

QUALITY TEST OF INTERPOLATION METHODS ON STEEPNESS REGIONS FOR THE USE IN SURFACE MODELLING

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Surface modelling has been a widely used methodology for interdisciplinary facilities in all kinds of earth-related studies. There are many interpolation methods applied for model generation using the measured points, samples, on the ground. The quality of the outcomes of an interpolation method is highly related to the accuracy, quantity, and distribution of the selected samples reflecting the topography within the study area. This study aims to examine the quality of four interpolation methods, namely the methods Kriging, Modified Shepard's, Inverse distance weighting, and Radial Basis Function, considering height differences between the neighbour stations. To check the quality of height components within the study area derived applying different interpolation models, four artificial surfaces with sudden height changes were created. The standard deviations used for comparison of the quality of interpolation models were determined using differences between the height values of control points. These values were set as true and interpolated values for the same points.

Keywords: digital elevation model, earth topography, high resolution observed data, interpolation methods, surface modelling

Test kvalitete metoda interpolacije na strmim predjelima za uporabu u modeliranju površine

Izvorni znanstveni članak

Modeliranje površine je široko korištena metodologija u svim vrstama istraživanja vezanih za tlo. Postoje mnoge metode interpolacije koje se koriste za razvijanje modela uporabom mjernih točaka, uzoraka, na tlu. Dobivena kvaliteta određene metode interpolacije uvelike ovisi o točnosti, količini i raspodjeli izabranih uzoraka koji odražavaju topografiju proučavanog područja. Cilj je ovoga rada ispitati kvalitetu četiriju metoda interpolacije, odnosno metoda Kriging, Modified Shepard's, inverzno ponderiranje udaljenosti i Radial Basis Function, uzimajući u obzir visinske razlike između susjednih mjesta. Za provjeru kvalitete visinskih komponenti u okviru proučavanog područja, dobivenih primjenom različitih modela interpolacije, kreirane su četiri umjetne površine naglih promjena visine. Odstupanja od standarda korištena za usporedbu kvalitete modela interpolacije određena su primjenom razlika između vrijednosti visina na kontrolnim točkama. Te su vrijednosti uzete kao prave i interpolirane vrijednosti za iste točke.

Ključne riječi: digitalni model elevacije, metode interpolacije, modeliranje površine, promatrani podaci visoke rezolucije, topografija tla

1

Introduction

The physical surface (topography) of the solid earth including the oceanic and continental lithospheres is a body of 3D nature. In most studies related to the description of topography of the solid earth in a curvilinear or Cartesian coordinate system a height value is assigned to each pair of horizontal coordinates of a Riemannian 2D space [1]. Therefore, Digital Elevation Models (DEMs) are a crucial instrument for modelling the topography, as part or whole, of the earth's surface. The outcomes of modelling should enable a variety of applications in earth and environmental sciences like Geographic Information System (GIS), infrastructure design, and engineering works including modelling of hydrology, water flow, and ground deformation [2 ÷ 7]. A digital elevation model defines the surface mathematically, usually determined for points of a regular area wide grid, applying a specific interpolation method to a set of selected uniform or non-uniform samples available within the study area [7]. The reached accuracy of a digital elevation model mainly depends on the quality, density, and distribution of the selected samples within the study area as well as on the mathematical model applied for interpolation of samples. The criteria for choice of the grid and mesh spanning are another important factor, which influences the accuracy of a DEM solution. The boundary of grid spanning must be in accordance with the samples available, in order to avoid any extrapolation [3, 7 ÷ 11]. Accordingly, surface modelling studies have become significant issue for

planning earth related facilities as well as being a helpful tool to prepare future network plans over them. Surface modelling process requires a wide criteria combination including adequate distribution of the selecting sample points, number and settlement of the control points, grid intervals and interpolation method [12, 13]. In order to generate surface as closely as the real object properties, mathematical functions should be used to represent the features to obtain required surface quality based on these criteria. With the most widely used name, it is called as interpolation method and this method is involved in several stages of modelling procedure. Interpolation methods can be categorized according to different criteria and they can be used for different purposes [7].

As stated in Gong et al. [3], in the previous researches, numerous investigations have been conducted both for theoretical analysis and experimental testing [14] to assess the quality of DEMs. In this experimental study, to have a better understanding of digital elevation model generation on steepness regions, interpolation methods used for terrain modelling have been investigated through the created artificial areas, where the sudden height differences were observed. Four artificial surfaces have been created by means of deterministic surface function configurations and regions of interest have been bound by differential heights. Each sub region was investigated by four different interpolation methods, namely Kriging, Modified Shepard's, Inverse Distance Weighting (IDW) and Radial Basis Function methods. The quality of the generated surface models was studied using the method of cross validation and the reference point settlement was produced by Jack-Knife method [15, 16, 17]. The reached

standard deviations of the models were then compared to each other depending on the height differences to study the quality of the applied methods.

2

Methodology

2.1

Interpolation Techniques

Interpolation is an approximation problem in mathematics and an estimation problem in statics [7]. Interpolation in digital terrain modelling is used to predict the unobserved point heights by using observation values of neighbouring points from distributed sampling reference points. There have been two common assumptions behind interpolation methods: (1) the terrain surface is continuous and smooth, (2) there is a high correlation between the neighbouring data points [7].

Interpolation methods can be classified in many ways including local/global, exact/approximate and deterministic/stochastic (geostatistical) methods [18]. The various methods can be used for interpolation of elevation such as, Inverse Distance Weighting (IDW), Kriging, Minimum Curvature, Neighbour, and Radial Basis Functions (RBF), Triangulation Moving Average, Local or Global Polynomial. Selected interpolation models must be realistic and well adjusted to the topographic structure of the surface [9, 19 ÷ 22].

In this study, four interpolation methods, namely Kriging, Modified Shepard's, IDW and RBF were used for estimating unknown point elevations.

2.2

Control and comparison of the produced DEM accuracy

To give a decision on an appropriate interpolation method depending on the surface characteristics, each method was investigated by applying it to the four different regions. The accuracy of the models and the results were analyzed for the given interpolation methods using Jack-Knife procedure. The differences between interpolated and known height values of the reference points were calculated and standard deviations computed from the interpolated and known heights of the surface models were used for assessing the quality of the interpolations. All DEMs were generated in 1×1 m grid interval to avoid effects of grid size.

In order to control the quantity of the generated surfaces, a portion of the original data set for an independent evaluation of the model was put aside. The heights of all points in the domain of x and y have been calculated using functions. Re-sampling of the reference points has been selected through this generated dataset, which was 5 % of the original data. However, the randomly selected points did not directly reflect the original data form, and then additionally 25 reference points were selected, where the height changes are sudden. For each created surface, totally 75 points served as reference points, and their heights were compared to the interpolated heights of four surfaces, namely (A) Kriging, (B) Modified Shepard's, (C) Inverse distance weighting and (D) Radial Basis Function. The absolute values of the differences were statistically analyzed.

For each test area, the standard deviation of each deterministic generated surface model was calculated by using height differences (d_i) between reference point and interpolated height of the same point for each relevant interpolation model, separately.

$$d_i = h_i - h'_i, \quad (1)$$

where h_i and h'_i are the reference height and the interpolated height, respectively.

We obtain the mean of height differences of points with

$$\mu = \frac{\sum d_i}{N}, \quad (2)$$

where μ expresses the mean of height differences of points and N gives the total number of used data points.

Considering the equations (1) and (2), we obtain the standard deviation σ of interpolated points derived applying a certain interpolation model:

$$\sigma = \sqrt{\frac{\sum (d_i - \mu)^2}{N - 1}}. \quad (3)$$

Accordingly, the average deviation of the points can be obtained from Eq. 4 given as follows:

$$\sigma_{ave} = \frac{\sum_{i=1}^N |d_i - \mu|}{N}. \quad (4)$$

2.3

Test areas

2.3.1

Artificial surfaces

The following deterministic functions (Fn) have been used to create artificial surfaces in the domain $x = [-50, 50]$ and $y = [-50, 50]$ from uniform distributed values.

$$\text{Fn (1): } z_1 = \max(\alpha \cdot (-0,01 \cdot x^2 - 0,01 \cdot y^2), 0) \quad (5)$$

$$\text{Fn (2): } z_2 = \max(75 - 0,04 \cdot x^2 - 0,04 \cdot y^2, 0) \quad (6)$$

$$\text{Fn (3): } z_3 = \left(\cos x + \sin\left(\frac{x}{10}\right) - \sin\left(\frac{y}{10}\right) \right) \cdot 5 + 20 \quad (7)$$

$$\text{Fn (4): } z_4 = \max(75 - x^2 - 0,02 \cdot (y - 10)^2, 0). \quad (8)$$

By using the above functions, 1000 points have been created. The reason of choosing them is to obtain different surface morphology in terms of capturing height changes to compare and select the appropriate interpolation methods. The shape of the surfaces created from Eq. (5), Eq. (6), Eq. (7) and Eq. (8) can be seen in Fig. 1. From Eq. (5), four different study regions were obtained in order to examine the height changes. The slopes of the peak and the bottom heights have been configured by a variable as so-called α , and the value of α alters

depending on the height differences, that is, as generated by sub cases. Depending on Eq. (5), the height differences in regions have been settled between 0 and 8 m ranges, and four artificially created regions have been defined. Each region has two different slope values, which increase the steepness of the terrain topography. From the other equations, complex surfaces that may represent near-real world topography were defined. The shapes of the surfaces can be listed as valley, con, randomly changed rough surface and narrowed con with respect to Eq. (5) to Eq. (8).

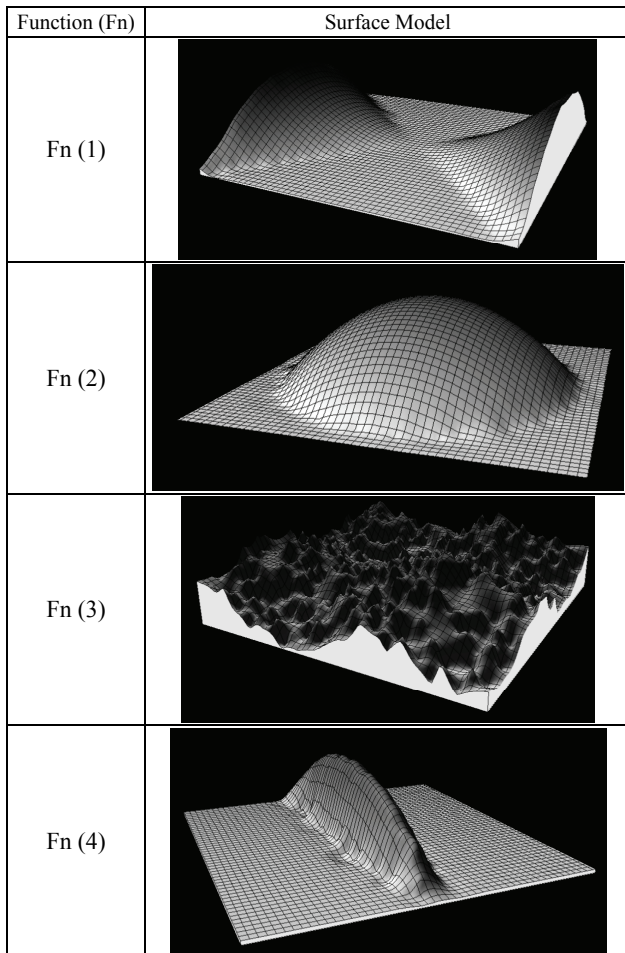


Figure 1 Illustration of the artificial surfaces derived by Eq. (5) to Eq. (8)

Illustrations of the study regions can be seen in Fig. 1. To test the accuracy of each interpolation technique for each sub area, the distribution of control points was selected by using Jack-Knife method and to check the quality and precision of the generated surfaces, 5 % of the total calculated points were selected randomly. These points served as reference points and their height values are stored to achieve a comparison with the heights of generated surfaces. In order to enhance statistical quantity of the generated surfaces, the standard deviation values of the differences between estimated height values and heights of reference points were used. The statistical results of the models were compared to each method depending on the height differences.

As seen from Eq. (5), four cases have been generated and the only value changed here is the heights of the surface points. Thus, it is aimed to determine the

performance analysis of the interpolation methods for the fixed located and scattered points. The distributions of the points are illustrated in Fig. 2. The first test area (created by Function (1)) consists of 975 surface points and 75 reference points, and the average height difference of the areas is 0,2 m; 0,6 m; 1,02 m and 1,43 m with respect to α values 0,05; 1,50; 2,50 and 3,5.

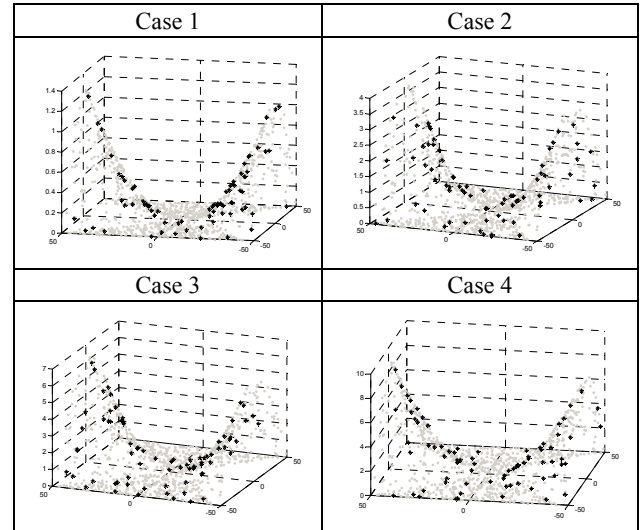


Figure 2 Creation of the artificial surface 1, Fn (1), for $\alpha = 0,05$ to 3,5, the slopes range from 2 % and 35 % with reference and ground points distributions (reference points are marked bold and ground points are marked lighter colour).

Table 1 Statistical information of the surfaces created by Fn (1), Eq. (5)

Function (Fn)	Case #	Min. / m	Max. / m	Mean / m
Fn (1)	1	0	1,222	0,204
	2	0	3,666	0,613
	3	0	6,111	1,021
	4	0	8,555	1,430
Fn (2)		0	74,631	23,601
Fn (3)		5,158	34,474	19,962
Fn (4)		0	74,631	33,048

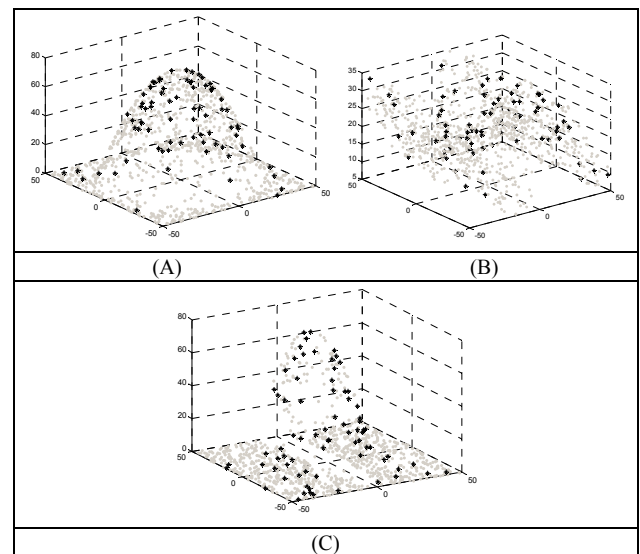


Figure 3 Creation of the artificial surfaces from (A) Fn (2), (B) Fn (3) and (C) Fn (4) (reference points are marked bold and ground points are marked lighter colour)

As seen in Fig. 2, distribution of the reference points has a randomly scattered pattern and in addition to this,

on the steepness sections of the area, additional reference points were selected and added into the data set for controlling the quality of the DEMs. This procedure was also applied to the other surfaces. Different to other surfaces, in the third surface model, the additional reference points were selected randomly rather than selecting on the steepness regions (see Fig. 3) in order to accommodate the surface topography. Tab. 1 presents the height characteristics of the created surfaces. Here, Functions (2) and (3) have similar height values, but they differ on the shapes of the surfaces by narrowing both sides of the con.

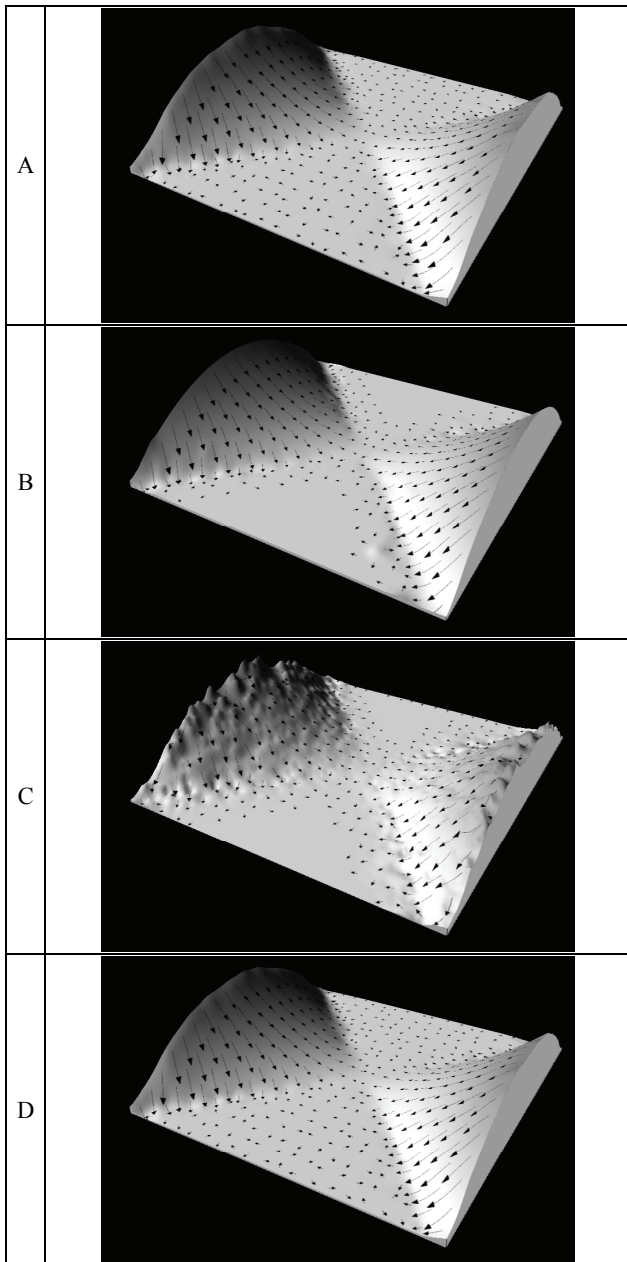


Figure 4 Interpolation of artificial surfaces produced by Fn (1) (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

3 Results

The evaluations of the DEM accuracies of the characteristic terrains derived from artificial surfaces are given in Tabs. 2, 3, 4 and 5 respectively.

The results for the surface model derived from Function (1) were investigated in four sub cases, namely; Cases 1, 2, 3 and 4. In Tab. 2, the results of the Function (1) are represented. Tab. 2 indicates that the Kriging and the RBF interpolation methods are the most accurate methods in Case 1. In Case 2, Modified Shepard's and RBF methods are the most accurate methods and in all cases of α and the accuracy obtained by IDW interpolation method is the lowest, which can be also traced in Fig. 5. In the other cases of Function (1), Kriging and RBF methods also give similar solutions. When comparing Case 4 with the other cases, it can be noted that the accuracy of DEM has become lower when height change is sudden relative to the other cases and the standard deviation of Case 4 has the highest value. Referring to Fig. 4, IDW presents a continuous surface but it is not smoothed with undulations.

Table 2 Statistical results (in meter) between the function given by Fn (1) and (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

	A Kriging	B Modified	C Inverse	D Radial
Case 1				
Mean	-0,000	-0,001	-0,001	-0,000
Min.	-0,030	-0,022	-0,091	-0,024
Max.	0,055	0,006	0,214	0,044
Stand. dev.	0,010	0,005	0,042	0,008
Ave. dev.	0,004	0,002	0,021	0,004
Case 2				
Mean	-0,010	-0,002	-0,002	-0,008
Min.	-0,228	-0,159	-0,411	-0,210
Max.	0,038	0,100	0,447	0,052
Stand. dev.	0,036	0,026	0,121	0,033
Ave. dev.	0,019	0,009	0,074	0,016
Case 3				
Mean	-0,007	-0,004	0,016	-0,005
Min.	-0,187	-0,177	-0,454	-0,167
Max.	0,049	0,074	0,402	0,074
Stand. dev.	0,032	0,027	0,137	0,029
Ave. dev.	0,016	0,011	0,088	0,013
Case 4				
Mean	-0,013	-0,000	0,018	-0,012
Min.	-1,113	-0,163	-1,197	-1,113
Max.	0,336	0,149	1,245	0,287
Stand. dev.	0,141	0,039	0,283	0,138
Ave. dev.	0,045	0,015	0,164	0,041

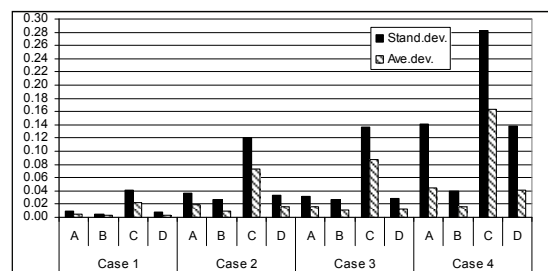


Figure 5 Standard deviations (m) and average deviations (m) of derived DEMs from Fn (1)

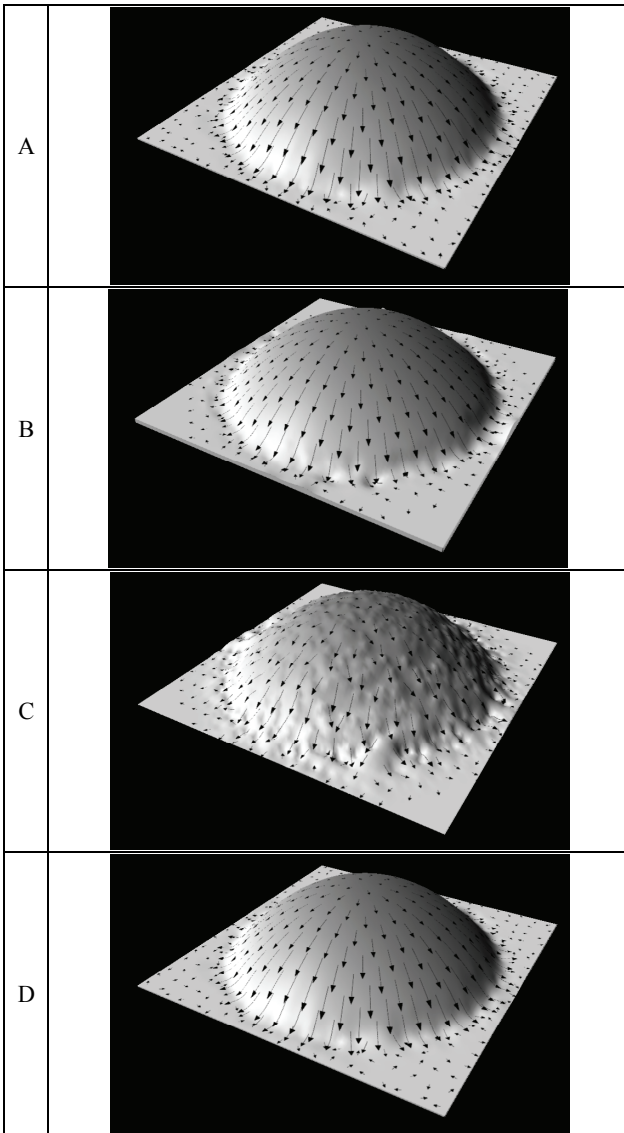


Figure 6 Interpolation of artificial surfaces produced by Fn (2) (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

Table 3 Statistical results between the function given by Fn (2) and (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

	A Kriging	B Modified	C Inverse	D Radial
Mean	-0,009	-0,043	0,626	-0,020
Min.	-2,443	-1,428	-6,620	-2,043
Max.	0,815	0,286	3,528	0,485
Stand. Dev.	0,464	0,254	1,782	0,360
Ave. dev.	0,194	0,111	1,228	0,149

The same investigation for Functions (2), (3) and (4) were performed by applying the interpolators. It can be noticed that similar solutions were obtained where sudden height changes occurred over the test areas. Referring to Fig. 9 it may be noticed that only one DEM derived from Function (3) gives similar solutions in all cases of interpolations. Although IDW presents the same statistical results, the smoothness of the produced surface is weak (Figs. 6, 7, 8). Besides, at the end of the hillsides of the surfaces, Modified Shepard's method gives sparse regions because of creating a range of values in the grid area rather than a single power point.

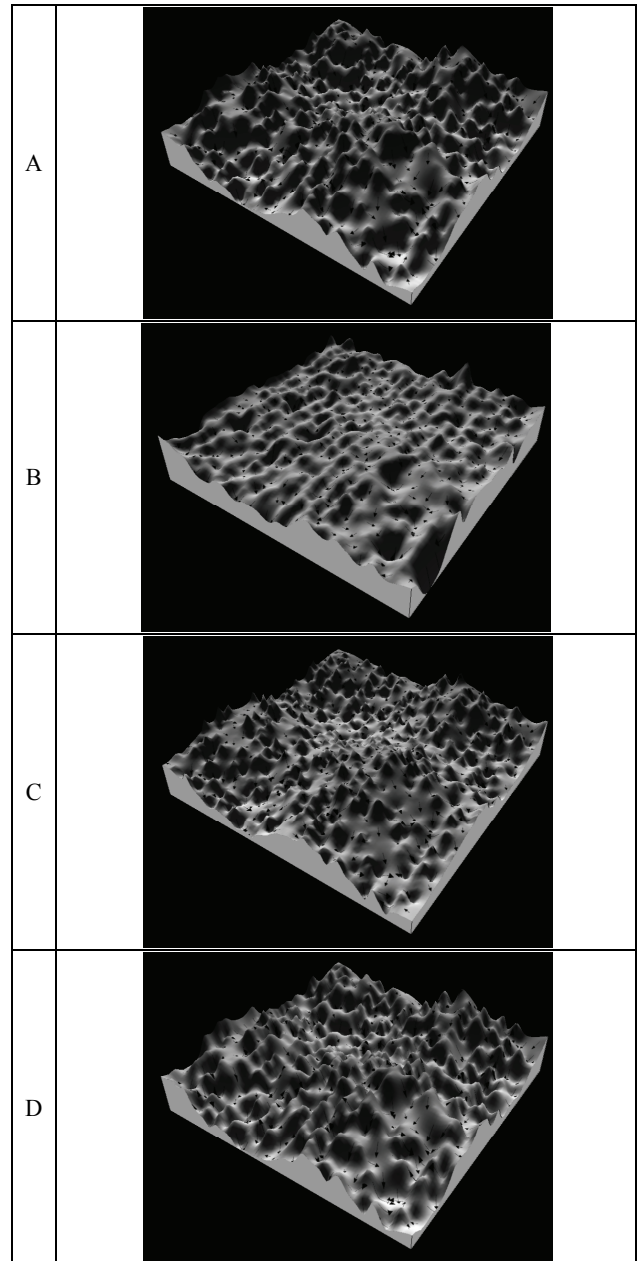


Figure 7 Interpolation of artificial surfaces produced by Fn (3) (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

In Tab. 3, the unique hill landscape form was investigated by using Function (2). The results indicate that the Modified Shepard's interpolation method supplies the most accurate DEM. The second most accurate ones are RBF and Kriging methods, respectively. Similar to the Function (1), IDW approach gave the highest standard deviation value and the constructed surface was irregular.

When comparing interpolation methods on randomly distributed partly hilly or mountainous surfaces (Fig. 7), IDW gave the lowest standard deviation value and Kriging and RBF followed this result (Tab. 4). Although IDW shows an accurate solution, by considering the form of the surface feature, Kriging and RBF offer more realistic solutions.

The last comparison was performed for Function (4), which has a narrowed con shape and can be detailed as a mountain range. The results presented in Tab. 5 show that IDW produces the highest standard deviation value, since

the surface detail failure can be seen in Fig. 8C. The previous higher standard deviation values are obtained from RBF, Modified Shepard's and Kriging methods, respectively.

Table 4 Statistical results (in meter) between the function given by Fn (3) (A) Kriging (B) Modified Shepard's (C) Inverse Distance to a power (D) Radial Basis Function

	A Kriging	B Modified	C Inverse	D RBF
Mean	0,358	0,489	0,571	0,228
Min.	-7,830	-15,245	-5,622	-9,137
Max.	7,094	14,344	6,486	8,453
Stand. dev.	3,459	4,536	3,294	3,414
Ave. dev.	2,690	3,106	2,760	2,530

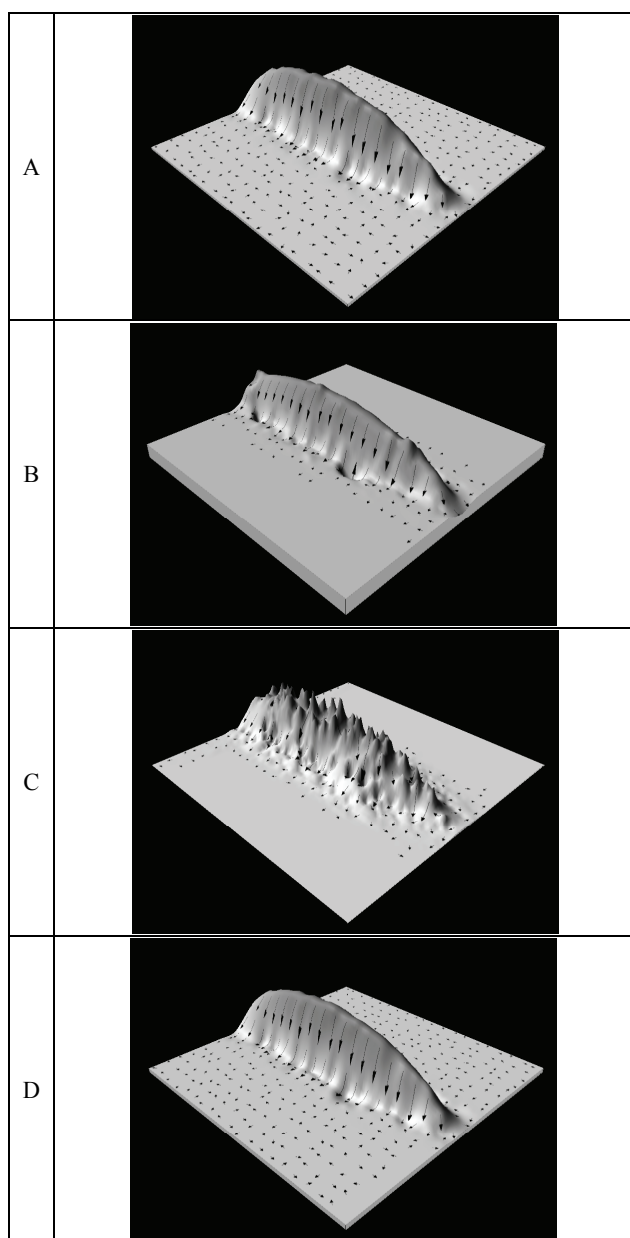


Figure 8 Interpolation of artificial surfaces produced by Fn (4) (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

Additionally, to make visual analyses of the derived DEMs, the perspective views of them were compared to the original data sets. The best form for representing topographic features of the test areas were obtained by

RBF and Kriging interpolation techniques. The IDW performs inaccurate surface details that differ from the original dataset.

Table 5 Statistical results between the function given by Fn (4) (A) Kriging (B) Modified Shepard's (C) Inverse Distance Weighting (D) Radial Basis Function

	A Kriging	B Modified	C Inverse	D Radial
Mean	0,463	-0,029	4,059	0,254
Min.	-8,416	-8,096	-15,568	-7,353
Max.	6,857	7,198	24,890	6,261
Stand. dev.	2,588	2,184	8,781	2,116
Ave. dev.	1,624	1,050	7,022	1,205

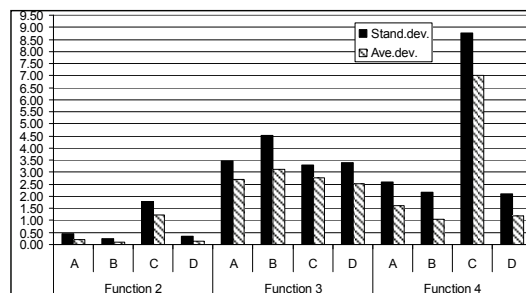


Figure 9 Standard deviation (m) and average deviation (m) of derived DEMs from Functions 2, 3 and 4.

4 Conclusion

Digital elevation models, which have significant importance for all kinds of planning, construction work, visualization, mapping, etc. facilities, should be taken into account with the surface modelling parameters in representation of the partial or whole of the earth surface. For high accuracy and more precise solutions in surface modelling, the landform types should be investigated separately due to the height changes of the points, which represent the surface characteristics or where the surface characteristics occurs.

It can be concluded that the landscape features of the terrain models affect the quality of the derived DEMs by means of model interpolators. Herein, four test areas have been created to compare and validate the interpolation methods as compatible with the original data sets. To understand the influence of the interpolation methods to surface modelling on steeper hill slope regions, topographic features of landscapes must be considered along with the interpolator characteristics.

In the presented experimental study, four interpolation methods have been examined depending on the created artificial surfaces. The selection of the features of the surface equations represents the rough regions in which the steepest of the surface topographies show an altered frequency. This study reveals that a comparison of the interpolation methods with terrain modelling procedure should be considered in order to determine the validity of the models classified by the terrain morphology. Results indicate that interpolation methods such as Kriging and RBF give more effective solutions where the heights change suddenly and significantly, but if the height differences are relatively lower in the study area, the standard deviations give similar solutions with respect to each interpolation methods.

5

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