DYNAMIC MODEL OF THE PROCESS OF CONTACT AND CONVECTIVE PAPER DRYING

Muhamed Bijedić, Husejin Duraković

Dynamic model of continual process of contact and convective paper drying is recommended, as well as numerical procedure for its solution. Mathematical basis of the model suggested is partial differential equation of unsteady heat transfer by conduction, in direction perpendicular to paper surface. The equation has been solved with reference to two boundary conditions, one for each side of paper web, by finite difference method with iterative improvement (method predictor-corrector). As model fitting parameters contact coefficient and effective thermal conductivity are used. These parameters had been changed until the best agreement was achieved among calculated and observed moisture contents and calculated values of heat consumed and moisture removed to corresponding quantities observed in the real industrial process.

Keywords: mathematical model, paper drying, contact drying, convective drying

Dinamički model procesa kontaktnog i konvektivnog sušenja papira

Predložen je dinamički model kontinuiranog procesa kontaktnog i konvektivnog sušenja papira, kao i numerički postupak za njegovo rješavanje. Matematička osnova predloženog modela je parcijalna diferencijalna jednadžba nestacionarnog prijenosa topline kondukcijom, u smjeru okomitom na površinu papirne trake. Jednadžba je zajedno s graničnim uvjetima, riješena metodom konačnih razlika (metod prediktor-korektor). Kao podešavajući parametri modela korišteni su kontaktni koeficijent i efektivna toplinska provodnost. Ovi parametri su mijenjani dok se nije postiglo najbolje slaganje između izračunatih i stvarnih sadržaja vlage u papiru i površinskih temperatura papirne trake. Model je potvrđen usporednjem vrijednosti dobivenih podešavajućih parametara s podacima iz literature, kao i izračunatih vrijednosti ušteda topline i izdvojenje vlage s odgovarajućim vrijednostima u stvarnom industrijskom procesu.

Ključne riječi: matematički model, sušenje papira, kontaktno sušenje, konvektivno sušenje

1 Introduction

Uvod

Drying section is not only critical sector in paper production, but also a big energy consumer. It is estimated that two thirds of overall energy requirements in paper production is related to its drying [1]. Computer simulation of paper drying section has proved to be useful tool, e.g. for optimizing of production and for better understanding of complex interaction of process parameters. Many models of various complexities have been published, from those simple ones using many empirical correlations, to those very complicated and elaborated [2,3].

In some models all transport processes in paper web are represented by complete parameters, e.g. by apparent thermal conductivity which includes not only conduction but also evaporation/condensation as well among the web. Approach with complete parameters gives very good results, while at the same time considerably decreases time of calculation comparing to the case where equations of mass and heat transfer have to be solved simultaneously [1]. In simpler models phenomena of internal mass transfer are not considered, but only evaporation at the surface of paper web.

Mathematical basis of the model is regularly equation of unsteady heat transfer by conduction. If heat transfer is regarded one-dimensional and perpendicular to paper web, then this equation may be solved using finite difference method [4]. Because only one-dimensional heat conduction is considered, continual paper web may be divided into small elements. By following step-by-step progression of such small element, in direction of the web movement, it is possible to simulate whole drying section of paper machine. During this procedure it is necessary to follow position of the element in order to apply corresponding boundary conditions.

In order to get reliable results by simulation program, comparisons have to be made between calculated and observed data at industrial paper drying machines. The following quantities are measured: the moisture content in paper web, the surface temperature of paper web, the moisture content in the air, the air temperature, and the cylinder surface temperature. Additional data are necessary, like machine velocity, steam pressures, basic gramage of paper, etc.

When experimental data are obtained for a paper-drying machine, simulation program has to be adjusted to the machine. For that purpose so-called fitting parameters are used. These parameters are changed until the best agreement between calculated and observed moisture contents and surface temperatures of paper web are achieved. Necessary precondition for fitting is that calculated initial and final moisture content in paper web correspond to experimentally measured values.
Temperature profile of paper web surface, being in contact with drying cylinder, is obtained by solving equation (1) with boundary condition given by equation (2). In order to obtain initial conditions for solving equation (1), temperature range of paper web, from the moment when it touches drying cylinder to the moment when it sets apart from drying cylinder, is divided into 100 temperature intervals $\Delta T$. In that way, 101 points with "known" temperature are obtained. In order to determine in which moment in time paper web reaches each particular temperature, equation (1) is rearranged into the form:

$$\Delta t = \Delta T \cdot \frac{\rho_e \cdot c_e}{\lambda_e \cdot \frac{\partial^2 T}{\partial z^2}}.$$  \hspace{1cm} (4)

where $\Delta t$ is time needed to paper web to warm up for temperature interval $\Delta T$.

Calculation starts by computing first temperature derivative with respect to paper thickness, $\left(\frac{\partial T}{\partial z}\right)_{z=0}$, at paper surface being in contact with drying cylinder, from boundary condition given by equation (2):

$$\left(\frac{\partial T}{\partial z}\right)_{z=0} = \frac{\alpha_{\text{conv}}}{\lambda_e \cdot \delta} \left( T_{p,0} - T_{c,d} \right),$$  \hspace{1cm} (5)

for each paper temperature, $T_{p,0}$. Then, temperature profile of paper web is calculated at thickness $\Delta z = 0.01 \delta$, from Taylor series truncated after first derivative:

$$T_{p,\Delta z} = T_{p,0} + \Delta z \cdot \left(\frac{\partial T}{\partial z}\right)_{z=0},$$  \hspace{1cm} (6)

where $\delta$ is paper web thickness. Afterwards, first temperature derivative with respect to paper thickness is calculated, at distance $\Delta z$ from paper surface being in contact with the cylinder, from equation:

$$\left(\frac{\partial T}{\partial z}\right)_{z=\Delta z} = \left(\frac{\partial T}{\partial z}\right)_{z=0} \cdot \frac{\lambda_{e,0}}{\lambda_{e,\Delta z}}.$$  \hspace{1cm} (7)

Now, there are enough data to calculate second temperature derivative with respect to paper thickness, $\left(\frac{\partial^2 T}{\partial z^2}\right)_{z=0}$, at paper surface being in contact with drying cylinder, from equation:

$$\left(\frac{\partial^2 T}{\partial z^2}\right)_{z=0} = \frac{\left(\frac{\partial T}{\partial z}\right)_{z=\Delta z} - \left(\frac{\partial T}{\partial z}\right)_{z=0}}{\Delta z}.$$  \hspace{1cm} (8)

When values of second temperature derivative with respect to paper thickness are obtained, corrected temperature profile of paper web is calculated at thickness
\[ \Delta z = 0.01 \delta \] from Taylor series truncated after second derivative:

\[ T_{p, \Delta z} = T_{p, 0} + \Delta z \left( \frac{\partial T}{\partial z} \right)_{z=0} + \frac{\Delta z^2}{2} \left( \frac{\partial^2 T}{\partial z^2} \right)_{z=0}. \] (9)

Then, temperature profile obtained from equation (9) is compared to temperature profile obtained from equation (6). If absolute difference between each individual temperature obtained from equations (6) and (9) is less than 10^{-5} calculation is continued by equation (4), but if it is not the case the procedure is repeated from equation (7), but now with temperature profile obtained from equation (9). This procedure is repeated until upper condition is satisfied. Afterwards, time is calculated from equation:

\[ \tau_i = \tau_{i-1} + \Delta \tau. \] (10)

If initial time is set to zero \( \tau_0 = 0 \), then after 100 steps final time \( \tau_f \) is obtained, for which particular element of paper web crosses over cylinder. Real time of retention of particular element of paper web is calculated from equation:

\[ \tau_i = \frac{l_i}{w}, \] (11)

where \( l_i \) is length of contact paper-cylinder, and \( w \) is velocity of paper web. If calculated time of retention, \( \tau_{en} \), and real time of retention, \( \tau_i \), differ it means that boundary condition from equation (2) is not satisfied, that is, corresponding heat flux cannot be transferred from cylinder to paper in given conditions. Since heat flux between drying cylinder and paper is function of contact coefficient, thermal conductivity, and temperature difference, then reason for disagreement between these times is just in them. However, temperature of drying cylinder is obtained by measurement, as well as temperature of paper web prior to and after cylinder. In that case assumed value of contact coefficient is not accurate enough, and exact value is obtained iteratively.

Namely, if calculated time of paper web retention is longer than real time it means that heat flux is not sufficient for paper web to warm up from initial to final measured temperature. Then, heat flux must be increased, and it is achieved by increasing contact coefficient. However, if calculated time is shorter than real time heat flux is too large, and contact coefficient has to be reduced.

It has been proved that step of change of contact coefficient of 5 W/(m^2K) is small enough for achieving agreement between calculated and real time of contact cylinder-paper among 0.1 second. By decreasing the step of contact coefficient higher accuracy may be achieved, but computing time is increased drastically.

Heat flux exchanged between drying cylinder and paper, W/m^2, is obtained from equation:

\[ q_{c, p} = \frac{1}{N} \sum_{i=1}^{N} \alpha_{conv} \left[ T_{c, d} - T_{p, 0} \right]. \] (12)

Temperature profile of paper web surface, being in contact with surrounding air, is obtained by solving equation (1) with boundary condition given by equation (3). Calculation starts by computing first temperature derivative with respect to paper thickness, \( (\partial T/\partial z)_{z=0} \), at paper surface being in contact with surrounding air, from boundary condition given by equation (3):

\[ \left( \frac{\partial T}{\partial z} \right)_{z=\delta} = \frac{m_w \cdot r/A - \alpha_{conv} \left( T_a - T_{p, d} \right)}{\lambda_e}, \] (13)

for each paper temperature, \( T_{p, \delta} \). Then, temperature profile of paper web is calculated at thickness \( \Delta z = 0.01 \delta \), from Taylor series truncated after first derivative:

\[ T_{p, \delta-\Delta z} = T_{p, \delta} + \Delta z \left( \frac{\partial T}{\partial z} \right)_{z=\delta}. \] (14)

 Afterwards, first temperature derivative with respect to paper thickness is calculated, at distance \( \Delta z \) from paper surface being in contact with surrounding air, from equation:

\[ \left( \frac{\partial T}{\partial z} \right)_{z=\delta} = \frac{\left( \frac{\partial T}{\partial z} \right)_{z=\delta-\Delta z} - \left( \frac{\partial T}{\partial z} \right)_{z=\delta}}{\Delta z}. \] (15)

Now, there are enough data to calculate second temperature derivative with respect to paper thickness, \( (\partial^2 T/\partial z^2)_{z=\delta} \), at paper surface being in contact with surrounding air, from equation:

\[ \left( \frac{\partial^2 T}{\partial z^2} \right)_{z=\delta} = \frac{\left( \frac{\partial T}{\partial z} \right)_{z=\delta-\Delta z} - 2 \left( \frac{\partial T}{\partial z} \right)_{z=\delta} + \left( \frac{\partial T}{\partial z} \right)_{z=\delta}}{\Delta z}. \] (16)

When values of second temperature derivative with respect to paper thickness are obtained, corrected temperature profile of paper web is calculated at thickness \( \Delta z = 0.01 \delta \), from Taylor series truncated after second derivative:

\[ T_{p, \delta-\Delta z} = T_{p, \delta} + \Delta z \left( \frac{\partial T}{\partial z} \right)_{z=\delta} + \frac{\Delta z^2}{2} \left( \frac{\partial^2 T}{\partial z^2} \right)_{z=\delta}. \] (17)
Then, temperature profile obtained from equation (17) is compared to temperature profile obtained from equation (14). If absolute difference between each individual temperature obtained from equations (14) and (17) is less than $10^{-5}$ calculation is continued by equation (4), but if it is not the case the procedure is repeated from equation (15), but now with temperature profile obtained from equation (17). This procedure is repeated until upper condition is satisfied. Afterwards, time is calculated from equation (10) until $\tau_N$ is obtained, which is compared to real time from equation (11). If calculated time of retention, $\tau_N$, and real time of retention, $\tau_I$, differ it means that boundary condition from equation (3) is not satisfied, that is, corresponding heat and mass flow rate can not be exchanged between paper and surrounding air in given conditions. Since heat flux between paper and surrounding air is function of contact coefficient, thermal conductivity, mass transferred, and temperature difference, then reason for disagreement between these times is just in them. However, temperature of surrounding air is obtained by measurement, as well as temperature and moisture content of paper web prior to and after cylinder. From the other side, calculation has proved that contribution of heat transferred by convection is far less comparing to heat transferred with mass. Therefore, if value of convective coefficient in equation (13) was varied in significant range, it would not have more significant influence on value of temperature derivative with reference to thickness, at paper surface being in contact with surrounding air. However, it has proved that value of this derivative is very sensitive to change of paper web thermal conductivity. In that case assumed value of thermal conductivity is not accurate enough, and exact value is obtained iteratively.

Namely, value of thermal conductivity had been changed around assumed value until agreement between calculated and real time of contact cylinder-paper was achieved among 0,1 second. When this condition is fulfilled calculation is returned to equation (5), because by changing the thermal conductivity temperature regime is changed at paper surface being in contact with drying cylinder. Calculation finishes when a pair of values $\alpha_{cont.}$ and $\lambda_e$ are found for which calculated and real retention times of paper on cylinder, for each side of paper web, differ less than 0,1 second.

Effective density of paper web is calculated from expression:

$$\rho_e = (1 - \varepsilon)\rho_s + \varepsilon \rho_w + (\varepsilon - \varepsilon_w)\rho_a,$$

where: $\varepsilon$ – paper porosity, $\rho_s$, $\rho_w$, $\rho_a$ – densities of cellulose fiber, water, and air, respectively.

Effective specific heat capacity of paper web is calculated from expression:

$$c_e = \frac{(1 - \varepsilon)\rho_s c_s + \varepsilon \rho_w c_w + (\varepsilon - \varepsilon_w)\rho_a c_a}{\rho_e},$$

where $c_s$, $c_w$, and $c_a$ are specific heat capacities of cellulose fiber, water, and air, respectively.

Heat flux exchanged between paper surface and surrounding air, W/m$^2$, by convection and mass transfer, is obtained from equation:

$$q_{p,a} = \frac{1}{N} \sum_{i=1}^{N} \left[ \rho_{conv.} \left[ T_{p,\delta} - T_a \right] + \dot{m}_a \cdot r / \Lambda \right].$$

Moisture removed at a cylinder, kg/m$^2$, is calculated from equation:
\[ m_{w} = B |u_{\text{imp.}} - u_{\text{out}} |. \] (21)

3 Results

Rezultati

Numerical values of contact coefficient, given in figure 2, correspond to those from literature [5,6]. Namely, at low velocities of paper machine (long contact times) temperature of paper web approaches temperature of the cylinder. For that reason larger values of contact coefficient are required in order to transfer the same quantity of heat. It is particularly emphasized at several last cylinders, as figure 2 indicates. Results obtained for effective thermal conductivity, given in figure 3, are uniform along whole machine, with average value about 0.4 W/(mK), which coincides to literature data [6,7].

In figure 4 values of heat flux cylinder-paper are given, and in figure 5 values of heat flux paper-air, for each cylinder of paper drying machine. In figure 6 cumulative values of heat exchanged are given, from cylinder to paper and from paper to air, while in figure 7 cumulative values of heat brought are given along with moisture removed, all in function of time. From these two figures one can see that during passage through the machine paper web exchanges 800 kJ/m² of heat with drying cylinders and 350 kJ/m² with surrounding air, while 0.17 kg/m² of moisture is removed from paper, for almost 1 second. Algorithm of solution of dynamic model of paper drying process is given in figure 8.
4 Conclusion

Zaključak

Recommended dynamic model has been tested at industrial process of paper drying in "Natron-Hayat" d.d. Maglaj, Bosnia and Herzegovina, on paper machine PS-1. Contact coefficient is one of fitting parameters, and its values are obtained by numerical solution of partial differential equation (1), with boundary condition (2). Therefore, its value is tightly connected to coefficient of thermal diffusivity, that is, to second fitting parameter – effective thermal conductivity of paper web.

The given model of drying may serve for detection and elimination of disadvantages of real industrial processes. Also, it may be useful for periodical inspection of drier performances. Results obtained by solution of the model are very reliable, because of large number of measurement points. Advantage of the model suggested is ability of its application to drying not only paper but also other porous materials whose thickness is small enough that heat transfer by conduction may be regarded as one-dimensional.

5 Symbols

Oznake

- $A$ - contact area paper-air, $m^2$
- $B$ - basic paper grammage, $kg/m^2$
- $c_d$ - specific heat capacity of air, $J/(kg·K)$
- $c_e$ - effective specific heat capacity of paper web, $J/(kg·K)$
- $c_s$ - specific heat capacity of cellulose fiber, $J/(kg·K)$
- $c_w$ - specific heat capacity of water, $J/(kgK)$
- $l$ - length of contact paper-cylinder, $m$
- $m_{w}$ - mass flow rate of moisture from paper to air, $kg/s$
- $m_w$ - moisture removed, $kg/m^2$
- $r$ - specific heat of evaporation of water, $J/kg$
- $T$ - temperature, $K$
- $T_0$ - air temperature at finite distance from paper surface, $K$
- $T_{c,d}$ - temperature of outer surface of drying cylinder, $K$
- $T_{p,0}$ - temperature of paper surface being in contact with cylinder, $K$
- $T_{p,δ}$ - temperature of paper surface being in contact with surrounding air, $K$
- $u_{imp.}$ - moisture content in paper, per dry basis, prior to cylinder, $kg/kg$
- $u_{out.}$ - moisture content in paper, per dry basis, after cylinder, $kg/kg$
- $w$ - velocity of paper web, $m/s$
- $z = 0$ - boundary plane cylinder-paper,
- $z = δ$ - boundary plane paper-air,

Greek letters

- $α_{cont.}$ - coefficient of heat transfer by contact, $W/(m^2·K)$
- $α_{conv.}$ - coefficient of heat transfer by convection, $W/(m^2·K)$
- $δ$ - paper web thickness, $m$
- $Δτ$ - time needed to paper web to warm up for temperature interval $ΔT$, $s$
- $ε$ - porosity, -
- $λ_e$ - effective thermal conductivity of paper web, $W/(m·K)$
- $ρ_a$ - density of air, $kg/m^3$
- $ρ_e$ - effective density of paper web, $kg/m^3$
- $ρ_s$ - density of cellulose fiber, $kg/m^3$
\( \rho_w \) density of water, kg/m³
\( \tau \) time, s
\( \tau_i \) real contact time paper-cylinder, s
\( \tau_N \) calculated contact time paper-cylinder, s

Description of variables appearing in flowchart in figure 8

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>COND</td>
<td>effective thermal conductivity of paper</td>
</tr>
<tr>
<td>COND0</td>
<td>initial value of effective thermal conductivity of paper</td>
</tr>
<tr>
<td>CONT</td>
<td>coefficient of heat transfer by contact</td>
</tr>
<tr>
<td>CONT0</td>
<td>initial value of coefficient of heat transfer by contact</td>
</tr>
<tr>
<td>DTA0</td>
<td>integration step at paper surface being in contact with drying cylinder</td>
</tr>
<tr>
<td>DTAUA0</td>
<td>vector of contact times paper-cylinder for given integration step (side A)</td>
</tr>
<tr>
<td>DTBD</td>
<td>integration step at paper surface being in contact with surrounding air</td>
</tr>
<tr>
<td>DTADB0</td>
<td>vector of contact times paper-cylinder for given integration step (side B)</td>
</tr>
<tr>
<td>DTDZA0</td>
<td>vector of first derivatives of temperature with reference to paper thickness at surface being in contact with drying cylinder</td>
</tr>
<tr>
<td>DTDZB0</td>
<td>vector of first derivatives of temperature with reference to paper thickness at surface being in contact with surrounding air</td>
</tr>
<tr>
<td>DTDZA1</td>
<td>vector of first derivatives of temperature with reference to paper thickness at finite distance from surface being in contact with drying cylinder</td>
</tr>
<tr>
<td>DTDZB1</td>
<td>vector of first derivatives of temperature with reference to paper thickness at finite distance from surface being in contact with surrounding air</td>
</tr>
<tr>
<td>D2TDZ2A0</td>
<td>vector of second derivatives of temperature with reference to paper thickness at surface being in contact with surrounding air</td>
</tr>
<tr>
<td>D2TDZ2B0</td>
<td>vector of first derivatives of temperature with reference to paper thickness at surface being in contact with drying cylinder</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>QCP</td>
<td>heat flux cylinder-paper</td>
</tr>
<tr>
<td>QPA</td>
<td>heat flux paper-air</td>
</tr>
<tr>
<td>T1A0</td>
<td>temperature of paper surface being in contact with drying cylinder in moment of first contact (obtained by measurement)</td>
</tr>
<tr>
<td>T1BD</td>
<td>temperature of paper surface being in contact with surrounding air in moment of coming across drying cylinder (obtained by measurement)</td>
</tr>
<tr>
<td>TNA0</td>
<td>temperature of paper surface being in contact with drying cylinder in moment of last contact (obtained by measurement)</td>
</tr>
<tr>
<td>TNBD</td>
<td>temperature of paper surface being in contact with surrounding air in moment of leaving drying cylinder (obtained by measurement)</td>
</tr>
</tbody>
</table>

TPA0 temperature of paper surface being in contact with drying cylinder
TPBD temperature of paper surface being in contact with surrounding air
T1PA1 temperature of paper at finite distance from surface being in contact with drying cylinder, calculated from (6)
T1PB1 temperature of paper at finite distance from surface being in contact with surrounding air, calculated from (14)
T2PA1 temperature of paper at finite distance from surface being in contact with drying cylinder, calculated from (9)
T2PB1 temperature of paper at finite distance from surface being in contact with surrounding air, calculated from (17)
TAU real contact time paper-cylinder (obtained by measurement)
TAUA calculated contact time paper-cylinder (side A)
TAUB calculated contact time paper-cylinder (side B)

6 References

Literatura


