This article focuses onto the process of fire-resistant coating production. That process belongs to the class of processes, considered statistically, which have only one, lower specification limit. Such processes are interesting in that they usually do not have the accompanied normal distribution. In order to bring the process under control, we monitored the fire-resistant coating production using the $\bar{X}$-R control chart. The process capability analysis was applied on one set of data analyzed using three different methods. In the first method the data were approximated using the 3-parameter Weibull distribution. In the second method, data were transformed using the Johnson transformation while in the third method, applied for the purpose of comparison of sets, data were approximated with normal distribution. In the accompanied experiment, the thickness of the fire-resistance coating was measured on seven steel girders. The results have been analyzed using the trial version of the Minitab software package. Using the Monte Carlo method, numerical simulations were performed in order to estimate the uncertainty of measurements of coating thickness.

Keywords: fire-resistant coating, measurement uncertainty, non-normal data, SPC
describes tolerance field range with reference to actual data dispersion, while \( C_{pk} \) index defines the process position with reference to requirement limits. Process capability indices are given with the following expressions:

\[
C_p = \frac{(USL - LSL)}{6\sigma_{within}},
\]

\[
C_{pk} = \min \left( \frac{(USL - \bar{x})}{3\sigma_{within}}, \frac{(\bar{x} - LSL)}{3\sigma_{within}} \right),
\]

where:
- \( USL \) – upper specification limit
- \( LSL \) – lower specification limit
- \( \bar{x} \) – arithmetic mean (central line of the control chart)
- \( \sigma_{within} \) – within-subgroup standard deviation.

Using (1) and (2), the standard deviation is estimated on the basis of data from control chart. Various control charts are used for detection of variations in the process and determining the amount of process standard deviation.

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Process standard deviation, is estimated from all data. Overall capability, on the other hand, is what the customer experiences; it accounts for the differences between subgroups. Capability indices that assess overall customer experiences; it accounts for the differences in process variation.

In the paper the Johnson and Box-Cox data transformation is also applied.

### 2.1 Process capability using a non-normal distribution

There is no generally accepted calculation of process capability indices in non normal distribution models [4]. If the data are skewed the estimated proportion of defective items may be extremely over or under estimated. In that case, it is better to either transform the data to make the normal distribution a more appropriate model, or choose a nonnormal probability model for the data.

Well-established data transformation systems are Box-Cox and Johnson transformations. Data are transformed using appropriate function so that transformed values are in line with the established model for normal distribution. These systems enable transformation of all major types of continued distributions to a normal distribution. Apart from data, it is also necessary to transform specification limits. Transformations today are extremely accessible, since they are built into practically all statistical programs.

If the data being analyzed are not normally distributed, \( C_{pk} \) will not be provided because it is based on the \( z \) formula [5]. The values of \( P_p \) and \( P_{pk} \) are not obtained based on the mean and the standard deviation but rather on the parameters of the particular distributions that the observations follow. If we elect to use normal option for the process capability analysis and the normality assumption is violated because the data are skewed in one way or another, the resulting values of \( C_{pk} \), \( C_p \), \( P_p \), \( P_{pk} \) would not reflect the actual process capability.

In this paper we have used the ISO method which is one of the two methods Minitab offers to calculate process capability statistics for nonnormal processes. This method, recommended by the International Organization for Standardization, calculates capability statistics from the 0.135; 50.0 and 99.865 percentiles of the specified distribution to model the data. The portion of the data distribution between the 0.135th and 99.865th percentiles corresponds to the 6σ spread calculated in capability analysis for normal data.

The ISO method calculates capability statistics as follows:

\[
P_p = \frac{\bar{x}_{0.5} - LSL}{x_{0.99865} - x_{0.00135}} = \frac{T}{6\sigma},
\]

\[
P_{pk} = \min \left( \frac{(USL - \bar{x}_{0.5})}{x_{0.99865} - \bar{x}_{0.5}}, \frac{(\bar{x}_{0.5} - LSL)}{\bar{x}_{0.5} - x_{0.99865}} \right),
\]

where:
- \( x_{0.99865} \) – the 99,865th percentile of the specified distribution to model the data.
- \( x_{0.00135} \) – the 0.135th percentile of the specified distribution to model the data.
- \( \bar{x}_{0.5} \) – the 50th percentile, or median, of the specified distribution to model the data.

In the paper the Johnson and Box-Cox data transformation is also applied.

### 3 Experimental part

Primary function of a fire-resistance coating on steel load-bearing elements is to prevent heating up of basic material as long as possible up to 500 °C. Therefore the coating functions as a heat insulator, i.e. provides heat insulation. Fire-resistance coating in its chemical composition represents antioxidants that expand, i.e. it increases its volume when affected by fire, whereby it rejects and keeps away the heat shock from the structure surface in the course of fire. Its volume can increase more than 100 times. Fire-resistance coatings enable the surface to be protected for 30, 60 or 90 minutes, depending on requirements [6]. In order to ensure protection for the required number of minutes, the coating needs to be applied in accordance with standards. If standards are followed, the coating should protect the structure during the chosen period.

In order to estimate the quality of applied fire-resistance coating, its production process was monitored using \( \bar{x} - R \) control chart, and the process capability analysis was subsequently conducted.

As a part of experiment we measured the thickness of fire-resistant coating on seven steel girders. Thickness of every girder was measured 125 times. Fire-resistance
coatings’ thicknesses were checked using Elcometre – 456, a device for measuring dry film thickness. Minimal required coat thickness on steel bearing elements was 2.65 mm. Elcometer works by inducing eddy current in metal surfaces. Process capability of fire-resistance coating application was estimated on the basis of results obtained by measuring overall thickness of fire-resistance coating. The results have been analyzed using trial version of Minitab statistical software package. In this paper uncertainty of measurement results of thick coating was also estimated.

The process considered has only one, lower specification limit, \( LSL = 2.65 \) mm. Due to their nature, such processes often do not have normal distribution [7]. The normality of data can be tested in several ways. In this article the normality of data was tested using probability plot (Fig. 1). It was established that the data are not normally distributed (\( P \) value < 0.005). The graph itself shows that the data are not normally distributed for a confidence interval of 95%. A significant portion of dots are scattered outside confidence limits. Moreover the Anderson-Darling null hypothesis for normality yielded an infinitesimal \( P \)-value of less than 0.005. Therefore we concluded that the data were not normally distributed. Since the data were not derived from normal distribution, they were compared to 14 distributions and the Johnson and Box-Cox data transformation was performed.

Minitab uses Anderson-Darling statistics to perform Goodness-of-fit test. In addition to graphs, there is also \( AD \) (Anderson-Darling) value and appropriate \( P \)-value for each individual distribution. \( P \)-value that exceeds defined significance limits and in this case amounts to \( \alpha = 0.05 \) indicates that the data are in line with this distribution.

In this case the data are well approximated by 3-parameter Weibull distribution, thus Johnson transformation can be applied onto them. Due to the nature of the data, other distributions and Box-Cox transformation did not give satisfactory results.

In order to bring the process under control, it was monitored using \( \bar{x} \)-\( R \) control chart (Fig. 2).

All points fall within the bounds of the control limits, and the points do not display any nonrandom patterns so we can say that the process is in control.

In this article the process capability analysis was conducted on one set of data, which were first approximated using the 3-parameter Weibull distribution, secondly transformed using the Johnson transformation and in the third approach, applied for comparison, data were approximated using normal distribution (Figs. 3a + 3c).

If the Weibull distribution process is estimated using process capability indices based on normal distribution, indices will be \( C_p = 1.42 \) and \( P_{pk} = 1.44 \) (Fig. 3a). If we conduct a process normal capability analysis, we will obtain a \( C_p \) and PPM calculated based on the normal z-transformation. Because the z-transformation cannot be used to calculate a process capability for nonnormal data
unless the data have been normalized, the results obtained would be misleading.

Calculation and diagram for the process capability based on the Weibull distribution model can be seen in Fig. 3b. For the Weibull distribution model only preliminary capability coefficient has been calculated, where \( P_{pk} = 2,15 \).

![Figure 3c Process Capability – Johnson Transformation](image_url)

Johnson Transformation (Fig. 4c) optimally selects one of the three families of distribution: SB, SL, and SU, where B, L and U refer to the variable being bounded, lognormal and unbounded respectively. The selected distribution function is then used to transform the data to follow normal distribution. Using the Johnson transformation we obtained data on preliminary process capability coefficients and based on diagrams it can be stated that there is a significant difference in results among data that primarily depend on the calculation method used. Such results may lead to misleading conclusions regarding the process capability, i.e. expected number of defects in the process.

When estimating capabilities for non normal sample data it is very important to first approximate these data using the distribution proven the most appropriate in the AD test. If there is possibility to choose among several distributions, it is necessary to choose the one with higher \( P \)-value \([8, 9]\).

4 Estimation of measurement uncertainty

In the process of the coating thickness measurement there are numerous values that significantly impact the uncertainty of measurement. The main sources of the uncertainty that contribute to the uncertainty of the measurement are listed as follows:

- Measured instrument used in the measurement process
- The standard for instrument fine tuning
- The repeatability and the renewability of the instrument positioning
- The geometry of the surface of the measured subject (the curve of the surface, the deviations of the flatness, harshness)
- The impact of the temperature.

The above stated main results can be expressed in algebra way, and they can be combined between themselves, for the purpose of obtaining the math model that describes the measuring. Generally, the uncertainty is calculated for a very specific measurement procedure. The specificity of the measuring procedure and the factors of impact must be uniquely defined before the determination of uncertainty.

4.1 Math measurement model

Mathematical model which incorporates all important values in the measurement process is:

\[
d = d_x + \delta d_a + \delta d_c + \delta d_m + \delta d_i + \delta d_p + \delta d_t.
\]

Where are:

- \( d \) – real coating thickness, µm
- \( d_x \) – measured coating thickness, µm
- \( \delta d_a \) – marginal instrument error
- \( \delta d_c \) – the correction for standard impact
- \( \delta d_m \) – the correction for the surface geometry of the measurement subject
- \( \delta d_i \) – the correction for the reading impact
- \( \delta d_p \) – the impact of the repeatability of the positioning
- \( \delta d_t \) – temperature impact.

Calculation of the uncertainty of measurement was done using the Monte Carlo method \([10]\). Probability density function of output value \( g(d) \) is shown in Fig. 4.

![Figure 4 Probability density function of output value g(d)](image_url)

Output value \( d \) is located inside the interval:

\((Y_{0.025} = 4,37 \mu m; Y_{0.975} = 4,41 \mu m)\) with \( P = 95 \% \).

Based on the evaluation of the uncertainty in measurement the extended uncertainty \( U \) in measurement has been determined in the procedure of coating thickness measuring in the amount of:

\[ U = 22 \mu m; k = 1,8; P = 95 \%, \]

where are:

- \( k \) – coverage factor
- \( P \) – probability.
Measured result cannot be described with only one value, as in reality there are many sources of uncertainty. Measured result is complete only if it contains the value assigned to the measured value and the uncertainty of the measurement assigned to that value. For the successful evaluation of the uncertainty of measurement the most important is the close connection of the math modelling measuring system and the measuring itself, or doing the experiment in the manner that all significant impacts on measured uncertainty are varied [11]. It is simple to conclude that the precision of each measurement is reflected in the evaluation of the uncertainty in measurement.

5 Conclusion

Failure to grasp nonnormality properly, often brings about decision fallacies and furthermore results in losing confidence when it comes to process capability analysis. There are two approaches when dealing with data that are not normally distributed. Of importance is to identify and resolve the causes of nonnormality or use tools that do not have any difficulty dealing with nonnormal data. Proper defining of causes and sources of nonnormality is required in order to be able to timely undertake certain activities to correct them, when possible. In this article we dealt with problems of applying normal distribution too lightly.

Measured coating thickness by far exceeds lower limit, meaning that more paint has been used, which again represents an unnecessary cost. Due to the too thick coating, it is possible that solvents are not dissolved from it, thus the coating lacks elasticity and has a long drying period, resulting in its cracking.

Based on conducted analysis one may argue that a high quality measurement system is essential for the detection and monitoring of process variations.

6 References

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