EuJAP | VOL. 7 | No. 2 | 2011
ORIGINAL SCIENTIFIC PAPER
UDK: 161/164

NORM PERFORMATIVES AND DEONTIC LOGIC

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ABSTRACT

Deontic logic is standardly conceived as the logic of true statements about the existence of obligations and permissions. In his last writings on the subject, G. H. von Wright criticized this view of deontic logic, stressing the rationality of norm imposition as the proper foundation of deontic logic. The present paper is an attempt to advance such an account of deontic logic using the formal apparatus of update semantics and dynamic logic. That is, we first define norm systems and a semantics of norm performatives as transformations of the norm system. Then a static modal logic for norm propositions is defined on that basis. In the course of this exposition we stress the performative nature of (i) free choice permission, (ii) the sealing legal principle and (iii) the social nature of permission. That is, (i) granting a disjunctive permission means granting permission for both disjuncts; (ii) non-prohibition does not entail permission, but the authority can declare that whatever he does not forbid is thereby permitted; and (iii) granting permission to one person means that all others are committed to not prevent the invocation of that permission.

Keywords: deontic logic, dynamic semantics, update semantics, imperatives, performatives.

It is a fundamental feature of norms that they are *imposed* and *adopted* (and *promulgated*, *reaffirmed*, and so on).[1] To say this is not to claim that norms are arbitrary: as human beings, we are prone to impose and adopt only certain norms and not others. Rather, it indicates that norms are not existing in and of themselves. We may feel compelled to condemn killing human beings, but until we actually do so there is no *norm* against it, only our revulsion (and inclination to condemn).

Because norms are not natural entities, the 'logic of norms' cannot be grounded in the logical structure of reality. To explain why $p \vee q$ and $\neg p \wedge \neg q$ cannot both be true, perhaps we appeal to a correspondence theory of truth. To explain why an object necessarily is not both green all over and red all over, we may appeal to its intrinsic properties and to the metaphysics of (secondary) qualities. Yet, to explain why

[1]This relation between the norm and its introduction is constitutive and not necessarily temporal: a person may adopt a norm *in* accepting blame, but the blame itself, as well as its acceptance, can only be understood as such by presupposing that the norm is (thereby) adopted.

two norms are conflicting, we cannot do so by means of an appeal to the impossibility of them co-existing. Despite our inclinations to accept only certain norms and not others, we may still decide to accept norms that turn out to be conflicting in rare or unforseen cases. Legal experts and judges commonly have to deal with the co-existence of conflicting norms. The tension between privacy and security is a familiar source of examples. Security issues may instigate us to endorse a more stringent security policy (e.g., one involving CCTV cameras), overlooking the potential conflicts of such a norm with the privacy norms to which we have committed ourselves already in legally binding international agreements. No doubt the ten commandments in the New Testament did not reckon with the possibility of IVF, cloning, HIV, ultrasound, peer-to-peer file sharing, and more—leading to a potential conflict, either among them, or with certain norms concerning those new phenomena. So, potential and actual quandaries in our systems of norms do not make it impossible that such norm systems be put forward and become real. Consequently, we cannot explain the 'conflicting' of two norms in terms of the impossibility of an *actual* norm system encompassing both norms. That is, the proposition stating that the one norm is binding and the proposition stating that the other norm is binding may both be true, and yet those norms need not be consistent.

What then, if anything, does it mean that norms are 'consistent'? Note that this question is not answered by the statement that two norms are consistent just in case the proposition that one complies with both is contingent. That statement, if true, would still only concern the question *which* norms are consistent. Moreover, it does not clarify what, if any, constraint *permissive* norms impose on consistency.

In some of his last works on deontic logic, von Wright proposed that the logic of norms is to be understood in terms of the practice of norm imposition. Although conflicts may occur in actual norm systems, an authority who *imposes* these norms on you is not being *rational*. Such an authority puts you in a potential quandary, making it impossible for you to comply with the norms it has put forward. Similarly, we may add, a person who accepts or endorses such norms (or the authority of the norm-giver) is not being rational either. In this way, von Wright proposes to explain the logic of norms by an appeal to the rationality of the practice of introducing (imposing and endorsing) norms.

> p and $\neg p$ are mutually contradictory. But why should Op and $O\neg p$ be deemed so? Answer: A norm-giver who demands that one and the same state of affairs both be and not be the case cannot have his demand satisfied. He is "crying for the moon." His issuing the norms is *irrational*. (von Wright 1996, 40)

The concept of consistency to be invoked in explaining norm consistency is therefore not that the norms can jointly *exist*, but that they can be subsequently *imposed* without thereby manifesting irrationality.[2]

Importantly, this account of norm consistency does not only offer an alternative *grounding* of the logic of norms: it also leads us to ask which logical validities obtain for normative language. Which acts of norm imposition can be subsequently performed without manifesting irrationality? The simple answer would be that these are all series of acts leading to a body of norms such that the proposition that those norms are jointly fulfilled is consistent. Von Wright rejects this answer, on the basis of an account of permission. Standard deontic logic defines permission as the absence of prohibition. That would mean that permission imposes no constraints on the possibility to comply with a body of norms. Intuitively, that seems clearly to get the facts wrong. But why? According to von Wright, in order to explain this we need to consider the act of *giving* permission.[3]

> If permission is not just absence of prohibition what *is* it then as something "positive"? A person, who *has* a permission may do, usually also not do, the permitted thing. But what does the person do who *gives* a permission?
>
> Giving permission is a kind of "binding one's hands." It is somewhat like giving a promise or like saying "you are free to do this, I am not going to interfere." One could also say that the permission-giver imposes a prohibition on himself not to prevent the permission-holder from availing himself of the permission. (von Wright 1999, 37)

Various authors have stressed the importance of the act of permission *giving* for our understanding of permission—and its logic. Kelsen (1949) writes: "One can give somebody a permission, confer upon somebody a right, only by imposing a duty upon somebody else." Permission giving therefore indirectly imposes a constraint on the possibility of complying with a body of norms: through the act, other parties are being given prohibitions against some way of acting. But insofar as these prohibitions are not part of the *content* of the permission that has been given, logically the *existence* of that permission does not entail the existence of a prohibition imposed through the act.

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[2]Note that it will not do to qualify this as a second order norm, i.e., as a norm that one ought not to impose norms that cannot be jointly fulfilled (von Wright (1999, 33) himself suggests this). That would only reintroduce the question concerning the nature of norm consistency at a second order level.

[3]See also von Wright (1963, 88 and further) on the intersubjective aspects of permission.

Von Wright argues that the relation between obligation and permission is thus misconstrued by standard deontic logic as a logical connection between certain statements about (existing) norms. Permission is not the absence of prohibition, but it is a distinct category of norms. As a consequence, the principle that "whatever is not prohibited is thereby permitted" has been misread as a statement of logic, whereas it is in fact a *meta-norm*—comparable to the legal formula *nullum crimen sine praevia lege poenali*. Its imposition is an act whereby a norm system is "closed off", i.e., determining the normative status of any action not yet covered by any of the preceding norm performatives.

In what follows, I present an attempt to characterise the logic of norm performatives. It purports to do for the logic of norms, roughly, what prescriptivism (e.g. Hare 1952) does for their content. As will be clear from the above, this involves at least separate acts of obligation and permission, a multi-actor approach to represent the positive element of permission, and an act of closure whereby permission is granted for anything that has not yet been prohibited. First, I introduce the concept of an effectivity relation and define some properties and operations in terms of it. Second, I state an update semantics for (conditional) norm performatives, with an aside concerning free choice permission. Third, I formulate a dynamic semantics for norm propositions (i.e., statements about the existing norms) in which we can study the 'static' consequences of a consistent 'dynamics' of imposing norms. Fourth and last, I define the 'sealing legal act' of permitting whatever has not been forbidden. Before embarking on all of this, I first make some preliminary clarifying remarks.

1. Preliminaries: actions, coalitions, performatives

What is the object of a norm? If you get permission to open the window, and opening the window in the present circumstances will cool down the room, does this mean that you have been given permission to cool down the room? In the framework presented in this paper, the permission does not contradict a prohibition against cooling down the room.[4] In von Wright's terminology, we may distinguish the 'result' of an action from its 'consequences'. The result of opening the window is that the window is open, whereas the cooling down of the room is a consequence. The consequences of an action are of course largely dependent on the contingent circumstances. Because of this, we had best define norms in terms of only the result of the permitted action.

[4]That is, unless we add a static principle that the 'coalition with zero members' has permission to do all that is causally necessary to happen. In that case, the agglomeration principle of 'additive closure' implies that whenever you get permission to open the window you also get permission to open the window plus everything that is causally necessary to happen as a consequence. See also the discussion on playable and closed world effectivity functions in Broersen *et.al.* 2009.

For characterising the positive aspect of permission, we need to logically relate the permissions and obligations of different actors. The framework presented here imports various elements from coalition logic (Pauly 2001). Norms will be attributed to *coalitions*, or groups, which are represented as arbitrary sets of actors. This allows for an elegant characterisation of intersubjective consistency of norm systems: if one coalition C gets permission to act in such a way that A will necessarily become true, then the complement of C (in the society of norm subjects) cannot subsequently be given permission to act in such a way as to prevent A from becoming true—at least not without the norm-giver thereby "crying for the moon".

Von Wright (1996) contrasts norm propositions (statements about the norm system) with norms proper, but he also uses the phrase 'norm enunciation' or 'norm expression' for the latter. As I understand him, no commitment to moral realism is intended by the distinction between norm propositions and norms proper. But insofar as it might be, I will distance myself from that idea and contrast norm propositions with norm *performatives*. So, rather than attributing deontic logic to an ontologically primary domain of norm-entities, I attribute this logic to a domain of norm imposing (or norm enunciating) actions. Deontic logic is a reflection of the rules in this practice—the existence of which may in turn be explained by reference to the 'human rationality' of the practice.

2. Effectivity relations

Pauly (2001) introduced Coalition Logic as a framework for reasoning about strategic ability. It is a non-normal modal logic that is interpreted on a model with a *dynamic effectivity function* replacing the accessibility relation in a Kripke model for normal modal logic. Here, we appropriate the dynamic effectivity functions, reformulating them as relations and defining update functions on them in a semantics for norm performatives. Before coming to this semantics, this section consists of the definitions of effectivity relations and the operations on them.

Given a domain $\mathscr{S}$ of states and a finite set N of agents, the effectivity relation E attributes sets of states (propositions) to sets of agents (called 'coalitions'), for any given states. Intuitively, if $\langle s, C, X\rangle \in E$, then coalition C 'can' (in some further to be specified sense) see to it that X (at the subsequent moment), at state s.

Definition 2.1 (Effectivity relation)**.** An effectivity relation is a set $E \subseteq \mathscr{S} \times \mathscr{P}ow(N) \times \mathscr{P}ow(\mathscr{S})$. The E-alternatives for C at s are $sE_C =_{\text{def}} \{X \mid \langle s, C, X\rangle \in E\}$.
Given a coalition C and sets of states X and Y, we define $E_{XCY} =_{\text{def}} \{\langle s, C, Y\rangle \mid s \in X\}$.

I write $\mathscr{X} \sqcap \mathscr{Y}$ for the pairwise intersection of members of $\mathscr{X}$ and $\mathscr{Y}$. The complement of any set A in its obvious domain is written $\overline{A}$. The effectivity function f_E can be reconstructed from the relation E by mapping each state s onto the function mapping each coalition C onto sE_C. The set E_{XCY} will be used later to represent the content of particular norm performatives, to the extent that C must, or may, see to it that Y under circumstances X. This will be made more clear later on.

In the formal analysis of effectivity functions, two important formal properties are outcome monotonicity and superadditivity (see Pauly 2001). Outcome monotonicity says that if a coalition is effective for some set X it is also effective for any larger set: if the coalition can ensure that the next state will be some state in X, then *eo ipso* it can ensure that the next state will be some state in $X \cup Y$. For instance, if the majority of U.S. voters can ensure that Obama is re-elected, then (trivially) they can also ensure that either Obama re-elected or Canada leaves the Commonwealth of Nations. Superadditivity relates the powers of coalitions to the powers of its members. If one party can ensure that the window is open and another party can ensure that the door is open, then together they can ensure that both the door and the window are open. This can only apply as a logical principle if the two coalitions are disjoint: e.g., if Sally is needed both for opening the window and for opening the door, then she will have to chose with which party to collaborate. The reason for calling this property *super*additive is that it leaves open the possibility that some larger coalition has powers that extend beyond the powers of its members. As Gärdenfors points out:

> ... the rights of a group G is, normally, not just the union of the rights of the individuals in G, but the group may agree on contracts and have other forms of collectivistic rights which essentially extend the power of the group beyond the individual rights. (Gärdenfors 1981, 344)

In keeping with the more process-oriented perspective on deontic logic, we define two *closure operations*: the outcome monotonic closure and the additive closure. The latter leaves open the abovementioned possibility of a group having abilities extending beyond the sum of powers of its members, but it is not a 'superadditive closure' since no such powers are included in the additive operation.

Definition 2.2 (Closure)**.** Given an effectivity relation E, its outcome monotonic closure is $E^{\uparrow}$:

$$X \in sE^{\uparrow}_C \text{ iff } Y \in sE_C \text{ for some } Y \subseteq X.$$

Its additive closure is E^{+}, which is defined by induction on the size of $C \cup D$:

$$(X \cap Y) \in sE^+_{C \cup D} \text{ iff } X \in sE^+_C \text{ and } Y \in sE^+_D, \text{ provided } C \text{ and } D \text{ are disjoint.}$$

To construct the additive closure we begin with the empty set, which partitions into disjoint sets $\emptyset$ and $\emptyset$—therefore it intersects all the things for which it is effective. Then we move to all singletons, partitioning into itself and the empty set; all pairs of agents; and so on until we reach the entire domain of agents in the last step.

Although perhaps formally complex, or involved, the meaning of additive closure is straightforward: when one coalition C can independently force the next state to be in the set X and another, disjoint coalition D can independently force the next state to be in the set Y, then by additive closure $C \cup D$ can force the next state to be in $(X \cap Y)$. If we close permission additively, then we make permissions independent of allegiance: if you are permitted to open the door and I am permitted to open the window, then by additive closure we are jointly permitted to vent the room.

Definition 2.3 (Properties)**.** In terms of the closures defined above, we define the following properties.

- E is *outcome monotonic* iff $E^{\uparrow} = E$;
- E is *additive* iff $(E^{\uparrow})^+ = E^{\uparrow}$;
- E is *regular* iff $\langle s, C, \emptyset \rangle \notin E^+$, for any s and C.

Intersubjective compatibility of coalitional power is called 'regularity' in coalition logic (Pauly 2001). This means that, if $X \in sE_C$, then $\overline{X} \notin sE_{\overline{C}}$. In words, if coalition C can guarantee an outcome in X in state s, then the others cannot simultaneously guarantee *avoiding* an outcome in X. It can easily be observed that, if some two disjoint coalitions C_1 and C_2 can force disjoint sets of outcomes, then in the additive closure their union can force the empty set. Regularity, in our framework, is the property of effectivity relations for which this is not the case.

Regularity is the formal representative of the idea that, in giving permission, the norm giver is "binding one's hands". A norm giver who makes the effectivity function irregular is neglecting the commitments undertaken earlier through his other acts of norm imposition. Maintaining regularity will therefore be the way we will understand the practice of consistent norm imposition, in the next section. The first proposition connects regularity to the definition given by Pauly (2001).

Proposition 2.1. *If there is some X and some C such that $X \in sE^{\uparrow}_{C}$ and $\overline{X} \in sE^{\uparrow}_{\overline{C}}$, then E is not regular.*

This proposition can easily be seen to be true, given the fact that if E meets the stated condition, then E^{+} will contain $\langle s, N, \emptyset \rangle$.

The next proposition states that we can characterise additivity equivalently in a different way. This also makes the connection with the definition of superadditivity in Coalition Logic (Pauly 2001) more evident.

Proposition 2.2. *E is additive iff $(*)$ for all s, disjoint C_1 and C_2, and X_1 and X_2, if $X_1 \in sE_{C_1}$ and $X_2 \in sE_{C_2}$, then $Y \in sE_{C_1 \cup C_2}$ for some $Y \subseteq (X_1 \cap X_2)$.*

Proof. $\Leftarrow$: Suppose that E satisfies condition $(*)$ but E is not additive. Then $E^{\uparrow} \subset (E^{\uparrow})^{+}$, since additive closure is an additive operation. Given the inductive definition of additive closure, there must be some coalitions A, B, C, with A and B disjoint and $C = A \cup B$, such that $\langle s, A, X \rangle$ and $\langle s, B, Y \rangle$ are members of $E^{\uparrow}$, but $\langle s, C, (X \cap Y) \rangle$ is not. By outcome monotonic closure, there must be $X' \supseteq X$ and $Y' \supseteq Y$ such that $\langle s, A, X' \rangle$ and $\langle s, B, Y' \rangle$ are members of E. On the basis of $(*)$ we can then conclude that there is some set $Z \subseteq (X' \cap Y') \in sE_{C}$. This clearly implies that $(X \cap Y) \in sE^{\uparrow}_{C}$, which contradicts our earlier assumption.

$\Rightarrow$: Suppose that E is additive but does not satisfy condition $(*)$. Then there are s, X_1, X_2, and disjoint A and B such that $X_1 \in sE_A$ and $X_2 \in sE_B$ but for no $Y \subseteq (X_1 \cap X_2)$, $Y \in sE_{A \cup B}$. If we now take the outcome monotonic closure of E, then this set will include $\langle s, A, X_1 \rangle$ and $\langle s, B, X_2 \rangle$ but not $\langle s, A \cup B, (X_1 \cap X_2) \rangle$. Then additive closure of $E^{\uparrow}$ will add that element, which contradicts our assumption that $E^{\uparrow} = (E^{\uparrow})^{+}$. $\square$

To come to a characterisation of updating with norm performatives, we need to introduce operations of adding particular norms to a given norm system. This is done by defining two operations on effectivity relations: *joining* two of them together and *merging* the powers incorporated in two of them. The first of these operations is, under special circumstances a form of adding possibilities to an effectivity relation while preserving additivity, as will be shown below.

Definition 2.4 (Operations). Let E_1 and E_2 be two effectivity relations. The *join* operation $\sqcup$ and the *merge* operation $\sqcap$ are defined in the following manner.

- $E_1 \sqcup E_2 =_{\text{def}} E_1 \cup E_2 \cup \{\langle s, C, (X \cap Y) \rangle \mid \exists D \subseteq C : X \in sE_{1D} \text{ and } Y \in sE_{2(C \setminus D)}\}$;
- $E_1 \sqcap E_2 =_{\text{def}} \{\langle s, C, X \rangle \mid X \in (sE_{1C} \sqcap sE_{2C})\}$;

Some light can be shed on these definitions in the form of some propositions. The next proposition states that intersection of two effectivity functions preserves outcome monotonicity.

Proposition 2.3. $(E_1^{\uparrow} \sqcap E_2^{\uparrow}) = (E_1 \sqcap E_2)^{\uparrow}$.

Proof. $\Rightarrow$: If $X \in s(E_1^{\uparrow} \sqcap E_2^{\uparrow})_C$, then there are $Y_1 \in sE_{1C}$ and $Y_2 \in sE_{2C}$ and $Y_1' \supseteq Y_1$ and $Y_2' \supseteq Y_2$ such that $X = (Y_1' \cap Y_2')$. Then $(Y_1 \cap Y_2) \in s(E_1 \sqcap E_2)_C$ and $(Y_1 \cap Y_2) \subseteq X$, so $X \in s(E_1 \sqcap E_2)^{\uparrow}_C$.

$\Leftarrow$: If $X \in s(E_1 \sqcap E_2)^{\uparrow}_C$, then there are $Y_1 \in sE_{1C}$ and $Y_2 \in sE_{2C}$ such that $(Y_1 \cap Y_2) \subseteq X$. Then $Y_1 \in sE^{\uparrow}_{1C}$ and $(Y_2 \cup (X \setminus (Y_1 \cap Y_2)) \in E^{\uparrow}_{2C}$, so $X \in s(E_1^{\uparrow} \sqcap E_2^{\uparrow})_C$. □

Merging effectivity relations preserves additivity, as is stated and proved next.

Proposition 2.4. *If E_1 and E_2 are additive, then so is $E_1 \sqcap E_2$.*

Proof. Using Proposition 2.2. Suppose that there are s, X, Y and disjoint C and D such that $X \in s(E_1 \sqcap E_2)_C$ and $Y \in s(E_1 \sqcap E_2)_D$ but for no $Z \subseteq (X \cap Y)$, $Z \in s(E_1 \sqcap E_2)_{C \cup D}$. Then the definition of $\sqcap$ tells us that there must be X_1, X_2, Y_1 and Y_2 such that $X = (X_1 \cap X_2)$ and $Y = (Y_1 \cap Y_2)$ and $X_1 \in sE_{1C}$, $X_2 \in sE_{2C}$, $Y_1 \in sE_{1D}$ and $Y_2 \in sE_{2D}$. But the assumption is that E_1 and E_2 are additive, so $(X_1 \cap Y_1) \in sE_{1(C \cup D)}$ and similarly for E_2. Then, applying the definition of $\sqcap$ once more, we conclude that $((X_1 \cap Y_1) \cap (X_2 \cap Y_2)) \in s(E_1 \sqcap E_2)_{C \cup D}$. Of course $((X_1 \cap Y_1) \cap (X_2 \cap Y_2)) \subseteq (X \cap Y)$. □

Next, we show that additivity is preserved by the join operation under certain circumstances. This is relevant, because the update semantics for permission performatives will be defined as being of this form.

Proposition 2.5. *If E_1 is additive and $E_2 = E_{XCY}$ for some X, C and Y, then $E_1 \sqcup E_2$ is additive.*

Proof. For brevity we define $E^* =_{\text{def}} (E_1 \sqcup E_2) \setminus (E_1 \cup E_2)$, so $E_1 \sqcup E_2 = E_1 \cup E_2 \cup E^*$.

Suppose that there are some s, X_1, X_2 and disjoint C_1 and C_2 such that $X_1 \in s(E_1 \sqcup E_2)_{C_1}$ and $X_2 \in s(E_1 \sqcup E_2)_{C_2}$ whereas for no $Y \subseteq (X_1 \cap X_2)$, $Y \in s(E_1 \sqcup E_2)_{C_1 \cup C_2}$. X_1 and X_2 must be members of either E_1, E_2 or E^*. We proceed by cases:

- $X_1 \in sE_{1C_1}$ and $X_2 \in sE_{1C_2}$: by additivity of E_1, $(X_1 \cap X_2) \in sE_{1(C_1 \cup C_2}$ and so $(X_1 \cap X_2) \in s(E_1 \sqcup E_2)_{C_1 \cup C_2}$;
- $X_1 \in sE_{2C_1}$ and $X_2 \in sE_{2C_2}$: impossible since E_2 is specific to only one coalition.
- $X_1 \in sE^*_{C_1}$ and $X_2 \in sE^*_{C_2}$: impossible because all coalitions in E^* overlap;
- $X_1 \in sE_{1C_1}$ and $X_2 \in sE^*_{C_2}$: by definition of $\sqcup$, there must be some X_3 and X_4 and disjoint C_3 and C_4, such that $(X_3 \cap X_4) = X_2$ and $(C_3 \cup C_4) = C_2$, and such that $X_3 \in sE_{1C_3}$ and $X_4 \in sE_{2C_4}$. But then it is also true that C_1 and C_3 are disjoint. So by additivity of E_1, we conclude that there is some $Z \subseteq (X_1 \cap X_3)$ such that $Z \in sE_{1(C_1 \cup C_3)}$. And since C_4 is also disjoint from $(C_1 \cup C_3)$, $(Z \cap X_4) \in sE^*_{(C_1 \cup C_3) \cup C_4}$. We observe that $(Z \cap X_4) \subseteq (X_1 \cap X_2)$, and $(C_1 \cup C_3) \cup C_4 = (C_1 \cup C_2)$. So for some subset of $(X_1 \cap X_2)$, it is the case that this set is an element of $sE^*_{C_1 \cup C_2}$ and hence an element of $s(E_1 \sqcup E_2)_{C_1 \cup C_2}$, which contradicts our assumption;
- $X_1 \in sE_{2C_1}$ and $X_2 \in sE^*_{C_2}$: impossible because any member of E^* concerns a coalition superset of the one coalition occurring in E_2;
- $X_1 \in sE_{1C_1}$ and $X_2 \in sE_{2C_2}$: by definition of $\sqcup$, $(X_1 \cap X_2) \in sE^*_{C_1 \cup C_2}$, which contradicts our assumption, given $E^* \subseteq E_1 \sqcup E_2$.

$\square$

Lastly, if E_2 is already contained in the outcome monotonic closure of some additive effectivity relation E_1, then joining E_2 to E_1 will not yield anything new in that outcome monotonic closure.

Proposition 2.6. *If E_1 is additive and $E_2 \subseteq E_1^{\uparrow}$, then $(E_1 \sqcup E_2)^{\uparrow} = E_1^{\uparrow}$.*

Proof. Let us abbreviate the set $\{\langle s, C, (X \cap Y)\rangle \mid \exists D \subseteq C : X \in sE_{1D} \text{ and } Y \in sE_{2(C \setminus D)}\}$ with E^*, so that $(E_1 \sqcup E_2) = (E_1 \cup E_2 \cup E^*)$. It is enough to prove that $E^* \subseteq E_1^{\uparrow}$. Suppose that this is not the case, so $\langle s, C, (A \cap B)\rangle \in E^* \setminus E_1^{\uparrow}$. Then there is some $D \subseteq C$ such that $\langle s, D, A\rangle \in E_1$ and $\langle s, (C \setminus D), B\rangle \in E_2$. But then both tuples are also members of $E_2^{\uparrow}$. By additivity of E_1, this means that $\langle s, C, (A \cap B)\rangle \in E_1^{\uparrow}$, which contradicts our assumption. $\square$

3. Update semantics for norm performatives

3.1. General introduction into update semantics

Philosophers of logic have often thought that the logical concepts—consistency, entailment, contradiction, and so on—must be explicated in terms of truth. Von

Wright claimed that this idea wrongly limits the applicability of such concepts.

> Deontic logic gets part of its philosophical significance from the fact that norms and valuations, though removed from the realm of truth, yet are subject to logical law. This shows that logic, so to speak, has a wider reach than truth. (von Wright 1957, vii)

The insistence that this is not a genuine application of logical concepts is, according to von Wright (1996, 45), "simply stubbornness". As indicated earlier, von Wright's reference to "norms and valuations" will be understood here as applying to norm performatives. So, I contend that logical concepts of consistency and entailment apply to norm performatives despite the fact that truth evaluation is not applicable to such performatives—or any other performatives for that matter.

Update semantics (Veltman 1996) explicates the logical concepts in other terms. The meaning of a sentence is explicated, not in terms of conditions under which the message conveyed by it is true, but in terms of the change in one's commitment slate (here: norm system) as a result of *accepting* the message conveyed by it. To each sentence is assigned a function that transforms an input commitment slate into a resulting, or 'updated' commitment slate. Consistency, entailment and validity are characterised in terms of this update function. A commitment slate entails, or 'supports', a sentence if, and only if, updating that commitment slate with the sentence has no effect. That is, the commitment slate then already incorporates the message conveyed by the sentence. A sequence of sentences entails a further sentence if, and only if, any commitment slate that we update subsequently with the sequence of sentences supports the further sentence. This account of entailment is more general than the classical one in the sense that it allows for a straightforward application to sentences to which truth evaluation does not apply, such as interrogatives (Groenendijk 1999) and imperatives (Mastop 2012).

In this section an update system for norm performatives is presented.[5] This system characterises the meanings of the norm performatives as such: its object language is a language of performatives. In the following section this update semantics is used to define a dynamic deontic logic. There the object language is 'constative' or descriptive, allowing us to reason *about* the updates effected by norm performatives, and its consequences for the norm system.

[5]Formal analyses of permission change have been provided by Kamp (1973), Lewis (1979) and van Rooy (2000), amongst others. Some earlier approaches to an update semantics of normative language are van der Torre and Tan (1998), Žarnić (2002) and Mastop (2012).

3.2. An update system

The goal in update semantics is to define an *update system*, consisting of a language (a space of message-conveying expressions), a space of possible norm systems, and an update function that assigns, in a systematic way, to every message-conveying expression in the language, an update function determining what norm system results from accepting the message conveyed by that expression in a given initial norm system.

We first define a norm system, then a language of norm performatives and third an update semantics.

Definition 3.1 (Norm system)**.** Given a set P of simple proposition letters, their valuation is an assignment $V : P \to \mathcal{P}ow(\mathcal{S})$. A norm system S is a tuple $\langle F_S, D_S, R_S \rangle$, such that $F_S \subseteq \mathcal{S}$ and D_S and R_S are effectivity relations. The empty norm system $\mathsf{0}$ is $\langle \mathcal{S}, D_0, R_0 \rangle$, where $D_0 = (\mathcal{S} \times \mathcal{P}ow(N) \times \{\mathcal{S}\})$ and $R_0 = \emptyset$. S is called *regular* iff D_S, R_S and $(D_S \sqcap R_S)$ are regular. S is called *possible* iff $F_S \neq \emptyset$.

A norm system has three parameters: a representation of the factual information available; an effectivity relation D representing the duties of coalitions at any given state; and an effectivity relation R for the rights of coalitions at any given state. The empty norm system is one in which there is no information (every state is possible), there are no rights, and only the trivial outcome is obligatory.[6] A norm system is regular if the *combination* of rights and duties is intersubjectively consistent. Every coalition must be able to combine one way to fulfil all of its duties with one right, without thereby making it logically impossible for the others to do the same thing.

Admittedly, this requirement is somewhat arbitrary. A libertarian might insist that every combination of *individual* choices for each agent of a way to fulfil its duties plus one right should be consistent. Moreover, depending on what rights we consider it might be argued that rights agglomerate. The definition of regularity follows a definition given by von Wright, cf. below.

We write $S^{\uparrow}$ for $\langle F_S, D_S^{\uparrow}, R_S^{\uparrow} \rangle$. When it comes to assessing whether a norm system supports some sentence, new duties the fulfilment of which entails fulfilling already existing duties should be considered as already supported. This way derived obligations and derived permissions are supported as well.

[6]This last feature can be compared to the deontic logic axiom that $O(p \vee \neg p)$. Here it is not an axiom but merely a feature of the 'null' state in which there are no substantial norms.

Definition 3.2 (Performative language)**.** Given a set P of propositional atoms, L_P is the usual propositional language based on P. The language L_1 is the smallest set containing L_P and $O_C(\phi|\psi)$ and $P_C(\phi|\psi)$ for all ϕ and ψ in L_P.

The language L_1 is interpreted below as a language of performatives. The propositional sentences are interpreted as informative of the facts concerning which state we are in. In the terminology of Dynamic Epistemic Logic (van Ditmarsch et al. 2007) they are 'public announcements'. The sentences $O_C(\phi|\psi)$ are performatives whereby the coalition C is obligated to see to it that ϕ under the circumstances that ψ. Roughly speaking, they are conditional imperatives. The sentences $P_C(\phi|\psi)$, correspondingly, are conditional permission grantings.

Definition 3.3 (Update semantics)**.** Given a norm system $\langle F_S, D_S, R_S\rangle$, the update of the norm system with a L_1 expression is defined in the following way.

$$\begin{array}{lcl}
\langle F_S,D_S,R_S\rangle[p] & = & \langle F_S \cap V(p), D_S, R_S\rangle \\
\langle F_S,D_S,R_S\rangle[\neg\phi] & = & \langle (F_S \setminus F_{S[\phi]}), D_S, R_S\rangle \\
\langle F_S,D_S,R_S\rangle[\phi \wedge \psi] & = & \langle (F_{S[\phi]} \cap F_{S[\psi]}), D_S, R_S\rangle \\
\langle F_S,D_S,R_S\rangle[O_C(\psi|\phi)] & = & \langle F_S, D_S \sqcap E_{||\phi||C||\psi||}, R_S\rangle \\
\langle F_S,D_S,R_S\rangle[P_C(\psi|\phi)] & = & \langle F_S, D_S, R_S \sqcup E_{||\phi||C||\psi||}\rangle
\end{array}$$

Here, $||\phi|| = F_{0[\phi]}$, for any L_P-expression ϕ.

In defining norm consistency I follow von Wright (1996) (cf. von Wright 1999, 34) who proposes that obligations and permissions are consistent if, and only if, *all* obligations plus *one* permission (hence, the set $D_S \sqcap R_S$) are jointly consistent.[7] Different definitions may be considered and motivated, and different definitions may be given.

Definition 3.4 (Logical concepts)**.** Given a norm system S, S *supports* α, written $S \Vdash \alpha$, if, and only if, $(S[\alpha])^{\uparrow} = S^{\uparrow}$. And α *entails* β if, and only if, $S[\alpha] \Vdash \beta$ for all S. Performative α is *consistent* in S iff $S[\alpha]$ is possible and regular.

The main purpose of this definition is to demonstrate that the logical concepts can be explicated in terms of performatives, without a prior explication of them in terms of truth conditions or truth preservation. Although the norm performatives still have propositional contents, elsewhere I have argued that a semantics of imperatives and permission sentences is possible in which they do not embed propositions (Mastop 2012).

[7] He in fact requires that they be not only consistent, but 'doable'. To come closer to that notion, the concept of violation, Definition 6.1, may be used.

A first simple example is the following. The two tuples—or permissive norms, if you like—$\langle s, C, ||\phi|| \rangle$ and $\langle s, \overline{C}, ||\neg\phi|| \rangle$ can happily coexist in R_S without implying that S is irregular. However, if we were to institute such norms by means of permission performatives, we would have to do so by means of updating some norm system S with $P_C(\phi|\psi)$ and $P_{\overline{C}}(\neg\phi|\chi)$, with the requirement that s is a member of both $||\psi||$ and $||\chi||$. The result of the update is a norm system S' such that $\langle s, N, \emptyset \rangle \in R_{S'}$. So the update does lead to irregularity, which means that the second update is inconsistent in its update context. It is the granting of the permissions which is inconsistent, not the coexistence of the permissive norms in themselves.

The propositions stated in the previous section show that updating preserves additivity of R_S and D_S. If we start out with a additive norm system, such as $\mathtt{0}$, then we will maintain additivity of our permissive and commissive norms.

3.3. Free choice permission

An ongoing discussion in the literature on imperative logic is whether or not disjunctive imperatives are granting a free choice. The issue has sometimes been presented as a matter of the validity of an inference from "$!(A \vee B)$" to "$!A$", instead of vice versa. The correct presentation of the issue, I believe, is that on a free choice account, the command $O_C((p \vee q)|r)$ compels C to make a *free* choice between seeing to it that p or seeing to it that q, if r—and so the subsequent prohibition of either seeing to it that p or seeing to it that q, if r, would be "crying for the moon" to repeat von Wright's phrasing. Similarly, the permission $P_C((p \vee q)|r)$ would grant C to a free choice between invoking permission for p and invoking permission for q, if r.

I am convinced that disjunctive norm performatives are free choice granting in this sense. Pragmatic accounts of free choice 'readings' of 'or', in terms of plausible speaker's intentions, generally only explain our *surprise* upon a later limitation of our choices, and not our *objection* that we had been granted a choice. If mom says "Clean up your room or do your homework" and you start doing your homework, then if dad later interrupts you and says "Stop doing your homework and clean up your room" you may protest and say: "But mom told me to do either". You may of course also decide to comply with dad's command, since doing so does not interfere with complying with mom's command, but that is not to say that the two commands are consistent.

The present framework in terms of effectivity relations easily admits for an incorporation of a free choice permission account of disjunctive norm performatives. The set $s(D_S)_C$ represents the alternative ways for C to fulfil all of its obligations at s. A free choice command diversifies this set, merging its elements with the contents of both disjuncts. In the familiar example by Ross (1941), in order to fulfil all of your obligations, X (post the letter plus any other obligations), or Y (burn the letter plus any other obligations). Adding a command that one not burn the letter would turn Y into $\emptyset$, thus breaching norm consistency. Even granting some disjoint coalition the permission to see to it that the letter is not burned would violate the free choice granted to the subject of the disjunctive command.

In order to accommodate a free choice semantics for disjunction, we specify an alternative way of defining $||\phi||$, comparable to alternative semantics (Rooth 1992) or inquisitive semantics (Ciardelli and Roelofsen 2011).

$$\begin{array}{rclrcl} ||p|| & = & \{V(p)\} & ||\neg p|| & = & \{\overline{V(p)}\} \\ ||\phi \vee \psi|| & = & ||\phi|| \cup ||\psi|| & ||\neg(\phi \vee \psi)|| & = & ||\neg\phi|| \sqcap ||\neg\psi|| \\ ||\phi \wedge \psi|| & = & ||\phi|| \sqcap ||\psi|| & ||\neg(\phi \wedge \psi)|| & = & ||\neg\phi|| \cup ||\neg\psi|| \end{array}$$

Furthermore, we define $E_{XC\mathscr{Y}} =_{\text{def}} \bigcup_{Y \in \mathscr{Y}}(E_{XCY})$. The definitions of the update semantics can then be left as stated above.

Without free choice, $\mathsf{O}[O_C((p \vee q)|r)][O_C(\neg p|r)]$ is regular (and possible). With the alternative definition of $||\phi||$, it is not regular. The first update introduces into D_S the items $\langle s, C', V(p)\rangle$ and $\langle s, C', V(q)\rangle$, for any $s \in V(r)$ and $C' \supseteq C$. The second update then requires the merging of D_S with items $\langle s, C', \overline{V(p)}\rangle$, for any $s \in V(r)$ and $C' \supseteq C$. Consequently, if $V(r)$ is not empty, we end up with an item $\langle s, N, (V(p) \cap \overline{V(p)})\rangle$, making the resulting norm system irregular. Furthermore, with the alternative semantics, we find that $P_C((p \vee q)|r) \Vdash P_C(p|r)$. So disjunctive permission granting is free choice permission granting.

In what follows, I will disregard the matter of free choice interpretations of 'or' in performatives, assuming the simpler semantics of the previous section.

4. Dynamic deontic logic

The update semantics only allows us to reason about norm systems in terms of the possibility to update it in some way. A dynamic semantics relates the update system to a language to reason about the static facts concerning the norm system.[8] Below we specify a model incorporating a norm system as well as an (unchanging) effectivity relation for ability.

Definition 4.1 (Model). A model M is a quadruple $\langle \mathscr{S}, S, A, V\rangle$, where S is a norm system, V is a valuation of proposition letters and A is an effectivity relation that is (i) additive, (ii) regular, and for all s and C, (iii) sA_C is nonempty. For ease of notation, $M[\alpha] =_{\text{def}} \langle \mathscr{S}, S[\alpha], A, V\rangle$, for any $\alpha \in L_1$.

[8] See a series of recent papers by Yamada for a developed account of dynamic deontic logic; e.g. Yamada (2008).

Next we define a language to reason, in a standard modal logic sense, about these models. Continuing to follow von Wright's terminology, we might call this a language of norm propositions, although perhaps these modal formulas come closer to his idea of "practical necessity" in von Wright 1999.

Definition 4.2 (Modal language). L_2 is the modal language based on the propositional atoms P and the sentential operators ∇, $\triangle_C$, $\Diamond_C$, $\Box_C$, and $[\alpha]$, for any $C \subseteq N$ and α in L_1.

$\Diamond_C \phi$ means that ϕ is permitted for C and $\Box_C \phi$ means that ϕ is obligatory for C. The modality ∇ represents the information present in F_S. For instance, if we accept a norm performative $O_C(p|q)$ and an informative statement q, then the proposition $\nabla \Box_C p$ will be true. This proposition can be rendered in natural language as "Given the known (acknowledged) facts, C ought to see to it that p". The dynamic modal sentence $[\alpha]\phi$ is to be read as "After accepting α in the norm system, ϕ is true".

Definition 4.3 (Dynamic semantics). Let M be a model and S its norm system. The satisfaction set of an L_2-expression in M is defined inductively below.

$$
\begin{array}{lll}
s \in ||\phi||^M & \text{iff} & s \in F_{0[\phi]}, \text{ for } \phi \in L_P \\
s \in ||\nabla\phi||^M & \text{iff} & F_S \subseteq ||\phi||^M \\
s \in ||\triangle_C \phi||^M & \text{iff} & \langle s, C, ||\phi||^M \rangle \in A^{\uparrow} \\
s \in ||\Diamond_C \phi||^M & \text{iff} & \langle s, C, ||\phi||^M \rangle \in R_S^{\uparrow} \\
s \in ||\Box_C \phi||^M & \text{iff} & \langle s, C, ||\phi||^M \rangle \in D_S^{\uparrow} \\
s \in ||[\alpha]\phi||^M & \text{iff} & s \in ||\phi||^{M[\alpha]}
\end{array}
$$

State s satisfies ϕ in M, written $M, s \models \phi$, iff $s \in ||\phi||^M$. Formula ϕ is valid in M, written $M \models \phi$, iff $||\phi||^M = \mathscr{S}$.

In the evaluation we make use of the outcome monotonic extensions of the effectivity relations. This is so in order to ensure that derived obligations can be expressed as well. When a person has an obligation to close all the windows in the building, then he also has an obligation to close some particular window in the building.

The semantics for obligation does not as such guarantee that the obligation operator is agglomerating. That is, it is possible that there are mutually exclusive ways to fulfil all of one's obligations, in which case we may find that $\Box_C p$ and $\Box_C q$ but not $\Box_C(p \wedge q)$.[9] Agglomeration is valid (in a model) just in case there

[9] If we accept the free choice semantics for disjunction it is possible to define an obligation operator such that only $\Box_C(p \vee q)$ would come out true in this situation.

is a unique minimal element to sD_C for every s and C. The update operation $\sqcap$ preserves this property for the effectivity relation D.

Proposition 4.1. *Let S be the norm system in M, with R_S and D_S additive. The update and dynamic semantics can be related as follows (with ϕ and ψ in L_P): (i) $M \models \nabla\phi$ iff $S \Vdash \phi$; (ii) $M \models \psi \to \Diamond_C\phi$ iff $S \Vdash P_C(\phi|\psi)$; (iii) $M \models \psi \to \Box_C\phi$ iff $S \Vdash O_C(\phi|\psi)$.*

Proof. For permission. If R_S is additive, then $M \models \psi \to \Diamond_C\phi$ iff $S \Vdash P_C(\phi|\psi)$.

$\Rightarrow$: The formula $\psi \to \Diamond_C\phi$ is valid on M iff $\langle s, C, ||\phi||^M\rangle \in R_S^{\uparrow}$ for all $s \in ||\psi||^M$. This is equivalent to $E_{||\phi||C||\psi||} \subseteq R_S^{\uparrow}$. The update semantics defines $R_{S[P_C(\phi|\psi)]} = (R_S \sqcup E_{||\phi||C||\psi||})$. From Proposition 2.6 we know that $(R_S \sqcup E_{||\phi||C||\psi||})^{\uparrow} = R_S^{\uparrow}$, in view of the additivity of R_S. Combining these facts, $R^{\uparrow}_{S[P_C(\phi|\psi)]} = R_S^{\uparrow}$, which means that $S \Vdash P_C(\phi|\psi)$.

$\Leftarrow$: Given the definition of support, $S \Vdash P_C(\phi|\psi)$ iff $(R_S \sqcup E_{||\phi||C||\psi||})^{\uparrow} = R_S^{\uparrow}$. From this it follows directly that $E_{||\phi||C||\psi||} \subseteq R_S^{\uparrow}$. As pointed out above, this implies that $\psi \to \Diamond\phi$ is valid on M. $\square$

The facts stated above form the core of the argument of this paper: the logical properties of norms as such are rather minimal. What are commonly considered to be the 'axioms of deontic logic' are in reality derived properties of norm systems. The properties in question are derived from the logic that is really inherent in the practice of norm imposition.

In the last two sections, we go into some possible further directions in which this approach to the logic of norms can be taken. The next section concerns the common dictum that 'ought implies can'. Until now we have limited the logic to 'ought implies may'. That is, there can be no conflict within the norm system itself if it is to be consistent. Yet, we may feel that no obligation can exist if its fulfilment is in any way impossible in the light of the actual practical possibilities of the subject of that obligation.

Then, in section 5, we consider the principle of *nullum crimen sine lege*: that everything that has not been forbidden is thereby permitted. This principle should be understood as a meta-norm, rather than a logical truth. So conceived, it can be formally represented by an update that 'closes off' the norm system, completing it with permissions.

5. Sealing legal principle

The sealing legal principle states that 'whatever is not forbidden is permitted'. As mentioned above, von Wright argued that this principle is not a logical truth but a performance whereby the norm system is closed. There are different possibilities for closing a norm system, an obvious alternative is 'whatever is not (expressly) permitted is forbidden'.

Understood as a statement, the sealing legal principle could be formally rendered as $\neg\Box_C\neg\phi \to \Diamond_C\phi$ or, considering regularity of permission, $\neg\Diamond_{\overline{C}}\neg\phi \to \Diamond_C\phi$. The latter formulation is known in coalition logic as 'maximality'. Considering the closure operations defined earlier, it might be thought that we could introduce yet another one: the maximal closure of E, obtained by means of the following equation.

$$X \in sE_C^{max} \text{ iff } X \in sE_C \text{ or } \overline{X} \notin sE_{\overline{C}}.$$

The problem with this is, that if neither $X \in sE_C$ nor $\overline{X} \in sE_{\overline{C}}$, we would add both, leading to an effectivity relation that is no longer regular: its additive closure would be such that $\emptyset \in sE_N$. Therefore, if we would characterise it as the 'maximality' closure condition, the sealing legal principle would not be norm consistent in all norm systems. If neither of us has a prohibition against either closing the door or keeping the door open, then giving us permission to do these things would imply giving the pair of us permission to a enforce a contradiction. What we need is a restricted variant of the closure such that norm consistency can be guaranteed.

The issue is not merely technical. It requires that we explicate what is meant by the formulation of the sealing legal principle. When the authority is closing off the norm system, he is evidently not giving us permission to the actions of *others*. That is, the authority is not granting everyone a *claim right* with respect to others if that right had not been explicitly denied beforehand.[10] This shows that we must interpret the sealing legal principle as limited by the abilities of the coalitions: 'whatever is not forbidden *that you can do*, is hereby permitted'.

We cannot characterise the sealing legal act as an operation on norm systems, because it has to be relative to the actual abilities of the agents. Therefore, we define it as an operation on models.

Definition 5.1 (Sealing Legal Act)**.** Let a model $M = \langle \mathscr{S}, \langle F, D, R\rangle, A, V\rangle$ be given. The sealing legal act ω is a closure operation on M such that $M[\omega] = \langle \mathscr{S}, \langle F, D, R^c\rangle, A, V\rangle$, such that:

$$R^c = (R^{\uparrow} \cup (A \cap R^{-})^{\uparrow})^{+}.$$

[10]See for instance von Wright (1963, 89) on the concept of a 'claim'.

R^- is the complementary effectivity relation to R, defined by $X \in sR^-_C$ iff for no D disjoint with C and no Y disjoint with X, $Y \in sR^{\uparrow}_D$.

Let us call an effectivity relation E maximal iff either $X \in sE_C$ or $\overline{X} \in sE_{\overline{C}}$, for each s, X and C. Then, under the assumption of maximality of the ability relation A, the sealing legal closure principle preserves regularity and ensures maximality of permissions.

Proposition 5.1. *If R is regular and A is regular and maximal, then R^c is regular and maximal.*

Proof. For regularity: First note that $R^{\uparrow}$ is regular and, because A is regular, the same is true for $(A \cap R^-)^{\uparrow}$. Given that R is regular, if $X \in sR^{\uparrow}_C$ then $Y \notin sR^-_{\overline{C}}$ for any $Y \subseteq \overline{X}$. Therefore, $\overline{X} \notin s(A \cap R^-)^{\uparrow}_{\overline{C}}$. So, the additive closure of $R^{\uparrow}$ and $(A \cap R^-)^{\uparrow}$ will be regular as well.

For maximality: If either $X \in sR_C$ or $\overline{X} \in sR_{\overline{C}}$, then the same is true for R^c. So suppose that neither $X \in sR_C$ nor $\overline{X} \in sR_{\overline{C}}$. By maximality of A, either $X \in sA_C$ or $\overline{X} \in sA_{\overline{C}}$. Suppose, without loss of generality, that $X \in sA_C$. By regularity of R, for no $Y \subseteq \overline{X}$ and for no D disjoint with C, $Y \in sR_D$. This is equivalent to $X \in sR^-_C$. Therefore $\langle s, C, X\rangle \in (A \cap R^-)$, so $X \in sR^c_C$. This proves that R^c is maximal. □

Note that, as the proof displays, the regularity and maximality of A is crucial for the regularity and maximality of R^c.

Several theorists of norm logic differentiate between *rights* and weaker *liberties*. We can see them as logically distinct, in the sense that a norm that effectively prohibits the invocation of a right can be said to be inconsistent with the right, whereas a liberty can be retracted by a later prohibition. For instance, the right to express your opinion precludes, by consistency, the command to refrain from voicing any criticism against the state. In the absence of a prohibition, you have a liberty to open your store on a Sunday. However, that liberty can be taken away by the government by means of an explicit prohibition.

This distinction can be expressed using the sealing legal principle. Coalition C has a *liberty* to X in state s, in model M, with norm system S iff (i) $X \notin sR^{\uparrow}_{SC}$, (ii) $X \in sR^c_{SC}$ and (iii) $D_S \sqcap \{\langle s, C, X\rangle\}$ is regular. Perhaps liberties, so understood, are somewhat like "tolerance" in the sense of von Wright (1963, 88). Note that we are not adding any claim rights to R by the sealing legal principle. So it is a maximisation of rights but at the same time a minimisation of claims.

6. Consistency and quandary freedom

Consistency as defined is, arguably, still a rather abstract concept. Even if the norm-giver is *consistent* in imposing norms, the norms may still be impossible to *execute* for a coalition, given its limited powers. As a final consideration, the framework is extended to deal with the real, practical limitations of agents and coalitions in complying with their obligations (and invoking their permissions).

To begin with, the following definition is intended to capture these practical impossibilities.

Definition 6.1 (Violation and quandary)**.** Given a model $M = \langle \mathscr{S}, S, A, V \rangle$, $X \in sA_C$ is a *violation* in M if, and only if, for no $Y \in sA_{\overline{C}}$, $(X \cap Y) \in s(R_S \sqcap D_S)_N$. State s is a *quandary* for C in M if, and only if, all $X \in sA_C$ are violations.

In imposing norms the minimal requirement is consistency: no conflict internal to the norm system. However, ideally we also want to avoid creating quandaries in people's law abiding conduct. Hamblin (1972) distinguishes different versions of this requirement of "quandary freedom", with varying strength. At the very least, we must avoid that N is in a quandary, for in that case the norms cannot but be violated. As a form of liberalism, we might demand that no singleton coalition (i.e., every individual actor) is ever in a quandary: no individual should be forced to collaborate with others in order to not violate the norms. A strategic version of quandary freedom would be that each coalition should be able (given some initial state) to avoid getting into a quandary at any point. Another version of quandary freedom is that, as long as the coalition obeys the norms, it does not enter into a quandary. This is not a very robust form of quandary freedom: the norm system works well as long as everybody acts in accordance with it, but it might not be prepared for cases in which the norms are violated. Lastly, the most demanding form of quandary freedom is that no quandary ever arises.

The definition above allows for a straightforward syntactic definition of these forms of quandary freedom. We extend the language with constants Q_C, for any $C \subseteq N$, expressing that coalition C is in a quandary. We add the semantic clause: $s \in ||\mathrm{Q}_C||^M$ iff s is a quandary for C in M. Now these forms of quandary freedom can be defined in our language as follows:

- Minimal QF: $\neg Q_N$;
- Individual QF: $\bigwedge_{i \in N} \neg \mathrm{Q}_{\{i\}}$;
- Strategic QF: $\bigwedge_{C \subseteq N} \triangle_C \neg \mathrm{Q}_C$;
- Legal QF: $\bigwedge_{C \subseteq N} (\neg \mathrm{Q}_C \rightarrow \triangle_C \neg \mathrm{Q}_C)$;

- Absolute QF: $\bigwedge_{C \subseteq N} \neg Q_C$.

A norm system can be evaluated, relative to a given model and state, in various respects: whether it prohibits moral wrongs well enough, to what extent it respects the autonomy and integrity of agents, and also how well it minimizes the occurrence of quandaries. One way to test a norm system in the latter respect would be to determine whether the formula for some version of quandary freedom is true in the current state, valid in the model, or perhaps true in all states that can be 'reached' from the current state.

7. Conclusion

Standard deontic logic is a logic of impersonal "ought to be"-statements and its semantics is often presented as distinguishing a set of 'ideal worlds' in a Kripke model. This paper addresses (what I believe to be) several shortcomings of that approach. Firstly, the logic of permission is only understandable in a social setting, in which the freedom of one agent contributes to the normative constraints for others. Furthermore, the validity of free choice permission requires another move away from standard deontic logic. But most importantly, the aim of this contribution has been to motivate and present a dynamic semantic perspective on the logic of norms: to be consistent in giving orders and permissions, the norm giver cannot prohibit what it has permitted before, or permit incompatible things to different parties. The inspiration for this view comes from von Wright's (1996; 1999) later work on the nature of deontic logic and its formalisation is a variant of the update semantics of Veltman (1996).

In the formal literature on 'dynamics' in semantics and logic, the original idea in dynamic semantics of dynamifying the standard conception of meaning (Groenendijk and Stokhof (1991), Veltman (1996)) has faded somewhat to the background in recent work on public announcements and dynamic epistemic logic and public announcement logic (see for instance Wang (2011)), assuming a standard static intensional semantics for the dynamic modalities. The present paper goes against this trend, arguing for the primacy of the update formalism as giving a semantic foundation for the dynamic logic.

A further study of the formalism has to come at a later stage. Apart from an axiomatisation of both the update semantics and of the dynamic logic, a natural direction to take would be an investigation into the formal analysis of rights and legal positions.[11] The discussions on the sealing legal principle and quandary freedom can be seen as a first step in this direction.

[11] Compare Lindahl (1977), or von Wright (1963, 88 and further). See also Mastop (2002).

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Received: December 21, 2011
Accepted: January 16, 2012

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