In uncertainty we trust: a median voter model with risk aversion

PAVEL A. YAKOVLEV, PhD
Duquesne University, Department of Economics and Quantitative Sciences, Pittsburgh
yakovlevp@duq.edu

Article**
JEL: D72, D78
UDC: 336

* I would like to thank Russ Sobel, Justin Ross, and two anonymous referees for their comments and suggestions.
** Received: June 13, 2010
Accepted: July 24, 2011
Summary
The principal-agent problem and uncertainty are some of the key factors affecting financial and political markets. Fear of the unknown plays an important role in human decision making, including voting. This article describes a theoretical model where voter risk aversion towards uncertainty gives political incumbents a significant advantage over their challengers, exacerbating the principal-agent problem between voters and legislators. The model presented predicts that a rise in voter uncertainty concerning the challenger allows the incumbent to deviate from the median voter’s policy preference without losing the election. This model reconciles the paradoxical coexistence of ideological shirking and high incumbent reelection rates without abandoning the elegant median voter framework.

Keywords: ideology, incumbency advantage, shirking, median voter, risk aversion, principal-agent problem

1 INTRODUCTION
The high reelection rates enjoyed by political incumbents suggest that either they do a superb job of representing voters’ interests or that they command a significant incumbency advantage over challengers. Each view has its supporters and critics. In this article, I focus on the documented prevalence of the incumbency advantage or monopoly power in the electoral systems (Garand and Gross, 1984; Collie, 1981; Ansolabehere, Brady and Fiorina, 1988; Gelman and King, 1990; Cox and Katz, 1996). This advantage has been linked to lower electoral responsiveness (Coyne and Fund, 1992; Rivers and Brady, 1991), weaker political competition (Levitt and Wolfram, 1996), greater pork-barrel spending (Fiorina, 1989), better knowledge of the electorate (Neuman, 1986), fiscal illusion and “love for stability” (Mourao, 2008 and 2010), and ideological shirking (Yakovlev, 2007). Therefore, it is not surprising that the incumbency advantage is often viewed as a threat to public sector efficiency.

Despite numerous empirical studies documenting the inefficiencies of the electoral systems, very few theoretical papers examine how inefficiencies may arise in a competitive median voter framework. This article attempts to fill this void by developing a model that explains how voter risk aversion towards uncertainty translates into incumbency advantage and enables incumbents to legislate farther away from their constituents’ interests or to pursue ideological shirking. The model is based on Black’s (1948) median voter framework and entails solving a computationally intensive, sequential Stackelberg game. The model shows that higher incumbency advantage, in terms of higher vote shares, enables incumbents to sacrifice some of their votes in exchange for ideological shirking and still get reelected. This conclusion is consistent with the empirical evidence on politicians deviating from the median voter without being voted out of office.¹ The theoretical

model developed in this article reconciles the paradoxical coexistence of ideological shirking and high incumbent reelection rates in the U.S. House of Representatives, for example, as documented by Yakovlev (2007).

2 VOTER RISK AVERSION AND IDEOLOGICAL SHIRKING

The incumbency advantage is the often-cited reason as to why incumbents seem to win the vast majority of elections. Voter risk aversion towards uncertainty could be one of the major sources of the incumbency advantage (Berger et al., 2000). As implied by the popular idiomatic expression “Better the devil you know than the devil you don’t”, a well-known but imperfect incumbent might be preferred by the voters to a relatively unknown but well-qualified challenger. This uncertainty-driven advantage enables incumbents to behave in a way that is inconsistent with their voters’ preferences and still get reelected. In other words, this asymmetric uncertainty allows incumbents to shirk, exacerbating the principal-agent problem in the political market where voters are the principal and incumbents are their agents. There are two not mutually exclusive types of shirking: ideological and non-ideological. This article focuses on the former. Ideological shirking pertains to misrepresenting constituents’ preferences. For example, Washington (2008) finds that the proportion of daughters significantly affects a male legislator’s propensity to vote liberally on reproductive rights despite his constituents’ ideology. Non-ideological shirking pertains to actions that avoid work or duties such as absenteeism, corruption, and so forth.

The idea that politicians are not simply vote-maximizing agents is not new. For instance, Sobel (1992) models politicians as rational utility-maximizing agents who may pursue other interests even if it may cost them reelection. Sobel concludes that incumbents find it optimal to give up some surplus votes (votes in excess of what is needed for reelection) in exchange for pursuing other objectives such as logrolling, special-interest legislation, or legislation closer to their own ideology. This idea is consistent with the theorem of minimal winning coalitions, which postulates that a political coalition will be as small as possible as long as it is winning or can dictate a decisive outcome (Riker, 1962). It is also possible that some incumbents may find it optimal to shirk to the full extent even if it puts them out of office. However, since the incumbent reelection rate in the U.S. Congress and Senate elections typically exceeds 70 percent (Benjamin and Malbin, 1992; Davidson and Oleszek, 1994; and Miller, 1999), it must be the case that most of the incumbents remaining in office do not have an insatiable appetite for shirking. Therefore, the model developed in this article describes the behavior of a typical

---

2 See Bender and Lott (1996) for a thorough and critical review of the shirking literature.

3 The formation of a minimal winning coalition here does not depend on the transaction costs between the members of the “selectorate” and the coalition since their decisions are independent of each other and are based only on their single-peaked preferences for the platforms proposed by the competing candidates.

4 A study by Biglaiser and Mezzetti (1997) finds that the incumbent expecting to lose the election is willing to pay more for the public project than its total value to the voters, suggesting that incumbents shirk more in their last office term.
incumbent, who finds it optimal to do some shirking as long as it does not put him or her out of office.

3 THE MODEL

This article’s theoretical model builds on the seminal work by Hotelling (1929), Black (1948), and Downs (1957). Hotelling’s (1929) competition or “linear city” model, known for its principle of minimum differentiation, can be thought of as a precursor to the median voter model. In Hotelling’s linear city model, two competing businesses locate next to each other in the middle of a one-dimensional market space in order to maximize their market shares. Hotelling’s principle of minimum differentiation extends naturally to a “political market” where two competing politicians propose the exact same policy most preferred by the median voter. The median voter theorem was first articulated in Duncan Black’s 1948 article, “On the Rationale of Group Decision-making” and popularized in Anthony Downs’s 1957 book, An Economic Theory of Democracy. I combine the median voter framework with voter risk-aversion specification borrowed from Chen and Niou (2007), who attempt to reconcile high incumbent reelection rates with legislative term limits.

As typical of the median voter models, I assume a one-dimensional policy space [0, 1] on which political candidates propose their platforms. Let voters’ preferences over policies be single-peaked and characterized by quadratic utility function $u_x(y) = -(y - x)^2$, where $x$ is the voter $x$’s most-preferred policy and $y$ is the actually policy implemented. Let some voter $x$ be identified by his most preferred policy value $x$ that is uniformly distributed on a one-dimensional policy space $[0, 1]$. Let the incumbent be the left-wing candidate and the challenger be the right-wing candidate, stated mathematically as $\mathbb{P}(x_{PI} < x_{PC}) < 1$, where $x_{PI}$ and $x_{PC}$ correspond to their respective historical policy platforms or their party’s platforms.

The next step is to specify how uncertainty about political candidates enters a voter’s utility function. Suppose that the incumbent and the challenger are associated with different uncertainty levels in the eyes of the voters. Voters may form a relatively good impression of the incumbent’s policy platform based on the observed performance in office, decreasing the uncertainty associated with the incumbent. The same cannot be said about the untested challenger, who might appear as a wildcard candidate. Therefore, the incumbent and the challenger will differ in their abilities to propose credible platforms in an election, giving rise to the incumbency advantage. When the challenger proposes some platform $x_{C}$, the voters form an expectation of that platform due to the uncertainty associated with whether or not the promised platform will be implemented. Like Berger et al. (2000), and Chen and Niou (2007), I assume that voters form an expectation of the challenger’s platform that can be represented by the random variable $\bar{x}_C$ with density function $g(\bar{x}_C)$. Then, voter $x$’s expected utility associated with platform $x_C$ becomes
\[ u(x_C) = \int_0^1 (\bar{x}_C - x)^2 g(\bar{x}_C) d\bar{x}_C = -V(x_C) - [m(x_C) - x]^2, \]

where \( m(x_C) \) is the mean of the random variable \( \bar{x}_C \) with variance \( V(x_C) \).

Like Bernhard and Ingberman (1985), I assume that \( m(x_C) = x_C \), which states that the mean of \( \bar{x}_C \) equals the proposed platform. Subsequently, \( V(x_C) \) is the extent of uncertainty with which voters perceive the candidate’s real position. Thus, voter utility is a function of uncertainty regarding the challenger’s true position \( V(x_C) \) and the distance between the voter’s and the candidate’s expected positions \( (x_C - x)^2 \). Voter utility decreases in both \( V(x_C) \) and \( (x_C - x) \). Like Chen and Niou (2007), I decompose \( V(x_C) \) into intrinsic uncertainty \( v_C \) and extrinsic uncertainty \( k_C (x_C - x_P)^2 \), where \( v_C \) is voter disutility from the intrinsic uncertainty that is associated with the challenger and \( k_C (x_C - x_P)^2 \) is voter disutility from the extrinsic uncertainty caused by the challenger’s strategic positioning. Intrinsic uncertainty represents voter’s risk premium for being indifferent between accepting a definite position \( x_C \) and accepting a random position \( \bar{x}_C \) with the mean value \( x \). Extrinsic uncertainty represents voter skepticism towards the platform proposed by the challenger. The farther \( x_C \) deviates from \( x_P \), the greater the voter disutility. Voter skepticism increases in \( k_C \), which represents the degree of voter aversion to extrinsic uncertainty. The more the challenger deviates from his historical or party platform \( x_P \), the greater the uncertainty with which he will be perceived by the voters. Voter risk aversion towards uncertainty limits the challenger’s ability to propose a vote-maximizing platform by making the voters skeptical of that platform. Voter utility for the challenger is then:

\[ u(x_C) = -v_C - k_C (x_C - x_P)^2 - (x_C - x)^2. \] (2)

Analogously, voter utility for the incumbent can also be described as a function of uncertainty \( V(x_I) \) regarding the incumbent’s true position and the distance between the voter’s and the candidate’s expected positions \( (x_I - x)^2 \). As with the challenger, \( V(x_I) \) can be decomposed into intrinsic uncertainty \( v_I \) and extrinsic uncertainty \( k_I (x_I - x_P)^2 \), where \( k_I \) is the degree of voter aversion towards extrinsic uncertainty. For obvious reasons, voters may experience some intrinsic uncertainty with the incumbent, but probably less than with the challenger \( (v_I < v_C) \). Additionally, voters dislike the deviation of the incumbent’s legislative platform \( x_I \) from the promised platform \( x_P \) because it represents ideological shirking or voter betrayal and, as such, can be modeled as extrinsic uncertainty. Additionally, rationally ignorant voters may not know the exact positioning of the incumbent’s policy platform, but may form a pretty accurate expectation of it on average (i.e. let the mean of random variable \( \bar{x}_I \) equal \( x_I \)). Hence, voter utility for the incumbent can be described as:

\[ u(x_I) = -v_I - k_I (x_I - x_P)^2 - (x_I - x)^2. \] (3)

The incumbency advantage stems from a lower amount of intrinsic uncertainty for the incumbent than for the challenger \( (v_I < v_C) \). This asymmetrically-distributed uncertainty allows the incumbent to deviate (shirk) from \( x_P \) and still win reelec-
tion because voters dislike uncertainty. Ideological shirking, defined as \( s_I = x_{pI} - x_I \) can be negative \((s_I < 0)\) or positive \((s_I > 0)\). That is, a left-wing incumbent in this model can legislate to the right or to the left of \( x_{pI} \) on the \([0, 1]\) policy space. By changing the definition of \( x_{pI} \) one can change which platform the incumbent is shirking from: the incumbents’ party platform, the core constituents’ platform, the platform espoused in the primaries, or the median voter platform. In this article, I assume that the incumbent ran and was elected on the platform most preferred by the median voter \((x_{pI} = 0.5)\). Therefore, ideological shirking represents deviation from the median voter platform. Since the median voter platform may not be efficient, it is possible that ideological shirking could be socially optimal in some situations.

Continuing with the model, let \( \bar{x} \) be the position of the voter who is indifferent to both incumbent and challenger. Replacing \( x \) with \( \bar{x} \) and setting the two utility functions equal to each other yields:

\[
-v_I - k_I (x_{pI} - x)^2 - (x_I - \bar{x})^2 = -v_C - k_C (x_C - x_{PC})^2 - (x_C - \bar{x})^2. \tag{4}
\]

Solving equation (4) for \( \bar{x} \) yields:

\[
\bar{x} = \frac{x_I^2 + k_I (x_{pI} - x_I)^2 - x_C^2 - v + k_C (x_{PC} - x_C)^2}{2(x_I - x_C)}. \tag{5}
\]

The share of votes obtained by the incumbent (the left-wing candidate) is determined by this cumulative density function or CDF:

\[
\int_0^{\bar{x}} f(x)dx = F(\bar{x}). \tag{6}
\]

The share of votes obtained by the challenger (right-wing candidate) is then:

\[
\int_{\bar{x}}^{1} f(x)dx = 1 - F(\bar{x}). \tag{7}
\]

The electoral competition between the incumbent and the challenger can be modeled as a strategic, sequential Stackelberg game in which the incumbent moves first and the challenger responds. In this static game, the incumbent legislates strategically to ensure his reelection for another term, after which he retires. Thus, the incumbent runs for reelection only once, but consumes the ego-rents from holding office and shirking in both terms. The amount of shirking is constrained by the reelection incentive only in the first term and is the focus of this model.

The Stackelberg game is solved by backward induction because the incumbent anticipates (endogenizes) the challenger’s response. First, I solve for the challenger’s platform \( x_C^* \) that maximizes his vote share \( (1 - \bar{x}) \):

\[
x_C^* = x_I + k_c x_I + \sqrt{(k_c + 1)(k_c (x_I^2 + x_{PC}^2) - 2 k_c x_{PC} x_I - k_c (x_I^2 - 2 x_I + x_{pI}^2) - v_I + v_c)} \tag{8}
\]

\[
(k_c + 1)
\]
The incumbent takes the challenger’s optimal policy platform \( x^*_C \) into account in his optimal policy platform \( x^*_I \). To find \( x^*_I \), I substitute the challenger’s optimal platform \( x^*_C \) for \( x_C \) in equation (5), set the expression for \( \bar{x} \) in equation (5) equal to \( 0.5 + \varepsilon \), \( \varepsilon \) being the tie-breaker or decisive vote percentage, and solve for \( x^*_I \). Assuming that all other parameters in the model are exogenous, the incumbent’s optimal policy platform is:

\[
x^*_I = \frac{(k_c + 1)}{(2k_I x_{PI} + 2\varepsilon + 1)^2} \left( \frac{(k_c + 1)(4 v_C - k_j k_c (v_c + x_{PI} (x_{PI} + 1) + x_{PC} (x_{PC} - 1) - v_I)}{-k_j (v_I - x_{PI} + x^2_{PI} + \varepsilon^2 + x_{PI} + \varepsilon) - v_I + k_c (\varepsilon + 1) + v_c - v_I - x_{PC} (1 - x_{PC})) - 8 \varepsilon ((1 + k_I) k_c x_{PC} - k_j x_{PI} (1 + k_c))} + k_c - k_j \right) (9)
\]

The solution in equation (9) ensures that the incumbent obtains the maximum amount of ideological shirking \( s_I = x_{PI} - x^*_I \) and still receives his ego-rents from holding office (i.e. reelection occurs with exactly \( \bar{x} = 0.5 + \varepsilon \) votes). A left-wing incumbent must be legislating or shirking to the left of \( x_{PI} \) on the \([0, 1]\) policy space if \( s_I > 0 \), and to the right of \( x_{PI} \) if \( s_I < 0 \).

Recall this article’s hypothesis that greater relative uncertainty \( v_C > v_I \) associated with the challenger increases the incumbency advantage, which is the amount of votes he can trade for shirking \( s_I \) without losing office. Equation (9) ensures that all votes in excess of \( \bar{x} = 0.5 + \varepsilon \) are turned into shirking. The partial derivatives of \( s_I \) with respect to \( v_I \), \( v_c \), and other variables would be useful in examining the model’s predictions, but because these equations are too cumbersome, the numerical solutions presented in the next section will be used to determine the signs of these derivatives.

### 4 NUMERICAL SOLUTIONS

The numerical solutions presented in this section are based on the model’s parameters held constant at \( x_{PI} = 0.5, x_{PC} = 0.6, v_c = v_I = 0.01, k_j = k_c = 1, \) and \( \varepsilon = 0.01 \), while varying one of them at a time to determine their partial effects on shirking.

The numerical solutions shown in figure 1 indicate that ideological shirking \( s_I \) is negatively related to the intrinsic uncertainty associated with the incumbent \( (ds_I/dv_I < 0) \) and positively related to the intrinsic uncertainty associated with the challenger \( (ds_I/dv_C > 0) \). The graphed solutions also reveal that shirking decreases

---

5 As discussed in the previous section, the incumbent maximizes votes only up to \( 0.5 + \varepsilon \) because the additional votes beyond that amount do not contribute anything to victory or to the ego-rents from holding office, but do decrease shirking. In reality, a political candidate may not be able exercise this level of precision and may seek additional or surplus votes as a buffer, which can also be captured by \( \varepsilon \).

6 The model can be generalized to include situations where the two candidates compete on the same side of the policy space, but the left-wing and right-wing characterization of political candidates is more intuitively appealing and realistic.

7 These parameters imply that the incumbent’s promised platform is centered on the median voter, the challenger’s party platform is slightly to the right of the median voter, both candidates are equal in intrinsic and extrinsic uncertainty; and tie-breaker or surplus votes \( (\varepsilon) \) are rather small.
more rapidly in $v_I$ than it increases in $v_c$, holding the other constant. The practical implication of these results is that incumbents can attain a higher level of votes (and shirking) by making the challengers look relatively more unreliable, wavering, or untrustworthy. This posits one possible explanation for the Republican opponents of the 2004 Democratic nominee, John Kerry, labeling him as a “flip-flopper.” Doing so may have been an attempt, by the Republicans, to increase the intrinsic uncertainty associated with this Democratic challenger.

**Figure 1**

*Shirking and intrinsic uncertainty*

![Figure 1](image1)

The numerical solutions shown in figure 2 indicate that ideological shirking is positively related to the extrinsic uncertainty associated with the challenger ($ds_I/dk_c > 0$) and negatively related to the extrinsic uncertainty associated with the incumbent ($ds_I/dk_I < 0$). The practical implication of these results is that incumbents’ shirking will increase with greater voter aversion ($k_c$) towards challengers who appear as ideologically extreme. This may explain why politicians often claim that their opponents are far outside the mainstream.

**Figure 2**

*Shirking and extrinsic uncertainty*

![Figure 2](image2)
The numerical solutions shown in figure 3 reveal that ideological shirking decreases with the incumbent’s desire to keep more surplus votes \((ds_I/\,de < 0)\). Every additional surplus vote won by the incumbent means one less vote that can be converted into shirking (recall the prior discussion on the theorem of minimal winning coalitions). The implication of this result is that if there is a close race or if the incumbent is rather uncertain about election outcomes, he may prefer to create a buffer in terms of surplus votes, which will limit his shirking potential.

**Figure 3**
*Shirking and surplus votes*

![Shirking and surplus votes graph](image)

Holding everything else the same, the numerical solutions shown in figure 4 indicate that ideological shirking increases as the challenger’s party platform \(x_{PC}\) moves farther away from the median voter \((ds_I/\,dx_{PC} > 0)\). The intuition here is that as the challenger’s party platform becomes more extreme, the challenger becomes less capable in proposing a vote-maximizing platform since voters dislike deviations from \(x_{PC}\). As a result, the incumbent can attain more shirking.

**Figure 4**
*Shirking and challenger’s party platform*

![Shirking and challenger’s party platform graph](image)

In contrast to the previous numerical solutions, the relationship between ideological shirking \(s_I\) and the incumbent’s promised policy platform \(x_{PI}\) is more complex,
but far more interesting (see the real part\(^8\) of the solution shown in gray in figure 5). Recall that ideological shirking is equal to \(x_{PI} - x_I^*\), where \(x_I^*\) is the incumbent’s optimal policy platform that is a function of all the other parameters in the model, including \(x_{PC}\). Assuming the same parameters as before (\(x_{PC} = 0.6\), \(v_I = v_C = 0.01\), \(k_I = k_C = 1\), and \(\varepsilon = 0.01\)), the numerical solutions shown in figure 5 reveal that ideological shirking is generally rising in \(x_{PI}\) but exhibits some non-linearity around the median voter.

**Figure 5**

*Shirking and incumbent’s promised platform*

\[\text{Notes: Gray line is the real part of the solution. Black line is the imaginary part of the solution.}\]

Moreover, ideological shirking changes from being negative (\(x_{PI} - x_I^* < 0\)) to being positive (\(x_{PI} - x_I^* > 0\)) as \(x_{PI}\) rises. In the negative shirking range, where \(x_{PI} < x_I^*\) and \(x_{PI}\) is closer to zero, the incumbent must legislate closer to the median voter to get reelected (\(x_I^*\) is to the right of \(x_{PI}\)). In the positive shirking range, where \(x_{PI} > x_I^*\), the incumbent chooses \(x_I^*\) to the left of \(x_{PI}\). As \(x_{PI}\) begins moving away from zero, \(x_I^*\) runs a bit ahead of \(x_{PI}\) until \(x_{PI}\) approaches 0.42, where the first policy reversal occurs as \(x_I^*\) changes direction and begins moving towards zero. At this point, shirking begins to behave non-linearly (at the beginning of the bulge in figure 5) as it decreases fast, initially, in absolute value. As \(x_{PI}\) continues to rise, shirking continues to decline, but at a decreasing rate, eventually converging to zero (\(s_I = x_{PI} - x_I^* = 0\)) around \(x_{PI} = 0.45 = x_I^*\) as the two variables cross paths on the \([0, 1]\) policy space. Soon after \(x_{PI}\) surpasses \(x_I^*\), the second and last policy reversal occurs: \(x_I^*\) changes direction and begins catching up with \(x_{PI}\). As \(x_I^*\) growth approaches \(x_{PI}\) growth, shirking peaks locally, where the bulge’s slope is equal to zero, and then declines briefly in \(x_{PI}\). Shirking stops declining and begins to rise again as the bulge turns into a line at \(x_{PI} = x_{PC} = 0.6\). From there on, shirking rises in \(x_{PI}\) in the linear fashion. The non-linear shirking range in figure 5 expands pro-
portionally in both directions as \( x_{pc} \) moves farther away from the median voter. Recall that the incumbent’s policy platform \( x_i^* \) and, by extension, shirking \( s_i \) are limited by \( x_{pi} \) and \( x_{pc} \) because voters dislike deviations from these platforms.

The solutions described above are comparable to the solutions obtained with other parameter values. Shirking remains non-zero for many other parameter values, indicating that the incumbency advantage can be rather strong and shirking persistent.

5 CONCLUSION

Bender and Lott (1996) offer a thorough review of the empirical literature on legislative shirking, which is yet to reach a consensus. This article contributes to the on-going debate by developing a theoretical model that reconciles high incumbent reelection rates and ideological shirking within the elegant median voter framework. The theoretical model developed in this article shows how greater voter uncertainty associated with challengers allows incumbents to deviate from the promised platform (i.e., shirk) and still be reelected. The presence of ideological shirking would indicate that the political market is plagued by the principal-agent problem, forcing the public to question whose interests the elected officials actually represent. The central conclusion of this article’s model is that the principal-agent problem is exacerbated when voters perceive incumbents to be less risky than challengers. The model predicts that the incumbency advantage and ideological shirking can be significantly diminished if challengers can decrease their intrinsic uncertainty. Another possible solution to the incumbency advantage is the adoption of legislative term limits, but the author of this article is skeptical of their efficacy.

It must also be noted that ideological shirking, depending on its direction, can either increase or decrease public sector inefficiency if the median voter outcome is inefficient. Future research should identify the cases when ideological shirking leads to welfare-enhancing outcomes.
LITERATURE


