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PARALLAX IN A DYNAMIC, NON-CONTACT & NON-DESTRUCTIVE, PRECISE LENGTH MEASUREMENT

Josip Stepanić, Srđan Medić, Živko Kondić

Original scientific paper

If a dimensional measurement on an object is to be performed during its non-perturbed motion, a specific approach is needed. In this article we consider length measurement of an object based on photographs of the object during its unperturbed motion. The main obstacle for determining measured length is the parallax which is generated as a convolution of contributions occurring in different parts of a measuring system. The quantitative expressions for the parallax are derived, and applied onto processing of the photos. As a result, we derive in a closed form the algorithm for quantitative estimates of the precisely measured length and accompanied standard deviation.

Keywords: *length measurement, motion, non-contact measurement, non-destructive measurement, parallax, perturbation*

Paralaksa u dinamičkom, beskontaktnom i nerazornom, preciznom mjerenju duljina

Izvorni znanstveni članak

Za dimenzijska mjerenja na objektima u nesmetanom gibanju potreban je specifični pristup. U ovom radu razmatramo mjerenje duljine objekata u neometanom gibanju pomoću snimljenih fotografija objekta. Glavna prepreka određivanju mjerene duljine je paralaksa koja je generirana kao niz doprinosa koji nastaju u različitim dijelovima mjernog sustava. Kvantitativni izraz za paralaksu je izveden i primijenjen na analizu fotografija. Kao rezultat, u zatvorenom obliku izveden je algoritam kvantitativne procjene mjerene duljine i pripadnog standardnog odstupanja.

Ključne riječi: *beskontaktno mjerenje, gibanje, mjerenje duljine, nerazorno mjerenje, paralaksa, smetnje*

1 Introduction

Length of an object, or some of its parts, is a standard characteristic used for many purposes such as monitoring, diagnostics, characterisation, classification, searches, etc. [1]. Length is estimated experimentally in a measurement process, which is prescribed in details owing to the overwhelming use of that characteristic. Among a variety of prescriptions, the condition that the measured object be static is often taken as a *conditio sine qua non*. Furthermore, for many types of objects, e.g. objects of smaller mass, it is assumed that they can be manipulated in space during measuring process as is needed.

However, there is a large class of objects for which dimensional objects are important, yet which cannot be manipulated and, moreover, which cannot be measured statically. In other words, these are objects which are in a motion that cannot be perturbed during measurement process.

Such a class includes objects which should be dimensionally measured relatively often but the stopping of which is too expensive to be performed. Examples are movable parts of large engines. Furthermore, such a class includes dimensional characterisation of living species which should be left undisturbed either because their stopping is too expensive and/or risky, or because a disturbance would induce changes in their shape, or other kind of behaviour that is to be characterised on the basis of a length and other types of measurements. Completely independent to these two examples of such a class of objects to be measured, one encounters the topic of repetitive measurements, which are suitably automated. In these cases one could also profit from non-contact and non-destructive measurements of length.

Therefore, the class considered is highly heterogeneous, differing in time-scales of observed motion, length-scales of measured quantities, distance between measurement sensor and measured object, etc.

Yet, one encounters a universal structure of a measurement system which is to be applied in order to obtain measured data, Fig. 1. It includes a sensing unit and the recording & processing unit as required elements of a minimal configuration of a measurement system [1, 2]. Along with these, there are plausible elements such as data-transfer unit or display for human operator.

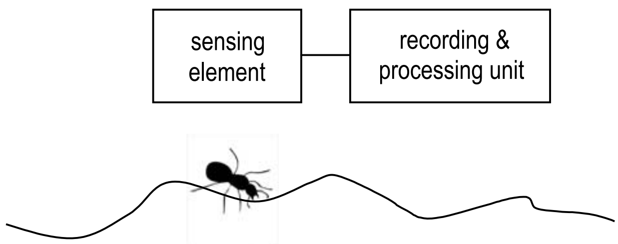

Figure 1 Structure of a minimal configuration of a measurement system for non-contact & non-destructive, dynamic measurements

In that structure, input for a sensing unit are registered rays which were previously emitted, such as light, microwaves, or other types of electromagnetic radiation, ultrasound, etc. Output of a sensing unit is formatted collection of data about object measured. That is, naturally, input to recording & sensing unit, while its output is estimate of a measured quantity (here: length) and accompanied standard deviation. We utilise here a standard deviation as a proper measure of measurement precision, while for a future work regarding more complete characterisation of the whole method a measurement uncertainty should be estimated.

Further in text we will consider that sensor uses light. That is determined merely for simplicity, and the complete procedure could be straightforwardly adapted to situations in which sensor uses microwaves or other parts of electromagnetic radiation, ultrasound, etc.

Because of that, to the output of the sensor, and input for recording & processing unit, we will refer as to the photo. It is realised as a collection of digital data.

In this article, we formulate the generic model for analysis of the set of photos of a distant object which makes possible reliable estimate of its length and accompanied standard deviation.

The second section contains description of a model. In the third section we derive a compact expression for a parallax the amount of which continuously varies over the photo. Fourth section contains derivation of statistical quantities, estimate of length and accompanied standard deviation based on the set of photos with extracted contribution of the parallax. Fifth section contains conclusions and guidelines for practitioners.

2 Formulation of the analysed model

Model formulated has generic characteristics of a non-contact & non-destructive measurement of length. It is simplified and deals with a single object of sufficient visibility, which performs uniform linear motion on the horizontal surface of uniform texture. Let v be the speed of the object relatively to the surface, and w its measured dimension. Let the object be thin, so that we think of it as of a rectangle of one side length L, and of undetermined length of the other side. Photo apparatus is statically mounted at a height h from the surface on which the object is moving. Its dimensions are assumed negligible. We parameterise motion of the object using rectangular coordinate system with the origin coincident with the orthogonal projection of a photo apparatus onto the surface. For simplicity, the object moves toward the origin, Fig. 2. We put x-axis to be coincident with the direction of the object. In the initial moment $t = 0$, the object is at distance d from the origin. If the object's speed is v, its trajectory is:

$$x(t) = d - v \cdot t. \tag{1}$$

Instead of simple form of (1), we can think of more general, but still linear motion in the form

$$x(t) = f(t).$$

The formalism presented here is still valid, yet precise form of relevant quantities changes accordingly.

Length s of projection of that object onto the plane of the photo is function of its true length L, and distances d and h. Since the recorded object length spans finite interval on the photo, its parts are seen under different angles, ranging from (Fig. 2) α_1 to α_2. Usually, their differences are neglected, equality $\alpha_1 = \alpha_2$ assumed, and relation $L = s/\cos \alpha_1$ obtained.

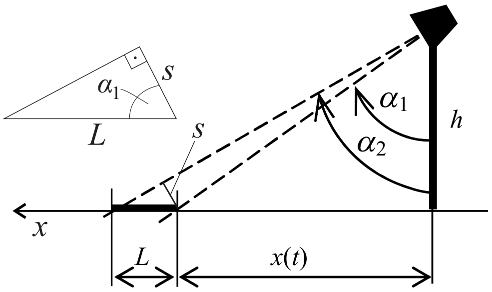

Figure 2 Modelled geometry

Before proceeding with exploitation of the stated model, it is on order to characterise it, i.e. address the meaning of its non-contactness and dynamic character from the point of view of more general measurement systems. Generally, non-contact measurements and contact measurements differ in precision and in locality. Contact measurements provide one with a relatively more precise measurement, but for a relatively smaller part of a measured object [3, 4]. In that sense they are suitable, or even requested for high precision measurements. By contact measurements we also consider the optical length measurements in laboratory, since these implie free manipulating relatively with the measured object, its positioning in a precise location on the measuring table, etc. In that sense, non-contact measurements are performed when contact measurements are not possible, such as are cases listed before. Therefore, it is expected that standard deviation of non-contact measurements would be larger than similar quantity of contact measurements if both were possible to conduct. Nevertheless, since non-contact measurement has no alternative in previously described cases in which it is applied, its results are considered to be acceptable outcomes.

Dynamic character of measurements is additional, even more stringent specification of measurements. This characteristic introduces additional error. On the basis of previous experience, dynamically introduced error is expected to be considerably larger than error introduced because of non-contactness. The dynamically induced error increases with speed of the object, as the finite speed during finite recording time (here: exposition) brings about geometrically caused blurring visible on the records (here: photos). So, we have occurrence of the error cause: the distance that measured object travelled during recording time. For relatively fast objects, that error can prevent any application of dynamic measurements. Furthermore, for objects with complex trajectory, e.g. solid objects with changes in orientation, it can be impossible to satisfy any reasonably imposed frequency of taking records (here: taking photos). While the precise values of quantities here discussed gradually lessen in time, following constant progress in digital electronics, the principle persists that motion of the object introduces significant contribution to the error in measurements.

When there is relative motion between the observer and the observed object, one usually thinks of aerial recording e.g. in photogrammetry. Moreover, recording using the method of synthetic aperture (e.g. SAR – Synthetic Aperture Radar) combines data from several records into one, coherent representation of a recorded object with enhanced resolution. It is, therefore, in order to analyse the differences between such approaches here.

The dynamic method analysed in this article utilises static recording facilities to obtain information regarding an object or being in motion. In that sense, relative velocity between them is unconstrained, both in magnitude, in average values and regarding its standard deviation in a given time interval. On the contrary, photogrammetry utilises sensing equipment mounted onto a platform in motion, so the relative velocity between the recorded object and the sensing equipment is known and can be changed according to need so that recording precision be maximised. In that sense, it is expected that the dynamic method of recording objects in motion has lower level of precision compared to methods with sensing/recording equipment in motion.

3 Parallax

However, in precise measurements of length, the variation in angle must be taken into account, so the correct expression for L is found starting from Fig. 3.

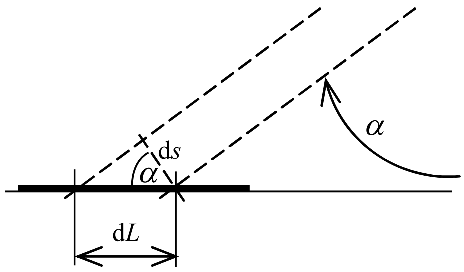

Figure 3 Geometrical considerations for an element dL of the object's length

In Fig. 3, geometrical considerations for an infinitesimally small length element dL of the object are shown. In Fig. 3, ds is the projection of dL onto the plane of photo apparatus. Since the length of the total projection is the sum of lengths of all infinitesimal projections, one has

$$s = C\cdot h\cdot \int_{\alpha_1}^{\alpha_2} \frac{\mathrm{d}\alpha}{\cos\alpha} = C\cdot h\cdot \ln\frac{1+\tan\left(\frac{\alpha}{2}\right)}{1-\tan\left(\frac{\alpha}{2}\right)}\Bigg|_{\alpha_1}^{\alpha_2}. \tag{2}$$

Factor C in (2) is a constant characterising transformation of the size of the projections of L onto plane of a photo in processing unit. It is assumed independent of variables x, v, d and parameters h and L. From Fig. 2 the following relations are seen:

$$L = h\cdot(\tan\alpha_2 - \tan\alpha_1), \tag{3}$$

$$x = h\cdot\tan\alpha_1. \tag{4}$$

Similarly to (2) one has

$$s_0 = C\cdot h\cdot \ln\left[\tan\left(\frac{\alpha_1}{2}+\frac{\pi}{4}\right)\right]. \tag{5}$$

In (5), s_0 is assumed or seen projection of $x(t)$ onto the photo. Combining (2) – (5), after straightforward but somewhat lengthy calculations, one gets

$$\frac{L}{h} = \mathrm{sh}\frac{s+s_0}{C\cdot h} - \mathrm{sh}\frac{s_0}{C\cdot h}, \tag{6}$$

in which hyperbolic sinus is obtained. In the limes of small objects, $(s + s_0)/C\cdot h << 1$, using $\mathrm{sh}(p) \approx p$ for any $p << 1$, one gets $L = s/C$ which is valid if photographed object is perpendicular to the line of its sight from the photo apparatus.

Note that functional relation between $x(t)$ and s_0 is obtained after changing $s_0 = 0$ in (6) and changing s into s_0:

$$\frac{x}{h} = \mathrm{sh}\frac{s_0}{C\cdot h}. \tag{7}$$

Expression (6) is nonlinear in s and x (which is implicit in the expression for s_0 in accordance with (4) and (5)), so the apparent length s of the object of fixed length w changes nonlinearly with its position in the photo, all for fixed photographic parameters. Therefore, expression (6) provides one with a quantitative expression for a parallax, change of measured quantity caused by a change of its position relatively to the observer, here a sensing unit.

4 Determining the mean value of length

Expression (6) is obtained for a sequence of different positions of the analysed object, so the set of photos recorded in moments t_1, t_2, ... t_N, brings about the following sequence of its apparent lengths $s_1 = s(t_1)$, $s_2 = s(t_2)$, ... $s_N = s(t_N)$, respectively. However, in realistic conditions the processing unit would provide one with somewhat different amounts for apparent lengths, because of existing yet here uncharacterised random errors ε_1, ε_2, ..., ε_N, respectively, so the final sequence of apparent lengths attributed to sequence of moments is:

$$t_i \to \tilde{s}_i = s_i + \varepsilon(t_i),\ i = 1,\ldots,N. \tag{8}$$

Naive estimate of the object's length would be a weighted mean of the sequence of values L obtained for sequence of apparent lengths s. Weights are determined in accordance with the fact that the smaller the s, the larger the error attributed to L. In case when the only data about the object is the assumption that its length L is constant, that naive approach would be the only possible [5]. Weights are chosen as $1/L'(s)$, the reciprocal values of derivative of L considered as a function of apparent length $\tilde{s}_i$, as given in (6), evaluated for measured element of (8). Summarily, expressions for mean value of length $\overline{L}$ and accompanied standard deviation σ_L, respectively, are as follows:

$$\overline{L} = \frac{\sum_{i=1}^{N}\frac{L(\tilde{s}_i)}{L'(\tilde{s}_i)}}{\sum_{i=1}^{N}\frac{1}{L'(\tilde{s}_i)}}, \tag{9}$$

$$\sigma_L = \sqrt{\frac{\left(\sum_{i=1}^{N}\frac{L(\tilde{s}_i)}{L'(\tilde{s}_i)} - \overline{L}\right)^2 \sum_{i=1}^{N}\frac{1}{L'(\tilde{s}_i)}}{\left(\sum_{i=1}^{N}\frac{1}{L'(\tilde{s}_i)}\right)^2 - \sum_{i=1}^{N}\frac{1}{\left[L'(\tilde{s}_i)\right]^2}}}. \tag{10}$$

Previous mentioning of random errors and expression (8) did not specify in what parts of a measurement system these errors originate.

Possible situations include random errors from: (i) global change in the environment (fluctuations in light falling on the recorder part of a horizontal surface), or (ii) local change caused by e.g. rounding the apparent length caused by finite dimension of a pixel in a photo. One can argue that the former origin of errors influences all pixels equally, and thus influences all measured quantities equally. For simplicity, we concentrate on that contribution to random errors ε_i. The later type of random errors can be treated in a similar way, yet that requires additional data about the sensing and processing procedures and corresponding instruments' characteristics.

However, since uniform linear motion of the object is assumed, then the corresponding kinematics brings about additional pieces of information regarding the object, making possible formulation of a qualitatively different mean. As is regularly encountered in practice, measurements of independent variable – here that is time t – are much more precise than measurements of dependent variables, i.e. s. Thus, it is opportune first to calculate trajectory of the object applying the least square method, e.g. linear regression, onto the sequence of values of s_0 as determined in different times, and subsequently to use the regression to estimate random contribution to error in s.

Therefore, first we estimate speed of the object using sequence of paired values (t_i, x_i) in order to obtain v from (1).

Using standard formulas for linear regression one obtains v, and $x(0) = d$. Then, according to the assumption (i) about global change cause of errors the error is found in the following way:

$$s_{0i} = C \cdot h \cdot \mathrm{Arsh}(d - v \cdot t_i), \qquad (11)$$

where $\mathrm{Arsh}(\cdot)$ is the inverse hyperbolic sinus function.

$$\varepsilon_i = \tilde{s}_{0i} - s_{0i}, \qquad (12)$$

and finally

$$L_i = L(\vec{s}_i - \varepsilon_i) = L[\tilde{s}_i - \vec{s}_{0i} + C \cdot h \cdot \mathrm{Arsh}(d - v \cdot t_i)]. \qquad (13)$$

Using set of estimates of length obtained by (13), one then proceeds in estimating the mean length by exploiting non-weighted expression for mean and correspondingly the non-weighted expressions for other statistical quantities.

5 Conclusions and guidelines for practicioners

Non-contact & non-destructive dynamic measurements have significant, yet uncharacterised potential. The full value of combining estimates of measurement quantities with the precise (or, at least assumed) form of relating them in time, is still far from being realised. One can argue in fact that such a potential is still relatively weakly utilised.

The procedure developed in this article incorporates all pieces of information about the measured system. Simultaneously, therefore, the procedure and thereby the obtained results, is a basis for further utilising of the stated approach.

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Authors' addresses

Prof. Josip Stepanić, Ph.D.
Faculty of Mechanical Engineering and Naval Architecture
University of Zagreb
I. Lučića 1
HR-10000 Zagreb, Croatia

Prof. Srđan Medić, Ph.D.
Karlovac University of Applied Sciences
I. Meštrovića 10
HR-47000 Karlovac, Croatia

Prof. Živko Kondić, Ph.D.
Varaždin University of Applied Sciences
J. Križanića 33/6
HR-42000 Varaždin, Croatia