MUSICAL COMPOSITION AND ELEMENTARY EXCITATIONS OF THE ENVIRONMENT

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SUMMARY

During the performance of a musical composition a special human environment comes into being. We aim to interpret the compositions as a specific class of human environment in order to utilise that for description of structure and dynamics of general human environment. In particular, we analysed four Bach's fugas considering their motifs and introducing the concept of the energy of a motif. We used bosonic excitations from theoretical physics as an analogical starting point for this concept, with the aim of proposing a new way to shed light onto more complex human environments. The aim is neither to reduce music to physical theories nor to define new theories of music, but to approach the investigation of complex environments in a new way, using already existing concepts.

KEY WORDS

elementary excitations, musical composition, motifs, Bach, bosonic heat bath

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1. INTRODUCTION

When a music composition is performing, the listener is situated in a specific environment. This environment is a set of the auditory influences. Other non-auditory influences are supposed as negligible and the auditory influences are limited as close as possible just to the composition performance.

During the history the rules of composing were changing. These rules are dependent on the cultural setting, individual characteristics of composers and listeners, modes of life, and many other facts. Different influences are non-linearly transformed into the music compositions. From the other side, music surely influences the humans. This influence is not reducible to some simple effects, but reflects complicated psycho-physical properties of the listeners and composers. Altogether, these two lines of reasoning argue that the interplay of the humans and music compositions is a complex system. Great number of different compositions thus give evidence that the performing of music builds one environment linked to the society.

In summary, a temporary and auditory environment, formed during music composition performance, is one of possible realisations of complex human environment.

The scope of this article is to present one simplified interpretation of the musical compositions. Main idea is to present a composition as a special type of environment. This may improve the understanding of general structure and dynamics of complex environments and human-environment relations.

One of the existing environment descriptions comes from theoretical physics, which is introduced and applied here. It presumes environment as a set of the potential and realised excitations [1]. Therefore, the first step is to analyse the composition as a collection of elementary excitations.

Here, the compositions are represented as the collections of the motifs as elementary but structured elements of the composition. This is a starting reduction of composition's characteristics. Furthermore, the motifs are statistically analysed, which is another one reduction. Speaking in terms of physics, it is the projection of composition onto a chosen set of motifs as simple units of interpretation. After that, a connection between the statistics of motifs and the notion of bosonic heat bath environment is established.

The aim of this article is neither to develop some theories about interpretation of composition and composing rules nor to reduce music composition to the simple notion of theoretical physics. On the contrary, the contribution is in connecting the composition with other types of complex human environment notions.

2. MUSICAL COMPOSITION AND BOSONIC HEAT BATH

The environments in physics are usually described as collection of mutually independent entities, so called normal modes [1]. They are determined uniquely by certain set of attributes, so called quantum numbers. Each mode has definite energy, that is a measure of easiness of creation of this mode, or the measure of interaction of this mode with other elements in environment. It depends on number of quanta in a given mode. There are two general types of modes: bosons
and fermions. Number of bosons in a given mode is unbounded and number of fermions in a given mode can not exceed one.

It is founded that the interaction of the elements that reside in the environment goes via exchange of elementary excitations of the environment, which are bosons. Regarding a musical composition performance as an environment and the motifs as elementary excitations of this environment, here is used the notion of bosonic heat bath for the description of statistics of basic elements of composition - motifs. Heat bath is a collection of bosons subjected to certain distribution over their energies.

First step in this analysis is to associate to every motif its energy. For this purpose each motif is represented by sequence

\[ m = (m_1, m_2, ..., m_N), \]  

where \( m_i \) is the height of the \( i \)th element in the sequence. This height is measured in whole tones or half-tones above the lowest tone in the motif. Single element denotes the tone of the shortest duration. Tones of longer duration are denoted as a repetition of the same element such number of times to span the duration of this tone by the shortest tones. The tone duration and tone heights are discrete. In analogy with bosons, to each motif is attributed its energy. This is performed through the expression

\[ E(m) = \sum_{i=1}^{N-1} (m_{i+1} - m_i)^2 + \omega^2 \sum_{i=1}^{N} (m_i - \bar{m})^2. \]  

Here is

\[ \bar{m} = \frac{1}{N} \sum_{i=1}^{N} m_i \]  

the average value of height in one motif including the duration of each tone. The first part in (2) formally corresponds to the kinetic energy in general physics and will be called kinetic part. The second part corresponds to potential energy and will be called potential part. Kinetic part takes into account tone height variations in a motif between neighbouring tones. Squared differences are introduced because of time inversion symmetry, and also because of analogy with general physics formulation. Potential part measures the deviation of all tones from average tone height of motif, and is taken from general physics considerations as the simplest form of the potential energy.

The rhythm is included in expression (2), although not so obviously. Pre-factor \( \omega^2 \) is introduced also by analogy with potential energy of harmonic force. Expression (2) is applied to all of identified motifs in a composition and for now \( \omega^2 \) is assumed as an undetermined but unique constant.

From the other side, in physics, the energies of the same bosons are discrete and given by formula

\[ E(n) = \hbar \omega (n + c), \]  

where \( \omega \) has the same meaning as in (2) where it represents the angular frequency of a mode, \( n \) is the number of quanta in this mode, while \( \hbar \) and \( c \) are constants that need to be determined.
These two formulations of energy calculations have to be adapted one to the other. Process of this adaptation gives the values of still undetermined constants. Namely, the formulas (2) and (4) enable to represent the motifs as a set of pairs of energy value and a number of bosons in a mode with that energy. However, one does expect ideal transformation of (2) into (4). Instead, it is necessary to split the energies of motifs into categories determined by $n$. Request to have the best coincidence of (2) and (4) will result in determination of constants $\omega$, $h$, $c$ and $\varepsilon$, where $\varepsilon$ is the width of energy interval of each category. The determination proceeds as follows: for some initial pair of $(\omega, \varepsilon)$ the energies $E$ are calculated for all motifs. Then these values are grouped into classes in such a way that the energy difference between the motifs in neighbouring classes are larger than $\varepsilon$. An index is associated with each class, and it figures as state index $n$. Then the set of pair of values of energy and index $(E, n)$ obtained for the set of motifs, is fitted onto linear equation. The coefficients in the linear fit give the constants $h$ and $c$. Additionally, the sum $S$ of squared difference between the determined energies of the motifs and the energies predicted from the linear fit is recorded. After that, the whole procedure starting from the initial setting of $(\omega, \varepsilon)$ is repeated. After dense enough coverage of the $(\omega, \varepsilon)$ plane, the values of $\omega$ and $\varepsilon$ which belong to the minimum in the $S$ are found and set for values representing a given motif set. In that way these values are considered found, and are used for determination of further quantities.

3. RESULTS OF ANALYSIS OF FOUR BACH'S FUGAS

Four Bach's fugas were analysed in order to evaluate the statistics of motifs: fuga in D-major (code BWV 850) for piano with 35 analysed motifs, fuga in c-minor (BWV 847) for piano with 49 analysed motifs, fuga in d-minor (BWV 565) for organ with 137 motifs and fuga in a-minor (BWV 543) for organ with 184 motifs.

Piano fugas are shorter than the organ fugas. Besides, organ fugas include a number of different motif structures. For example, in both organ fugas about 30 different forms of motifs were recognised, compared with about 5 in both piano fugas. The divisioning of forms is performed with taking into account the number of notes, different rhythmic structure and different duration of notes. Furthermore, in organ fugas the classification of motifs is somewhat ambiguous. It probably depends on person analysing the compositions. However, we assume that rather low percentage of motifs is not straightforwardly classified.

![Figure 1](image_url). The beginning of fuga BWV 847 with three motifs designated.
In next step the energy of each motif is obtained as a function of parameter $\omega$. The motifs are ordered ascendingly and grouped in categories of the still undetermined width $\varepsilon$. Then the motif's energies are equalised with the relation (4). The criterion of best accordance is that the sum of squares of differences of the energy given by (2) and the energy given by (4) be minimal. This minimisation is performed numerically until the implicit self-consistency is achieved resulting with parameters $\omega$, $\eta$ and $c$, which are given in Table 1.

**Table 1.** Parameters resulted from analysis of fugas.

<table>
<thead>
<tr>
<th>Title</th>
<th>$\omega$</th>
<th>$\eta$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuga in D-major</td>
<td>0.59</td>
<td>4.82 $\pm$ 0.05</td>
<td>2.3 $\pm$ 0.1</td>
</tr>
<tr>
<td>Fuga in c-minor</td>
<td>0.54</td>
<td>6.19 $\pm$ 0.09</td>
<td>0.63 $\pm$ 0.06</td>
</tr>
<tr>
<td>Fuga in d-minor</td>
<td>0.56</td>
<td>3.36 $\pm$ 0.04</td>
<td>-1.6 $\pm$ 0.3</td>
</tr>
<tr>
<td>Fuga in a-minor</td>
<td>0.81</td>
<td>1.99 $\pm$ 0.03</td>
<td>1.53 $\pm$ 0.01</td>
</tr>
</tbody>
</table>

The results of described fitting procedure are illustrated in Figure 2. The motifs’ energies are represented by energy values for harmonic oscillator’s bosons given by (4), which depends linearly on number of quanta $n$. Different energies are attributed to different number of motifs.

![Figure 2](image-url)

**Figure 2.** Dependence of energies of motifs on number of quanta in a given mode for fuga BWV 847. Graphs show coincidence of energies calculated by equations (2) and (4).

4. **DISCUSSION**

Firstly let us emphasise a double reduction that has been introduced. The first one is the reduction of each composition to a set of motifs. Obviously in this way some of the composition's characteristics are missed. The second reduction is in attributing to each motif a single value of energy. This drops many interesting properties of motifs. The reduction of a whole composition to a set of independent motifs, and then each and every motif to just one number is the initial step in trying to find the similarity between elementary excitations in bosonic heat bath, the concept originating in theoretical physics, and human environment in which musical composition is performed.

In analysing the compositions, only their internal properties were considered. Listener is not included. The aim of the analysis is to describe elementary
excitations of the surroundings when the composition is performed. These preliminary results show partial similarity with bosonic elementary excitations. Despite good agreement, many details have to be resolved.

Motifs are considered as the independent elements. But in compositions there are also larger forms present, such as sentences and themes. In these the interplay of different motifs is rather important. Especially when the composition has two parts performed simultaneously. Another simplification is that $\omega$ is presumed constant over the whole composition and that the form of energy is the same for all parts of composition.

The meaning of parameters obtained remains presently unanswered. One of the reasons is too small number of the studied cases.

5. CONCLUSION AND FUTURE DEVELOPMENT

Connecting the motifs in music composition with bosons as elementary excitations of the environment the new concept of interpretation of the music composition as an auditory environment is introduced. In this preliminary work the starting point was the definition of energy calculation procedure. The results obtained show that it is possible to represent the energies in the same manner as in the case of bosonic excitations of the environment.

Such an interpretation represents first step toward the interpretation of human environment and the interaction of humans with elementary excitations of the environment.

To establish a firmer foundation of proposed procedure it remains to do much work. It is necessary to study a greater number of compositions to obtain a more reliable statistics. Then, step by step, more elements ignored till now should be regarded and calculated. One of them is the interaction of motifs if they sound in the same time or if they follow each other. Then, other substructures of the compositions should be analysed, because they form another type of excitations, possibly at different energy scales, as is the case in real physical systems. After the description of music composition as human environment, next work may be concentrated to investigation of human-environment interaction.

Again, our purpose is not to make an ultimate theory of music in society. The long-range objective is to gain some knowledge or descriptiveness of the complex environments using well-known concepts in new situations.

6. REFERENCE

GLAZBENO DJELO I ELEMENTARNA POBUĐENJA OKOLINE

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SAŽETAK

Izvođenjem glazbenog djela ostvaruje se posebna vrsta ljudske okoline. U ovom radu je nastojimo interpretirati kao posebnu klasu ljudske okoline, radi upotrebe takvog pristupa u opisu strukture i dinamike opće ljudske okoline. Posebno, analizirali smo četiri Bachove fuge promatrajući njihov rastav na motive i pridjeljujući motivima energiju. Ta energija dobro se daje opisati relacijom koja potječe od bozonskih pobuđenja u teorijskoj fizici, što je polazna točka pristupa rasvijetljavanja složenijih ljudskih okolina. Namjera nije svesti glazbu na fiziku, niti uvoditi nove teorije glazbe, već pristupiti istraživanju složenih okolina na nov način koristeći već postojeće koncepte.

KLJUČNE RIJEČI

elementarna pobuđenja, glazbeno djelo, motivi, Bach, bozonska kupka