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CONTINUITY OF THE TYCHONOFF FUNCTOR τ

ABSTRACT. Let **C** *be a class of the inverse systems* $X = \{X_\lambda, f_{\alpha\beta}, A\}$. *We say that a functor* **F** *is* **C**-*continuous if* $F(\lim \underline{X})$ *is homeomorphic with* $\lim F(\underline{X})$.

In the present paper the continuity of Tychonoff functor τ *is investigated.*

Section Two contains some theorems concerning the non-emptyness and w-compactness of the limit of inverse systems of w-compact spaces.

Section Three is the main section. Some theorems concering C-continuity of the Tychonoff functor τ *are proved, where C is a class of the inverse systems of w-compact,* τ*-compact, H-closed or R-clased spaces.*

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0. INTRODUCTION

0.1. The set of all continuous, real-valued (bounded) function on a topological spaces X will be denoted by C(X) ($C^*(X)$).

Unliess otherwise stated, no separation axioms will be assumed.

0.2. A set $A \subseteq X$ is regularly closed (open) if $A = \overline{\text{Int}\, A}$ ($A = \text{Int}\, \overline{A}$).

0.3. A set $A \subseteq X_1$ is said to be zero-set if there is an $f \in C(X)$ such that $A = f^{-1}(0)$. The zero-set of f is denoted by Z(f) or by $Z_X(f)$.

A cozero-set is a complement of zero-set.
It is well-known [3] that

(i) $z(f) = z(|f|) = z(f^n) = z(|f| \wedge 1)$

(ii) Evry zero-set is G_δ

(iii) $z(fg) = z(f) \cup z(g)$

(iv) $z(f^2 + g^2) = Z(|f| + |g|) = z(f) \cap z(g)$

(v) The countable intersection of zero-set is zero-set.

0.4. Two subsets A and B of X are said to be completely separtated in X if there exists a function $f \in C_*(X)$ such that $f(x) = 0$ for all $x \in A$, and $f(x) = 1$ for all $x \in B$.

0.5. A space X is said to be completely regular [3] provided that it is Hausdorff space such that each closed set $F \subseteq X$ and each $x \notin F$ are completely separated.

0.6. A space X is said to be almost regular [9] if for each regularly closed $F \subset X$ and each $x \in X\backslash F$ there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$.

0.7. By cf (A) we denote the cofinality of the well-ordered set A i.e. the smallest ordinal which is cofinal in A.

0.7. We say that a space X is quasicompact if every centred family of closed subsets of X has a non-empty interesection.

0.8. A space X is functionally Hausdorff of for each distinct points x and y of X there is a continuous function $f : X \longrightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$. Each functionally Hausdorff space is Hausdorff.

0.9. It follows that in a functionally Hausdorff space X for each distinct points x and y there are cozero-sets U_x and U_y such $x \in U_x - \{ y \}$ and $y \in U_y \subseteq X - \{ x \}$.

0.10. If U is a cozero-set containing $x \in X$, there exist a cozero-set $V \ni x$ such that $x \in V \subset \overline{V} \subset U$. Namely, if $f : X \longrightarrow [0,1]$ is a function such that $x \in f^{-1}([0,1]) = U$, then we define a function $F : [0,1] \to [0,1]$ such that $F(y) = 0$ for $y \leq f(x) / 2$ and $F(y) = ((2y - f(x)) : (2 - f(x))$ for $y > f(x) / 2$. Now, let $G = Ff$. We have $\overline{G^{-1}(0,1)} \subseteq U$.

0.11. If $\underline{X}$ is functionally Hausdorff, then $\{x\} = \cap \{\overline{U} : U$ is the cozero-set containing $x \in X\}$. The proof holds from 0.8., 0.9. and 0.10.

1. FUNCTOR τ

Let X be a topological space. We define an equivalence relation ρ on X such that $x \rho y$ iff $f(x) = f(y)$ for each $f \in C(X)$. Let $\tau(X) = X/\tau$ be a set of all equivalence classes equiped with the smaltest topology in which are continuous all functions g such that $g. \tau_X \in C(X)$, where $\tau_X : X \longrightarrow X/\tau$ is the natural projections. In [3:41] is actually proved that $\tau(X)$ is completely regular.

By $[x]$ we denote the equivalence class containing $x \varepsilon X$.

1.1. LEMMA. If $f : X \longrightarrow Y$ is a continuous mapping into a completely regural space Y, then there exist a continuous mapping $g : \tau(x) \longrightarrow Y$ such that $f = g . \tau_X$.

P r o o f. If $x \int y$ then must be $f(x) = f(y)$ since $f(x) \neq f(y)$ implies that there is $f' \in C(Y)$ such that $f'(x) = 0$, $f'(y) = 1$. This is in contradiction with $x \rho y$ since $ff' \in C(X)$. This means that for $x' \varepsilon \tau(X)$ one cane define $g(x') = f(x)$, $x \in x'$.

1.2. COROLLARY. If $f : X \longrightarrow Y$ is a continuous mapping, then ther exists a continuous mapping $\tau(f) : \tau(X) \longrightarrow \tau(Y)$ such that $\tau(f)\,\tau_X = \tau_Y\, f$.

1.3. LEMMA. If X is functionally Hausdorf, then $\tau_X : X \dashrightarrow \tau(X)$ is one-to-one.

P r o o r. Trivial.

An open set $U \subseteq X$ is τ-open is U is the union of the cozero-sets.

We say that a space X is w-compact [4] (quasi-H-closed) if for each centred family $\{U_\mu : \mu \in M\}$ of τ-open (open) sets $U_\mu \subseteq X$ the set $\cap \{\overline{U}_\mu : \mu \in M\}$ is non-empty.

1.4. THEOREM. If X is w-compact, then $\tau(x)$ is a compact space ($= T_2$ quasi-compact).

P r o o f. It suffices to prove that $\tau(X)$ is quasi-H-closed since each regular H-closed is compact. Let $\{U_\mu : \mu \in M\}$ be a centred family of open sets in $\tau(X)$. This means U_μ is τ-open in X. It follows that $\cap \{\overline{U}_\mu : \mu \in M\} \neq 0$, where $\overline{U}_\mu$ is a closure in X. Let x $\in \{\overline{U}_\mu : \mu \in M\}$. From the continuity of τ_X we have $\tau_X(x) \in \cap \{\overline{U}_\mu : \mu \in M\}$ where now $\overline{U}$ is a closure in $\tau(X)$. The proof is completed.

A space X is said to be τ-compact [4] iff each cover $\{U_\mu : \mu \in M\}$ of X consisting of the cozero-sets U_μ has a finite subcover.

1.5. THEOREM. If X is τ-compact, then $\tau(X)$ is compact.

P r o o f. Trivial since each open set in $\tau(X)$ is τ-open in X.

A space X is said to be perfectly w-compact (τ-compact, H-closed, R-closed) if $\tau_X^{-1}(y)$ is copmact for each $y \in \tau(X)$ i.e. every equivalence class $[y]$ is compact.

2. INVERSE SYSTEMS OF W-COMPACT AND τ - COMPACT SPACES

We start with the following theorem.

2.1. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of τ -compact (w-compact) functionally Hausdorff spaces X_α. If X_α, $\alpha \in A$, are non-empty, then $X = \lim \underline{X}$ is non-empty. Moreower, if $f_{\alpha\beta}$ are onto, then the projections $f_\alpha : X \text{--->} X_\alpha$, $\alpha \in A$, are onto mappings.

P r o o f. From 1.2. it follows that $\underline{X}_\tau = \{\tau(X_\alpha), \tau(f_{\alpha\beta}), A\}$ is an inverse systems. In view of Lemma 1.3. there is a mapping $\tau : \underline{X}$ ---> $\underline{X}_\tau$ such that $\tau = (\tau_{X_\alpha} : X \text{--->} \tau(X_\alpha))$ and τ_{X_α}, $\alpha \in A$, is identity mapping. The mapping τ induces a mapping $\lim \tau : \lim \underline{X}$ ---> $\lim X_\tau$ which is 1-1. This means that $\lim \underline{X} \neq 0$ iff $\lim \underline{X}_\tau \neq$ 0. Since $\underline{X}_\alpha$ is the inverse system of compact spaces $\tau(X_\alpha)$, we have $\lim \tau(\underline{X}) \neq 0$. The proof is completed.

Since each quasi-H-closed space is w-compact, we have

2.2. THEOREM. LET $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of functionally Hausdorff non-empty quasi-H-closed spaces X_α. Then $X = \lim \underline{X}$ is non-empty.

We say that a regular (almost regular) space X is R-closed (AR-closed) if it is closed in each regular (almost regular) space in which it can be embedded [9]. Each completely regular R-closed (AR-closed) space X is compact since $X \subset \beta X$ [2]. If X is R-closed, Y regular, and $f : X \text{--->} Y$ a continuous mapping then Y is R-closed.

2.3. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of non-empty functionally Hausdorff R-closed spaces X_α. Then $X = \lim \underline{X}$ is non-empty.

P r o o f. The space $\tau(X_\alpha)$ is completely regular R-closed i.e. a Hausdorf compact space. See the proof of Theorem 2.1.

We say that a mapping $f : X \longrightarrow Y$ is τ-open if $f(U)$ is τ-open for each τ-open set $U \subset X$.

2.4. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of w-compact functionally Hausdorff spaces X_α If the projections $f_\alpha : \lim \underline{X} \longrightarrow X_\alpha$, $\alpha \in A$, are τ-open, then $X = \lim \underline{X}$ is functionally Hausdorff and w-compact.

P r o o f. Let $U = \{U_\mu : \mu \in M\}$ be a maximal centred family of τ-open sets in X. For each $\alpha \in A$ let $U_\alpha = \{f_\alpha(U_\mu) : \mu \in M\}$. We prove that U_α is the maximal centred family of τ-open sets in X_α (f_α is τ-open!). Suppose that V_α is τ-open in X_α such that $V_\alpha \cap f_\alpha(U_\mu)$ is non-empty for each $U_\mu \in U_\alpha$. This means that $f_\alpha^{-1}(V_\alpha)$ is τ-open set wich meets each U_μ. From the maximality of U it follows that $f_\alpha^{-1}(U_\alpha) \in U$ i.e. $V_\alpha \in U_\alpha$. Hence, U_α is maximal. From the w-compactness of X_α it follows that $Y_\alpha = \cap \{\overline{f_\alpha(U_\mu)} : U_\mu \in U\}$ is non-empty. From the maximality of U_α it follows that U_α contains all neighborhoods of all $y_\alpha \in Y_\alpha$ From 0.11. it follows that $Y_\alpha = \{y_\alpha\}$, where $y_\alpha \in X_\alpha$. For each $\alpha \in A$ let W_α be a family of all τ-open sets containing y_α. From the maximality of $U_\beta, \beta \geq \alpha$ it follows that U_β contains $f_{\alpha\beta}^{-1}(U_\alpha) = \{f_{\alpha\beta}^{-1}(U) : U \in U_\alpha\}$

This means that $f_{\alpha\beta}(y_\beta) = y_\alpha$, $\beta \geq \alpha$. Hence $y = (y_\alpha : \alpha \in A\}$ is a point of X. It is readily seen that $y \in \cap \{\overline{U} : U \in \quad\}$. The proof is completed since it is clear that X is functionally Hausdorff.

2.5. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of perfect w-compact (τ-compact, H-closed, R-closed) spaces X_α. A space $X = \lim \underline{X}$ is non-empty iff the spaces X_α, $\alpha \in A$, are non-empty.

3. CONTINUITY OF THE FUNCTOR τ

Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system and let τ be a Tychonoff functor described in Section One. From 1.2. it follows that $\tau(\underline{X}) = \{\tau(X_\alpha), \tau(f_{\alpha\beta}), A\}$ is an inverse system. Let C be a class of the inverse systems. We say that the functor τ is C - c o n t i n u o u s if $\tau(\lim X)$ is homeomorphic to $\lim \tau(\underline{X})$ for each $\underline{X}$ in C. The functor τ is said to be continuous if τ is C-continuous for each class C.

3.1. LEMMA. If $\underline{X}$ is an inverse system, then there exists a continuous mapping $\tau_1 : \tau(\lim \underline{X}) \text{---}> \lim \tau(\underline{X})$.

P r o o f. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system and let $\tau(\underline{X}) = \{\tau(X_\alpha), \tau(f_{\alpha\beta}), A\}$. From 1.2. it follows that there is $\tau_{1_\alpha} : \tau(\lim \underline{X}) \text{---}> (X_\alpha)$ such that $\tau_{X_\alpha} f_\alpha = \tau_{1_\alpha} \tau$, where $\tau : \lim \underline{X} \text{---}> \tau(\lim \underline{X})$. It is readily seen that $\tau_{1_\alpha} = \tau(f_{\alpha\beta}) . \tau_{1_\beta}$, $\beta \geq \alpha$. This means that the mappings τ_{1_α}, $\alpha \in A$, induce a continuous mapping $\tau_1 : \tau(\lim \underline{X}) \text{---}> \lim \tau(X)$. The proof is completed.

3.2. LEMMA. $\lim \tau = \tau_1 \tau$

P r o o f. From the definition of τ_1 it follows $\tau_{1_\alpha} = f'_\alpha \tau_1$, where

f'_α : $\lim \tau$ ($\underline{X}$) ---> $\tau(X_\alpha)$ is a projection. Moreower, $\tau_{X_\alpha} f_\alpha = \tau_{1_\alpha} \tau$ and $\tau_{X_\alpha} f_\alpha = f'_\alpha$. $\lim \tau$. It follows that $\tau_{1_\alpha} \tau = f'_\alpha \lim \tau$ and $\tau_{1_\alpha} \tau$ $= f'_\alpha . \tau_1 . \tau$ i.e. $\lim \tau = \tau_1 \tau$. Q.E.D.

3.3. THEOREM. Let C be the class of all inverse systems $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ such that X_α, $\alpha \in A$, $X = \lim \underline{X}$ is w-compact (τ-compact) functionally Hausdorf. If the projections $f_\alpha : X \text{--->} X_\alpha$, $\alpha \in A$, are onto, then the Tychonoff functor τ is C -cointinuous.

P r o o f. From Lemma 1.3. it follows that each τ_{X_α} , $\alpha \in A$, is 1-1. This means that $\lim \tau$ is 1-1. Since $\lim \underline{X}$ is functionally Hausdorf we infer by 1.3. that $\tau : \lim \underline{X} \text{--->} \tau (\lim \underline{X})$ is 1-1. It follows that $\tau_1 : \tau (\lim \underline{X})$ ---Y $\lim \tau$ (X) is one-to-one. Since $\lim \tau$ ($\underline{X}$) and τ ($\lim \underline{X}$) are compact (1.4.THEOREM) we infer that $\tau_1$is a homeomorphism. The proof is completed.

3.4. COROLLARY. Let C be the class of all inverse systems an in Theorem 2.4. Then the Tychonoff functor τ is C -continuous.

3.5. REMARK. In [4] is proved that if $\{X_\alpha : \alpha \in A\}$ is a family of w-compact spaces X_α, then $\prod X_\alpha$ is w-compact an $\tau (\prod X_\alpha) = \prod \tau (X_\alpha)$.

3.6. THEOREM: Let H be a class of the inverse systems $\underline{X} = \{X_\alpha, f_{\alpha\beta} A\}$ such that X_α $\alpha \in A$, $X = \lim \underline{X}$ are functionally Hausdorff H-closed (R-closed). If the projections $f_\alpha : X \text{--->} X_\alpha$ $\alpha \in A$, are onto mappings, then the functor τ is H -continuous.

P r o o f. The spaces $\tau(X_\alpha)$, $\alpha \in A$, and the spaces τ ($\lim \underline{X}$), $\lim \tau$ (X) are compact (See the proof of 2.3. and 3.3.).

In [14] it is proved that lim $\underline{X}$ is H-closed if X_α are H-closed, $f_{\alpha\beta}$ open and that $f_{\alpha\beta}$ are onto if $f_{\alpha\beta}$ are open onto. Hence, from 3.6. we obtain.

3.7. THEOREM. Let H be a class of the inverse system of H-closed functionally Hausdorff spaces X_α and open onto mappings $f_{\alpha\beta}$ Then the functor τ is H -continuous.

From [6] it follows that lim $\underline{X}$ is R-closed (AR-closed) f X_α are R-closed (AR-closed) and if $f_{\alpha\beta}$ are open-closed. By similar method of proof we have.

3.8. THEOREM. Let R be a class of the inverse systems of R-closed (AR-closed) functionally Hausdorf spaces X_α and open-closed onto mappings $f_{\alpha\beta}$. Then the functor τ is R -continuous.

We say that an inverse system $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ is factorisable (or f-system) [10] it for each continuous mapping f : lim $\underline{X}$---> [0,1] there exists a continuous mapping $g_\alpha : X_\alpha$ ---> [0,1] such thaf $f = g_\alpha f_\alpha$, where f_α : lim $\underline{X}$ ---> X_α is the natural projection.

3.9. LEMMA. If $\underline{X}$ is an f-system, then the mapping $\tau_1 : \tau$ (lim $\underline{X}$) ---> lim τ (X) is one-to-one.

P r o o f. Let [x] and [y] be two distinct points of τ (lim $\underline{X}$), where x, y $\in$ lim_X. This means that there exists an f : lim $\underline{X}$ ---> [0,1] such that f (x) = 0 and f (y) = 1. Since $\underline{X}$ is f-system there is an $\alpha \in A$ and $g_\alpha : X_\alpha$ ---> [0,1] such that $f = g_\alpha f_\alpha$. It follows that $[f_\alpha (x)] \neq [f_\alpha(y)]$ since $g_\alpha f_\alpha (x) = 0$ and $g_\alpha f_\alpha (y)=1$. This means that $\tau_1([x]) \neq \tau_1([y])$. The proof is completed.

3.10. THEOREM. Let W be a class of the inverse f-system $X = \{X_\alpha, f_{\alpha\beta}, A\}$ such that all X_α and $X = \lim \underline{X}$ are w-compact (H-closed, τ-compact, R-closed, AR-closed). Then the Tychonoff functor τ is W -cotinuous.

P r o o f: From 1.4. Theorem it follows that $\tau(\lim \underline{X})$ and $\lim \tau(\underline{X})$ are compact. By virtue of 3.5. Lemma it follows that τ_1 is the homemorphism Q.E.D.

3.11. LEMMA. [11]. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be a well-ordered inverse system such that $w(X_\alpha) < \tau$ and $cf(A) > \tau > \aleph_0$. If $f_{\alpha\beta}$ are perfect (open or X is continuous) then $w(\lim X) < \tau$.

We close this Section with the following

3.12. THEOREM. Let C be a class of the inverse systems $\underline{X}$ as in 3.11. If $\lim \underline{X}$ is w-compact (τ-compact, H-closed, $\bar{R}$-closed, AR-closed) adn if the projections $f_\alpha : X \to \alpha \in A$, are onto, then the functor τ is C -continuous.

P r o o f. In view of Theorem 3.10. it suffices to prove that $\underline{X}$ is an f-system. Let $X = \lim \underline{X}$ and let $f : X \to [0,1]$ be a real-valued function. For each $z \in [0,1]$ let N_z be a countable family of open sets such that $\cap \{U : U \in N_z\} = \{z\}$. We can asume that $N = \{N_z : z \varepsilon [0,1]\}$ is countable. It is readily seen that for each $U_i \in f^{-1}(N)$ there exist an $\alpha \in A$ and open $U_{\alpha_i} \subseteq X_{\alpha_i}$ such that $U_i = f_{\alpha_i}^{-1}(U_{\alpha_i})$ [7] (See also [12]). Since the cardinality $|N| \le \aleph_0$ and $cf(A) > N_0$ there exist an $\alpha \in A$ such that $\alpha > \alpha_i$, $i \in N$. Let Y_z be a set $\cap \{U_\alpha : f_\alpha^{-1}(U_\alpha) \in f_\alpha^{-1}(N_z)\}$. It is clear that $Y_z \cap Y_{z'} = \emptyset$ iff $z \ne z'$ and that $X_\alpha = \cup \{Y_z : z \in [0,1]\}$. This means that for each $x_\alpha \in X_\alpha$ there is only one $z \in [0,1]$ such that $x_\alpha \in Y_z$. Put $g_\alpha(x_\alpha) = z$. We define $g_\alpha : X_\alpha \to [0,1]$ such that $f = g_\alpha f_\alpha$. In order to complete the proof we prove that g_α is continuous. Let $x_\alpha \in X_\alpha$ and let $g_\alpha(x_\alpha) = z$. For each neighborhoods $V \varepsilon N_z$ there is a neighborhood U_α of x_α such that $f_\alpha^{-1}(U_\alpha) = V$. This means that $g_\alpha(U_\alpha) = V$. The proof is completed.

4. CONNECTEDNESS OF THE LIMIT SPACE

We start with following theorem

4.1. THEOREM: A topological space X is connected iff $\tau(X)$ is connected.

P r o o f. If X is connected, then τ (X) is connected since τ_X : X ---> τ (X) is continuous surjection. Conversely, let $\tau(X)$ be connected. If X is disconnected, then there exist two disjoint open sets U, V $\subseteq$ X such that X = U $\cup$ V.

Let g : X ---> $[0,1]$ be a mapping such that g (x) = 0 if x $\in$ U and g (x) = 1 if x $\in$ V. Clearly, g is continuous. From the definition of τ (X) it follows that $\tau_X(U) \cap \tau_X(V) = \emptyset$ and $\tau_X(U) \cup \tau_X(V) = \tau(X)$, where τ_x (U) is the image of U. Let f : τ (X) ---> $[0,1]$ be a mapping such that $f[\tau_X(U)] = 0$, $f[\tau_X(V)] = 1$. Clearly, f τ =g. Since g $\in$ C (X), from the definition fo τ (X) it follows that is continuous i.e. f $\in$ C (τ X)). This means that $\tau_x(U) = f^{-1}(0)$ and $\tau_x(V) = f^{-1}(1)$ i.e. τ_x (U) and τ_x (V) are disjoint open sets in τ (X). This contradiction with the connectedness of τ (X). The proof is completed.

4.2. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system such that the functor τ is $\underline{X}$-continuous. The space X = lim $\underline{X}$ is connected iff lim τ $\underline{X}$ is connected.

P r o o f. The space τ (lim $\underline{X}$) is connected since it is homemorphic with lim τ $\underline{X}$. From 4.2. it follows that lim $\underline{X}$ is connected iff τ (lim $\underline{X}$) is connected. Q.E.D.

Now, from Theorems 4.1. and 4.2. and from theorems of Section Three we obtain the following theorems.

4.3. THEOREM. Let X be an inverse system as in Theorem 2.4. Then X = lim X is connected iff X_α $\alpha \in A$, are connected.

4.4. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system such that X_α, $\alpha \in A$, X = lim $\underline{X}$ are functionally Hausdorff H-closed (R-closed). If the projections f_α : X ---> X_α, $\alpha \in A$, are onto mappings, then X is connected iff X_α, $\alpha \in A$, are connected.

4.5. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of H-closed functionally Hausdorf spaces X_α and open onto mappings $f_{\alpha\beta}$. The space $X = \lim \underline{X}$ is connected iff X_α, $\alpha \in A$, are connected.

4.6. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of R-closed (AR-closed) functionally Hausdorf spaces X_α and open-closed onto mappings $f_{\alpha\beta}$. The space $X = \lim \underline{X}$ is connected iff the spaces X_α, $\alpha \in A$, are connected.

4.7. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse f-system such that all X_α and $X = \lim \underline{X}$ are w-compact (τ-compact, H-closed, R-closed, AR-closed). X is connected iff X_α, $\alpha \in A$, are connected.

REFERENCES:

(1) Arhangel'skij A.V., Ponomarev V.I., Osnovy ob{~ej topologii v zada~ah i upra'nenijah, Nauka, Moskva, 1974.
(2) Engelking R., General Topology, PWN, Warszawa, 1977.
(3) Gillman L. and Jerison M., Rings of Continuous Functions, D. van Nostrand Comp., New York, 1960.
(4) Ishii T., Some results on w-compact spaces, (in russian), UMN 35 (1980), 61-66.
(5) Lon~ar I., Applications of Θ-clsed and μ-closed sets, Zbornik radova Fakulteta organizacije i informatike Vara'din 8 (1984), 237-254.(6) ...., Inverse systems of R-closed spaces, Math. Balkan., 2(1988), 172 - 181.
(7) ...., Lindelöfov broj i inverzni sistemi, Zbornik radova FOI Vara'din, 7 (1983), 245-252.
(8) Sigal M.K. and Asha Mathur, A note on almost completely

regular spaces, Glasnik matemati~ki 6 (26) (1971), 345-349.
(9) ...., On minimal almost regular spaces, Glasnik matemati~ki 6 (26) (1971), 179-185.
(10) [~epin E.V.:, Funktory i nes~etnye stepeni kompaktov, UMN 36 (1981), 3-62.
(11) Tka~enko M.G., Some results on inverse spectra I., Commentationes math. univ. carol. 22 (1981), 621-633.
(12) ...., Some results on inverse spectra II., Commentationes math. univ. carol. 22 (1981), 819-841.
(13) ...., Cepi i kardinaly, DAN SSSR 239 (1978), 546-549.
(14) Vinson T.O. and Dickman R.H., Inverse limits and absolutes of H-closed spaces, Proc. Amer. Math. Soc. 66 (1977), 351-358.

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Lončar I. Neprekidnost Tihnovljevog funktora

SADRŽAJ

U radu je istrazivana neprekidnost Tihonovljevog funktora τ. Pri tome kazemo da je funktor F C-neprekidan ako su prostori $F(\lim X)$ i $\lim F\underline{X}$ homeomorfni, gdje je C klasa inverznih sistema $\underline{X}$ = = $\{X_\alpha, f_{\alpha\beta}, A\}$.
U odjeljku 1. dana je definicija i osnovna svojstva funktora τ. Drugi odjeljak sadrzi teoreme o nepraznosti i w-kompaktnosti limesa inverznih sistema w-kompaktnih prostora.
Teoremi iz drugog odjeljka sluze za dokazivanje teorema o C-neprekidnosti funktora τ, gdje je C klasa inverznih sistema w-kompaktnih (τ-kompaktnih, H-zatvorenih ili R-zatvorenih) prostora.