CONTINUITY OF THE TYCHONOFF FUNCTOR $\tau$

Abstract. Let $C$ be a class of the inverse systems $X = \left\{ X_\lambda, f_{\alpha \beta} \right\}$. We say that a functor $F$ is $C$-continuous if $F(\lim X)$ is homeomorphic with $\lim F(X)$.

In the present paper the continuity of Tychonoff functor $\tau$ is investigated.

Section Two contains some theorems concerning the non-emptyness and $w$-compactness of the limit of inverse systems of $w$-compact spaces.

Section Three is the main section. Some theorems concerning $C$-continuity of the Tychonoff functor $\tau$ are proved, where $C$ is a class of the inverse systems of $w$-compact, $\tau$-compact, $H$-closed or $R$-closed spaces.

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0. INTRODUCTION

0.1. The set of all continuous, real-valued (bounded) function on a topological spaces $X$ will be denoted by $C(X)$ ($C(X)$).

Unless otherwise stated, no separation axioms will be assumed.

0.2. A set $A \subseteq X$ is regularly closed (open) if $A = \text{Int} A$ ($A = \text{Int} \ A$).

0.3. A set $A \subseteq X$ is said to be zero-set if there is an $f \in C(X)$ such that $A = f^{-1}(0)$. The zero-set of $f$ is denoted by $Z(f)$ or by $Z_X(f)$.
A cozero-set is a complement of zero-set.
It is well-known \[3\] that

(i) \( z(f) = z(\{ x \mid f(x) \} ) = z(f^n) = z(\{ x \mid f(x) \wedge 1 \} ) \)

(ii) Every zero-set is \( G_\delta \)

(iii) \( z(fg) = z(f) \cup z(g) \)

(iv) \( z(f^2 + g^2) = Z(\{ x \mid f(x) + g(x) \} ) = z(f) \cap z(g) \)

(v) The countable intersection of zero-set is zero-set.

0.4. Two subsets \( A \) and \( B \) of \( X \) are said to be completely separated in \( X \) if there exists a function \( f \in C(X) \) such that \( f(x) = 0 \) for all \( x \in A \), and \( f(x) = 1 \) for all \( x \in B \).

0.5. A space \( X \) is said to be completely regular \[3\] provided that it is Hausdorff space such that each closed set \( F \subseteq X \) and each \( x \notin F \) are completely separated.

0.6. A space \( X \) is said to be almost regular \[9\] if for each regularly closed \( F \subseteq X \) and each \( x \in X \setminus F \) there exist disjoint open sets \( U \) and \( V \) such that \( x \in U \) and \( F \subseteq V \).

0.7. By \( cf(A) \) we denote the cofinality of the well-ordered set \( A \) i.e. the smallest ordinal which is cofinal in \( A \).

0.7. We say that a space \( X \) is quasicompact if every centred family of closed subsets of \( X \) has a non-empty intersection.

0.8. A space \( X \) is functionally Hausdorff of for each distinct points \( x \) and \( y \) of \( X \) there is a continuous function \( f : X \rightarrow [0,1] \) such that \( f(x) = 0 \) and \( f(y) = 1 \). Each functionally Hausdorff space is Hausdorff.

0.9. It follows that in a functionally Hausdorff space \( X \) for each distinct points \( x \) and \( y \) there are cozero-sets \( U_x \) and \( U_y \) such \( x \in U_x \) \( \setminus \) \( \{ y \} \) and \( y \in U_y \subseteq X \setminus \{ x \} \).
0.10. If $U$ is a cozero-set containing $x \in X$, there exist a cozero-set $V \ni x$ such that $x \in V \subseteq \overline{V} \subseteq U$. Namely, if $f : X \to [0,1]$ is a function such that $x \in f^{-1}([0,1]) = U$, then we define a function $F : [0,1] \to [0,1]$ such that $F(y) = 0$ for $y \leq f(x) / 2$ and $F(y) = ((2y - f(x) : (2 - f(x))$ for $y > f(x) / 2$. Now, let $G = Ff$. We have $G^{-1} (0,1) \subseteq U$.

0.11. If $X$ is functionally Hausdorff, then $\{x\} = \cap \{\overline{U} : U$ is the cozero-set containing $x \in X\}$. The proof holds from 0.8., 0.9. and 0.10.

1. FUNCTOR $\tau$

Let $X$ be a topological space. We define an equivalence relation $\rho$ on $X$ such that $x \rho y$ iff $f(x) = f(y)$ for each $f \in C(X)$. Let $\tau(X) = X/\tau$ be a set of all equivalence classes equipped with the smallest topology in which are continuous all functions $g$ such that $g \circ \tau_x \in C(X)$, where $\tau_x : X \to X/\tau$ is the natural projections. In [3:41] is actually proved that $\tau(X)$ is completely regular.

By $[x]$ we denote the equivalence class containing $x \in X$.

1.1. LEMMA. If $f : X \to Y$ is a continuous mapping into a completely regular space $Y$, then there exist a continuous mapping $g : \tau(x) \to Y$ such that $f = g \circ \tau_x$.

Proof. If $x \not\rho y$ then must be $f(x) = f(y)$ since $f(x) \neq f(y)$ implies that there is $f' \in C(Y)$ such that $f'(x) = 0$, $f'(y) = 1$. This is in contradiction with $x \rho y$ since $f' \in C(X)$. This means that for $x' \in \tau(X)$ one can define $g(x') = f(x)$, $x \in x'$.

1.2. COROLLARY. If $f : X \to Y$ is a continuous mapping, then there exists a continuous mapping $\tau(f) : \tau(X) \to \tau(Y)$ such
that $\tau(f) \tau_X = \tau_Y f$.

1.3. LEMMA. If $X$ is functionally Hausdorff, then $\tau_X : X \to \tau(X)$ is one-to-one.

Proof. Trivial.

An open set $U \subseteq X$ is $\tau$-open if $U$ is the union of the cozero-sets.

We say that a space $X$ is w-compact [4] (quasi-H-closed) if for each centred family $\{U_\mu : \mu \in M\}$ of $\tau$-open (open) sets $U_\mu \subseteq X$ the set $\bigcap \{U_\mu : \mu \in M\}$ is non-empty.

1.4. THEOREM. If $X$ is w-compact, then $\tau(X)$ is a compact space ($= T_2$ quasi-compact).

Proof. It suffices to prove that $\tau(X)$ is quasi-H-closed since each regular H-closed is compact. Let $\{U_\mu : \mu \in M\}$ be a centred family of open sets in $\tau(X)$. This means $U_\mu$ is $\tau$-open in $X$. It follows that $\bigcap \{\overline{U}_\mu : \mu \in M\} \neq 0$, where $\overline{U}_\mu$ is a closure in $X$. Let $x \in \{\overline{U}_\mu : \mu \in M\}$. From the continuity of $\tau_X$, we have $\tau_X(x) \in \bigcap \{\overline{U}_\mu : \mu \in M\}$ where now $\overline{U}$ is a closure in $\tau(X)$. The proof is completed.

A space $X$ is said to be $\tau$-compact [4] if each cover $\{U_\mu : \mu \in M\}$ of $X$ consisting of the cozero-sets $U_\mu$ has a finite subcover.

1.5. THEOREM. If $X$ is $\tau$-compact, then $\tau(X)$ is compact.

Proof. Trivial since each open set in $\tau(X)$ is $\tau$-open in $X$.

A space $X$ is said to be perfectly w-compact ($\tau$-compact, H-closed, R-closed) if $\tau_X^{-1}(y)$ is compact for each $y \in \tau(X)$, i.e., every equivalence class $[y]$ is compact.
2. INVERSE SYSTEMS OF W-COMPACT AND $\tau$ - COMPACT SPACES

We start with the following theorem.

2.1. THEOREM. Let $X = \{X_{\alpha}, f_{\alpha \beta}, \alpha \in A\}$ be an inverse system of $\tau$-compact (w-compact) functionally Hausdorff spaces $X_{\alpha}$. If $X_{\alpha}, \alpha \in A$, are non-empty, then $X = \lim X$ is non-empty. Moreover, if $f_{\alpha \beta}$ are onto, then the projections $f_{\alpha} : X \longrightarrow X_{\alpha}, \alpha \in A$, are onto mappings.

Proof. From 1.2. it follows that $X = \{\tau(X_{\alpha}), \tau(f_{\alpha \beta}), \alpha \in A\}$ is an inverse system. In view of Lemma 1.3. there is a mapping $\tau : X \longrightarrow X_{\tau}$ such that $\tau = (\tau_{X_{\alpha}} : X \longrightarrow \tau(X_{\alpha}))$ and $\tau_{X_{\alpha}}, \alpha \in A$, is identity mapping. The mapping $\tau$ induces a mapping $\lim \tau : \lim X \longrightarrow \lim X_{\tau}$ which is 1-1. This means that $\lim X \neq 0$ iff $\lim X_{\tau} \neq 0$. Since $X_{\alpha}$ is the inverse system of compact spaces $\tau(X_{\alpha})$, we have $\lim \tau(X) \neq 0$. The proof is completed.

Since each quasi-H-closed space is w-compact, we have

2.2. THEOREM. LET $X = \{X_{\alpha}, f_{\alpha \beta}, \alpha \in A\}$ be an inverse system of functionally Hausdorff non-empty quasi-H-closed spaces $X_{\alpha}$. Then $X = \lim X$ is non-empty.

We say that a regular (almost regular) space $X$ is R-closed (AR-closed) if it is closed in each regular (almost regular) space in which it can be embedded [9]. Each completely regular R-closed (AR-closed) space $X$ is compact since $X \subset \beta X$ [2].

If $X$ is R-closed, $Y$ regular, and $f : X \longrightarrow Y$ a continuous mapping then $Y$ is R-closed.

2.3. THEOREM. Let $X = \{X_{\alpha}, f_{\alpha \beta}, \alpha \in A\}$ be an inverse system of
non-empty functionally Hausdorff R-closed spaces $X_\alpha$. Then $X = \lim\limits_\alpha X$ is non-empty.

Proof. The space $\tau (X_\alpha) \tau$ is completely regular R-closed i.e. a Hausdorff compact space. See the proof of Theorem 2.1.

We say that a mapping $f : X \to Y$ is $\tau$-open if $f(U)$ is $\tau$-open for each $\tau$-open set $U \subset X$.

2.4. THEOREM. Let $X = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of w-compact functionally Hausdorff spaces $X_\alpha$. If the projections $f_\alpha : \lim\limits_\alpha X \to X_\alpha$, $\alpha \in A$, are $\tau$-open, then $X = \lim\limits_\alpha X$ is functionally Hausdorff and w-compact.

Proof. Let $U = \{U_\mu : \mu \in M\}$ be a maximal centred family of $\tau$-open sets in $X$. For each $\alpha \in A$, let $U_\alpha = \{f_\alpha(U_\mu) : \mu \in M\}$. We prove that $U_\alpha$ is the maximal centred family of $\tau$-open sets in $X_\alpha$ ($f$ is $\tau$-open!). Suppose that $V_\alpha$ is $\tau$-open in $X_\alpha$ such that $V_\alpha \cap f_\alpha(U_\mu)$ is non-empty for each $U_\mu \in U$. This means that $f_{\alpha\beta}^{-1}(V_\alpha)$ is $\tau$-open set which meets each $U_\mu$. From the maximality of $U$ it follows that $f_{\alpha\beta}^{-1}(U_\alpha) \in U_\beta \mu$. Hence, $U_\alpha$ is maximal. From the w-compactness of $X_\alpha$ it follows that $Y_\alpha = \cap \{f_\alpha(U_\mu) : U_\mu \in U\}$ is non-empty. From the maximality of $U_\alpha$ it follows that $U_\alpha$ contains all neighborhoods of all $y_\alpha \in Y_\alpha$. From 0.11, it follows that $Y_\alpha = \{y_\alpha\}$, where $y_\alpha \in X_\alpha$. For each $\alpha \in A$, let $W_\alpha$ be a family of all $\tau$-open sets containing $y_\alpha$. From the maximality of $U_\beta$, $\beta \geq \alpha$ it follows that $U_\beta$ contains $f_{\alpha\beta}^{-1}(U_\alpha) = \{f_{\alpha\beta}^{-1}(U_\mu) : U_\mu \in U_\alpha\}$.

This means that $f^{-1}_{\alpha\beta}(y_\beta) = y_\alpha$, $\beta \geq \alpha$. Hence $y = (y_\alpha : \alpha \in A)$ is a point of $X$. It is readily seen that $y \in \cap \{U : U \in \}$. The proof is completed since it is clear that $X$ is functionally Hausdorff.
2.5. THEOREM. Let \( X = \{X_\alpha, f_{\alpha\beta}, A\} \) be an inverse system of perfect \( w\)-compact (\( \tau\)-compact, \( H\)-closed, \( R\)-closed) spaces \( X_\alpha \). A space \( X = \lim X \) is non-empty iff the spaces \( X_\alpha, \alpha \in A \), are non-empty.

3. CONTINUITY OF THE FUNCTOR \( \tau \)

Let \( X = \{X_\alpha, f_{\alpha\beta}, A\} \) be an inverse system and let \( \tau \) be a Tychonoff functor described in Section One. From 1.2. it follows that \( \tau(X) = \{\tau(X_\alpha), \tau(f_{\alpha\beta}), A\} \) is an inverse system. Let \( C \) be a class of the inverse systems. We say that the functor \( \tau \) is \( C \)-continuous if \( \tau(\lim X) \) is homeomorphic to \( \lim \tau(X) \) for each \( X \) in \( C \). The functor \( \tau \) is said to be continuous if \( \tau \) is \( C \)-continuous for each class \( C \).

3.1. LEMMA. If \( X \) is an inverse system, then there exists a continuous mapping \( \tau_1 : \tau(\lim X) \rightarrow \lim \tau(X) \).

Proof. Let \( \tau = \{\tau(X_\alpha), f_{\alpha\beta}, A\} \) be an inverse system and let \( (X_\alpha) = \lim X \). From 1.2. it follows that there is \( \tau_1 : \tau(\lim X) \rightarrow (X_\alpha) \) such that \( \tau_1 \alpha x = \tau_1 \alpha \tau_{\alpha\beta} \). It is readily seen that \( \tau_1 \alpha = \tau_1 \beta_{\alpha\beta} \). This means that the mappings \( \tau_{\alpha\beta} \), \( \alpha \in A \), induce a continuous mapping \( \tau_1_\alpha : \tau(\lim X) \rightarrow \lim \tau(X) \). The proof is completed.

3.2. LEMMA. \( \lim \tau = \tau_1 \tau \)

Proof. From the definition of \( \tau_1 \) it follows \( \tau_1 \alpha = f_\alpha \tau_1 \), where
f' : \lim_{\alpha} \tau(X) \rightarrow \tau(X_{\alpha}) \text{ is a projection. Moreover, } \tau_X f_{\alpha} = \tau_1 \tau_{\alpha} \text{ and } \tau_X f_{\alpha} = f'_{\alpha} \lim \tau. \text{ It follows that } \tau_1 \tau = f'_{\alpha} \lim \tau \text{ and } \tau_1 \tau = f'_{\alpha} \tau_1 \tau \text{ i.e. } \lim \tau = \tau_1 \tau. \text{ Q.E.D.}

3.3. THEOREM. Let C be the class of all inverse systems \( X = \{X_{\alpha}, f_{\alpha\beta} : \alpha, \beta \in A \} \) such that \( X_{\alpha}, \alpha \in A, X = \lim X \) is w-compact (\( \tau \)-compact) functionally Hausdorff. If the projections \( f_{\alpha} : X \rightarrow X_{\alpha}, \alpha \in A, \) are onto, then the Tychonoff functor \( \tau \) is C-continuous.

Proof. From Lemma 1.3. it follows that each \( \tau_X \), \( \alpha \in A \), is 1-1. This means that \( \lim \tau \) is 1-1. Since \( \lim X \) is functionally Hausdorff we infer by 1.3. that \( \tau : \lim X \rightarrow \tau(\lim X) \) is 1-1. It follows that \( \tau_1 : \tau(\lim X) \rightarrow \lim \tau(X) \) is one-to-one. Since \( \lim \tau(X) \) and \( \tau(\lim X) \) are compact (1.4. THEOREM) we infer that \( \tau_1 \) is a homeomorphism. The proof is completed.

3.4. COROLLARY. Let C be the class of all inverse systems an in Theorem 2.4. Then the Tychonoff functor \( \tau \) is C-continuous.

3.5. REMARK. In [4] is proved that if \( \{X_{\alpha} : \alpha \in A\} \) is a family of w-compact spaces \( X_{\alpha} \), then \( \prod X_{\alpha} \) is w-compact an \( \tau(\prod X_{\alpha}) = \prod \tau(X_{\alpha}). \)

3.6. THEOREM: Let H be a class of the inverse systems \( X = \{X_{\alpha}, f_{\alpha\beta} : \alpha, \beta \in A \} \) such that \( X_{\alpha}, \alpha \in A, X = \lim X \) are functionally Hausdorff H-closed (R-closed). If the projections \( f_{\alpha} : X \rightarrow X_{\alpha}, \alpha \in A, \) are onto mappings, then the functor \( \tau \) is H-continuous.

Proof. The spaces \( \tau(X_{\alpha}), \alpha \in A, \) and the spaces \( \tau(\lim X), \lim \tau(X) \) are compact (See the proof of 2.3. and 3.3.).
In [14] it is proved that \( \lim \ X \) is H-closed if \( X_\alpha \) are H-closed, \( f_{\alpha\beta} \) open and that \( f_{\alpha\beta} \) are onto if \( f_{\alpha\beta} \) are open onto. Hence, from 3.6. we obtain.

3.7. THEOREM. Let \( H \) be a class of the inverse system of H-closed functionally Hausdorff spaces \( X_\alpha \) and open onto mappings \( f_{\alpha\beta} \). Then the functor \( \tau \) is H-continuous.

From [6] it follows that \( \lim \ X \) is R-closed (AR-closed) if \( X_\alpha \) are R-closed (AR-closed) and if \( f_{\alpha\beta} \) are open-closed. By similar method of proof we have.

3.8. THEOREM. Let \( R \) be a class of the inverse systems of R-closed (AR-closed) functionally Hausdorff spaces \( X_\alpha \) and open-closed onto mappings \( f_{\alpha\beta} \). Then the functor \( \tau \) is R-continuous.

We say that an inverse system \( \mathcal{X} = \{ X_\alpha, f_{\alpha\beta}, A \} \) is factorisable (or f-system) [10] if for each continuous mapping \( f : \lim \ X \rightarrow [0,1] \) there exists a continuous mapping \( g_\alpha : X_\alpha \rightarrow [0,1] \) such that \( f = g_\alpha f_\alpha \), where \( f_\alpha : \lim \ X \rightarrow X_\alpha \) is the natural projection.

3.9. LEMMA. If \( \mathcal{X} \) is an f-system, then the mapping \( \tau_1 : \tau (\lim \ X) \rightarrow \lim \tau (X) \) is one-to-one.

Proof. Let \([x]\) and \([y]\) be two distinct points of \( \tau (\lim \ X) \), where \( x, y \in \lim \ X \). This means that there exists an \( f : \lim \ X \rightarrow [0,1] \) such that \( f (x) = 0 \) and \( f (y) = 1 \). Since \( \mathcal{X} \) is f-system there is an \( \alpha \in A \) and \( g_\alpha : X_\alpha \rightarrow [0,1] \) such that \( f = g_\alpha f_\alpha \). It follows that \( [f_\alpha (x)] \neq [f_\alpha (y)] \) since \( g_\alpha f_\alpha (x) = 0 \) and \( g_\alpha f_\alpha (y) = 1 \). This means that \( \tau_1([x]) \neq \tau_1([y]) \). The proof is completed.
3.10. THEOREM. Let \( W \) be a class of the inverse \( f \)-system \( X = \{X_\alpha, f_{\alpha\beta}, A\} \) such that all \( X_\alpha \) and \( X = \lim X \) are \( w \)-compact (H-closed, \( \tau \)-compact, R-closed, AR-closed). Then the Tychonoff functor \( \tau \) is \( W \)-continuous.

Proof: From 1.4. Theorem it follows that \( \tau (\lim X) \) and \( \lim \tau (X) \) are compact. By virtue of 3.5. Lemma it follows that \( \tau_1 \) is the homeomorphism Q.E.D.

3.11. LEMMA. [11]. Let \( X = \{X_\alpha, f_{\alpha\beta}, A\} \) be a well-ordered inverse system such that \( w (X_\alpha) < \tau \) and \( \text{cf}(A) > \tau > \aleph_0 \). If \( f_{\alpha\beta} \) are...
perfect (open or $X$ is continuous) then $w(\lim X) < \tau$.

We close this Section with the following

3.12. THEOREM. Let $C$ be a class of the inverse systems $X$ as in 3.11. If $\lim X$ is $w$-compact ($\tau$-compact, $H$-closed, $R$-closed, AR-closed) and if the projections $f_{\alpha}: X \rightarrow \alpha \in A$, are onto, then the functor $\tau$ is $C$-continuous.

Proof. In view of Theorem 3.10, it suffices to prove that $X$ is an $f$-system. Let $X = \lim X$ and let $f: X \rightarrow [0,1]$ be a real-valued function. For each $z \in [0,1]$ let $N_z$ be a countable family of open sets such that $\cap \{U: U \in N_z\} = \{z\}$. We can assume that $N = \{N_z: z \in [0,1]\}$ is countable. It is readily seen that for each $U \in f^{-1}(N)$ there exist an $\alpha \in A$ and open $U_{\alpha_1} \subseteq X$, such that $U_{\alpha_1} = f^{-1}(U)$. (See also [12]). Since the cardinality $|N| \leq \aleph_0$ and $cf(A) > \aleph_0$, there exist an $\alpha \in A$ such that $\alpha > \alpha_1$, $i \in N$. Let $Y = \{Y_z: z \in [0,1]\}$. This means that for each $x_{\alpha} \in X_{\alpha}$ there is only one $z \in [0,1]$ such that $x_{\alpha} \in Y_z$. Put $g_{\alpha}(x_{\alpha}) = z$. We define $g_{\alpha}: X_{\alpha} \rightarrow [0,1]$ such that $f = g_{\alpha}f_{\alpha}$. In order to complete the proof we prove that $g_{\alpha}$ is continuous. Let $x_{\alpha} \in X_{\alpha}$ and let $g_{\alpha}(x_{\alpha}) = z$. For each neighborhoods $V \in N_z$ there is a neighborhood $U_{\alpha}$ of $x_{\alpha}$ such that $f_{\alpha}^{-1}(U_{\alpha}) = V$. This means that $g_{\alpha}(U_{\alpha}) = V$. The proof is completed.

4. CONNECTEDNESS OF THE LIMIT SPACE

We start with following theorem

4.1. THEOREM: A topological space $X$ is connected iff $\tau(X)$ is connected.
Proof. If \( X \) is connected, then \( \tau(X) \) is connected since \( \tau_X : X \rightarrow \tau(X) \) is continuous surjection. Conversely, let \( \tau(X) \) be connected. If \( X \) is disconnected, then there exist two disjoint open sets \( U, V \subseteq X \) such that \( X = U \cup V \).

Let \( g : X \rightarrow [0,1] \) be a mapping such that \( g(x) = 0 \) if \( x \in U \) and \( g(x) = 1 \) if \( x \in V \). Clearly, \( g \) is continuous. From the definition of \( \tau(X) \) it follows that \( \tau_X(U) \cap \tau_X(V) = \emptyset \) and \( \tau_X(U) \cup \tau_X(V) = \tau(X) \), where \( \tau_X(U) \) is the image of \( U \). Let \( f : \tau(X) \rightarrow [0,1] \) be a mapping such that \( f[\tau_X(U)] = 0 \), \( f[\tau_X(V)] = 1 \). Clearly, \( f \tau = g \).

Since \( g \in C(X) \), from the definition for \( \tau(X) \) it follows that is continuous i.e. \( f \in C(\tau(X)) \). This means that \( \tau_X(U) = f^{-1}(0) \) and \( \tau_X(V) = f^{-1}(1) \) i.e. \( \tau_X(U) \) and \( \tau_X(V) \) are disjoint open sets in \( \tau(X) \). This contradiction with the connectedness of \( \tau(X) \). The proof is completed.

4.2. THEOREM. Let \( \{X_{\alpha}, f_{\alpha\beta} : A\} \) be an inverse system such that the functor \( \tau \) is \( X \)-continuous. The space \( X = \lim X \) is connected iff \( \lim \tau X \) is connected.

Proof. The space \( \tau(\lim X) \) is connected since it is homomorphic with \( \lim \tau X \). From 4.2. it follows that \( \lim X \) is connected iff \( \tau(\lim X) \) is connected. Q.E.D.

Now, from Theorems 4.1. and 4.2. and from theorems of Section Three we obtain the following theorems.

4.3. THEOREM. Let \( X \) be an inverse system as in Theorem 2.4. Then \( X = \lim X \) is connected iff \( X_{\alpha} \), \( \alpha \in A \), are connected.

4.4. THEOREM. Let \( X = \{X_{\alpha}, f_{\alpha\beta} : A\} \) be an inverse system such that \( X_{\alpha}, \alpha \in A, X = \lim X \) are functionally Hausdorff H-closed (R-closed). If the projections \( f_{\alpha} : X \rightarrow X_{\alpha}, \alpha \in A \), are onto
mappings, then $X$ is connected iff $X_\alpha$, $\alpha \in A$, are connected.

4.5. THEOREM. Let $X = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of H-closed functionally Hausdorff spaces $X_\alpha$ and open onto mappings $f_{\alpha\beta}$. The space $X = \lim X$ is connected iff $X_\alpha$, $\alpha \in A$, are connected.

4.6. THEOREM. Let $X = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of R-closed (AR-closed) functionally Hausdorff spaces $X_\alpha$ and open-closed onto mappings $f_{\alpha\beta}$. The space $X = \lim X$ is connected iff the spaces $X_\alpha$, $\alpha \in A$, are connected.

4.7. THEOREM. Let $X = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse f-system such that all $X_\alpha$ and $X = \lim X$ are w-compact ($\tau$-compact, H-closed, R-closed, AR-closed). $X$ is connected iff $X_\alpha$, $\alpha \in A$, are connected.

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Lončar I. Neprekidnost Tihnovljevog funkторa

SADRŽAJ

U radu je istrazivana neprekidnost Tihnovljevog funkторa τ. Pri tome kazemo da je funktor F C-neprekidan ako su prostori F(lim X) i limF X homeomorfni, gdje je C klasa inverznih sistema X = = \{X_\alpha, f_\alpha\beta, A\}.

U odjeljku 1. dana je definicija i osnovna svojstva funkторa τ. Drugi odjeljak sadrži teoreme o nepraznosti i w-kompaktnosti limesa inverznih sistema w-kompaktnih prostora. Teoremi iz drugog odjeljka služe za dokazivanje teorema o C-neprekidnosti funkторa τ, gdje je C klasa inverznih sistema w-kompaktnih (τ-kompaktnih, H-zatvorenih ili R-zatvorenih) prostora.