S-curve modelling in early phases of construction projects

Nives Ostojić-Škomrlj, Mladen Radujković

The methodological procedure for forecasting cost distribution over time is given for the project realization phase using cost s-curves for three different types of structures: building, tunnel, and motorway. Three different approaches are used, and their results are correlated and presented in form of mathematical regression expressions and appropriate diagrams. The proposed methodology can be used for cash flow forecasting during all phases of construction projects, specially in the earliest phase in which detailed information about the project is scarce.

Key words: construction planning, building engineering, tunnels, motorways, S-curve, regression models, cost scheduling

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Model progonziranja S-krivulja u ranim fazama građevinskih projekata

U radu je prikazan metodološki postupak kojim se definira način prognoziranja raspodjele troškova u vremenu za fazu izvršenja projekta pomoću troškovne S-krivulje za tri različita tipa građevina: visokogradnja, tuneli i autocesta. Primjenjena su tri različita pristupa, a rezultati sva tri pristupa i njihove integracije prikazani su u obliku regresijskih matematičkih izraza i odgovarajućih dijagrama. Predložena metodologija može se koristiti za prognoziranje troškovne dinamike građevinskih projekata u svim njihovim fazama, uključivo i najraniju fazu kada ne postoji dovoljno detaljnih podataka.

Ključne riječi: planiranje izgradnje, visokogradnja, tuneli, autoceste, S-krivulja, regresijski modeli, prognoza dinamike troškova

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Prognosierungsmodell von S-kurven in der Frühphase von Bauprojekten


Schlüsselwörter: Bauplanung, Hochbau, Tunnels, Autobahnen, S-Kurve, Regressionsmodelle, Prognose der Kostendynamik
1. Introduction

On construction projects, the study of cost distribution over time is a topic of high significance for clients and contractors alike. The information about costs gains its full significance when it is complemented with information about the time period in which the costs are incurred. The cost scheduling, arising from harmonized plans and cost estimates approved by client and contractor, is agreed upon on many construction projects. This is an information of great importance for all participants in construction projects, and has therefore been the subject of many studies. Some of these studies focus on the principles regulating these variables in various phases of the project, i.e. on specific types of construction projects and, at that, the objective is to determine the theoretical cumulative S-curve that will best represent the standard distribution of costs over time. S-curves represent a cumulative flow of money over a time period. In these curves, the time “t” is presented on the abscissa, and the costs “v” are given on the ordinate. Possibilities for modelling cumulative costs over time at the project realization phase, based on three different approaches, are presented in full detail in this paper. Influence parameters and distribution principles must be defined for selected types of construction projects, under conditions of different organisation scenarios, specific realization requirements, client’s decision, external and internal risks, and project structure influences. It is known that all these parameters greatly vary from project to project, which results in an extremely complex combination of cases that can not be analyzed individually. This is why a basis providing results in conditions without extreme influences, i.e. in standard project realization conditions, has been developed by random selection of projects. Cost scheduling in special and extreme cases of realization can be defined in scope of a new study based on specific situations and, at that, the results obtained in the course of the present study can be used as a fundamental basis for such new research.

2. Methodological procedure, methods and research plan

The study of cost distribution over time by selected groups of construction projects was conducted by the S-curve modelling in order to determine a theoretical curve that presents, in the most appropriate way, cost relationships under standard conditions of project realization. At that, three modelling approaches were used (Figure 1.). Each of these approaches offers several curve alternatives depending on the earlier defined organizational influences and parameters. The idea consists in the realization of three different research approaches, and in merger of their results during preparation of the planned S curve for selected groups of construction projects. In all similar studies conducted so far, the results were obtained based on one approach only.

In the first approach, all alternative proposals relating to the theoretical cost-time relationship were analyzed by groups of typical construction projects based on data found in international and domestic literature. Following inspection of this literature, the results defining the cost curves by groups of projects were selected (first way of cost curve preparation).

In the second procedure, the modelling was conducted by detailed analysis of data about initially planned and real cost-time relationship for completed construction projects that have been classified by typical groups. The comparison between “as planned” and “as realized” resulted in cost curves according to the studied groups of projects, in the second way of cost curve preparation. The analyses conducted in this respect also revealed information about specific influences that can change the cost-curve geometry.

In the third approach, the cost-time modelling was conducted through creation of standardized critical path models for normal conditions of realization, according to typical groups of construction products. Standard critical path plans were created by expert planners who prepared, based on practical experience, balanced plans with relatively uniform resource-use profiles, which resulted in the third way of cost curve preparation.

Figure 1. Methodology for S-curve shaped analyses
Electronic addresses of all authors are available to the interested reader.

**Table 1.** An overview of mathematical models for S-curve analysis, based on literature data

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bromilow</td>
<td>1978</td>
<td>[ Y = \left[ a + a_{(w/t)} + a_{(w/t)^2} + a_{(w/t)^3} \right] C ]</td>
</tr>
<tr>
<td>Hudson</td>
<td>1978</td>
<td>[ Y = S \left[ x + C x^2 - Cx - (6x^3 - 9x^2 + 3x) \right] K ]</td>
</tr>
<tr>
<td>Peer</td>
<td>1982</td>
<td>[ Y = 0.0089 + 0.26981 t + 2.36949 t^2 - 1.39030 t^3 ]</td>
</tr>
<tr>
<td>Tucker</td>
<td>1988</td>
<td>[ Y = a \left[ 1 - e^{-((x-d)/y)} \right] ]</td>
</tr>
<tr>
<td>Miskawi</td>
<td>1989</td>
<td>[ Y = 3^2 \left[ 2 \sin(x(1-t)/2) \sin(xt) \right] \ln \left( t+0.5 \right) \left( a+t \right) - 2t^4 + 3t^2 ]</td>
</tr>
</tbody>
</table>
| Boussabain i Elhag   | 1996 | \begin{align*}
za 0 \leq x \leq 1/3 & \quad Y = 9x^2/4 \\
za 1/3 \leq x \leq 2/3 & \quad Y = 3x/2 - 1/4 \\
za 2/3 \leq x \leq 1 & \quad Y = 9x/2 - 9x^2/4 - 5/4
\end{align*} |
a number of reasons, and so the "quest" for a universal solution is still under way. A typical diagram showing a cumulative cost/time analysis of a construction project is here reduced to the relationship of $v = 100\%$ and $t = 100\%$. It is shaped as a letter S and has for that reason been named the S-curve. The Y variable represents the cumulative percentage of funds invested in construction, while the X variable represents the cumulative percentage of time needed for completion of the construction process. At the beginning of the project, when mobilization is in progress and when production resources are being organized, the costs are accumulating at a slow rate. Later on, when most resources have been engaged, the cost accumulates rapidly at a nearly constant rate (relatively flat line in the centre of the presentation). As the end of the project approaches, when builders complete their work, the cost accumulation decreases. Most mathematical models are based on this formulation. Three groups of construction projects have been adopted in this study: buildings, tunnels, motorway sections.

The basic conclusion that was reached during consultation of literature, and analysis of mathematical models proposed in literature, is that the models proposed by Tucker, Miskawia, Boussabain, and Elhaga, and partly the Huson DHSS model developed in British Department of Health and Social Security, are favourable for forecasting the cost dynamics for building, tunnelling and motorway construction projects. In case of literature curves are considered in the coordinate system 100%/100%, the focus is on curve geometry, and so the results obtained can be used as proper basis even outside of the environment in which they were obtained.

If all favourable curves, classified by groups, are taken from literature, and if their best approximation is made using the least-square method, then this results in curves that describe the cumulative percentage of funds invested in construction for each group of structures. Depending on the type of construction project, mathematical expressions for each of the literature-based S-curves are as follows:

1. **Buildings**
   
   \[
   Y_{\text{buildings}} = 0.00308724 + 0.29833094 x + 0.0129652 x^2 + 0.0012448 x^3 - 0.000000260 x^4 + 0.00000001 x^5 
   \]  
   \[ (1) \]

2. **Tunnels**
   
   \[
   Y_{\text{tunnels}} = 0.0022540801 + 0.3013256588 x + 0.0190451810 x^2 + 0.0000154346 x^3 - 0.0000022972 x^4 + 0.0000000093 x^5 
   \]  
   \[ (2) \]

3. **Motorway**
   
   \[
   Y_{\text{motorway}} = 0.0022540801 + 0.3013256588 x + 0.0190451810 x^2 + 0.0000154346 x^3 - 0.0000022972 x^4 + 0.0000000093 x^5 
   \]  
   \[ (3) \]

### 2.2. S-curve models based on projects in Croatia

Data from 78 projects realized in the Republic of Croatia [1], classified into one of three categories mentioned in Section 2.1, were adopted for purposes of this study. The first category contains information about building construction projects, number of storeys: GF+4 (29 buildings). The second category is formed of data from tunnel construction projects (22 structures), while the third category contains data from civil engineering projects, namely motorway sections – only the route not including structures (27 sections). The projects included in this study were built over the past fifteen years, until 2010, and the main part of the study was realized during preparation of a doctoral thesis [2].

The data on construction projects were compiled through surveys and by interviewing participants in construction work: contractors, supervising engineers, and clients. During compilation of data, the key elements were:

- plan and parts of cost estimate (bill of quantities)
- initial construction price
- construction time defined in contract
- real construction price
- real construction time
- reasons for non-respect of construction price
- reasons for non-respect of construction time
- monthly payment of completed work

In addition to basic information about the structures – type of structure, time of construction, contract-based and real construction costs, contract-based and real construction time – the survey also included collection of data about monthly (invoiced) construction costs, which served as basis for preparation of realistic cost-time curves for each of the completed projects. As expected, it came out that in most cases there is a discrepancy between the planned and real construction time, and between the contract-based and real construction costs. This is why the information about the influence of risk and changes was also collected in the course of the survey. The approach from previous studies [12, 13, 14] was applied in risk classification and analysis. The information about the source of risk, initiators, influences, and effects, was collected. Cost curves by project were defined by comparing initial plans and actual realization, and by analyzing deviations and action of individual risks over time. The alternative in which risk action is included depicts the real situation, although it should be noted that the analysis of actual cases has shown that the effects of some risks can be alleviated by better project management. This is why an additional analysis, with an assumed alternative project management that would alleviate risks, was conducted for each example. Thus, alternative cost curves, including subjective evaluations of the author of the paper, were obtained.

Based on regression analysis of the sample (as in Section 2.1), mathematical expressions for S-curves of completed structures (including the risk action) can be presented as follows:

\[
Y_{\text{P+4}} = -0.0337623726 + 0.4038721032 x - 0.0073410793 x^2 + 0.0011063571 x^3 - 0.00000169230 x^4 + 0.0000000719 x^5 
\]

\[ (4) \]
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- for completed tunnels

\[ y_{\text{tunnels}} = -0.1374996879 + 0.7939777374 x - 0.0251265572 x^2 + 0.0013158131 x^3 - 0.0000176112 x^4 + 0.0000000717 x^5 \]

- for completed motorway sections

\[ y_{\text{motorway}} = -0.1162250238 + 0.3217680288 x + 0.0653865104 x^2 - 0.0033289281 x^3 + 0.0000703337 x^4 - 0.0000006271 x^5 + 0.000000002 x^6 \]

Mathematical expressions for S-curves of completed structures (with alleviated risk action) can be presented as follows:

- for completed buildings, GF+4

\[ y_{\text{GF+4}} = -0.0208063363 + 0.3755043598 x - 0.0045063985 x^2 + 0.0008679441 x^3 - 0.0000126806 x^4 + 0.000000506 x^5 \]

- for completed tunnels

\[ y_{\text{tunnels}} = -0.1628419754 + 0.7232842752 x - 0.0179868217 x^2 + 0.006969348 x^3 - 0.000073496 x^4 + 0.000000236 x^5 \]

- for completed motorway sections, route without structures

\[ y_{\text{motorway}} = -0.1480284072 + 0.4328567594 x - 0.0728915853 x^2 + 0.0037707534 x^3 + 0.000000790817 x^4 - 0.0000000329 x^5 \]

2.3. Standardized plan models prepared by experts

The need for preparing a standardized theoretical S-curve model is expressed in the introductory part of the paper. In this case the planner prepares a construction time schedule and a corresponding S curve based on some previously defined parameters (size of structure, cost estimate, technological realization of works). The procedure for preparing the time schedule of activities and financial schedule is the same for all three cases (building, tunnel, motorway section). Based on known data about work standards and work quantities for each of the three types of construction projects under study, a standard construction time schedule, presented via a Gantt chart, was prepared. Cost estimates were prepared for the said projects, and average prices of work were defined based on five bids. By integrating the progress schedule and cost estimate prices, we obtain cost plans, i.e. theoretical "S" curves for all three types of structures (absolute values of "t" and "v" are reduced to the scale of 100/100 for simplicity of calculation and to enable comparison). The standardized plans assume competent project management during realization, which also implies minimum expected negative risk action ranging from 0 to 5 % for time, and amounts to 0 % for costs.

According to analyses [2], mathematical expressions for S-curves of completed structures can be presented as follows:

- the following logistic curve (S-curve) proved to be the most favourable for the standardized model of a building (GF+4):

\[ y_{\text{GF+4}} = \frac{1}{1 + \left(0.612158x(0.919930)^x\right)} \]

- the best results were provided by the fifth degree regression polynomial for the standardized tunnel construction model:

\[ y_{\text{tunnels}} = 0.2397713305 - 0.7117306856 x + 0.1250470242 x^2 - 0.0049603919 x^3 + 0.00000905844 x^4 - 0.00000007270 x^5 + 0.0000000021 x^6 \]

3. Integrated procedure for defining prognostic S-curves of cost-time relationships

Mathematical expressions for the definition of cost-time relationships can be used in the modelling of prognostic S-curves for various types of structures (buildings, tunnels, motorway sections). In this study, the results were obtained by integration of three different approaches:

- in the first approach, models were defined based on proposals for presentation of theoretical cost-time relationships, as given in available literature,
In the second approach, the modelling was conducted by detailed analysis of the planned and realized cost-time relationships, for a number of completed construction projects. In the third approach, the modelling was conducted by preparation of standardized models for project groups, which resulted in standardized curves showing cost-time relationships.

In order to define final prognostic S-curves, individual results obtained by three different above-described procedures (S-curve models) must be integrated for each of the three types of structures under study. Final prognostic S-curve models for each of the three types of typical projects are determined in the last step of the integration procedure. An example for tunnels is shown in Figures 2. and 3.

Several statistical packages for data analysis, mostly SPSS and Statistica, were used in the solution finding process. As a basis for defining the final prognostic S-curve, three S-curves, obtained using three different approaches, were grouped for each of the three types of structures. The sum of squared deviations from original values was minimized using the regression analysis and the least-square method and, in this way, the most favourable regression model was determined. Best results were obtained by means of the sixth degree regression polynomial. Confidence intervals for \( p = 0.95 \), and the corresponding S-curve models, were defined in the last step. Depending on the type of construction project, mathematical expressions for standardized prognostic S-curves are:

For buildings:

\[
Y_{\text{buildings}} = -0.0643212823 + 0.562316845 \cdot x_1 - 0.0278540885 \cdot x_1^2 + 0.0016474856 \cdot x_1^3 - 0.00002349997 \cdot x_1^4 + 0.0000001165 \cdot x_1^5 - 0.0000000001 \cdot x_1^6 \tag{13}
\]

The corrected coefficient of determination amounted to 0.968. The standard assessment error was 1.02.

For tunnels:

\[
Y_{\text{tunnels}} = 0.0806494622 + 0.0216949007 \cdot x_1 + 0.0388645143 \cdot x_1^2 - 0.0006037768 \cdot x_1^3 + 0.0000086934 \cdot x_1^4 - 0.0000000909 \cdot x_1^5 + 0.0000000004 \cdot x_1^6 \tag{14}
\]

The corrected coefficient of determination amounted to 0.901. The standard assessment error was 1.027.

For motorways:

\[
Y_{\text{motorway}} = 0.0571888747 - 0.0645925803 \cdot x_1 + 0.0756879341 \cdot x_1^2 - 0.0029910596 \cdot x_1^3 + 0.0000570109 \cdot x_1^4 - 0.0000004830 \cdot x_1^5 + 0.0000000015 \cdot x_1^6 \tag{15}
\]

The corrected coefficient of determination amounted to 0.968. The standard assessment error was 2.4.

### 4. S-curve calculation example

The possibility of using results obtained during the study was tested on five examples for each type of structures under study. An example of calculation of prognostic S-curves for the construction of structures is given in the appendix for the following input data (Table 2):
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Type of structure under construction: building (GF+4), office-residential building, total gross area: 4,200 m².
Estimated time of construction: 13 months
Estimated cost of construction: HRK 24,676,320
Start of construction: 1 April

The data for the input variable x must be prepared in the first step, i.e. this variable must be reduced to the percent value as related to the total number of available days. The estimated Y value, and the bottom and top confidence limits, were defined for the input variable X using the Statistica program package. Once these data are translated into absolute values using study results presented in Section 3, the real amounts of the prognostic S-curve are calculated, and the top and bottom limits of assessment are given, with the 95 % probability (Table 3, Figure 4.).

5. Conclusion

The objective of this study was to define prognostic S-curves for typical types of construction projects so as to enable simulation and prognosis of project realization cost dynamic already in earliest project design phases, based on a small number of input parameters. The study was made in four steps during which S-curves were proposed for selected types of structures, based on consultation of international and domestic literature, analysis of data from completed projects, and plans prepared by expert planners. The integration of these three approaches resulted in definition of a prognostic S-curve of cost dynamics (scheduling) during the project realization phase, for three types of structures.

In all three cases, the sixth degree polynomial regression was proven to be the best prognostic model for integration of results from the three above described procedures. The proposed methodology is based on the definition of the prognostic cash flow model with best assessment, bottom limit assessment, and top limit assessment. The top and bottom limits show 95 % reliability limits in relation to cost and time.

The results obtained in this study can be used for an early prognosis of costs to be incurred during realization of a project. The proposed formula enables rapid and simple analysis, and the output lies within the range presented in international research and domestic practice. The proposed methodology can also be used to make prognostic curve models for other types of structures. However, in such a case
the potential user has to repeat a part of the procedure, use his own input data about completed structures, and engage experts for creation of standardized curves. The proposed methodology depicts a prognostic cost curve for good to average project realization conditions. It is obvious that breakdowns or stoppages, influences of considerable risks, poor organization of work, frequent changes, etc., will greatly modify the form of a prognostic curve, in which case each project get a specific dynamics, represented by an irregularly shaped S-curve [14].

REFERENCES