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Robustness Criteria for Safety Enhancement of Ship's Structural Components

Original scientific paper

The paper describes an application of the probabilistic event oriented system analysis to the design of ship structural components with the aim to evoke the possible usage of reliability theories in the design practice. The basic concepts of events analysis and of system robustness are summarized at the beginning. The system robustness takes on the entropy concept in probability and information theory in order to ensure the most uniform distribution of safety of failure events of structural components in ship's service. At the end, the paper presents in details an example of robustness maximization of a typical ship structural component under number of design failure criteria. The example corroborates that it is possible to find structural configuration with more uniform distribution of safety on the basis of robustness criteria.

Keywords: *robustness, entropy, reliability, structural design, ship structures*

Kriteriji robustnosti za povećanje sigurnosti sastavnica brodskih konstrukcija

Izvorni znanstveni rad

Članak opisuje primjenu vjerojatnosne analize usmjerene događajima na projektiranje sastavnica brodskih konstrukcija u cilju prikaza mogućeg korištenja teorija pouzdanosti u praksi projektiranja. Na početku članka su ukratko opisani osnovni koncepti analize događaja i robustnosti sustava. Robustnost je prikazana preko entropije definirane u teoriji vjerojatnosti i teoriji informacija s ciljem da se osigura naj-jednolikijom razdioba sigurnosti u odnosu na moguća oštećenja strukturnih sastavnica u službi broda. Na kraju članak detaljno prikazuje primjer maksimizacije robustnosti tipične strukturne sastavnice brodskog trupa podvrgnute projektnim kriterijima oštećenja. Primjerom se potvrđuje da je moguće naći strukturnu konfiguraciju s jednolikijom razdiobom sigurnosti na osnovi kriterija robustnosti.

Ključne riječi: *robustnost, entropija, pouzdanost, projektiranje konstrukcija, brodske konstrukcije*

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1 Introduction

More rational approach to safety assessment of ship structures by employing probability theory has resulted in involvement of reliability methods in the rules of classification societies and thereby in the design practice [1]. The development of ship reliability methods also has opened new possibilities for enhancement of structural design process in combination with other theories [2]. The application of the event oriented system analysis (EOSA) [3, 4] to ship structures presented in this paper relies on the belief that a joint application of the theory of ship structures, classification rules, probability theory, statistics and information theory can be beneficial for safety of ship operations. The EOSA starts with considering the probabilities of random operational and failure events with the aim to estimate their uncertainties. The uncertainties of operations are put in a function of random design variables with respect to failure criteria involving loads, responses, geometry, material properties, corrosion etc. The aim of the EOSA is to consider structural behaviour as a system of events and define measures for structural robustness and redundancy by involving all relevant operational modes in service. These measures then provide the tool for maximization of the certainties of operations. The probabilistic system analysis is based on statistical data about physical components, environmental effects and their interactions [5, 6]. In addition to the mere probabilistic analysis the aim of the EOSA is to account for all known, observable, or at least important lifetime events and their relations.

2 Event oriented system analysis

The observable outcomes associated with a component of an engineering system can be denoted as events. Distinguishable exclusive and inclusive events or common cause events are random events denoted as basic events. The basic event may happen, when denoted A_i, or not, when denoted $\bar{A}_i,\ i = 1,2,...,n_e$. The two possibilities are sometimes called simple alternatives. The n_e is the total number of basic events, not necessarily equal to the number of components n_c. The quantitative methods of system analysis require component operational data such as the probability of operation $R_i = p(A_i)$ and the probability of failure $p_{f,i} = p(\bar{A}_i) = 1 - p(A_i)$.

The concept of entropy is known from earlier in the information theory [7]. The entropy of a single stochastic event A expresses unexpectedness of event and has a basic role in theory defined as:

$$H(A) = -\log p(A) \qquad (1)$$

The ship structures consist of a large a number of structural components. The behaviour of structural components is described by compound random events E_i consisting of a certain combination of random events. Here, combinations such as for example buckling and yielding failures of a plating or stiffener could be accounted for. To investigate behaviour of a component, or an entire structure, one has to include all, or at least relevant compound events and their probabilities. Such a collection of events constitutes a system of events. Systems of events are usually described as finite schemes [8]:

$$\boldsymbol{S} = \begin{pmatrix} E_1 & E_2 & \dots & E_N \\ p(E_1) & p(E_2) & \dots & p(E_N) \end{pmatrix} \qquad (2)$$

where $p(E_i)$, $i=1,2,\dots,N$ are probabilities of occurrence of compound events and N is the total number of all events constituting a system of events $\boldsymbol{S}$.

The EOSA categorizes events according to their status: operational, E_i^o, and failure events E_i^f. The probabilities of events can hopefully be estimated through reliability approaches such as first-order reliability method (FORM), advanced first-order reliability method (AFORM) or by Monte Carlo simulation and Bayesian methods [5, 6].

A system $\boldsymbol{S}$ of N_o operational events and N_f failure events, where $N = N_o + N_f$ can be written as:

$$\boldsymbol{S} = \begin{pmatrix} E_1^o \dots & E_{N_o}^o & E_{N_o+1}^f \dots & E_{N_o+N_f}^f \\ p(E_1^o) & p(E_{N_o}^o) & p(E_{N_o+1}^f) \dots & p(E_{N_o+N_f}^f) \end{pmatrix} \qquad (3)$$

The EOSA applies the entropy concept to assess the effects of the number of events and dispersion of their probabilities on uncertainties of operational and failure modes. The uncertainty of a whole system of events $\boldsymbol{S}$ is by definition [9]:

$$H(\boldsymbol{S}) = -\sum_{i=1}^{N} p_i \log p_i \qquad (4)$$

Events can be grouped into subsystems according to their operational $\boldsymbol{O}$ or failure $\boldsymbol{F}$ statuses:

$$\boldsymbol{O} = \boldsymbol{S}^o = \begin{pmatrix} E_1^o & \dots & E_N^o \\ p(E_1^o) & \dots & p(E_{N_o}^o) \end{pmatrix} \qquad (5)$$

$$\boldsymbol{F} = \boldsymbol{S}^f = \begin{pmatrix} E_{N_o+1}^f & \dots & E_{N_o+N_f}^f \\ p(E_{N_o+1}^f) & \dots & p(E_{N_o+N_f}^f) \end{pmatrix} \qquad (6)$$

where $\boldsymbol{S}^o$ is operational and $\boldsymbol{S}^f$ is failure subsystem of events.

The system $\boldsymbol{S}$ can be presented as a sum of operational and failure subsystems as shown:

$$\boldsymbol{S} = \begin{pmatrix} E_1 & \dots & E_N \\ p(E_1) & \dots & p(E_N) \end{pmatrix} = (\boldsymbol{S}^o + \boldsymbol{S}^f) = (\boldsymbol{O} + \boldsymbol{F}) \qquad (7)$$

The overall reliability of the system corresponds to all of the outcomes when the system is operating and can be calculated as the probability of the subsystem of operational modes $p(\boldsymbol{O})$:

$$R(\boldsymbol{S}) = p(\boldsymbol{O}) = \sum_{i=1}^{N_o} p(E_i^o) \qquad (8)$$

The appropriate failure probability of the system corresponds to all of the outcomes when the system fails and can be calculated as the probability of the subsystem of failure modes $p(\boldsymbol{F})$:

$$P_f(\boldsymbol{S}) = p(\boldsymbol{F}) = \sum_{i=N_o+1}^{N_o+N_f} p(E_i^f) \qquad (9)$$

If all compound events and their probabilities are known, i.e. $\Sigma p(E_i) = 1$, the system of events is a complete one and the uncertainty of a system can be easily calculated as Shannon entropy [7]:

$$H(\boldsymbol{S}) = -\sum_{i=1}^{N} p(E_i)\log p(E_i) = \sum_{i=1}^{N} p(E_i)\log\frac{1}{p(E_i)} \qquad (10)$$

If there are some events whose probabilities cannot be calculated or adequately assumed, the system of events is considered as incomplete, i.e. $\Sigma p(E_i)<1$. The entropy of an incomplete system of events is calculated as unconditional Renyi's entropy of order α [10]:

$$H^\alpha(\boldsymbol{S}) = \frac{1}{1-\alpha}\log\left(\sum_{i=1}^{N} p^\alpha(E_i) / \sum_{i=1}^{N} p(E_i)\right) \qquad (11)$$

where $-\infty \le \alpha \le +\infty$, $\alpha \ne 1$.

Uncertainty measure of either incomplete or complete system of events follows from the Renyi's entropy for $\alpha = 1$ and is denoted as Renyi/Shannon's entropy [3], as shown:

$$H^1(\boldsymbol{S}) = -\frac{\sum_{i=1}^{N} p(E_i)\log p(E_i)}{p(\boldsymbol{S})} = \frac{H(\boldsymbol{S})}{p(\boldsymbol{S})} \qquad (12)$$

All logarithms applied to entropy calculations are usually of base two. The uncertainties are expressed in *bits*. For $\alpha = 0$ from the Renyi's entropy follows that maximum entropy as shown:

$$H(\boldsymbol{S})_{\max} = \log\left[N / p(\boldsymbol{S})\right] \qquad (13)$$

The relation of probability preservation holds either for complete or for incomplete systems $\boldsymbol{S}$:

$$p(\boldsymbol{S}) = p(\boldsymbol{O}) + p(\boldsymbol{F}) = \sum_{i=1}^{N} p(E_i) \qquad (14)$$

It may be also noted that the sequence of the events within the system or within the subsystems is irrelevant for reliability and uncertainty considerations. The EOSA can be applied to any relations among subsystems, inclusive or exclusive and with dependent or independent events under the condition of adequate partitioning of a system of events. Due to their complexity, the ship structural components will usually be modelled as incomplete systems of events [11].

2.1 Uncertainty associated with subsystems of events

The entropy of a whole system of events (4) has not been used in structural design since it does not provide enough information for comparison and validation of different operational and failure events. However, partitioning of systems of events to subsystems of interest (5, 6) and related calculation of conditional entropies have showed potential for application in structural design [12, 13]. The uncertainty of a subsystem ***Si*** can be expressed as the Shannon's entropy only of a partial probability distribution of the system ***S*** under the condition that the subsystem ***Si*** occurs. Such conditional entropy does not depend on the system probability $p(\boldsymbol{S})$, being independent of whether the system ***S*** is complete or incomplete. The conditional entropy can be calculated:

$$H_{m_i}(\boldsymbol{S}/\boldsymbol{S}_i) = -\sum_{j=1}^{m_i} \frac{p(E_{ij})}{p(\boldsymbol{S}_i)} \log \frac{p(E_{ij})}{p(\boldsymbol{S}_i)} \tag{15}$$

where $\boldsymbol{S}_i$ is a subsystem of m_i events of the same status.

The maximal attainable conditional entropy of the subsystem *Si* is for m_i equally probable events:

$$H_{m_i}(\boldsymbol{S}/\boldsymbol{S}_i)_{\max} = \log m_i \tag{16}$$

Relative uncertainty of systems ***S*** with same number of events is calculated as:

$$h(\boldsymbol{S}) = \frac{H_N(\boldsymbol{S})}{H_N(\boldsymbol{S})_{\max}} \tag{17}$$

The average number of equally probable events, denoted $F_N(\boldsymbol{S}) = 2^{H_N(\boldsymbol{S})}$, may be useful for practical purposes. The system ***S*** under the condition that it is operational ***O*** or failed ***F*** can be presented respectively, as finite schemes as it is shown:

$$\boldsymbol{S}/\boldsymbol{O} = \begin{pmatrix} E_1^o/\boldsymbol{O} & E_2^o/\boldsymbol{O} & \cdots & E_{N_o}^o/\boldsymbol{O} \\ \dfrac{p(E_1^o)}{p(\boldsymbol{O})} & \dfrac{p(E_2^o)}{p(\boldsymbol{O})} & \cdots & \dfrac{p(E_{N_o}^o)}{p(\boldsymbol{O})} \end{pmatrix}$$

$$\boldsymbol{S}/\boldsymbol{F} = \begin{pmatrix} E_{N_o+1}^f/\boldsymbol{F} & E_{N_o+2}^f/\boldsymbol{F} & \cdots & E_{N_o+N_f}^f/\boldsymbol{F} \\ \dfrac{p(E_{N_o+1}^f)}{p(\boldsymbol{F})} & \dfrac{p(E_{N_o+2}^f)}{p(\boldsymbol{F})} & \cdots & \dfrac{p(E_{N_o+N_f}^f)}{p(\boldsymbol{F})} \end{pmatrix}$$

Uncertainty of system ***S***, under the condition that the system is operating is as shown:

$$H_{N_o}(\boldsymbol{S}/\boldsymbol{O}) = -\sum_{i=1}^{N_O} \frac{p(E_i^o)}{p(\boldsymbol{O})} \cdot \log \frac{p(E_i^o)}{p(\boldsymbol{O})} \tag{18}$$

Uncertainty of system ***S*** under the condition that the system is failing is as shown:

$$H_{N_f}(\boldsymbol{S}/\boldsymbol{F}) = -\sum_{i=N_0+1}^{N_o+N_f} \frac{p(E_i^f)}{p(\boldsymbol{F})} \cdot \log \frac{p(E_i^f)}{p(\boldsymbol{F})} \tag{19}$$

The entropy of the operational modes in (18) and of the failure modes in (19) depends only on the states of the subsystem of operational and failure modes, and not on any other state of the system. The maximal attainable entropy of system ***S*** under the condition that the system is operating is:

$$H_{N_o}(\boldsymbol{S}/\boldsymbol{O})_{\max} = \log N_o \tag{20}$$

The maximal attainable entropy of system ***S*** under the condition that the system is failed is:

$$H_{N_f}(\boldsymbol{S}/\boldsymbol{F})_{\max} = \log N_f \tag{21}$$

The operational and failure modes are of utmost interest both for the engineering system designers and for the system users. The subsystems of operational and failure modes can be considered on different levels of the hierarchical representation of the basic events with respect to their importance in system design. The uncertainties of operational and failure modes and their relations can be applied in the assessment of system performances. Following guidelines can be intuited:

- Higher entropy of operational modes is a consequence of a more uniform distribution of probabilities of operations and can indicate the increase of the system's operational abundance.
- Higher entropy of failure modes is a consequence of more uniform distribution of probabilities of failures and can be related to the increase of the endurance to failures, that is, increase of the system robustness.

2.2 Robustness definition

EOSA can be applied to modelling of a component of ship structure as a system ***S*** composed of events E_i with associated probabilities $p(E_i)$. Description of a system includes functional levels and functional states. Initial or intact structure is viewed as the first functional level. After failure of one or more structural components, the system transits from the first level to subsequent levels. On each level one or more functional states are possible. States represent systems (subsystems) composed of modes in which a structural component performs its functions with full or with reduced capacity.

Robustness of a structural system requires concern in normal operations under working conditions if there are several operational and failure modes. The EOSA comprehends robustness as the system's capability to respond to all possible random failures uniformly [9]. A robust behaviour is intuited when the system can provide more adequate failure modes to adverse demands with equal failure probabilities. When the system responds to all demands uniformly, there is a high uncertainty about which of the failure modes could occur. The EOSA relates robustness only to the uncertainty of the conditional entropy of a subsystem of failure events [9]:

$$ROB(\boldsymbol{S}/\boldsymbol{S}^f) = ROB(\boldsymbol{S}) = H_{N_f}(\boldsymbol{S}/\boldsymbol{S}^f) \tag{22}$$

3 Events in ship structures

Traditionally, ship structures are analyzed within empirical, semi-empirical and partially theoretically based rules of classification societies. Ship's hull basically consists of watertight hull plating and supporting substructures in the bottom, framing, decks, bulkheads, superstructures, etc. Ship structures subjected to random environmental and operational conditions are defined in the design process by their topology, geometry, scantlings of components and by material properties that are altogether considered as random variables when reliability methods [5, 6] and EOSA [9, 12, 13] are being applied.

Typical failure modes considered in structural analysis of steel ships can be divided into three groups [14]: large plastic deformations (yield), buckling and rupture (fatigue). Each of these types of failure includes several different damage states that differ according to seriousness with respect to the survivability criteria. Failure modes are determined for critical locations in ship structure based on knowledge from mechanics, theory of structures and engineering experience incorporated in the rules. Appropriate mathematical models define single failure modes in the space of relevant basic variables $\mathbf{X}$. Basic variables $\mathbf{X} = (X_1,...,X_n)$, where n is number of stochastic variables characterize structural behaviour by limit state functions $g(\mathbf{X})$. Limit state functions are determined by traditional deterministic approach with mandatory identification and quantification of uncertain parameters.

For each structural component in ship structures, there are normally more failure states related to it. Complex structure collapses when several important components are fully or partially damaged in the sequence, gradually reducing the load carrying capability also involving possible redistribution of loads on remaining components [6]. The following example will illustrate the application of EOSA to the assessment of scantlings of a longitudinal deck stiffener on a tanker based on robustness maximization criterion according to the failure criteria defined by the rules of DNV classification society for stiffened panels [15].

4 Example: robustness assessment of a tanker deck longitudinal

Deck stiffener of a tanker 'Barents Sea' (Figure 1) built in *Brodosplit Shipyard* (Table 1) was considered.

Table 1 **Main characteristics of the tanker *Barents Sea***
Tablica 1 **Glavne značajke tankera *Barents Sea***

L_{oa} = 182.5 m – length overall
L_{pp} = 174.8 m – length between perpendiculars
D = 17.5 m – moulded depth
B = 31.4 m – moulded breadth
T = 12.20 m – scantling draught
Z_D = 16.14 m^3 – hull girder section modulus (deck)
v = 15 kn – speed
C_b = 0.80 – displacement coefficient
DWT (Δ) = 47400 tdw – deadweight
z_{NL} = 7.552 m – distance of NL from baseline

Deck stiffener is located at midship section (Figure 1). It is a typical longitudinal stiffener built of HP profile (Figure 2) with h_w = 220 mm, t_w = 11.5 mm and effective plate flange width b_e = 800 mm determined according to [15]. Deck plating thickness is t_p = 14 mm.

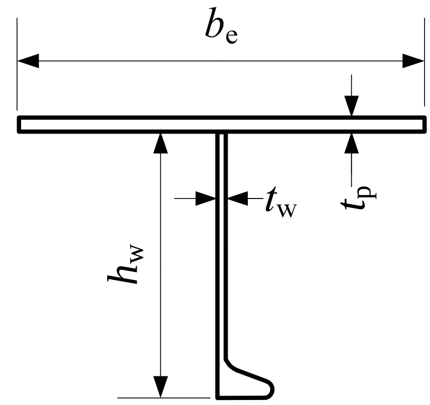

Figure 2 **Deck stiffener**
Slika 2 **Uzdužnjak palube**

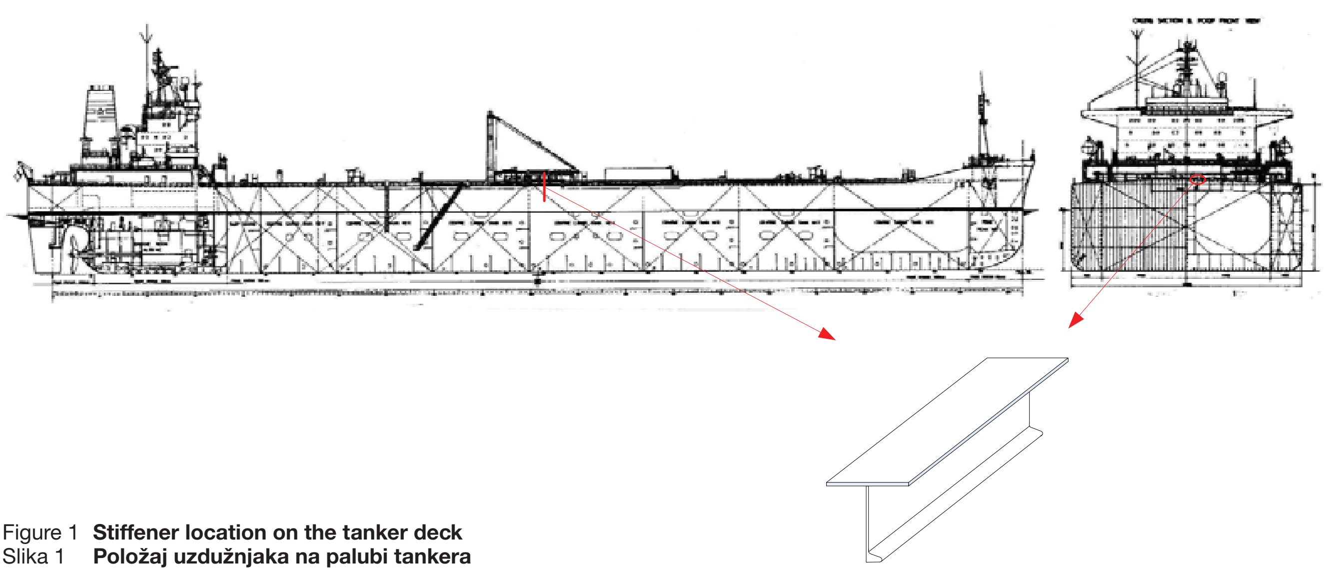

Figure 1 **Stiffener location on the tanker deck**
Slika 1 **Položaj uzdužnjaka na palubi tankera**

Still-water bending moment (sagging condition), M_{S1}, was taken from the loads and stability manual of the ship and amounts to $M_{S1} = -296250$ kNm. Wave bending moments for sagging (M_{W1}) and hogging (M_{W2}) in the area 0.4 to 0.65L from aft perpendicular are: $M_{W1} = -1446252$ kNm and $M_{W2} = 1332070$ kNm. Calculated deck pressure is: $p_2 = 13.6$ kN/m² [15].

These values are assumed to be the mean values of random load variables. Statistical data of load random variables (Table 2) can be found in [16, 17].

Table 2 **Characteristics of load random variables in robustness calculation**
Tablica 2 **Svojstva slučajnih varijabli opterećenja za proračun robustnosti**

			Mean	**Distribution**	**COV**
Still-water bending moment (sagging)	M_{S1}	kNm	296250	Normal	0.4
Still-water bending moment (hogging)	M_{S2}	kNm	37570	Normal	0.4
Wave bending moment (sagging)	M_{W1}	kNm	1446252	Gumbel	0.09
Wave bending moment (hogging)	M_{W2}	kNm	1332070	Gumbel	0.09
Deck pressure	p_2	kN/m²	13.6	Normal	0.09

Table 3 **Characteristics of geometry random variables in robustness calculation**
Tablica 3 **Svojstva slučajnih geometrijskih varijabli za proračun robustnosti**

			Mean	**Distribution**	**COV**
Plating thickness	t_p	mm	14.0	Normal	0.01
Web height	h_w	mm	22.0	Normal	0.02
Web thickness	t_w	mm	11.5	Normal	0.02
Cross sectional area	A	cm²	32.3	-	
Moment of inertia (without deck plating)	I	cm⁴	1542.0	-	
Effective flange width	b_e	mm	800.0	Normal	0.01
Section modulus (with b_e)	W_u	cm³	326.3	Log-Normal	0.04
Span	l	m	5.08	-	
Spacing	b	m	0.8	-	
Midship section modulus (deck)	W_D	m³	16.14	Log-Normal	0.04

Table 4 **Material characteristics (mild shipbuilding steel)**
Tablica 4 **Svojstva materijala (običan brodograđevni čelik)**

			Mean	**Distribution**	**COV**
Yield stress	σ_F	N/mm²	235.0	Log-Normal	0.06
Young's modulus	E	N/mm²	206000	Normal	0.01

4.1 Failure types and limit state functions

According to the DNV rules the failure types for a deck longitudinal are: 1. torsional buckling, 2. local buckling, 3. yield due to pressure and 4. fatigue damage. Stresses are calculated according to the DNV rules with included uncertainties of random variables. Stress in the deck due to bending moments for sagging σ_{a1} and hogging, σ_{a2}, is: $\sigma_{a1} = 127.9$ N/mm² and $\sigma_{a2} = 84.8$ N/mm². These stresses should not be greater than $0.6\sigma_F$. Critical buckling stress (lateral) is: $\sigma_{c1} = 196.0$ N/mm². Critical buckling stress (torsion) is: $\sigma_{c2} = 179.3$ N/mm². Critical buckling stress (local web buckling) is: $\sigma_{c3} = 234.7$ N/mm².

For critical buckling stresses, σ_{ci} the following condition must be fulfilled: $\sigma_c \geq (\sigma_a/\eta)$, where factor η is defined as $\eta = 0.85$ for lateral buckling, $\eta = 0.9$ for torsional buckling and $\eta = 1.1$ for local web buckling. Stress in the deck due to pressure on the deck: $\sigma_p = 76.4$ N/mm². These calculated values are taken as mean values of random variables, following log-normal distribution with COV 0.07 according to [17]. Fatigue damages for structural detail considered in the example were calculated by *ShipRight* FDA program [18].

Limit states functions for failure types in ship structures are calculated according to:

$$g(\mathbf{X}) = x_u W \sigma_{cr} - x_s M_s - x_w x_s M_w \tag{23}$$

where x are the uncertainties of adequate random variables for load and strength (Table 5). These are also modelled as random variables according to [19]. W is corresponding section modulus and σ_{cr} is critical stress of considered failure type. The following limit state functions can be written according to the failure types considered in this example:

$$g_1 = W_D \cdot \sigma_{c2} \cdot x_u - 1.11 \cdot M_{S1} \cdot x_{SW} - 1.11 \cdot M_{W1} \cdot x_w \cdot x_S \quad (24a)$$

$$g_2 = W_D \cdot \sigma_{c3} \cdot x_u - 0.9 \cdot M_{S1} \cdot x_{SW} - 0.9 \cdot M_{W1} \cdot x_w \cdot x_S \quad (24b)$$

$$g_3 = \min\left[(225 f_1 - 130 f_{2d}) \cdot x_u - \sigma_p,\ (160 f_1 \cdot x_u - \sigma_p)\right] \quad (24c)$$

$$g_4 = \overline{T} - \tau \quad (24d)$$

where f_1 = 1.0 for mild shipbuilding steel, $f_{2d} = 5.7\ (M_{S1} + M_{W1}) / W_D$, assumed service period of shipτ = 20 years, and $\overline{T}$ = 107 years is the fatigue lifetime as calculated by *ShipRight FDA*.

Table 5 **Random uncertainty variables x [19]**
Tablica 5 **Slučajne varijable neizvjesnosti x [19]**

		Mean	Distribution	COV
Uncertainty of still-water bending moment	x_s	1.0	Normal	0.05
Uncertainty of wave bending moments due to linear analysis approach	x_w	0.9	Normal	0.15
Other uncertainties in wave bending moments	x_{sw}	1.15	Normal	0.03
Uncertainties in strength determination	x_u	1.0	Normal	0.15

4.3 EOSA of the deck longitudinal

For the purpose of simplicity it is assumed here that the stiffener has no reduced carrying capacity at all. Then the considered system of events is a typical series system, since occurrence of any failure event causes the system to fail. The series system describing stiffener's behaviour has $n = 4$ basic operational events A_i representing functional status of undamaged stiffener. There are also four complement events $\overline{A}_1$ = tor.buckling, $\overline{A}_2$ = local buckling, $\overline{A}_3$ = yield and $\overline{A}_4$ = fatigue, representing damaged (i.e. failed) stiffener. Number of compound events is then: $N = 2^n = 16$. Reliability indexes β_{Ai} and probabilities of failure $p_f(A_i)$, $i = 1,2,\ldots,4$, are calculated by AFORM [5, 6] procedure on a computer:

$\beta_{A1} = 1.339$; $p_f(A_1) = 0.902 \times 10^{-1}$
$\beta_{A2} = 3.441$; $p_f(A_2) = 0.289 \times 10^{-3}$
$\beta_{A3} = 3.348$; $p_f(A_3) = 0.406 \times 10^{-3}$
$\beta_{A4} = 5.281$; $p_f(A_4) = 0.639 \times 10^{-7}$

System of events ***S*** describing behaviour of the stiffener will be modelled as series system of events. It can be presented by a finite scheme as follows:

$$S = \begin{pmatrix} E_1^o & E_2^f & \ldots & E_{16}^f \\ p\left(E_1^o\right) & p\left(E_2^f\right) & \ldots & p\left(E_{16}^f\right) \end{pmatrix}$$

By AFORM procedure the probabilities of compound events can be calculated. Joint events of 3 or more basic events can be neglected according to [6] due to small probabilities of occurrences. Considering that, the stiffener can now be modelled as a system of 11 compound events and presented by the finite scheme as follows:

where E_i are compound events of the following statuses:
E_1^0 – operational event (undamaged stiffener)
$E_{1,1}$ – failure (torsional buckling)
$E_{2,2}$ – failure (local buckling)
$E_{3,3}$ – failure (yield)
$E_{4,4}$ – failure (fatigue)
$E_{1,2}$ – failure (torsional and local buckling)
$E_{1,3}$ – failure (torsional buckling and yield)
$E_{1,4}$ – failure (torsional buckling and fatigue)
$E_{2,3}$ – failure (local buckling and yield)
$E_{2,4}$ – failure (local buckling and fatigue)
$E_{3,4}$ – failure (yield and fatigue)

System's failure probability ***S*** is (9): $p_f(S) = 0.09$. Reliability of systems ***S*** (8): $R(S) = p\left(E_1^o\right) = 0.91$. Entropies (uncertainties) of a system are calculated according to (10): $H(\boldsymbol{S})$ = 0.445 (3.459; 0.128). where values in brackets represent maximum and relative entropies respectively. System robustness is (22): *ROB* (***S***) = 0.804. Maximum attainable robustness (21): $ROB\,(\boldsymbol{S})_{max}$ = log (11) = 3.322. Relative robustness can be expressed as: *rob* (***S***) = *ROB* (***S***) / $ROB\,(\boldsymbol{S})_{max}$ = 0.242.

4.4 Robustness analysis

Robustness of the deck longitudinal is analyzed as a system of events in order to determine maximum achievable robustness, i.e. the most uniform distribution of failure probabilities (Figure 3) of interest in tanker design [20]. The following design constraints were applied:

- Constant weight, i.e. constant cross sectional A = 144.3 cm^2
- Reliability value at least as reliability of the initial model ($R \geq 0.91$).

A study was conducted to investigate robustness as a function of plate flange width b_e (stiffener spacing). Additional constraints applied in this study were:

- 600 mm < b_e < 900 mm and
- 12 mm < t_p < 16 mm.

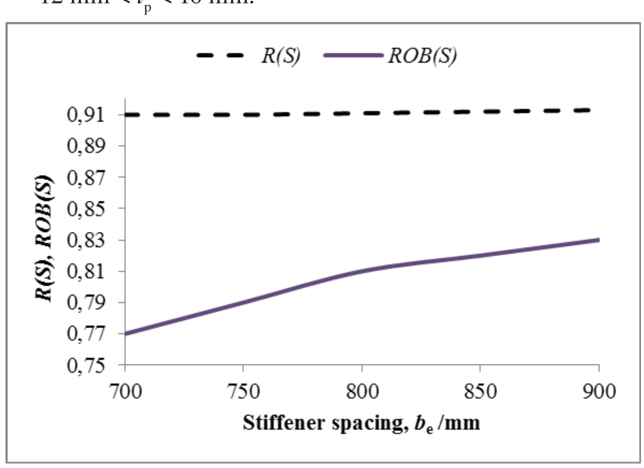

Figure 3 **Comparison of robustness and reliability of ship structural component**
Slika 3 **Usporedba robustnosti i pouzdanosti sastavnice brodske konstrukcije**

The study next calculates the maximally attainable robustness of the stiffener depending on two design variables: web height h_w

and web thickness t_w, within reasonable engineering assumptions formulated by the following design constraints (Figure 4):

- A = const.,
- 210 mm < h_w < 230 mm,
- R > 0.91.

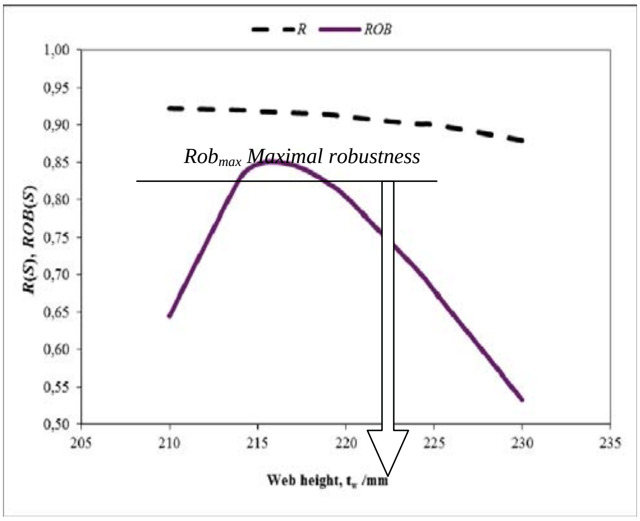

Figure 4 **Maximum robustness (ROB) of a tanker deck stiffener**
Slika 4 **Maksimalna robustnost (ROB) uzdužnjaka palube tankera**

The maximal robustness ROB_{max}=0.85 is obtained for stiffener web height of 216 millimetres (Figure 4).

Conclusion

The application of the event oriented system analysis to ship structures requires the knowledge about all the events and their probabilities of occurrence as well as their relation to the set of design variables and limit state functions. When some probabilities cannot be determined, or their influence can be neglected, the structural behaviour can be modelled by incomplete systems of events. The current state of applications of probabilistic reliability methods in shipbuilding allow the assessments of probabilities of operational and failure modes as well the systemic analysis of their interactions. The usefulness of the event oriented system analysis is in the way the uncertainties of operational and failure modes are treated and applied for design purposes in improvement of structural system robustness and redundancy.

The example indicated that the criterion of conditional entropy of subsystem of failure events may provide more uniform distribution of safety at the same level of reliability of ship structural components that is interpretable as the increase, even as the maximum, of the structural robustness (Figures. 3 and 4). Thus, the property of robustness in terms of the EOSA distinguishes different structural configurations of the same weight and of required reliability level with more uniform distribution of probabilities of failure. Moreover, modelling of ship structural components by events and using the event oriented system analysis appears as a complex but feasible task that provides more information about behaviour of ship structures under complex service conditions with a number of operational states and failure types.

References

[1] AYYUB, B.M. et. al.: "Reliability-based Design of Ship Structures: Current Practice and Emerging Technologies", SNAME Technical Report MM1C76, July, 1998.
[2] LUA, J.; HESS, P.E.: "Hybrid Reliability Predictions of Single and Advanced Double-Hull Ship Structures", Journal of Ship Research, Volume 47, Number 2, 1 June 2003 , p. 155-176(22).
[3] ŽIHA, K.: "Event Oriented System Analysis", Probabilistic Engineering Mechanics 2000;15(3), p.261-275.
[4] ŽIHA, K.: "Redundancy and Robustness of Systems of Events", Probabilistic Engineering Mechanics 2000;15(4), p.347-357.
[5] MADSEN, H., KRENK, S., LIND, N.C.: "Methods of Structural Safety", Prentice Hall Englewood Cliffs NJ, 1986.
[6] DITLEVSEN, O., MADSEN, H.: "Structural reliability methods", New York, J. Wiley, 1996.
[7] SHANNON, C.E., WEAVER, W.: "The mathematical theory of communication", Urbana University of Illinois Press, 1949.
[8] KHINCHIN, A.I.: "Mathematical Foundations of Information Theory", New York, Dover Publications, 1957.
[9] ŽIHA, K.: "Event-oriented Analysis of Series Structural Systems", Structural Safety 2001;23, p.1-29.
[10] RENYI, A.: "Probability theory", Amsterdam, North-Holland, 1970.
[11] BLAGOJEVIĆ, B., ŽIHA, K.: "On the assessment of redundancy of ship structural components", OMAE 2008, The 27th International Conference on Offshore Mechanics and Arctic Engineering, Estoril, Portugal, 15-20 June, 2008.
[12] ŽIHA, K.: "Event-oriented analysis of fail-safe objects", Transactions of FAMENA 2003;27(1), p.11-22.
[13] ŽIHA, K.: "Redundancy based design by event oriented analysis", Transactions of FAMENA 2003; 27(2), p. 1-12.
[14] HUGHES, O.F., PAIK, J.K.: "Ship Structural Analysis and Design", SNAME, 2010.
[15] …: Det Norske Veritas, "Rules for Classification of Ships", 2003.
[16] ATUA, K., ASSAKKAF, I., AYYUB, M.: "Statistical Characteristics of Strength and Load Random Variable of Ship Structures", Proc. of the 7th Specialty Conference on Probabilistic Mechanisms and Structural Reliability, ASCE, Massachusetts, August 1996.
[17] LEHETA, H.W., MANOSUR, A.E.: "Reliability-based Method for Optimal Structural Design of Stiffened Panels, Elsevier Science, Marine Structure, 10, 1997, p.323-352.
[18] …: "ShipRight FDA", Program Manual v.2.4 B, London, 1999.
[19] MANSOUR, A.E., HOVEM, L.: "Probability-based Ship Structural Safety Analysis, Journal of Ship Research, Vol.38, No.4, Dec.1994, p. 329-339.
[20] ČUDINA, P.: Design Procedure and Mathematical Models in the Concept Design of Tankers and Bulk Carriers, Brodogradnja, Vol.59, No.4, 2008, pp. 323-339.