

# ACTS OF REQUESTING IN DYNAMIC LOGIC OF KNOWLEDGE AND OBLIGATION

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## ABSTRACT

Although it seems intuitively clear that acts of requesting are different from acts of commanding, it is not very easy to state their differences precisely in dynamic terms. In this paper we show that it becomes possible to characterize, at least partially, the effects of acts of requesting and compare them with the effects of acts of commanding by combining dynamified deontic logic with epistemic logic. One interesting result is the following: each act of requesting is appropriately differentiated from an act of commanding with the same content, but for each act of requesting, there is another act of commanding with much more complex content which updates models in exactly the same way as it does. We will also consider an application of our characterization of acts of requesting to acts of asking yes-no questions. It yields a straightforward formalization of the view of acts of asking questions as requests for information.

**Keywords:** request, command, yes-no question, dynamified deontic logic, epistemic logic

## 1. Introduction

Acts of requesting seem undoubtedly different from acts of commanding. As Searle and Vanderveken have clearly stated, a request “allows for the possibility of refusal” (Searle and Vanderveken 1985, 199), but a command “commits the speaker to not giving him [= the commandee (the present author’s clarification)] the option of refusal” (op. cit., 201). Of course this does not mean that it is impossible to refuse to obey a command; but “when one refuses to obey an order or command, one cannot say that one refuses the order or command but rather that one refuses to *obey* it” because “[s]trictly speaking, one can only accept or refuse a speech act that allows for the option of acceptance or refusal” (op. cit., 195). Thus “one can say literally ‘I refused the offer’ or ‘I refused the invitation’ ”(ibid.), but one cannot say “I refused the command.”

But what does this difference amount to in dynamic terms? In what way is the situation after an act of requesting different from the situation after an act of commanding? And what effects does an act of requesting bring about if it does not exclude the possibility of refusal? The purpose of this paper is to answer these and other related questions concerning the distinction between requesting and commanding by developing a dynamic logic in which effects of acts of requesting and commanding can be compared. For this purpose we will extend  $\text{DMDL}^{+III}$  (“Dynamified” Multi-agent Deontic Logic + alethic modality III) developed in Yamada (2008a), by adding epistemic operators to it. Since an act of requesting allows for the possibility of refusal, an agent who makes a request will be in need of knowing whether it will be granted or refused, and an appropriate response to a request should address this question. One interesting result is the following: each act of requesting is appropriately differentiated from an act of commanding with the same content, but for each act of requesting, there is another act of commanding with much more complex content which updates models in exactly the same way as it does. We will also consider an application of our analysis of acts of requesting to acts of asking yes-no questions. It yields a straightforward formalization of the view of acts of asking questions as requests for information.

The structure of this paper is as follows. In Section 2, we review the development of dynamified deontic logics that leads to  $\text{DMDL}^{+III}$  closely, and show how acts of commanding and acts of promising are modeled in  $\text{DMDL}^{+III}$ . In Section 3, we add epistemic operators to  $\text{DMDL}^{+III}$ , and briefly examine what more can be said about acts of commanding and promising with their help. We then show how the workings of acts of requesting can be captured in the extended logic  $\text{DMEDL}$  (Dynamified Multi-agent Epistemic Deontic Logic) in Section 4. In Section 5, we first compare acts of requesting with acts of commanding further in order to show how each act of requesting is differentiated from an act of commanding with the same content (this illustrates the first part of the above mentioned result), and then show how our analysis of acts of requesting can be applied to the formalization of the notion of questions as requests for information by modeling acts of asking yes-no questions. In Section 6, we prove the second part of the above mentioned result: for each act of requesting, there is another act of commanding with much more complex content which updates models in exactly the same way as it does. Then we conclude with a brief discussion of the implications of this result and further research possibilities in Section 7.

Before proceeding to the next section, we would like to make a disclaimer here in order to make our goal clear. When we talk about acts of requesting and commanding in this paper, we have acts of commanding and requesting performed in a natural language in mind. But we will not deal directly with the semantics of natural language sentences used in performing these acts, but rather with the

dynamic nature of the performed acts themselves. We will try to characterize what acts of commanding and acts of requesting are in terms of the effects they bring about. In doing so, we will not aim to capture the pragmatic mechanisms that explain, for example, how the utterance of one and the same sentence of natural language counts as the performance of an act of commanding in one context and that of an act of requesting in another, either. We will rather aim to capture what the act of commanding and the act of requesting accomplish when they are performed in the respective contexts, and retrospectively elucidate each act as the kind of act that accomplishes those kinds of things.

## 2. Acts of commanding and acts of promising in DMDL<sup>+III</sup>

DMDL<sup>+III</sup> is one of the “dynamified” logics inspired by the development of systems of DEL (Dynamic Epistemic Logic). In this section, we first give a brief look at PAL (Public Announcement Logic), the simplest system that falls under DEL, and illustrate how it dynamifies static epistemic logic. Then we closely review the development of ECL (Eliminative Command Logic), the simplest logic that deals with acts of commanding. It dynamifies static deontic logic just like DEL dynamifies static epistemic logic. As DMDL<sup>+III</sup> is a refinement of ECL, most of the concepts necessary for understanding DMDL<sup>+III</sup> can be explained in simpler forms in reviewing the development of ECL. After that, we will show how DMDL<sup>+III</sup> refines ECL in two steps.

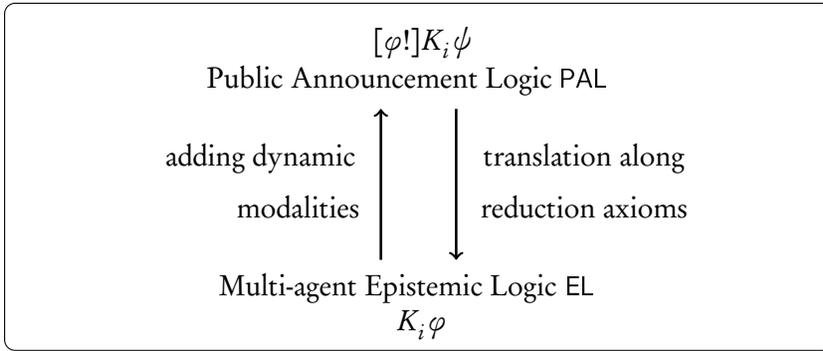
### 2.1. A brief look at PAL

The development of PAL is illustrated in Figure 1 on Page 62 in the form of a diagram. As the upward arrow in Figure 1 indicates, PAL is obtained by adding dynamic modalities, which represent public announcements, to EL. EL is a multi-agent variant of the standard epistemic logic, and the formula of the form  $K_i\varphi$  means that the agent  $i$  knows that  $\varphi$ . The formula of the form  $[\varphi!]\xi$  of PAL means that  $\xi$  holds after every truthful public announcement that  $\varphi$ , and thus the formula of the form  $[\varphi!]K_i\psi$  means that the agent  $i$  knows that  $\psi$  after every truthful public announcement that  $\varphi$ .

Given a model  $M$  for EL and a world  $w$  of  $M$ , the public announcement modality  $[\varphi!]$  is interpreted by the following clause in the truth definition for the language of PAL:

$$M, w \models_{\text{PAL}} [\varphi!]\xi \text{ iff } M, w \models_{\text{PAL}} \varphi \text{ implies } M_{\varphi!}, w \models_{\text{PAL}} \xi ,$$

where  $M_{\varphi!}$  is the “updated” model for EL obtained from  $M$  by replacing the epistemic accessibility relation  $R_i$  for each agent  $i$  with its subset



**Figure 1:** The development of PAL

$R_i - \{\langle x, y \rangle \in R_i \mid M, x \models_{\text{PAL}} \varphi \text{ and } M, y \models_{\text{PAL}} \neg\varphi\} - \{\langle x, y \rangle \in R_i \mid M, x \models_{\text{PAL}} \neg\varphi \text{ and } M, y \models_{\text{PAL}} \varphi\}$ .<sup>1</sup> Note that the truth of the formula of the form  $[\varphi!]\xi$  at  $w$  in  $M$  is defined in terms of the truth of the content  $\varphi$  of the announcement  $\varphi!$  at  $w$  in  $M$  and the truth of its subformula  $\xi$  at  $w$  in the updated model  $M_{\varphi!}$ . Thus the public announcement of the form  $\varphi!$  is interpreted as the type of the events that change the situation  $(M, w)$  into  $(M_{\varphi!}, w)$ . If  $\varphi$  is a formula of EL and no operator of the form  $K_i$  occurs in  $\varphi$ , the formula of the form  $\varphi \rightarrow [\varphi!]K_i\varphi$  is valid. This means that if  $\varphi$  is a non-modal formula, everyone comes to know that  $\varphi$  after every truthful public announcement that  $\varphi$ .<sup>2</sup> An interesting counterexample to the unqualified version of this principle is an announcement of the so-called “Moore formula”  $(\varphi \wedge \neg K_i\varphi)$ .

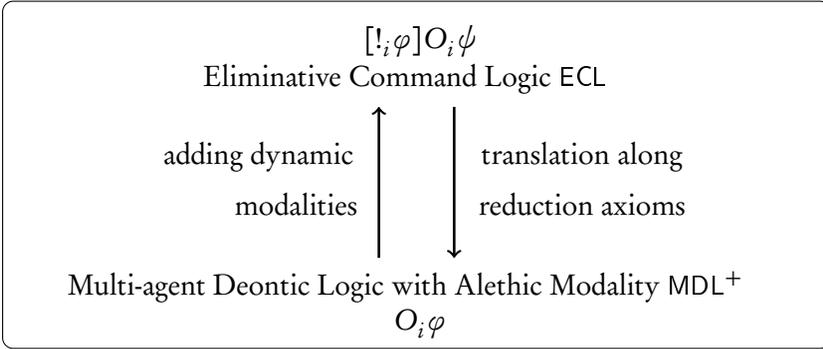
PAL is axiomatized by adding a set of so called “reduction axioms” and the necessitation rule for each announcement modality to the proof system of EL. As the downward arrow in Figure 1 indicates, the reduction axioms enable us to define translation function  $t$  that takes any formula  $\varphi$  from PAL and yields a formula  $t(\varphi)$  of EL that is provably equivalent to  $\varphi$ . This translation in turn enables us to derive the completeness of PAL from the completeness of EL.

<sup>1</sup>This way of updating is usually called “link-cutting”. Another way of updating, called “world elimination”, eliminates every non- $\varphi$  worlds from the domain of the model and restricts  $R_i$  to the new domain. For more on PAL and DEL, see van Ditmarsch, van der Hoek, and Kooi (2007). It gives a detailed state-of-the-art textbook exposition of the major systems of DEL as well as a useful historical overview of their development.

<sup>2</sup>Although there can be various artificial agents to which this applies, it seems too strong to be true of agents like us, as there is a possibility of disbelief on the side of the audience. There may be people who are so sceptical that they do not always believe public announcements. This gap, which is a gap between an illocutionary act (announcing that  $\varphi$ ) and a perlocutionary act (getting addressees to know that  $\varphi$ , or convincing them that  $\varphi$ ), can be avoided if we reinterpret  $\varphi!$  as a type of event in which agents simultaneously and publicly learn that  $\varphi$ . Then we can be said to have a theory of (group) learnability in the form of DEL. For more on the gap, see Yamada (2008b).

## 2.2. Acts of commanding in ECL

Inspired by the development of PAL and other dynamic epistemic logics, a series of dynamified deontic logics including DMDL<sup>+</sup> are developed. Eliminative Command Logic ECL developed in Yamada (2007a) is the simplest one in the series. Figure 2 on Page 63 shows the diagram of the development of ECL. Just like PAL, ECL is obtained by adding dynamic modalities, which represent types of acts of commanding, to the static base logic MDL<sup>+</sup> (Multi-agent Deontic Logic + alethic modality) and is axiomatized by adding a set of reduction axioms and necessitation rules for command modalities to the proof system of MDL<sup>+</sup>.



**Figure 2:** The development of ECL

The formula of the form  $O_i\varphi$  means that it is obligatory upon agent  $i$  to see to it that  $\varphi$ . Although indexing of deontic operators with a set of agents is not standard in deontic logic, we need to be able to distinguish agents to whom commands are given from other agents if we are to use deontic logic to reason about how acts of commanding change situations. For this purpose, the language of MDL<sup>+</sup> has a separate deontic operator  $O_i$  for each agent  $i$ . The language and the models of MDL<sup>+</sup> are defined as follows.<sup>3</sup>

**Definition 2.1.** Take a countably infinite set  $A_{\text{prop}}$  of proposition letters and a finite set  $I$  of agents, with  $p$  ranging over  $A_{\text{prop}}$  and  $i$  over  $I$ . The multi-agent deontic language  $\mathcal{L}_{\text{MDL}^+}$  is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \Box\varphi \mid O_i\varphi .$$

We use standard abbreviations  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$ , and  $\Diamond$ . In addition, we abbreviate  $O_i\neg\varphi$  as  $F_i\varphi$ , and  $\neg O_i\neg\varphi$  as  $P_i\varphi$ .

<sup>3</sup>The definition of the models in this paper is slightly different from that of Yamada (2007a), but there is no substantial difference.

**Definition 2.2.** By an  $\mathcal{L}_{\text{MDL}^+}$ -model, we mean a tuple  $M = \langle W^M, A^M, \{D_i^M \mid i \in I\}, V^M \rangle$  where:

- (i)  $W^M$  is a non-empty set (heuristically, of ‘possible worlds’ or ‘states’)
- (ii)  $A^M \subseteq W^M \times W^M$
- (iii)  $D_i^M \subseteq A^M$  for each agent  $i \in I$
- (iv)  $V^M$  is a function that assigns a subset  $V^M(p)$  of  $W^M$  to each proposition letter  $p \in \text{Aprop}$ .

Based on these definitions, the truth definition for the formulas of  $\mathcal{L}_{\text{MDL}^+}$  is given in a completely standard way by associating alethic modality  $\Box$  with  $A^M$  and each deontic modality  $O_i$  with  $D_i^M$ . Thus the formula of the form  $O_i\varphi$ , for example, is interpreted by the following clause:

$$M, w \models_{\text{MDL}^+} O_i\varphi \text{ iff for any } v \text{ such that } \langle w, v \rangle \in D_i^M, M, v \models_{\text{MDL}^+} \varphi .$$

Note that the following axiom, called ‘‘Mix’’ is shown to be valid according to these definitions:

$$P_i\varphi \rightarrow \Diamond\varphi .$$

This means that what is permitted is possible. The so-called axiom D of the following form, however, is not valid:

$$O_i\varphi \rightarrow P_i\varphi .$$

Since we may receive conflicting commands from different authorities, we cannot assume D axiom to be valid, as we will see later.

Note also that no restrictions are imposed upon the alethic accessibility  $A^M$ . Since further restrictions do not affect the discussion in this paper, we will not bother to add them. Thus,

**Definition 2.3.** The proof system for  $\text{MDL}^+$  contains the following axioms and rules:

- (Taut) all instantiations of propositional tautologies over the present language
- ( $\Box$ -Dist)  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  ( $\Box$ -distribution)
- ( $O_i$ -Dist)  $O_i(\varphi \rightarrow \psi) \rightarrow (O_i\varphi \rightarrow O_i\psi)$  ( $O_i$ -distribution)
- (Mix)  $P_i\varphi \rightarrow \Diamond\varphi$  (Mix Axiom)
- (MP)  $\frac{\varphi \quad \varphi \rightarrow \psi}{\psi}$  (Modus Ponens)
- ( $\Box$ -Nec) If  $\varphi$  is proved, infer  $\Box\varphi$  ( $\Box$ -necessitation)
- ( $O_i$ -Nec) If  $\varphi$  is proved, infer  $O_i\varphi$  . ( $O_i$ -necessitation)

The soundness and the completeness of this proof system can be proved in an entirely standard way.

Now let's move on to ECL. As we have seen in Figure 2, the language of ECL is obtained by adding dynamic modalities, which represent types of acts of commanding, to the language of the static base logic  $\text{MDL}^+$ . Thus,

**Definition 2.4.** Take the same countably infinite set  $\text{Aprop}$  of proposition letters and the same finite set  $I$  of agents as before, with  $p$  ranging over  $\text{Aprop}$  and  $i$  over  $I$ . The language of eliminative command logic  $\mathcal{L}_{\text{ECL}}$  is given by:

$$\begin{aligned} \varphi &::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \Box\varphi \mid O_i\varphi \mid [\pi]\varphi \\ \pi &::= !_i\varphi \ . \end{aligned}$$

An expression of the form  $!_i\varphi$ , which we will call a command term, represents the type of acts of commanding given to a commandee  $i$  to the effect that  $i$  should see to it that  $\varphi$ , and the formula of the form  $[!_i\varphi]\psi$  means that  $\psi$  holds after  $i$  is commanded to see to it that  $\varphi$ . Note that command terms are not formulas.<sup>4</sup>

The truth definition for this language is given with reference to an  $\mathcal{L}_{\text{MDL}^+}$ -model by extending the truth definition for  $\mathcal{L}_{\text{MDL}^+}$  mutatis mutandis with the following clause for the new formulas:

$$M, w \models_{\text{ECL}} [!_i\varphi]\psi \text{ iff } M_{!_i\varphi}, w \models_{\text{ECL}} \psi \ ,$$

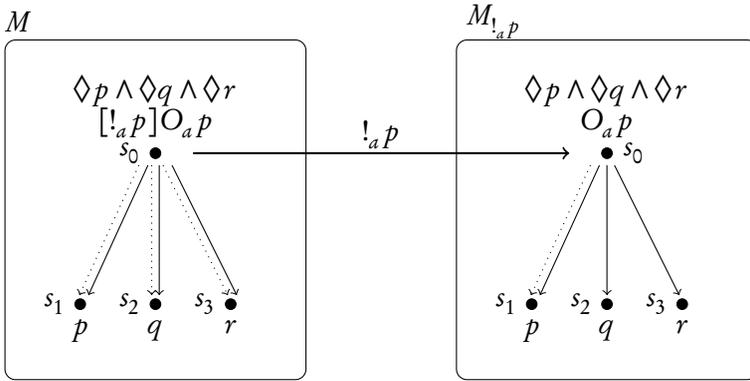
where  $M_{!_i\varphi}$  is the updated  $\mathcal{L}_{\text{MDL}^+}$ -model obtained from  $M$  by replacing only the deontic accessibility relation  $D_i^M$  for the agent  $i$  with its subset  $D_i^{M_{!_i\varphi}} = \{\langle x, y \rangle \in D_i^M \mid M, y \models_{\text{ECL}} \varphi\}$ . Thus, the update by the act of commanding of the type  $!_i\varphi$  only cuts the arrows of deontic accessibility for the agent  $i$  which arrive in non- $\varphi$ -worlds in  $M$ ; it does not cut any arrows of deontic accessibility for other agents.

Note that the truth of the formula of the form  $[!_i\varphi]\psi$  at  $w$  in  $M$  is defined in terms of the truth of its subformula  $\psi$  at  $w$  in the updated model  $M_{!_i\varphi}$ . This fits the intended meaning of  $[!_i\varphi]\psi$ , namely that  $\psi$  holds after  $i$  is commanded to see to it that  $\varphi$ . Note also that  $D_i^{M_{!_i\varphi}} \subseteq D_i^M$ . This guarantees that updated models will always be  $\mathcal{L}_{\text{MDL}^+}$ -models.

Figure 3 on Page 66 gives an image of how an act of commanding works in an example taken from Yamada (2007a). Imagine the following situation. You are working in an office shared with your boss and a few other colleagues on a hot day

<sup>4</sup>This means that they can be neither premises nor conclusions of inferences by themselves.

in summer. There is a window, but it is closed now. There is an air conditioner, but it is not running now. The temperature is rising and it now is at 30 degrees Celsius. The model-state pair  $(M, s_0)$  represents this situation.



**Figure 3:** Your boss's command

Let  $p$  stand for the proposition that the window is open,  $q$  for the proposition that the air conditioner is running, and  $r$  for the proposition that the temperature is above 30 degree Celsius. The presence of formulas near the states indicates that they hold in these states, and the absence of proposition letters near the states (but not the absence of non-atomic formulas) indicates that they do not hold in these states. The solid arrows represent the alethic accessibility, and the dotted arrows represent the deontic accessibility for you, here represented by  $a$ .<sup>5</sup> Thus you can open the window, or turn on the air conditioner, or even ignore the heat by concentrating on your work. All these alternatives are possible and permitted for you in  $(M, s_0)$ .

But now you hear your boss's voice. She commanded you to open the window. The pair  $(M_{!_a p}, s_0)$  represents the situation you are in after your boss's act commanding. All the alternatives that were possible in  $(M, s_0)$  are still possible in  $(M_{!_a p}, s_0)$ . But in order to obey your boss's command, you have to open the window. It becomes the only permissible alternative to you now. This effect of her command is modeled by cutting the arrows of deontic accessibility for you that arrive in non- $p$ -states in  $M$ . Thus we have  $M_{!_a p}, s_0 \models_{\text{ECL}} O_a p$ , and this in turn means that we have  $M, s_0 \models_{\text{ECL}} [!_a p] O_a p$ . The thick arrow from  $(M, s_0)$  to  $(M_{!_a p}, s_0)$  is an imaginary arrow in the sense that it is neither in  $M$  nor in  $M_{!_a p}$ ,

<sup>5</sup>For the sake of simplicity, arrows of deontic accessibility relations for other people and the reflexive arrows for alethic accessibility are omitted.

but it helps us to understand your boss's command of the form  $!_a p$  as the event that takes you from  $(M, s_0)$  to  $(M_{!_a p}, s_0)$ .

Note that the treatment of acts of commanding in ECL is based on a simplifying assumption that command issuing agents have suitable authority over commandees. We follow this treatment in this paper.<sup>6</sup> With the help of this simplifying assumption, the following result is obtained:

**Proposition 2.1** (The CUGO Principle). *If  $\varphi$  is a formula of  $\text{MDL}^+$  and is free of modal operators of the form  $O_i$ ,  $[!_i \varphi]O_i \varphi$  is valid.*

This principle means that, though not without exceptions, commands usually generate obligations (hence ‘‘CUGO’’). It partially characterizes the effects of acts of commanding.<sup>7</sup> As we have said in Section 1, the purpose of this paper is to give a similar kind of characterization to acts of requesting by extending a refinement  $\text{DMDL}^+_{III}$  of ECL.

Note that the unqualified version of the CUGO principle is not valid. A bit of terminology is of some help here. Let  $O$  be some modal operator. We call the pair  $\langle x, y \rangle$  of worlds an  $O$ -arrow and say  $y$  is  $O$ -accessible from  $x$  if  $\langle x, y \rangle$  is in the accessibility relation  $R$  that interprets  $O$ . Then we can say that the update by an act of commanding of the type  $!_i \varphi$  cuts every  $O_i$ -arrow that arrives in  $\neg \varphi$ -worlds in  $M$ . This guarantees that every  $O_i$ -arrow that remains after this update arrives in a world in which  $\varphi$  holds in  $M$ . But this does not guarantee that  $\varphi$  holds there in the updated model  $M_{!_i \varphi}$ . If  $O_i$  occurs in  $\varphi$ ,  $\varphi$  might be false at some world  $O_i$ -accessible from  $w$  in  $M_{!_i \varphi}$ .

ECL is axiomatized by adding a set of so-called ‘‘reduction axioms’’ and the necessitation rule for each command operator to the proof system of  $\text{MDL}^+$ . Thus,

**Definition 2.5.** The proof system for ECL contains all the axioms and all the rules of the proof system for  $\text{MDL}^+$ , and in addition the following reduction axioms and rules:

- (RA1)  $[!_i \varphi]p \leftrightarrow p$  where  $p \in \text{Aprop}$
- (RA2)  $[!_i \varphi]\top \leftrightarrow \top$
- (RA3)  $[!_i \varphi]\neg \psi \leftrightarrow \neg[!_i \varphi]\psi$
- (RA4)  $[!_i \varphi](\psi \wedge \chi) \leftrightarrow ([!_i \varphi]\psi \wedge [!_i \varphi]\chi)$

<sup>6</sup>The standard method used in order to treat preconditions for action like this is to introduce a function  $pre$  that assigns to each event term  $e$  its precondition  $pre(e)$ . For more on this, see Baltag, Moss, and Solecki (1998) or van Ditmarsch, van der Hoek, and Kooi (2007).

<sup>7</sup>This characterization is partial because acts of commanding involve other effects as well. For more on this, see Sections 6 and 7.

- (RA5)  $[\!:_i\varphi]\Box\psi \leftrightarrow \Box[\!:_i\varphi]\psi$   
 (RA6)  $[\!:_i\varphi]O_j\psi \leftrightarrow O_j[\!:_i\varphi]\psi$  where  $i \neq j$   
 (RA7)  $[\!:_i\varphi]O_i\psi \leftrightarrow O_i(\varphi \rightarrow [\!:_i\varphi]\psi)$   
 ( $[\!:_i\varphi]$ -Nec) If  $\psi$  is proved, infer  $[\!:_i\varphi]\psi$  .

The crucial axiom here is RA7. The formula on the left hand side,  $[\!:_i\varphi]O_i\psi$ , states that  $O_i\psi$  holds after the update. The formula on the right hand side specifies the necessary and sufficient condition for this in terms of what holds before the update. Take an arbitrary  $\mathcal{L}_{\text{MDL}^+}$ -model  $M$  and a world  $w$  of  $M$ . In order for  $O_i\psi$  to hold in  $w$  in the updated model  $M_{!_i\varphi}$ ,  $\psi$  must hold in every world  $O_i$ -accessible from  $w$  in  $M_{!_i\varphi}$ . But those worlds are exactly the  $\varphi$ -worlds in  $M$  that are  $O_i$ -accessible from  $w$  in  $M$ . In order for  $\psi$  to hold in those worlds after the update,  $[\!:_i\varphi]\psi$  has to hold in those world before the update. Thus  $O_i(\varphi \rightarrow [\!:_i\varphi]\psi)$  has to hold in  $w$  in  $M$ .

Note that the first two axioms enable us to eliminate command operators prefixed to proposition letters and  $\top$ . The remaining axioms enable us to reduce the length of the subformula to which command operators are prefixed step by step. Thus these axioms enable us to define translation function that takes any formula of ECL and yields a formula of  $\text{MDL}^+$  which is provably equivalent to the original formula. This translation in turn enables us to derive the completeness of ECL from that of  $\text{MDL}^+$ .

### 2.3. Conflicting commands in ECL and ECLII

Now we can move on to refinements. The following results about ECL are reported in Yamada (2007a):

**Proposition 2.2** (The Dead End Principle).  $[\!:_i(\varphi \wedge \neg\varphi)]O_i\xi$  is valid.

**Proposition 2.3** (The Restricted Sequential Conjunction Principle). If  $\varphi$  and  $\psi$  are formulas of  $\text{MDL}^+$  and free of modal operators of the form  $O_i$ ,  $[\!:_i\varphi][\!:_i\psi]\xi \leftrightarrow [\!:_i(\varphi \wedge \psi)]\xi$  is valid.

The dead end principle means that if an agent receives a command with contradictory content, everything comes to be obligatory upon him. The situation of this kind is usually called “deontic explosion”. Since the updated by  $!_i(\varphi \wedge \neg\varphi)$  cuts every  $O_i$ -arrow that arrives in a  $\neg(\varphi \wedge \neg\varphi)$ -world, it cuts every  $O_i$ -arrow, and so  $D_i^{M_{!_i(\varphi \wedge \neg\varphi)}}$  becomes empty. Thus  $O_i\xi$  becomes vacuously true in every world after the update by  $!_i(\varphi \wedge \neg\varphi)$ .

If we put  $\neg\varphi$  in the place of  $\psi$  in the restricted sequential conjunction principle, we get deontic explosion again. Situations of this kind can arise in real life as

an agent might receive such a pair of commands from different command issuing authorities.<sup>8</sup>

MDL<sup>+</sup>II and ECLII refine MDL<sup>+</sup> and ECL respectively in order to deal with conflicting commands in a more satisfactory way by indexing deontic operators and deontic accessibility relations by the set  $I \times I$  of pairs of agents (Yamada 2007b). Thus the formula of the form  $O_{(i,j)}\varphi$  from the static base logic MDL<sup>+</sup>II means that it is obligatory upon an agent  $i$  with respect to the authority  $j$  to see to it that  $\varphi$ , and the formula of the form  $[\!_{(i,j)}\!] \psi$  of the dynamified logic ECLII means that  $\psi$  holds after an authority  $j$ 's act of commanding an agent  $i$  to see to it that  $\varphi$ . The definitions of the languages, the models, the relations of truth in models, and the proof systems for MDL<sup>+</sup>II and ECLII are given in the same way as those for MDL<sup>+</sup> and ECL except for the indexing by  $I \times I$ .

Since  $I$  is a finite set, indexing by  $I \times I$  is just an instance of indexing by a finite set. Thus MDL<sup>+</sup>II and ECLII are just another instantiations of MDL<sup>+</sup> and ECL respectively, hence all the results obtained for MDL<sup>+</sup> and ECL apply to MDL<sup>+</sup>II and ECLII *mutatis mutandis*. In particular, the CUGO principle now reads:

**Proposition 2.4** (The CUGO Principle). *If  $\varphi$  is a formula of MDL<sup>+</sup>II and is free of modal operators of the form  $O_{(i,j)}$ ,  $[\!_{(i,j)}\!]\varphi$  is valid.*

With the help of this principle, we now have:

$$(M_{\!_{(a,b)}\!}p)_{\!_{(a,c)}\!}\neg p, \omega \models_{\text{ECLII}} (O_{(a,b)}p \wedge O_{(a,c)}\neg p) .$$

This is the situation  $a$  will be in after  $a$  receives a command from an authority  $c$  to the effect that  $a$  should see to it that  $\neg p$  after  $a$  receives a command from another authority  $b$  to the effect that  $a$  should see to it that  $p$ . Since it is not possible to obey both commands in this example,  $a$  has to decide which command to obey.

Note that this combination of incompatible commands does not generally produce deontic explosion. In  $(M_{\!_{(a,b)}\!}p)_{\!_{(a,c)}\!}\neg p$ ,  $p$ -worlds that are  $O_{(a,b)}$ -accessible in  $M$  (if any) and  $\neg p$ -worlds that are  $O_{(a,c)}$ -accessible in  $M$  (if any) will remain  $O_{(a,b)}$ -accessible and  $O_{(a,c)}$ -accessible respectively, since the update by  $\!_{(a,b)}\!$  only cuts  $O_{(a,b)}$ -arrows arriving in  $\neg p$ -worlds in  $M$  and the update by  $\!_{(a,c)}\!\neg p$  only cuts  $O_{(a,c)}$ -arrows arriving in  $p$ -worlds in  $M$ . Deontic explosions occur only when incompatible commands are given to one and the same agent by one and the same authority or an authority issues a command having contradictory content. If the

<sup>8</sup>Thus D Axiom cannot be included in the proof system of MDL<sup>+</sup>.

command issuing authority is rational, such a situation will be avoided; otherwise, obedience could not be expected.<sup>9</sup>

Note also that similar conflicts can arise between requests as well as between a request and a command. So, we will follow this treatment in developing our system later.

## 2.4. DMDL<sup>+III</sup>

DMDL<sup>+III</sup> refines ECLII further in order to model acts of promising along with acts of commanding (Yamada, 2008a). In the case of acts of commanding, obligations commandees owe are created by command issuing authorities (commanders, for short).<sup>10</sup> But in the case of acts of promising, obligations owed by agents who give promises (promisers, for short) are created by promisers themselves. Moreover, agents to whom promises are given (promisees, for short) will be entitled to rely on promisers to do what they have promised to do. In order to deal with this complexities, deontic operators and their corresponding accessibility relations are indexed by the set  $I \times I \times I$  in the static base logic MDL<sup>+III</sup>. As before, indexing by  $I \times I \times I$  is just indexing by a finite set, and thus MDL<sup>+III</sup> is yet another instantiation of MDL<sup>+</sup>. But this time DMDL<sup>+III</sup> includes more than ECL does. It deals not only with acts of commanding but also with acts of promising.<sup>11</sup>

In MDL<sup>+III</sup> and in DMDL<sup>+III</sup>, the formula of the form  $O_{(i,j,k)}\varphi$  means that it is obligatory upon agent  $i$  with respect to  $j$  in the name of  $k$  to see to it that  $\varphi$ . The agent  $i$  here is the agent who owes the obligation (sometimes called an obligor),  $j$  is the agent to whom the obligation is owed (sometimes called an obligee), and  $k$  is the creator of the obligation. As we will see shortly, they need not be distinct.

In DMDL<sup>+III</sup>, the formula of the form  $[Com_{(i,j)}\varphi]\psi$  means that  $\psi$  holds after an agent  $i$  commands an agent  $j$  to see to it that  $\varphi$ , and the formula of the form  $[Prom_{(i,j)}\varphi]\psi$  means that  $\psi$  holds after an agent  $i$  promises an agent  $j$  that she ( $i$ ) will see to it that  $\varphi$ . Note that the order of the parameters in the command term  $Com_{(i,j)}\varphi$  is changed from that of the term  $!_{(i,j)}\varphi$  of ECLII. In  $Com_{(i,j)}\varphi$ ,  $i$  is the commander and  $j$  is the commandee.

In the truth definition for the language of DMDL<sup>+III</sup>, the added dynamic formulas

<sup>9</sup>Weakening of some inference rules are proposed in the literature on deontic logic in order to avoid deontic explosions. Yamada (2008b) suggests the possibility of representing moral conflicts without triggering deontic explosions by modeling moral principles as command issuing authorities.

<sup>10</sup>Note that the use of “commander” here is not meant to be understood as referring to military rank.

<sup>11</sup>Thus, it is not called ECLIII.

are interpreted by the following clauses:

$$\begin{aligned} M, \omega \models_{\text{DMDL}^+} [Com_{(i,j)}\varphi]\xi &\text{ iff } M_{Com_{(i,j)}\varphi}, \omega \models_{\text{DMDL}^+} \xi \\ M, \omega \models_{\text{DMDL}^+} [Prom_{(i,j)}\varphi]\xi &\text{ iff } M_{Prom_{(i,j)}\varphi}, \omega \models_{\text{DMDL}^+} \xi, \end{aligned}$$

where

- (1)  $M_{Com_{(i,j)}\varphi}$  is the  $\mathcal{L}_{\text{MDL}^+}$ -model obtained from  $M$  by replacing  $D_{(j,i,i)}^M$  with its subset  $\{\langle x, y \rangle \in D_{(j,i,i)}^M \mid M, y \models_{\text{DMDL}^+} \varphi\}$ , and
- (2)  $M_{Prom_{(i,j)}\varphi}$  is the  $\mathcal{L}_{\text{MDL}^+}$ -model obtained from  $M$  by replacing  $D_{(i,j,i)}^M$  with its subset  $\{\langle x, y \rangle \in D_{(i,j,i)}^M \mid M, y \models_{\text{DMDL}^+} \varphi\}$ .

Thus the update by  $Com_{(i,j)}\varphi$  only cuts  $O_{(j,i,i)}$ -arrows arriving in  $\neg\varphi$ -worlds in the original model  $M$ , and the update by  $Prom_{(i,j)}\varphi$  only cuts  $O_{(i,j,i)}$ -arrows arriving in  $\neg\varphi$ -world in the original model  $M$ .

Again, we have:

**Proposition 2.5.** The CUGO Principle: *If  $\varphi$  is a formula of  $\text{MDL}^+$  and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Com_{(i,j)}\varphi]O_{(j,i,i)}\varphi$  is valid.*

And in addition to this, we have:

**Proposition 2.6.** The PUGO Principle: *If  $\varphi$  is a formula of  $\text{MDL}^+$  and is free of modal operators of the form  $O_{(i,j,i)}$ ,  $[Prom_{(i,j)}\varphi]O_{(i,j,i)}\varphi$  is valid.*

These principles partially capture how acts of commanding and promising work.

Note the differences between the obligations generated. In the case of the obligation generated by an act of commanding the commandee  $j$  owes the obligation created by the commander  $i$ , but in the case of the obligation generated by an act of promising the promiser  $i$  owes the obligation created by the promiser  $i$  herself, and the promisee  $j$  is the agent whom the obligation is owed. This difference enables us to consider the obligations created by acts of promising as representing the commitments of the promisers. This point will be of some importance when we analyze acts of requesting.<sup>12</sup>

We are now in a position to extend  $\text{MDL}^+$  and  $\text{DMDL}^+$ .

<sup>12</sup>There may be room for disagreement over whether the notion of the agent whom the obligation is owed make sense with respect to the obligation created by an act of commanding. But we will not pursue this point here.

### 3. The securing of uptake in DMEDL

We add epistemic operators to  $\text{MDL}^{+III}$ . For the sake of simplicity, we ignore alethic modality. Thus we define:

**Definition 3.1.** Take a countably infinite set  $\text{Aprop}$  of proposition letters, and a finite set  $I$  of agents, with  $p$  ranging over  $\text{Aprop}$ , and  $i, j, k$  over  $I$ . The language  $\mathcal{L}_{\text{MEDL}}$  of the Multi-agent Epistemic Deontic Logic MEDL is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_i\varphi \mid O_{(i,j,k)}\varphi$$

**Definition 3.2.** By an  $\mathcal{L}_{\text{MEDL}}$ -model, we mean a tuple  $M = \langle W^M, \{E_i^M \mid i \in I\}, \{D_{(i,j,k)}^M \mid i, j, k \in I\}, V^M \rangle$  where:

- (i)  $W^M$  is a non-empty set (heuristically, of ‘possible worlds’ or ‘states’)
- (ii)  $E_i^M$  is an equivalence relation such that  $E_i^M \subseteq W^M \times W^M$
- (iii)  $D_{(i,j,k)}^M \subseteq W^M \times W^M$
- (iv)  $V^M$  is a function that assigns a subset  $V^M(p)$  of  $W^M$  to each proposition letter  $p \in \text{Aprop}$ .

The truth definition (defining the relation  $\models_{\text{MEDL}}$ ) and the definition of the proof system (defining the relation  $\vdash_{\text{MEDL}}$ ) can be given in an entirely standard way. Since MEDL is a simple fusion of the deontic fragment of  $\text{MDL}^{+III}$  and the multi-agent variant of the standard epistemic logic, there is a complete axiomatization of it.

We dynamify MEDL into DMEDL (Dynamified MEDL) by adding the dynamic modalities indexed by action terms of the forms  $\text{Com}_{(i,j)}\varphi$ ,  $\text{Prom}_{(i,j)}\varphi$  and  $\text{Req}_{(i,j)}\varphi$ . The formula of the form  $[\text{Com}_{(i,j)}\varphi]\psi$  and the formula of the form  $[\text{Prom}_{(i,j)}\varphi]\psi$  are interpreted in exactly the same way as in  $\text{DMDL}^{+III}$ , except with reference to  $\mathcal{L}_{\text{MEDL}}$ -models and with  $\models_{\text{DMDL}^{+III}}$  replaced with  $\models_{\text{DMEDL}}$ . Thus we again have the DMEDL versions of the CUGO principle and the PUGO principle.

Before moving on to the analysis of acts of requesting, we take a brief look at what more we can say about acts of commanding and promising in DMEDL. One immediate consequence of having epistemic operators is the fact that we can now talk about the knowledge agents have about effects of speech acts. When a command is successfully given, for example, the commandee must know what command she has been given. Unless the force and the content is understood, no illocutionary act can be successfully performed, since the effects of illocutionary acts depend on the agreement on (and so the understanding of) what has been performed. Surprisingly, the following principles are valid in DMEDL:

**Proposition 3.1.** The CUGU Principle: *If  $\varphi$  is a formula of DMEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Com_{(i,j)}\varphi]K_j O_{(j,i,i)}\varphi$  is valid.*

**Proposition 3.2.** The PUGU Principle: *If  $\varphi$  is a formula of DMEDL and is free of modal operators of the form  $O_{(i,j,i)}$ ,  $[Prom_{(i,j)}\varphi]K_j O_{(i,j,i)}\varphi$  is valid.*

These principles state that acts of commanding and acts of promising usually generate knowledge of the effects captured in the CUGO principle and the PUGO principle respectively on the side of addressees.

We call these principles “CUGU” and “PUGU” because we believe that these principles characterize what Austin calls “the securing of uptake”. According to Austin, “the securing of uptake” means “bringing about the understanding of the meaning and of the force of the locution”. It is the “effect” that “must be achieved on the audience if the illocutionary act is to be carried out.” And so, “the performance of an illocutionary act involves the securing of uptake” (Austin, 1955, 117-118). In the case of an act of commanding, the understanding of the force means the understanding of the commander’s locution as an act of commanding and the understanding of the meaning of her locution includes the understanding of what is commanded. The CUGU principle partially characterizes what these understanding amount to. The same thing can be said of the PUGU principle as well.

We said “surprisingly” above because no epistemic update operation is required for these results. Take any model  $M$ , any world  $w$  of  $M$ , and any proposition letter  $p$  for example. After an act of commanding of the form  $Com_{(i,j)}p$  is performed in a situation  $(M, w)$ ,  $O_{(j,i,i)}p$  holds in any world  $v$  in the updated model  $M_{Com_{(i,j)}p}$  as every  $O_{(j,i,i)}$ -arrow arriving in a non- $p$ -world of  $M$  is eliminated in  $M_{Com_{(i,j)}p}$ , and every  $p$ -world of  $M$  remains to be a  $p$ -world in  $M_{Com_{(i,j)}p}$ . But if  $O_{(j,i,i)}p$  holds in any world in the updated model  $M_{Com_{(i,j)}p}$ , it holds in any world  $K_j$ -accessible from any world of  $M$ . Thus  $K_j O_{(j,i,i)}p$  holds in any world in  $M_{Com_{(i,j)}p}$ .

We need to note, however, that the same thing holds for any agent  $i \in I$ . It means that everyone comes to know that  $O_{(j,i,i)}p$  in  $w$  in  $M_{Com_{(i,j)}p}$ . This is natural when we consider a small everyday situation like the situation of the shared office on the hot summer day we considered earlier. But even in a small everyday situations like this, there are many ways in which only some of the agents come to know what speech act is performed by a particular person at a particular time.

Here it is important to understand how everyone comes to know  $O_{(j,i,i)}p$  in  $w$  in  $M_{Com_{(i,j)}p}$ . Although the update by  $Com_{(i,j)}p$  does not affect any epistemic acces-

sibility relations, it makes  $O_{(j,i,i)}p$  true in any worlds epistemically accessible for each agent. In that sense, the independence of each agent's epistemic accessibility relation from that of others does not fully model the privacy of knowledge in the context of dynamified modal logics. The standard way to model the distinction between agents who know what happens and those who do not is to introduce the so-called "event models", in which (un)certainty of each agent as regards what has happened is modeled, and define the update operation called "product update". If we do this for MEDL, our current update by  $Com_{(i,j)}\varphi$  will be modeled as the special case of product update by the event model in which every agent knows that an event of the type  $Com_{(i,j)}\varphi$  happens. Although it is possible to extend MEDL by introducing event models and product update, we will not pursue this possibility here as there are many things yet to be done before making life more complicated.<sup>13</sup>

#### 4. Acts of requesting in DMEDL

Now we move on to the analysis of acts of requesting. As we have seen in Section 1, an act of requesting allows the possibility of refusal. As a consequence of this, the following principle is not valid even if no operators of the form  $O_{(j,i,i)}$  occur in  $\varphi$ :

$$[Req_{(i,j)}\varphi]O_{(j,i,i)}\varphi .$$

In this respect, acts of requesting stand in sharp contrast to acts of commanding, for which we have the CUGO principle. But it is also clear that it would not be without any problems if an agent who has been requested to do something (a requestee, for short) gives no response. Although it is not obligatory upon the requestee to do what is requested, it is obligatory upon her to decide whether she should do what is requested. Moreover, she has to let the agent who has made the request (the requester, for short) know her decision.

If the requestee  $j$  decides that she should do what is requested, and the requested action is not the kind of thing to be done on the spot, she can promise the requester  $i$  that she ( $j$ ) will do what is requested. As the PUGU principle indicates, the requester  $i$  will know that  $O_{(j,i,i)}\varphi$ . If the requestee  $j$  decides that she ( $j$ ) should reject the request, she ( $j$ ) should let the requester  $i$  know that  $\neg O_{(j,i,i)}\varphi$ .

Now what about the case in which what is requested can be done on the spot. If the requestee  $j$  decides that she should do what is requested, she might do it on the spot without saying anything. Whether we should count this as the third alternative way of responding to an act of requesting, or consider it as skipping

<sup>13</sup>I have benefitted from a discussion with Johan van Benthem on this point. For more on the product update, see Baltag, Moss, and Solecki (1998) or van Ditmarsch, van der Hoek, and Kooi (2007).

to the sequel of an implicit promise might be a matter of opinion. We take the formulation with the three options.<sup>14</sup> Thus the clause for the formula of the form  $[Req_{(i,j)}\varphi]\xi$  reads:

$$M, \omega \models_{\text{DMEDL}} [Req_{(i,j)}\varphi]\xi \text{ iff } M_{Req_{(i,j)}\varphi}, \omega \models_{\text{DMEDL}} \xi \quad ,$$

where  $M_{Req_{(i,j)}\varphi}$  is the  $\mathcal{L}_{\text{MEDL}}$ -model obtained from  $M$  by replacing  $D_{(j,i,i)}^M$  with its subset  $\{\langle x, y \rangle \in D_{(j,i,i)}^M \mid M, y \models_{\text{DMEDL}} (\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)\}$ .<sup>15</sup>

This interpretation supports the following principles.

**Proposition 4.1.** The RUGO Principle: *If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Req_{(i,j)}\varphi]O_{(j,i,i)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  is valid.*

**Proposition 4.2.** The RUGU Principle: *If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Req_{(i,j)}\varphi]K_j O_{(j,i,i)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  is valid.*

We are now in a position to define the proof system for DMEDL. We first list three sets of reduction axioms.

**Theorem 4.1** (Reduction axioms for acts of commanding). *The following axioms are valid in DMEDL.*

- (C1)  $[Com_{(i,j)}\varphi]p \leftrightarrow p$
- (C2)  $[Com_{(i,j)}\varphi]\top \leftrightarrow \top$
- (C3)  $[Com_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[Com_{(i,j)}\varphi]\psi$
- (C4)  $[Com_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow [Com_{(i,j)}\varphi]\psi \wedge [Com_{(i,j)}\varphi]\chi$
- (C5)  $[Com_{(i,j)}\varphi]K_l\psi \leftrightarrow K_l[Com_{(i,j)}\varphi]\psi$
- (C6)  $[Com_{(i,j)}\varphi]O_{(l,m,n)}\psi \leftrightarrow O_{(l,m,n)}[Com_{(i,j)}\varphi]\psi \quad \text{if } \langle l, m, n \rangle \neq \langle j, i, i \rangle$
- (C7)  $[Com_{(i,j)}\varphi]O_{(j,i,i)}\psi \leftrightarrow O_{(j,i,i)}(\varphi \rightarrow [Com_{(i,j)}\varphi]\psi)$

<sup>14</sup>Traum (1999, 195) also talks about similar obligations as effects of acts of requesting, but he includes only the options of accepting or refusing.

<sup>15</sup>The formulation with two options can be obtained by using  $\{\langle x, y \rangle \in D_{(j,i,i)}^M \mid M, y \models_{\text{DMEDL}} (K_i O_{(j,i,i)}\varphi \vee K_i \neg O_{(j,i,i)}\varphi)\}$  instead.

**Theorem 4.2** (Reduction axioms for acts of Promising). *The following axioms are valid in DMEDL.*

- (P1)  $[Prom_{(i,j)}\varphi]p \leftrightarrow p$   
(P2)  $[Prom_{(i,j)}\varphi]\perp \leftrightarrow \perp$   
(P3)  $[Prom_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[Prom_{(i,j)}\varphi]\psi$   
(P4)  $[Prom_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow [Prom_{(i,j)}\varphi]\psi \wedge [Prom_{(i,j)}\varphi]\chi$   
(P5)  $[Prom_{(i,j)}\varphi]K_l\psi \leftrightarrow K_l[Prom_{(i,j)}\varphi]\psi$   
(P6)  $[Prom_{(i,j)}\varphi]O_{(l,m,n)}\psi \leftrightarrow O_{(l,m,n)}[Prom_{(i,j)}\varphi]\psi$  if  $\langle l, m, n \rangle \neq \langle i, j, i \rangle$   
(P7)  $[Prom_{(i,j)}\varphi]O_{(i,j,i)}\psi \leftrightarrow O_{(i,j,i)}(\varphi \rightarrow [Prom_{(i,j)}\varphi]\psi)$

**Theorem 4.3** (Reduction axioms for acts of Requesting). *The following axioms are valid in DMEDL.*

- (R1)  $[Req_{(i,j)}\varphi]p \leftrightarrow p$   
(R2)  $[Req_{(i,j)}\varphi]\perp \leftrightarrow \perp$   
(R3)  $[Req_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[Req_{(i,j)}\varphi]\psi$   
(R4)  $[Req_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow [Req_{(i,j)}\varphi]\psi \wedge [Req_{(i,j)}\varphi]\chi$   
(R5)  $[Req_{(i,j)}\varphi]K_l\psi \leftrightarrow K_l[Req_{(i,j)}\varphi]\psi$   
(R6)  $[Req_{(i,j)}\varphi]O_{(l,m,n)}\psi \leftrightarrow O_{(l,m,n)}[Req_{(i,j)}\varphi]\psi$  if  $\langle l, m, n \rangle \neq \langle j, i, i \rangle$   
(R7)  $[Req_{(i,j)}\varphi]O_{(j,i,i)}\psi \leftrightarrow O_{(j,i,i)}((\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi) \rightarrow [Req_{(i,j)}\varphi]\psi)$

As before, the first two axioms of each group enable us to eliminate dynamic operators prefixed to proposition letters and  $\top$ . The remaining axioms enable us to reduce the length of the subformula to which dynamic operators are prefixed step by step.

Now we define:

**Definition 4.1** (The proof system for DMEDL). The proof system for DMEDL is comprised of

- (1) all the axioms and rules of the proof system for MEDL,
- (2) all the reduction axioms for acts of commanding,
- (3) all the reduction axioms for acts of promising,
- (4) all the reduction axioms for acts of requesting, and in addition,
- (5) the necessitation rules for the dynamic operators  $[Com_{(i,j)}\varphi]$ ,  $[Prom_{(i,j)}\varphi]$ , and  $[Req_{(i,j)}\varphi]$ .

Since the above three sets of reduction axioms jointly enable us to define translation function that takes any formula from the language of DMEDL and yields the formula of MEDL that is provably equivalent to the original formula, we can derive the completeness of DMEDL from the completeness of MEDL. Thus we have:

**Theorem 4.4** (The completeness of DMEDL). *The proof system defined above completely axiomatizes DMEDL.*

## 5. Commanding, requesting, and asking questions in DMEDL

In this section, we first review the CUGO principle and the RUGO principle.

**The CUGO Principle** If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Com_{(i,j)}\varphi]O_{(j,i,i)}\varphi$  is valid.

**The RUGO Principle** If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Req_{(i,j)}\varphi]O_{(j,i,i)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  is valid.

In the following discussions, we assume that  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ , unless stated otherwise.

As we have seen, the CUGO principle is valid while  $[Req_{(i,j)}\varphi]O_{(j,i,i)}\varphi$  is not. This fact enables us to understand clearly the sense in which acts of commanding do not allow for the the option of refusal. It becomes obligatory upon the agent  $j$  to see to it that  $\varphi$  after the act of commanding of the form  $Com_{(i,j)}\varphi$  as the CUGO principle states, but not after the act of requesting of the form  $Req_{(i,j)}\varphi$ . Moreover, the RUGO principle enables us to understand in what sense the option of refusal is allowed for in the act of requesting of the form  $Req_{(i,j)}\varphi$ . Seeing to it that  $K_i \neg O_{(j,i,j)}\varphi$  is one of the three ways of meeting the obligation of the form  $O_{(j,i,i)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$ . In that sense refusal is a legitimate response to an act of requesting but not to an act of commanding.

We then move on to acts of asking yes-no questions and examine how the RUGO principle works in modeling them. The notion of question as a kind of imperative or request can be found in various authors including Åqvist (1965), Searle (1979), Hintikka (1981), and Searle and Vanderveken(1985).<sup>16</sup> Our analysis can be applied to the formalization of the notion of questions as requests for information in a

<sup>16</sup>There are various approaches to questions. Groenendijk and Stokhof (1997) offers a detailed survey of the field, and argues against Searle and Vanderveken's approach, arguing for the semantic approach to imperative sentences. Since we are not dealing with the semantics of natural language imperative sentences, "a priori there is no clash between" their semantic approach and our analysis as they notes (op. cit., 1074). For more recent works, see Minică (2011).

straightforward manner. Thus we can define the term that represents the type of the acts in which  $i$  asks  $j$  whether  $\varphi$  is the case or not,  $Ask\text{-}if_{(i,j)}\varphi$ , as an abbreviation for  $Req_{(i,j)}(K_i\varphi \vee K_i\neg\varphi)$ .<sup>17</sup>

Then by the RUGO principle, we have:

$$[Ask\text{-}if_{(i,j)}\varphi]O_{(j,i,i)}((K_i\varphi \vee K_i\neg\varphi) \vee K_i O_{(j,i,j)}(K_i\varphi \vee K_i\neg\varphi) \vee K_i \neg O_{(j,i,j)}(K_i\varphi \vee K_i\neg\varphi)).$$

Here  $i$  is the agent who asks the question and  $j$  the agent whom the question is asked. We will refer to them as “the requester” and “requestee”, and examine how well we can treat the situation after the act of this type as the situation in which information is requested.

Now, after the requester  $i$ 's act of asking, if the requestee  $j$  knows the answer and is willing to answer, she ( $j$ ) can meet the generated obligation by saying “yes” or “no” immediately, since doing so is to see to it that  $(K_i\varphi \vee K_i\neg\varphi)$ . Then the requester  $i$  will know that  $\varphi$  or know that  $\neg\varphi$  accordingly. If the requestee  $j$  is willing to answer but needs to consult books, maps, databases, or whatever in order to do so, she ( $j$ ) can promise the requester  $i$  that she ( $j$ ) will answer it later. Then, as the PUGU principle indicates, the requester  $i$  will know that the requestee  $j$  has committed herself ( $j$ ) to letting her ( $i$ ) know that  $\varphi$  or know that  $\neg\varphi$ . Thus the requestee  $j$  has seen to it that  $K_i O_{(j,i,j)}(K_i\varphi \vee K_i\neg\varphi)$ . If the requestee  $j$  cannot answer or decides not to answer for some reason or other, she ( $j$ ) has to let the requester  $i$  know that she ( $j$ ) will not commit herself ( $j$ ) to letting her ( $i$ ) know the answer. Doing so is to see to it that  $K_i \neg O_{(j,i,j)}(K_i\varphi \vee K_i\neg\varphi)$ . Thus the RUGO principle captures what  $j$  has to do after an yes-no question is asked in a natural way.<sup>18</sup>

## 6. Requesting and commanding again

So far, we have seen that the CUGO principle and the RUGO principle captures how differently acts of commanding and acts of requesting change situations fairly well. But now observe that the following principle is an instantiation of the CUGO principle:

**Proposition 6.1.** *If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ , the following formula is valid:*

$$[Com_{(i,j)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)]O_{(j,i,i)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi).$$

<sup>17</sup>The author owes this idea to the discussion with Berislav Žarnić.

<sup>18</sup>As Grice's discussion of the examinee's answer (1969, 106) suggests, however, this model does not work nicely for questions asked by the examiner in an oral exam. If we combine DMEDL with the dynamic logic of propositional commitments developed in Yamada (2012), we will be able to model such a question as a command to the effect that the commandee should commit herself to the truth or falsity of  $\varphi$ .

Moreover, we can prove the following result:

**Theorem 6.1.** *For each act of requesting, there is an act of commanding with much more complex content which updates models of DMEDL in exactly the same way as it does.*

*Proof.* By the definitions of updated models, we have:

$$M_{Req(i,j)\varphi} = M_{Com(i,j)(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)} \cdot$$

□

Does this mean that acts of requesting are acts of commanding?

We do not think so. As we have seen, an act of requesting of the form  $Req(i,j)\varphi$  and an act of commanding of the form  $Com(i,j)\varphi$  change the situation in clearly different ways from each other. The identity of the model updated by the act of requesting of the form  $Req(i,j)\varphi$  and the model updated by the act of commanding of the form  $Com_{(i,j)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  just means that it is possible to mimic each act of requesting by an act of commanding which has a related but carefully crafted much more complex content. But even an act of commanding of the form  $Com_{(i,j)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  is different from an act of requesting of the form  $Req(i,j)\varphi$  in that seeing to it that  $K_i \neg O_{(j,i,j)}\varphi$  is a way of obeying  $Com_{(i,j)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  while it is a way of refusing  $Req(i,j)\varphi$ .<sup>19</sup>

This consideration, however, reminds us of the following fact:

**Fact 1.** *There are other differences between acts of requesting and acts of commanding, and DMEDL does not deal with them.*

This is not surprising. As Sbisà (2001, 1792) points out, the use of language in communication is “multi-dimensional . . . , ranging from cognitive to emotional facets, from actional to affective ones, from social to the subjective”, and DMEDL is not meant to give a comprehensive account of such a complex phenomenon.

As Searle and Vanderveken (1985, 201) point out, for example, an agent who issues a command invokes a position of institutional authority, whereas an agent who makes a request does not. This difference enables us to understand why it is sometimes wise for a person not to issue a command but to make a request even

<sup>19</sup>“Commands” of the form  $Com_{(i,j)}(\varphi \vee K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$  could be used as a way of pretending that a commander has control of his men, but requests of the form  $Req_{(i,j)}\varphi$  could not.

in a situation in which she is in a suitable position of authority over the addressee. Invoking her position of authority overtly can be impolite and offensive.<sup>20</sup> In order to deal with the differences of this kind we need to extend our language and models. To do so, however, will not amount to abandoning what we have developed but to extending it, and we believe that DMEDL has successfully isolated one important dimension in which the workings of acts of requesting, commanding, and promising are compared.

## 7. Concluding remarks

Fact 1 seems to require us to further reflect on what are captured by the CUGO principle, the PUGO principle, and the RUGO principle. The existence of other differences DMEDL ignores suggests a possibility that there is a class of illocutionary acts whose members are differentiated from each other only by those differences. According to Searle and Vanderveken (1985, 201), the difference between acts of commanding and acts of ordering consists in the fact that the position of power an act of ordering invokes need not be institutionalized while the position of power an act of commanding invokes must be institutionally authorized. This in turn suggests that the characteristic the CUGO principle captures, though stated in reference to acts of commanding, is not specific to acts of commanding but is shared by acts of commanding and acts of ordering. And indeed there seems to be a sub-class of directive acts that share this characteristic, namely the class of directive illocutionary acts that do not allow for the option of refusal. This class seems to include at least telling in the directive sense, requiring, and demanding as well as commanding and ordering. Similarly, the characteristic the RUGO principle captures seems to be shared by acts of asking in the directive sense. Whether there are any commissive acts other than promising that share the characteristic the PUGO principle captures, however, does not seem clear and requires further investigation.

The CUGU principle, the PUGU principle, and the RUGU principle, on the other hand, seem to capture the common characteristic shared by all illocutionary acts in their respective specific forms, namely the necessity of the securing of uptake. The understanding to be secured is often considered as the understanding of the intention of the speaker, but the above principles requires something more objective or public, namely the understanding of the changes brought about in the deontic aspects of the situations.

The way these principles are shown to hold was, however, slightly too easy. As we have seen, we need to model the differences in the (un)certainty of agents as

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<sup>20</sup>Geis (1995) emphasizes the importance of the matters of “face” in criticizing the standard theory of speech acts.

regards what has happened. Since standard technique of doing this is available, our next step will be to extend DMEDL by introducing the product update.

The above reflection also suggests another interesting possibility of further research. The CUGO principle and other principles of “command logic” in fact enable us to reason at the level of higher generality than that of acts of commanding. We can reason generally about the class of directive acts that do not allow for the option of refusal. Other classes of illocutionary acts, of course, may be studied in this way as well.

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