e.g., Bloomberg) use this as their workhorse model for commodity options. This is likely due to its simplicity and ability to forecast option premiums after appropriate "tweaks" are put in place. Black proposes that futures prices follow a stochastic process as described below:

$$dF = \sigma F dz \tag{1}$$

where F stands for futures price, σ for volatility, and dz is an increment of Brownian motion.

The implication is that futures prices are unbiased expectations of terminal futures prices (ideally equal to the spot price at expiration), and the stochastic process followed by futures prices is a geometric Brownian motion.

The option premium V is equal to the present value of the expected option payoff under a risk-neutral distribution for terminal prices. For example, for a call option with strike K, volatility σ, risk-free interest rate r, and time left to maturity T:

$$V\left(K, F_0, T, \sigma, r\right) = e^{-rT} \int_0^\infty Max\left(F_T - K, 0\right) f\left(F_T; F_0, \sigma, r, T\right) dF_T \,. \tag{2}$$

Because delta hedging options on futures do not require a hedger to pay the full value of the futures contract due to margin trading, a risk-neutral terminal distribution for futures prices is equivalent to a risk-neutral terminal distribution for a stock that pays a dividend yield equal to the risk-free interest rate:

$$\ln F_T \sim N\left(\ln F_0 - \frac{1}{2}\sigma^2, \sigma^2 T \right). \tag{3}$$

Thus, Black's model postulates that the distribution of terminal futures prices, conditional on information known at time zero, is lognormal with the first four moments fully determined by the current futures price and volatility parameter σ. In particular, the first four moments of the risk-neutral terminal distribution are equal to:

$$\tilde{\mu} = F_0 \quad \tilde{\sigma}^2 = F_0{}^2\left(e^{\sigma^2 t} - 1\right) \quad SKEW = (e^{\sigma^2 t} + 2)\sqrt{e^{\sigma^2 t} - 1} \quad KURT = e^{4\sigma^2 t} + 2e^{3\sigma^2 t} + 3e^{2\sigma^2 t}. \tag{4}$$

For example, if a futures price is \$2.50, volatility is 30 percent, and there are 160 days left to maturity, the standard deviation of the terminal distribution would be \$0.50, skewness would be 0.60, and kurtosis would be 3.64. Therefore, the standard Black's model implies that the expected distribution of terminal prices would be positively skewed and leptokurtic. When complaints are raised that Black's model imposes normality restrictions, it is the logarithm of the terminal price that the critique refers to.

The standard way to check if Black's model is an appropriate pricing strategy is to exploit the fact that for a given futures price, strike price, risk-free interest rate, and time to maturity, the model postulates a one-to-one relationship between the volatility coefficient and the option premium. Thus, the pricing function can be inverted to infer the volatility coefficient from an observed option premium. Such coefficients are referred to as *implied volatility* (IV) and the principal testable implication of Black's model is that implied volatility does not depend on how deep in-the-money or out-of-money an option is. If the logarithm of the terminal price is not normally distributed, then Black's model is not appropriate, and implied volatility will vary with option moneyness – a flagrant violation of the model's assumptions. Black's model gives us a pricing formula for European options on futures. Prices of American options on futures that are assumed to follow the same stochastic process as in Black's model must also account for the possibility of early exercise. For that reason, their prices cannot be obtained through a closed-form formula, but must be estimated through numerical methods such as the Cox, Ross and Rubinstein (CRR) (1979) binomial trees.

Implied volatility curves for storable commodity products are almost always upward sloping. As an example consider the December 2006 corn contract. The futures price on June 26, 2006 was \$2.49/bu. As seen in Figure 1, the implied volatility curve associated with calculating IV using various December option strikes is strongly upward sloping, with the implied volatility coefficients for the highest strike options close to 15 percentage points higher than the implied volatility for options with lower strikes.

Geman (2005) calls this phenomenon an "inverse leverage effect," after the "leverage effect" proposed to explain downward-sloping implied volatility curves for individual company stocks. However, this is a complete misnomer. As Black (1976b) explains, the leverage effect arises from the fact that as stock price declines, the ratio of a company's debts to equity value, its leverage, increases. If the volatility of company assets is constant, then as the equity share of assets declines, volatility in equity will increase. While the leverage effect has the coherent causal model to justify the term, nothing similar exists for "inverse leverage effect."

We can gain further insight as to how Black's model performs if we plot the implied volatility curve for a single contract at different time-to-maturity horizons. As an example, consider December corn contracts in the years 2004 and 2006. As Figure 2 shows, three distinct patterns are noticeable. First, except when options are very near maturity, we always see an upward-sloping implied volatility curve. Second, implied volatility of at-the-money options, i.e., options that have the strike price equal to the current futures price, rises almost linearly until the end of June, declines throughout the summer months, and then starts rising again. Finally, near maturity, volatility skews give way to symmetric volatility smiles. The implied volatility coefficient measures volatility on an annual basis, and the variance of the terminal price, conditional on time remaining to maturity, is $\sigma^2(T-t)$. So if uncertainty about the terminal price is uniformly resolved as time passes, implied volatility will not decrease, but will stay the same. Likewise, when the same amount of uncertainty needs to be resolved in a shorter time interval, implied volatility will increase. Therefore, linear increases in implied volatility from distant horizons up until June are best interpreted not as increases in day to day volatility of futures price changes, but as a market consensus that the conditional variance of terminal prices is not much reduced before June.

While CRR binomial trees preserve the basic restrictions of Black's model, i.e., normality of log-prices terminal distribution, Rubinstein (1994; 1998) shows how that can be relaxed to allow for non-normal skewness and kurtosis. To illustrate the effect of skewness and kurtosis on Black's implied volatility I used Edgeworth binomial trees (Rubinstein, 1998). This allows for pricing options that exhibit skewed and leptokurtic distributions of terminal log-prices. As can be seen in Figure 3, zero skewness and no excess kurtosis (S=0, K=3) corresponds to a flat IV curve, i.e., CRR implied volatility estimated from options premiums is the same no matter what strike is used to infer it, just like Black's model would have it. A leptokurtic distribution will cause so-called "smiles", i.e., options with strikes further away from the current futures price will produce higher implied volatility coefficients. Positive skewness creates an upward-sloping curve, and negative skewness a downward-sloping IV curve.

Faced with the inability of Black's model to explain observed option premiums, researchers and traders have pursued three different approaches to address this issue:

1) Start from the end: relax the assumptions concerning risk-neutral terminal distributions of underlying futures prices, i.e., allow for non-lognormal skewness and kurtosis. As long as delta hedging is possible at all times (i.e., markets are complete), it is still possible to calculate option premiums as the present value of expected option payoffs. Examples of this approach include Jarrow and Rudd (1982), Sherrick et al. (1996), and Rubinstein (1998). While the formulas that derive option premiums as discounted expected payoffs assume that options are

European, one can still price American options using implied binomial trees calibrated to the terminal distribution of choice (Rubinstein, 1994).

2) Start from the beginning: start by asking what kind of stochastic process is consistent with a non-normal terminal distribution. By introducing appropriate stochastic volatility and/or jumps, one might be able to fit the data just as well as by the approach above. Examples of this approach are Kang and Brorsen (1995), Hilliard and Reis (1998), and Ji and Brorsen (2009).

3) "Tweak it so it works good enough" approach: if one is willing to sacrifice mathematical elegance, the coherence of the second approach, and insights that might emerge from the first approach, and if the only objective is the ability to forecast day-ahead option premiums, one can simply tweak Black's model. An example of such an approach would be to model the volatility coefficient as a quadratic function of the strike. Even though it makes no theoretical sense (this is like saying that options with different strikes live in different universes), this approach will work good enough for many traders. A seminal article that evaluates the hedging effectiveness of such an approach is Dumas, Fleming and Whaley (1998).

In this article I take the first approach, and modify Black's model by modifying the terminal distribution of the futures price. Instead of a lognormal, I propose a generalized lambda distribution developed by Ramberg and Schmeiser (1974) and introduced to options pricing by Corrado (2001). An alternative would be to use Edgeworth binomial trees, but preliminary analysis showed that such an approach may not be adequate for situations where skewness and kurtosis are rather high. In addition, Edgeworth trees work with the skewness of terminal log-prices, while I prefer to have implied parameters for the skewness of terminal futures prices directly, not their logarithms. In addition, the GLD pricing model allows for a higher degree of flexibility in terms of skewness and kurtosis, i.e., its parameters are easy to imply from observed options prices and it is straightforward to develop a closed-form solution for pricing options. While these are all favorable characteristics, it is in fact the ability to gain additional economic insight that truly justifies yet another option pricing model. GLD allows us to get an explicit estimate of skewness and kurtosis of the terminal distributions, and that knowledge can be used to make a strong connection between the economics of supply and financial models for pricing options on commodity futures.

2.2 Theory of Storage and Time-Series Properties of Commodity Spot and Futures Prices

Deaton and Laroque (1992) used a rational expectations competitive storage model to explain nonlinearities in the time series of commodity prices: skewness, rare but dramatic substantial increases in prices, and a high degree of autocorrelation in prices from one harvest season to the next. The basic conclusion of their work was that inability to carry negative inventories introduces a non-linearity in prices that manifests itself in the above characteristics.

This is an example of theory being employed in an attempt to replicate patterns of observed price data. In a similar fashion, but subtly different, Williams and Wright (1991) postulate that the moments of expected price distributions at harvest time vary with the current (pre-harvest) price and available carryout stocks, as shown in Figure 4. According to them, when observed at annual or quarterly frequency, spot prices exhibit positive autocorrelation which emerges because storage allows unusually high or low excess demand to be spread out over several years. Furthermore, the variance of price changes depends on the level of inventory. When stocks are high, and the spot price is low, the abundance of stored stocks serves as a buffer to price changes, and variance is low. When stocks are low, and thus spot price is high, stocks are nearly empty and unable to buffer price changes. Finally, the third moment of the price change distribution also varies with inventories. Since storage can always reduce the downward price pressure of a windfall harvest, but cannot do as much for a really bad harvest, large price increases are more common than large decreases. The magnitude of this cushioning effect of storage depends on the size of the stocks. In conclusion, one should expect commodity prices to be mean-stationary, heteroskedastic and with conditional skewness, where both the second and third moments depend on the size of the inventories.

Testing the theory proceeds with this argument: if we can replicate the price pattern using a particular set of rationality assumptions, then we cannot refute the claim that people indeed behave in such manner. That is the road taken by Deaton and Laroque (1992) and Rui and Miranda (1995). However, since in the spot price series we only see the realizations of prices, not the conditional expectations of them, we cannot use spot price data to directly test what the market *expected* to happen. As such, predictions from storage theory focused on the scale and shape of expected distributions of new harvest spot prices have remained untested. In this paper I use options data to infer the conditional expectations of terminal futures prices, and therefore test the following prediction of the theory of storage:

- The lower inventories are, the more positive the skewness of the conditional harvest futures price distribution will be.

This is tested using an options pricing formula based on the generalized lambda distribution to calibrate the skewness and kurtosis of expected (conditional) harvest futures price distributions. Implied parameters from the model are then used to test the hypotheses above, as described in detail in section 4.

2.3 The Role of Weather for Intra-year Resolution of Price Uncertainty

As demonstrated in section 2.1, a very small share of uncertainty concerning the terminal price of a new crop futures contract is resolved before June. A large part of uncertainty is resolved between late June and early October. The reason lies in corn physiology and the way weather stress impacts corn throughout the growing season. In the major corn producing areas of the U.S., corn is planted starting the last week of April. It takes about 80 days after planting for a plant to reach its reproduction stage, also known as corn silking. At this juncture, the need for nutrients is highest, and moisture stress has a large impact on final yield. Weather continues to play an important role through the rest of the growing cycle, as summarized by Figure 5, taken from Shaw et al. (1988).

Every month during the growing, the United States Department of Agriculture (USDA) publishes updated forecasts of corn yield per acre. At the beginning of the growing season, before corn starts silking, these forecasts are dominated by the long-run trend that reflects improvements in plant genetics. As can be seen in Figure 6, June forecasts of final yield deviated from trend value essentially the same in both the record yield year 2004/2005, when final yield was 15 bushels above the trend, and the major draught year of 1988/89, when final yields were 32 bushels below the trend. However, uncertainty is quickly resolved in July and August. As shown in Figure 7, whereas June forecasts deviated from final estimates from the low of -11 percent in 1994/95 to the high of 45 percent in 1988/89, the September estimate deviations ranged only from -7 percent to 12 percent.

A testable hypothesis that emerges from these stylized facts concerns the fundamental role of seasonality in uncertainty resolution, as well as pronounced negative skewness in deviations of final yields from trend values. In other words, do seasonal yield deviations contribute to a positive skewness of terminal price distribution and the dynamics of skewness throughout the marketing year? In particular, we might expect implied skewness to decrease throughout the growing season.

2.4 Option Pricing Formula Using Generalized Lambda Distribution

The generalized lambda distribution was developed by Ramberg and Schmeiser (1974), and its properties were described further by Ramberg et al. (1979). It was introduced to options pricing by Corrado (2001) who derived a formula for pricing options on non-dividend paying stocks. Here I review the properties of GLD and adopt Corrado's formula to options on futures.

GLD is most easily described by a percentile function[1] (i.e., inverse cumulative density function):

[1] F here stands for futures price, not for cumulative density function.

$$F(p)=\lambda_1+\frac{p^{\lambda_3}-(1-p)^{\lambda_4}}{\lambda_2}. \tag{5}$$

For example, to say that for $p=0.90, F(p)=4.5$ means that the market expects with a 90 percent probability that the terminal futures price will be lower than or equal to \$4.50/bu.

GLD has four parameters: λ_1 controls location, λ_2 determines variance, and λ_3 and λ_4 jointly determine skewness and kurtosis. In particular, the mean and variance are calculated as follows:

$$\begin{aligned}\mu &= \lambda_1 + A/\lambda_2 \\ \sigma^2 &= \left(B-A^2\right)/\lambda_2^2\end{aligned} \tag{6}$$

with $A=\frac{1}{1+\lambda_3}-\frac{1}{1+\lambda_4}$ and $B=\frac{1}{1+2\lambda_3}+\frac{1}{1+2\lambda_4}-2\beta(1+\lambda_3,1+2\lambda_4)$, where $\beta(\ )$ stands for complete beta function. We see that the λ_3 and λ_4 parameters influence both location and variance, however, λ_1 influences only the first moment, and λ_2 influences only the first two moments, i.e., skewness and kurtosis do not depend on λ_1 and λ_2.

The skewness and kurtosis formulas are:

$$\begin{aligned}\alpha_3 &= \frac{\mu_3}{\sigma^3}=\frac{C-3AB+2A^3}{\lambda_2^2\sigma^3} \\ \alpha_4 &= \frac{\mu_4}{\sigma^4}=\frac{D-4AC+6A^2B-3A^4}{\lambda_2^4}\end{aligned} \tag{7}$$

with $C=\frac{1}{1+3\lambda_3}-\frac{1}{1+3\lambda_3}-3\beta(1+2\lambda_3,1+\lambda_4)+3\beta(1+\lambda_3,1+2\lambda_4)$

and $D=\frac{1}{1+4\lambda_3}+\frac{1}{1+4\lambda_3}-4\beta(1+3\lambda_3,1+\lambda_4)-4\beta(1+\lambda_3,1+3\lambda_4)+6\beta(1+2\lambda_3,1+2\lambda_4)$.

A standardized GLD has a zero mean and unit variance, and has a percentile function of the form:

$$F(p)=\frac{1}{\lambda_2(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right) \tag{8}$$

where $\lambda_2(\lambda_3,\lambda_4)=sign(\lambda_3)\times\sqrt{B-A^2}$.

From here, we can move more easily to an options pricing environment. We wish to make GLD an approximate generalization of the log-normal distribution so we keep the mean and the variance the same as in (4), while allowing skewness and kurtosis to be separately determined by the λ_3 and λ_4 parameters. Therefore, the percentile function relevant for option pricing will be

$$F(p)=F_0\left(1+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)\right). \tag{9}$$

Note that this is equivalent to (5) with $\lambda_1=F_0+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)$

and $\lambda_2=\frac{\lambda_2(\lambda_3,\lambda_4)}{\sqrt{e^{\sigma^2 t}-1}}$. This will guarantee that the first two moments of the terminal distribution will be $\tilde{\mu}=F_0 \quad \tilde{\sigma}^2=F_0^{\,2}\left(e^{\sigma^2 t}-1\right)$, just like in Black's model.

The pricing formula for European calls is

$$V(K,F_0,T,\sigma,r,\lambda_3,\lambda_4)=e^{-rT}\int_0^\infty Max(F_T-K,0)\,dp(F). \tag{10}$$

As shown by Corrado (2001), we can simplify this through a change-of-variable approach where $F(p)=F_T$:

$$\int_0^\infty Max(F_T-K,0)\,dp(F)=\int_K^\infty (F_T-K)\,dp(F)=\int_{p(K)}^1 \left(F(p)-K\right)dp. \tag{11}$$

Here $p(K)$ stands for the cumulative density function, evaluated at *K*. While there is no closed-form formula for the function, values can be easily found with numerical approaches by using the percentile function.

Integrating $F(p)$ we get

$$G_1=\int_{p(K)}^1 F(p)\,dp=F_0\left(p+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_3+1}p^{\lambda_3+1}+\frac{(1-p)^{\lambda_4+1}}{\lambda_4+1}+\frac{1}{\lambda_4+1}p-\frac{1}{\lambda_3+1}p\right)\right)\Bigg|_{p(K)}^{1}$$

$$=F_0\left(1-p(K)+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{p(K)-p(K)^{\lambda_3+1}}{\lambda_3+1}+\frac{1-p(K)-(1-p(K))^{\lambda_4+1}}{\lambda_4+1}\right)\right)$$

with the final European call pricing formula being:

$$V(K,F_0,T,\sigma,r,\lambda_3,\lambda_4)=F_0e^{-rt}G_1-e^{-rt}KG_2 \tag{12}$$

where G_1 is defined above and $G_2=1-p(K)$.

In a similar way it can be shown that the price for a put is

$$V_P(K,F_0,T,\sigma,r,\lambda_3,\lambda_4)=e^{-rt}K(1-G_2)-F_0e^{-rt}(1-G_1). \tag{13}$$

3 Econometric Model

The first thing I seek to explain is inter-year variation in implied skewness. As argued in the second section, skewness will likely be impacted by weather once corn silking starts. Therefore, if we are to infer an impact of storage on skewness across many years, each with its own weather peculiarities, we should choose the time before the reproductive growth phase starts, i.e., no later than the third week of June. If we were to choose skewness observed much earlier than that, we would risk falling in the endogeneity trap. Before a marketing year is close to an end, consumption can react to changes in futures price, possibly even in options premiums, thus increasing or decreasing carryout stocks. It would make little sense then to use expected ending stocks-to-use as a predetermined explanatory variable and implied skewness as a dependent variable. To avoid this problem, expected ending stocks-to-use ratio, as reported in the June edition of the World Agricultural Supply and Demand Estimates (WASDE) report,[2] is employed as the explanatory variable for storage adequacy.

If the supply of corn is not completely inelastic to prices, we would expect rational producers to react to tighter expected storage stocks and higher new crop prices with an increase in planted acreage, so acreage response is the second variable we need to include in the model. To control for elastic supply, I use the measure of change between intended plantings, as reported in the Prospective Plantings report[3] published at the end of March, and the actual acreage planted in the previous marketing year.

In addition to supply side covariates, we need to address possible asymmetries in uncertainty of demand. Corn is used as livestock feed, an industrial sweetener and as an input in ethanol production. All three of these derived demand categories are likely impacted by macroeconomic shocks. Therefore, as a measure of demand uncertainty I use the June-to-June change in the national unemployment rate as published by the Bureau of Labor Statistics.

The final econometric model has the following form:

$$IS_{t,T} = \alpha + \beta_1 E_t \Delta h_T + \beta_2 E_t \left[S_T / D_t \right] + \beta_3 \Delta U_t$$

where $IS_{t,T}$ stands for implied skewness at time *t* for a contract expiring at time *T*. The change in acreage planted is h_T. Since in June we only observe intended plantings, expected change in acreage is used in the model. Expected ending stocks-to-use is $E_t \left[S_T / D_t \right]$ and ΔU_t is the June-to-June change in unemployment rate. Theory predicts that all coefficients except the constant should be negative. A stronger acreage response

[2] The WASDE report is published by the World Agricultural Outlook Board, an inter-agency body at the United States Department of Agriculture. Historical WASDE reports can be accessed at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1194.

[3] Prospective Plantings is a government report produced annually by the National Agricultural Statistics Service, an agency of the United States Department of Agriculture. Historical Prospective Plantings reports can be accessed at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1136.

and higher carryout stocks relative to demand imply more ability to buffer adverse weather shocks, and will thus reduce skewness. Likewise, a more unstable macroeconomic environment will decrease demand for fuel and possibly even meat, thus reducing upward pressure on corn prices.

I did not perform an explicit econometric analysis of intra-year dynamics of skewness, but I do present detailed visualizations in section 5 that support the argument that implied skewness will decline during the reproductive and grain filling growth phases for corn.

4 Data

Commodity futures for corn as well as options on futures are traded on the Chicago Mercantile Exchange (formerly the Chicago Board of Trade). A dataset comprising all recorded transactions, i.e., times and sales data for both futures and options on futures, for the period 1995 through 2009, was obtained. It includes data for both the regular and electronic trading sessions. The total number of transactions exceeds 30 million. Options data were matched with the last preceding futures transaction. LIBOR interest rates were obtained from the British Bankers' Association and represent the risk-free rate of return. Overnight, 1 and 2 weeks, and 1 through 12 months of maturity LIBOR rates for the period 1995 through 2009 were used to obtain the arbitrage-free option pricing formulas. In particular, each options transaction was assigned the weighted average of interest rates with maturities closest to the contract traded. To avoid serial correlation in residuals from estimating implied coefficients in subsequent steps, the data frequency was reduced to not less than 15 minutes between transactions for the same options contract. This resulted in 11,139 data files, each containing between 200 and 500 recorded transactions for a particular trading day for a given commodity. For each data point I separately estimate implied volatility using CRR binomial trees with 500 steps. Then, for each data point, the price of a European option using Black's formula is calculated using the same parameters (futures price, interest rate, time to maturity) as that recorded for the American option. In addition, volatility is set equal to the one implied for American options. These "artificial" European options are then used in calibration and econometric analysis.

The implied skewness used in the econometric analysis is calculated as a simple average over 10 business days following the June WASDE report. Due to high incidence of limit-move days and days with high intraday price changes, year 2008 is excluded from the sample. Including 2008 would render the calculation of higher moments unreliable. Descriptive statistics of variables used in the econometric analysis are given in Table 1, and corn supply/demand balance sheets are in Table 2.

Figure 8 presents a scatter diagram of expected ending stocks-to-use vs. implied skewness. Note the inverse relationship between these variables and the beneficial impact of the acreage response. For example, in the summer of 1996, carryout stocks-to-use were only 4.03 percent, two standard deviations below the average for 1995-2009. However, skewness was below the mean, due to a 12.2 percent increase in expected acreage, which is 2.2 standard deviations above the average increase of 1.4 percent. Similarly, in 2007 carryout stocks were only 8.56 percent of demand, but a massive acreage increase of 15.5 percent, by far the largest in this sample, reduced the skewness below the mean. It is instructive to look at 2006 as well. Although ending stocks were bountiful at 19.67 percent of demand, a reduction in acreage of 4.6 percent made for the third largest skewness in the sample.

5 Estimation Procedure and Results

For each contract, for each trading day, I separately estimate the parameters σ, λ_3, and λ_4 in the GLD option pricing formula. In particular, I minimize the squared difference in option premiums calculated with the GLD formula, and prices of European options as implied by Black's model.

To proceed, I first need a starting value for the implied volatility of an option with a strike price closest to the underlying futures price. The starting values for the λ_3 and λ_4 parameters were chosen to correspond to the skewness and kurtosis of the terminal futures price with the restriction that the logarithm of the terminal price is normally distributed with variance equal to $\sigma^2 t$, where σ^2 is the square of the starting value for the implied sigma parameter. Excel Solver is used to run the minimization problem, utilizing a FORTRAN compiled library (.dll file) created by Corrado (2001) that estimates GLD European call prices. A formula for the GLD European put option was then programmed in Visual Basic for Applications.

Estimated lambda parameters are employed to calculate implied skewness and kurtosis. GLD option prices seem to work rather well, with an average absolute pricing error about 3/8 of a cent per bushel, and a maximum pricing error usually reaching not more than 2 cents (this occurs for the least liquid and most away from the money options). While there may be issues regarding the robustness of implied parameters with respect to starting values, the implied parameters seem to be rather stable from one day to the next. For Dec '07 corn, for example, the skewness estimated between June 11 and June 25, 2005 varies between 1.15 and 1.26. For that year, the average absolute pricing error was 7/8 of a cent per bushel, with a maximum pricing error of 7.9 cents.

For all years in the sample, the implied skewness is 1.2 to 3 times higher than it would be if the logarithm of the terminal futures price was really expected to be normal. Implied kurtosis is 1.2 to 1.6 times higher than that predicted by Black's model. We thus see that deviations from Black's model are particularly pronounced in implied skewness.

The effects of a “weather scare” on implied skewness is demonstrated in Figure 9. For each contract month, skewness was averaged over 15 years (1995-2009) for the matching time to maturity horizons. Here we clearly see that skewness does not decrease nearly linearly as is the implication of Black’s model. Instead, we see three distinct phases. First, before the growing season, skewness is very stable and rather high. Next, during the growing season, skewness decreases from late June through October. Finally, over the last 90 days to option maturity, skewness declines concavely. It is fascinating to see exactly the same pattern in all five contract months. This illustrates the rationale for using skewness in June to search for the effect of storage – this is the time just before the skewness starts declining.

Simple linear regression is estimated for the period 1995-2009 using implied skewness as the dependent variable, with constant, expected ending stocks-to-use, expected planned change in planted acreage, and changes in the unemployment rate as predetermined explanatory variables. Regression statistics are reported in Table 3. Due to very low degrees of freedom (10), we have to rely on t-table for critical values, and use a one-tail test for the stocks-to-use coefficient.

An increase in stocks-to-use by 1 percentage point reduces skewness by 0.015, and this coefficient estimate is statistically significant at the 95 percent confidence level. To put this number in perspective, the difference between the lowest and the highest ending stocks-to-use recorded in the sample reduces skewness from 1.47 to 1.24, which is 47 percent of the difference between the highest and the lowest recorded skewness in the sample. Coefficients for demand uncertainty and acreage response are also statistically significant, have the expected sign, and exhibit much less noise than storage variable.

6 Conclusions and Further Research

An options pricing model based on a generalized lambda distribution provides a useful heuristic in thinking about determinants of the shape of terminal futures price conditional distributions. Results indicate that crop inventories and plant physiology play a significant role in determining the expected asymmetry of the terminal distribution. In particular, results reveal that implied skewness is much more persistent than implied by Black’s model. In years with low implied volatility, implied skewness remains much higher than would be the case under the lognormality restriction, and dynamics are dominated not by time to maturity, but by temporal patterns in resolution of uncertainty regarding crop yields.

Further research will focus on extending this analysis to soybeans and wheat for which times and sales data are also available. The U.S. is a major world player in corn, with 55.6 percent of world exports. That is higher than 45.3 percent of world exports of soybeans, and especially than 17.7 percent in wheat. Extending the analysis to other crops will identify the effect of trade and non-overlapping growing seasons in different countries on the magnitude, inter-year differences, and intra-year dynamics on implied higher moments of the terminal price distribution.

Thus far, the literature has focused on evaluating the impacts of government reports on implied volatility coefficients. The model presented here allows us to extend this to higher moments and examine how reports (i.e., information) influence the entire distribution of prices, not just the second moment. For example, we could use weekly crop progress reports to explain inter-year differences in the evolution of skewness through the summer months.

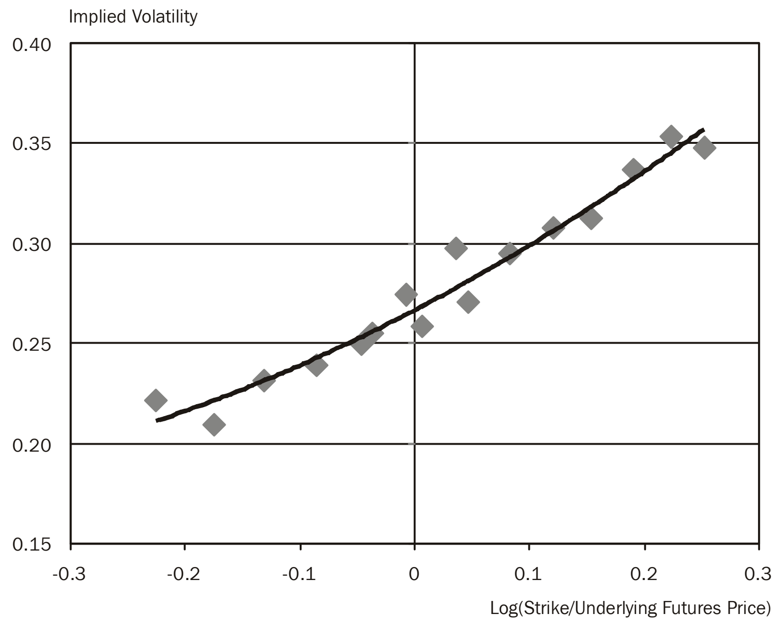
In the absence of high frequency data, many researchers use end-of-day reported prices for futures and options to evaluate implied higher moments. By re-estimating this model using only end-of-day data it is possible to examine the amount of noise and possible direction of bias such an approach brings to estimates of implied higher moments.

What happens when storage is not available to partially absorb the shocks to supply? It would be interesting to use the GLD option pricing model to examine the evolution and determinants of higher moments of non-storable commodities. Further research is needed to examine the impact of durability of production factors for commodities that are themselves not storable.

Finally, impacts of market liquidity and trader composition on the levels and stability of implied higher moments is a promising new area for research. With careful design of the analysis, we may be able to find a way to separate the part of the option price that is due to implied terminal price distributions from additional premium influences incurred due to hedging pressure or lack of market liquidity.

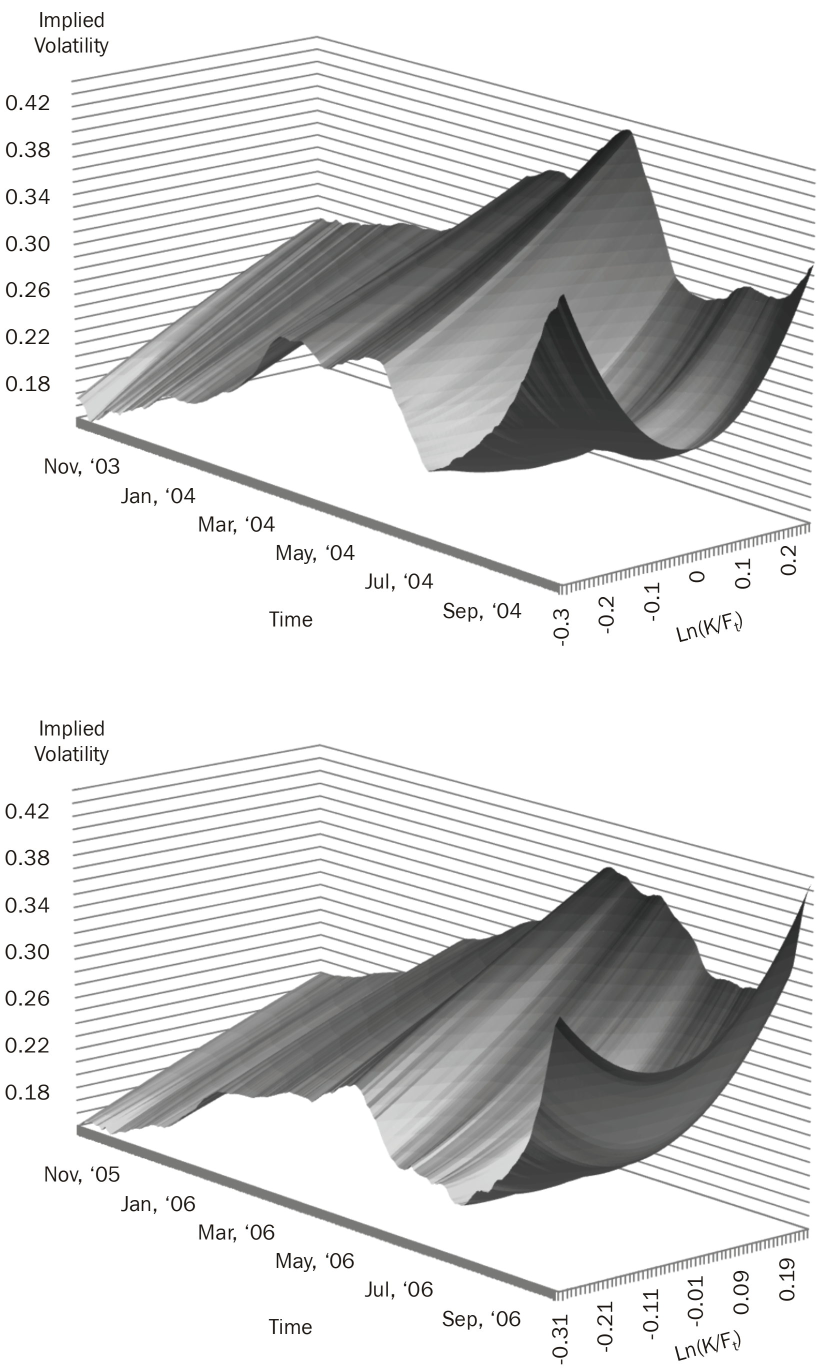
Appendix

Figure 1 **Typical Pattern for Implied Volatility Coefficients for Options on Agricultural Futures**



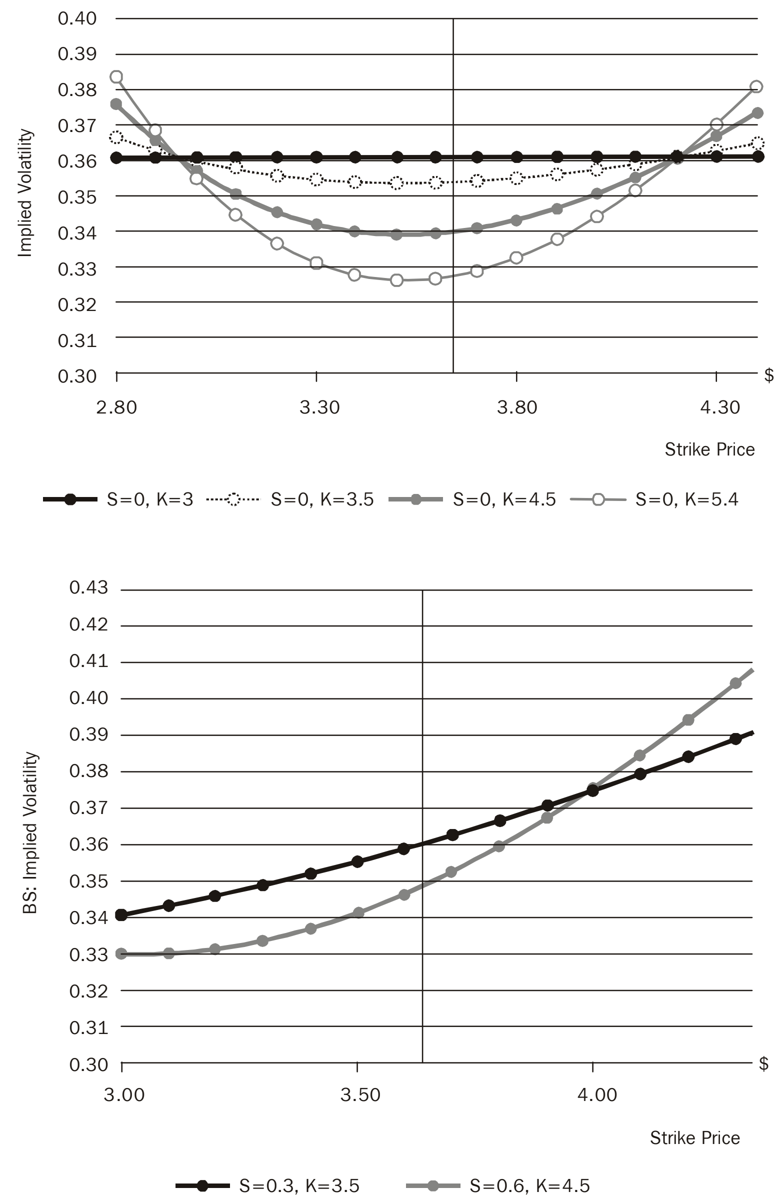
Notes: Implied volatility coefficients are estimated for options on Corn December 2006 futures contract, on 6/26/2006 using Cox, Ross and Rubinstein's binomial tree with 500 steps. Underlying futures price was $2.49/bu. Dots represent implied volatility coefficients for each strike, and smooth line is fitted quadratic trend.

Figure 2 **Evolution of Implied Volatility Curve for Options on Dec' 04 and Dec '06 Corn Futures**



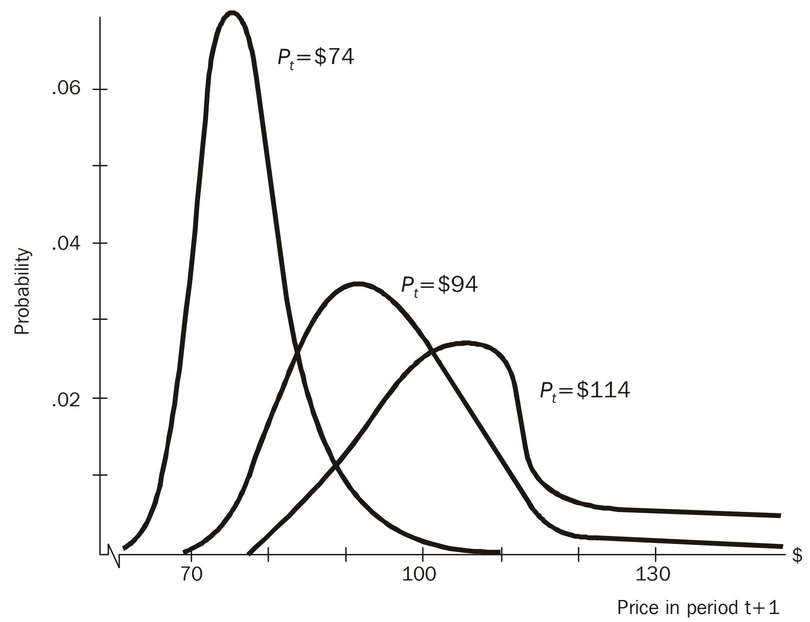
Notes: For each day, implied volatility is estimated for each traded option using 15-minute interval data. Quadratic trend curve is fitted to produce implied volatility curve. 30-day moving average is calculated to increase smoothness of the volatility surface and make it easier to see principal characteristic of the IV curve evolution. Z-axis shows option moneyness calculated as logarithm of the ratio between option strike (K) and underlying futures price F_t. When option strike price is equal to current futures price moneyness is zero.

Figure 3 **Effects of Excess Kurtosis and Positive Skewness on Implied Volatility**



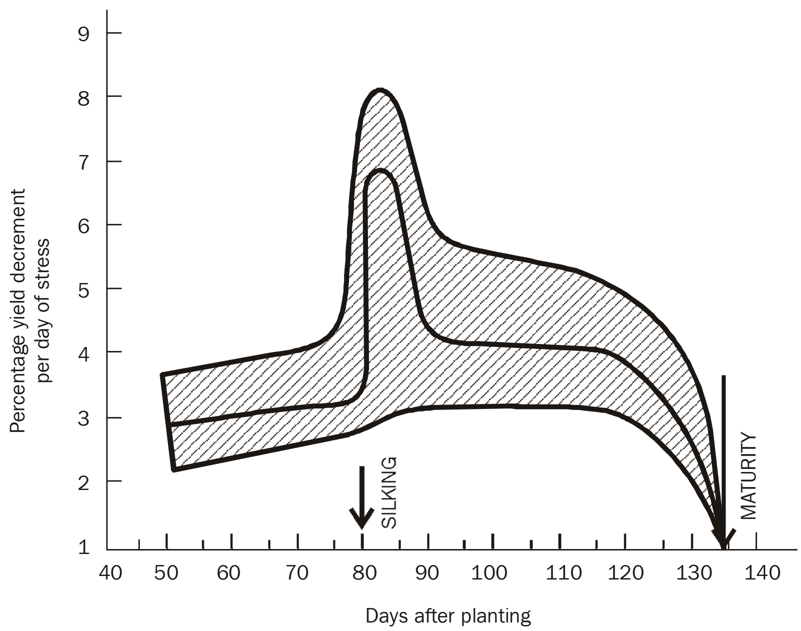
Notes: S stands for skewness, K for kurtosis of terminal futures log-prices. Option premiums are calculated via Rubinstein's Edgeworth binomial trees that allow for non-normal skewness and kurtosis, and implied volatility is inferred using Cox, Ross and Rubinstein's binomial tree which assumes normality in terminal futures prices. The black line in the above diagram with S=0 and K=3 corresponds to assumptions of Black's model, and in such a scenario implied volatility curve is flat across all strikes. Excess kurtosis (K>3) creates convex and nearly symmetric "smiles", and positive skewness produces an upward-sloping implied volatility curve.

Figure 4 **New Crop Price Distributions Conditional on Storage Adequacy**



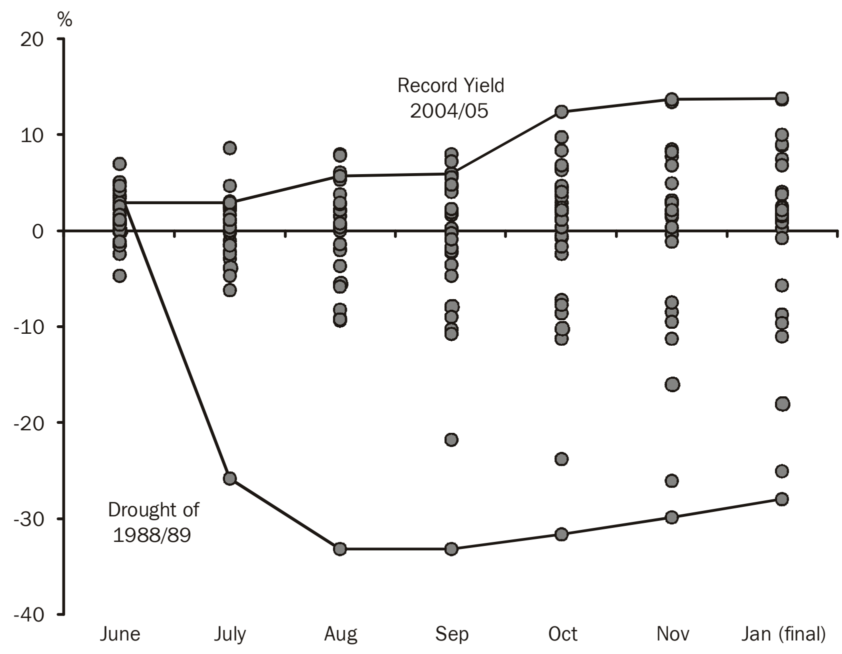
Notes: Reproduced from Williams and Wright (1991). Conditional price distributions obtained from rational expectations competitive equilibrium model with storage. Time frequency is one year, i.e., t+1 represents next harvest. New crop price distributions are conditional on information known after old crop carryout stocks have been determined, but before weather shock is revealed. Price and quantity are standardized to make non-stochastic equilibrium at ($100, 100 units). Higher prices at time *t* reflect lower carryout stocks, and correspond to higher skewness of new crop price distribution.

Figure 5 **Weather Stress in the Corn Crop**



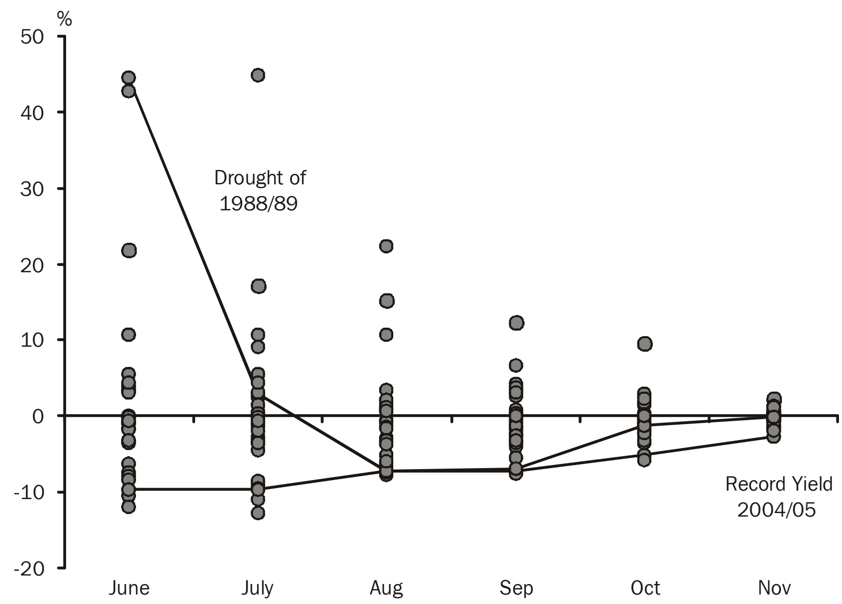
Notes: Reproduced from Shaw et al. (1988). This figure shows the relationship between the age of corn crop and percentage yield reduction due to one day of moisture stress. Outer lines show boundaries of experimental results, while the middle line shows the average.

Figure 6 **Monthly Projected Corn Yield 1980-2008 - Deviation from Trend**



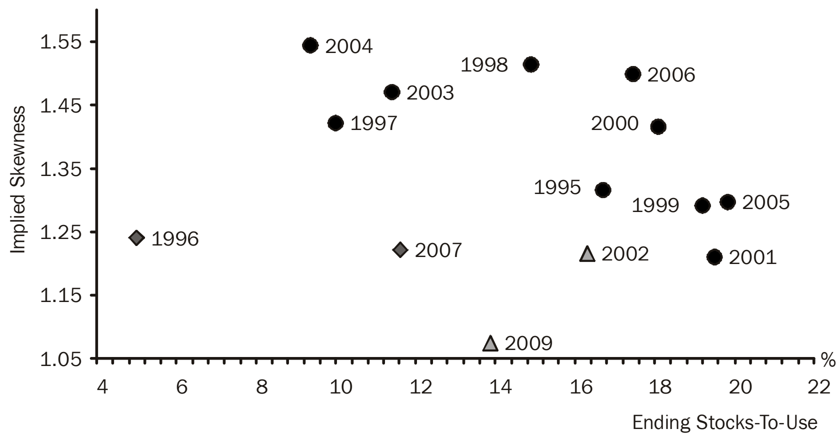
Notes: For each year, trend yield was calculated as simple linear regression over previous years, starting in 1960. Monthly projected yields were obtained from WASDE reports either directly or by calculations based on projected planted area and expected production size.

Figure 7 **Monthly Projected Corn Yield 1980-2008 - Deviation from Final Estimate (January)**



Notes: For each month, projected yield was obtained from WASDE reports. Final estimates are taken from WASDE reports published in January.

Figure 8 **Relationship between Implied Skewness and Expected Ending Stocks-to-Use**



Notes: Years with increase in intended cultivated acreage of 5 or more percent are marked with green rhombs. Years with June-to-June increase in unemployment rate of 1 percent or more are marked with blue triangles.

Figure 9 **Options on Corn Futures: Dynamics of Implied Skewness**

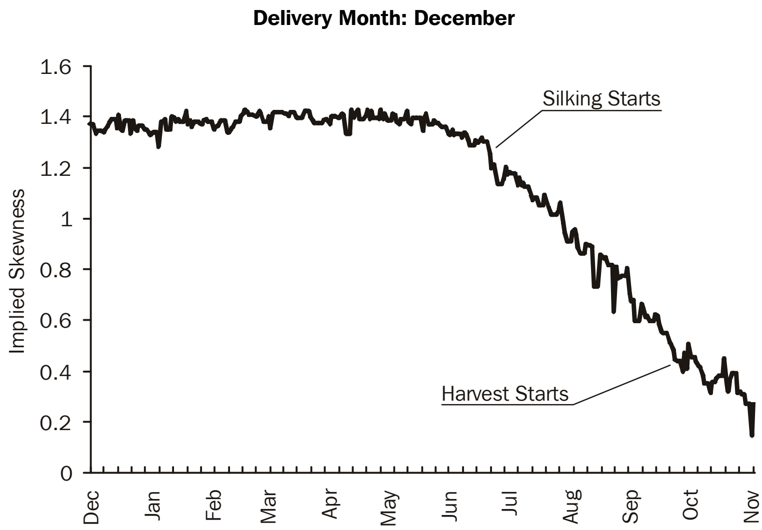


Figure 9 **Continued**

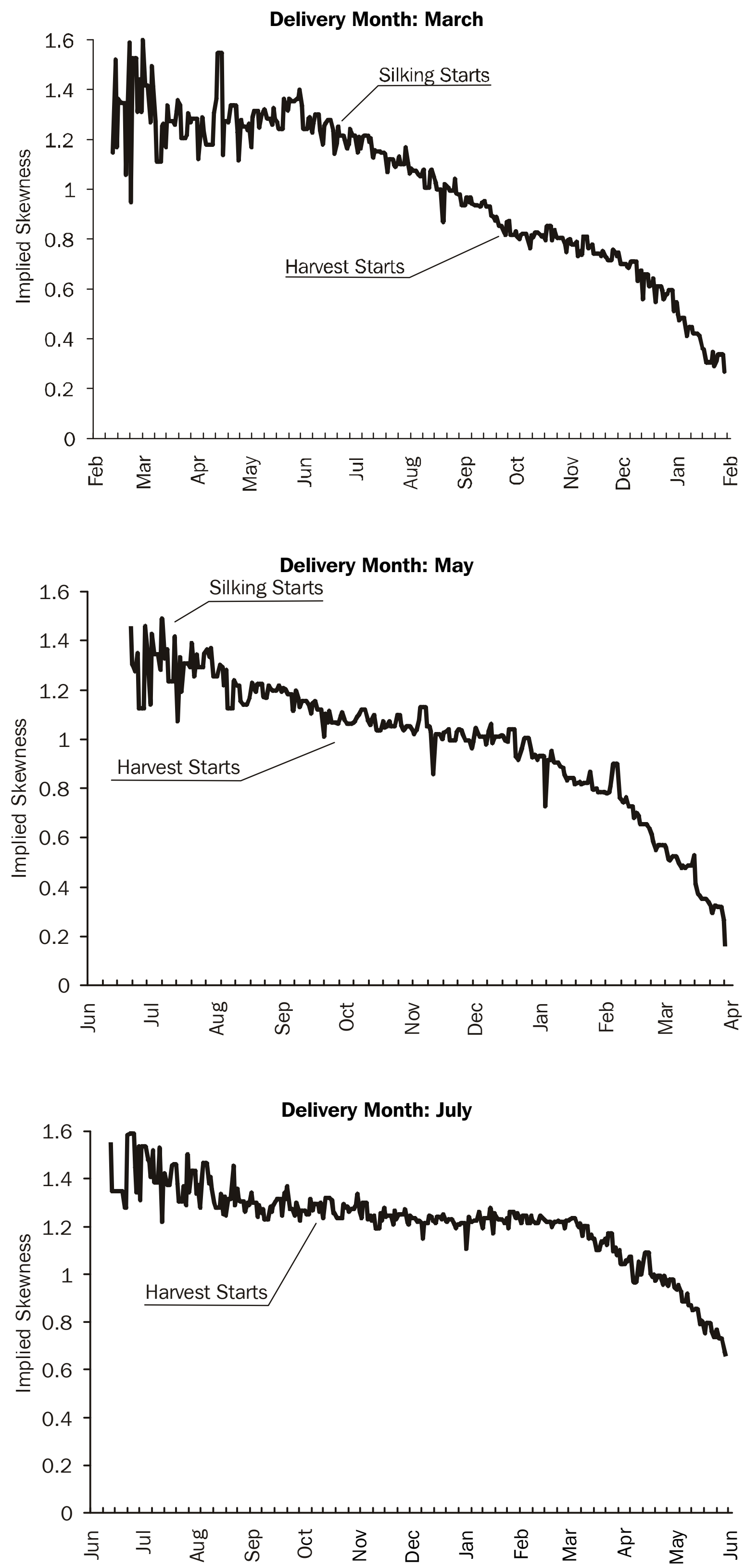
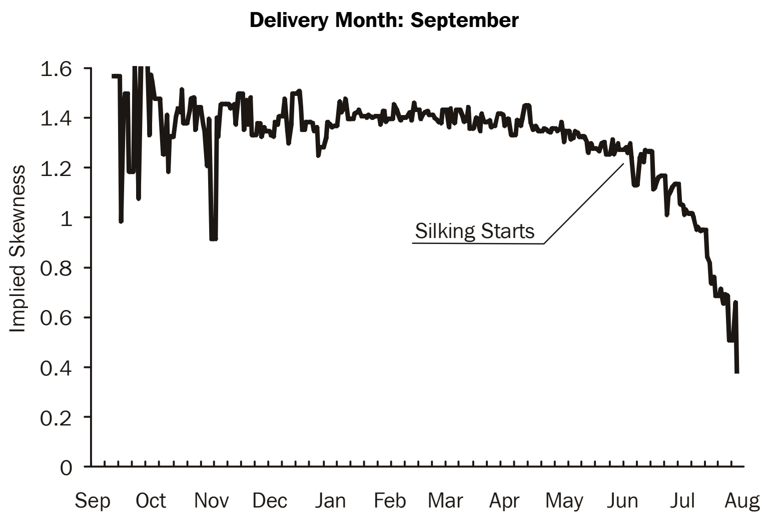


Figure 9 **Continued**



Notes: Implied skewness is estimated for each contract and for each trading day separately using generalized lambda distribution pricing model for options on commodity futures. Graphs show averages taken over 1995-2009 for each contract and each time to maturity. Corn silking is the reproductive stage of corn growth where weather starts having major impact on final grain yield. In the major U.S. corn growing area corn usually starts silking in the last days of June, and corn harvest normally begins in late September.

Table 1 **Determinants of Implied Skewness: Descriptive Statistics**

Variable	Mean	Standard deviation	Min	Max
Implied skewness	1.33	0.14	1.07	1.54
Ending stocks-to-use (%) WASDE June projection	14.4	5.36	4.03	21.23
Intended acreage planted – percentage change	1.37	5.89	-4.84	15.48
Unemployment rate change	0.17	0.23	-0.7	4.00

Notes: Implied skewness was calculated for December corn contracts as average for implied parameters over 10 trading days following June WASDE report. On average, 100-150 data points were used in estimating implied parameters for each trading day in the stated periods.

Table 2 **Corn Supply/Demand Balance Sheet 1995-2009**

		Marketing year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
SUPPLY	Exp.	Exp. acres planted	73.3	79.0	81.4	80.8	78.2	77.9	76.7	78.0	79.0	79.0	81.4	78.0	90.5	86.0	85.0
		Exp. acreage change (%)	**-7.4**	**11.0**	**2.4**	**0.7**	**-2.5**	**0.6**	**-3.5**	**2.9**	**-0.1**	**0.4**	**0.6**	**-4.6**	**15.6**	**-8.1**	**-1.2**
		Exp. yield	119.7	126.0	131.0	129.6	131.8	137.0	137.0	135.8	139.7	145.0	148.0	149.0	150.3	148.9	153.4
	Realized	Acres planted	71.2	79.5	80.2	80.2	77.4	79.5	75.8	79.1	78.7	80.9	81.8	78.3	93.6	86.0	86.5
		Acres harvested	65.0	73.1	72.7	72.6	70.5	72.4	68.8	69.3	70.9	73.6	75.1	70.6	86.5	78.6	79.6
		Harvested %	91.3	91.9	90.6	90.5	91.1	91.1	90.8	87.6	90.1	91.0	91.8	90.2	92.4	91.4	92.0
		Yield	113.5	127.1	127.0	134.4	133.8	137.1	138.2	130.2	142.2	160.4	147.9	149.1	151.1	153.9	164.7
		Production	7,374	9,293	9,366	9,761	9,437	9,968	9,507	9,008	10,114	11,807	11,112	10,535	13,074	12,101	13,110
		Beginning stocks	1,558	426	883	1,308	1,787	1,718	1,899	1,596	1,087	958	2,114	1,967	1,304	1,624	1,673
		Imports	16	13	9	19	15	7	10	14	14	11	9	12	20	15	8
		Total supply	8,948	9,732	10,258	11,088	11,239	11,693	11,416	10,618	11,215	12,776	13,235	12,514	14,398	13,740	14,792
DEMAND	Exp.	Exp. total demand	8,600	8,820	9,000	9,360	9,480	9,645	9,725	9,535	10,405	10,560	11,060	11,525	12,960	12,140	13,190
		Exp. ending stocks	347	909	1,259	1,727	1,759	2,048	1,621	1,084	806	2,215	2,176	987	1,433	1,600	1,603
		Exp. stocks-to-use (%)	**4.03**	**10.3**	**13.9**	**18.4**	**18.5**	**21.2**	**16.7**	**11.4**	**7.7**	**20.9**	**19.6**	**8.5**	**11.1**	**13.2**	**12.2**
	Realized	Feed & residual	4,696	5,360	5,505	5,472	5,664	5,838	5,877	5,558	5,798	6,162	6,141	5,598	5,938	5,205	5,159
		Food/Seed/Ind.	1,598	1,692	1,782	1,846	1,913	1,967	2,054	2,340	2,537	2,686	2,981	3,488	4,363	4,993	5.938
		Ethanol	N/A	N/A	N/A	N/A	N/A	N/A	N/A	996	1,168	1,323	1,603	2,117	3,026	3,677	4,568
		Exports	2,228	1,797	1,504	1,981	1,937	1,935	1,889	1,592	1,897	1,814	2,147	2,125	2,436	1,858	1,987
		Total demand	8,522	8,849	8,791	9,299	9,514	9,740	9,820	9,490	10,232	10,662	11,269	11,211	12,737	12,056	13,084
		Ending stocks	426	883	1,467	1,789	1,725	1,953	1,596	1,128	983	2,114	1,966	1,303	1,661	1,684	1,708
		Stocks-to-use (%)	5.0	10.0	16.7	19.2	18.1	20.1	16.3	11.9	9.61	19.8	17.5	11.6	13.0	14.0	13.1
		Avg. farm price	3.24	2.71	2.43	1.94	1.82	1.85	1.97	2.32	2.42	2.06	2.00	3.04	4.20	4.06	3.55

Notes: Acres planted and harvested are measured in million acres, yield in bushels per acre, beginning and ending stocks, imports, exports and other demand categories are measured in million bushels. Average farm price is measured in U.S. dollars per bushel. Corn marketing year starts on September 1 of the current calendar year, and ends on August 31 the following calendar year. Expected acres planted are based on the Prospective Plantings report published at the end of March preceding the marketing year. Expected total demand, ending stocks, and stocks-to-use are taken from the June WASDE report. For example, marketing year 2001/02 (denoted in the table simply as 2001) started on 09/01/2001, and ended on 08/31/2002. For that year, expected acres planted were published on 03/31/2001 and expected total demand, ending stocks and stocks-to-use were taken from the WASDE report published on 06/12/2002. Variables used in econometric analysis are bolded.

Table 3 **Determinants of Implied Sskewness: Regression Rresults**

Explanatory variables	**Dependent variable: GLD implied skewness**
Constant	1.55 (0.10)
Ending stocks-to-use (%)	-1.28 (0.64)
Intended acreage planted percentage change	-1.52 (0.58)
Unemployment percentage change	-0.07 0.02
Degrees of freedom	10
Mean root square error	0.075
R^2	0.66

Notes: Critical t-statistic for 10 d.f. for 95 percent is 1.81 for one-tail tests and 2.22 for two-tail tests. All coefficients are statistically significant at a 95 percent confidence level (ending stocks-to-use coefficient is significant at 95 percent using one-tail test or 90 percent using two-tail test).

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