CREATION OF OPTIMAL PERFORMANCE OF AN INVESTMENT PROJECT

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Abstract
The selection of an investment project is formulated as a multi-criteria decision-making problem. This paper presents a case in which the decision-maker uses nine criteria or rather attributes (Net Present Value, Internal Rate of Return, Payback Period, Accounting Rate of Return, Cumulative Cash Flows, Return on Investment, Net Profit Margin, Interest Coverage Ratio and Current Ratio).

Individual utility functions are constructed for each attribute separately, as well as a global utility function representing a weighted sum of individual utility functions. For every attribute a finite set of ordered pairs or utility points is determined, taking into account the decision-maker’s assessment. The given points are then approximated by the utility function.

Finally, according to the decision-maker’s assessment the optimization problem is solved with the purpose of achieving an optimal performance for each project. By way of negotiation the performances on offer approach the optimal performance of the project with the purpose of realising an agreement between the decision-maker and the investor.

Keywords: Investment Project, Multi-Criteria Decision-Making, Utility Function, Negotiation

1. INTRODUCTION

Choosing an investment project presents a problem which includes two parties interested in reaching an agreement, the decision-maker (top management of the company) and the investor (management of the investment center in the company). A business situation as such occurs when the decision-maker has to choose the optimal investment project between multiple projects on offer in the company. In line with the conducted selection, the decision-maker will be willing to invest either his/her own resources or resources obtained through a bank loan in the chosen project. For this reason both of the participants have the same goal, the approval of the project, which is understandable given that the same business subject is considered. Where they differ is in the efficiency indicator performances of the investment project. The decision-maker sets the conditions regarding the performance of the project which he/she expects the investor will accept.
Thus the problem is transferred to two-participant negotiation problem. In most cases more than one investment projects are submitted for approval (which is a realistic expectation regardless of the capital budget limitations set by the company), for which reason the decision-maker negotiates with each of the investors.

Each of the projects can have several different performances.

In this paper the investor proposes efficiency indicator performances in such a way that neither of the performances is dominant over other performances.

That means that the result for one indicator cannot be improved without making the result of another one worse (Pareto efficiency). For every performance offered in a project a utility function is constructed for each attribute. The optimization problem is the maximization of the score, of global utility function, for each of the projects (alternatives). The decision-maker starts the negotiation procedure with the optimal project performance.

This paper presents an integrated modelling concept that brings together multi attribute utility theory (MAUT) and negotiation concepts. MAUT model enables the consideration of factors that have different measures and different relative importance to the decision.

The rest of paper is organized as follows. In Section 2 an objective hierarchy is presented and a multi-criteria optimization model formulated. Section 3 shows how to construct decision-maker specific utility functions. The approach is demonstrated with an illustrative example involving five projects and a three attributes in Section 4.

2. INVESTMENT PROJECT SELECTION

Investment project selection is a classical multi-criteria decision-making (MCDM) issue. Financing of investment projects represents a process of identifying and selecting investments in a long-term asset, that is, the asset which entails the prospect of realizing economic gains in the period exceeding one year. This paper analyses investment projects whose duration amounts to 20 years classifying them as processes of long-term investment planning. It is necessary, in the first place, to choose the criteria on the basis of which the assessment of the investment project will be carried out. Let us suppose we have \( m \) projects (alternatives) and the variables of the decision denote as \( x_i, i=1,\ldots,m \). The variables are binary meaning that if \( x_i=1 \) the alternative \( i \) is accepted and if \( x_i=0 \) the alternative \( i \) is not accepted. As it is necessary to approve exactly one project, we have a limitation \( \sum_{i=1}^{m} x_i = 1 \). The choice of criteria or attributes is conducted on the basis of consultations with the decision-maker. In this paper, having conducted consultations with several experts from the field of business finances, nine criteria were shortlisted. Let \( t \) be the life span of the project, \( NNT_n \) the net cash flow of the investment project in the \( n \) year and \( r \) the discount rate. In that case:
1. **Net present value (NPV)** presents the sum total of net cash flows of the investment project reduced to the present value by the discounting. If \( I_0 \) is the initial investment in the project, then \( NPV \) net present value of the investment project equals:

\[
NPV = - I_0 + \sum_{n=1}^{I} \left( \frac{NNT_n}{1 + r} \right) n
\]

If by \( NPV_i \) we mark the net present value of the alternative \( i \), we get the following objective function:

\[
f_1(x) = \sum_{i=1}^{m} NPV_i x_i
\]

2. **Internal rate of return (IRR)** is the discount rate which reduces the net present value of the investment project to zero (the rate in question is the maximally acceptable profitability rate, the biggest rate the investment project can accept). It is calculated in the following fashion:

\[
NPV = - I_0 + \sum_{n=1}^{I} \left( \frac{NNT_n}{1 + IRR} \right) n = 0
\]

If by \( IRR_i \) we mark the internal rate of return of the alternative \( i \), we get the following objective function:

\[
f_2(x) = \sum_{i=1}^{m} IRR_i x_i
\]

3. **Payback period (PBP)** presents the number of periods (years) in which it is necessary to realize such a net cash flow of the investment project so as to retrieve the total value of the realized investment within the scope of the life span of the project. If \( I_n \) is the value of the investment in the \( n \) year of the investment project’s life span, than \( t_p \) (PBP) payback period of the investment project is calculated as follows:

\[
\sum_{n=0}^{I} I_n = \sum_{n=0}^{I_p} NNT_n
\]

If by \( PBP_i \) we mark the payback time of the alternative \( i \), we get the following objective function:

\[
f_3(x) = - \sum_{i=1}^{m} PBP_i x_i
\]

4. **Accounting rate of return (ARR)** represents the ratio of the average value of all future accounting net gain/losses of the enterprise during the life span of the investment project and the net value of the investment realised in the same time span. The average value of all future accounting net gain/losses of
the enterprise results from dividing the sum of all future accounting net gain/losses of the enterprise with the total number of years in which they have been realised (i.e. the duration of the life span of the investment project). If $\pm ND$ is the accounting net gain/loss (+/–) of the enterprise in the year $n$, then accounting rate of return $ARR$ equals:

$$ARR = \frac{\sum_{n=1}^{t} ND_n}{n}$$

(7)

If by $ARR_i$ we mark the accounting rate of return of the alternative $i$, we get the following objective function:

$$f_4(x) = \sum_{i=1}^{m} ARR_i x_i$$

(8)

5. **Cumulative cash flows (CCF)** represent the final sum or cumulation of the future net cash flows of the investment project in the last year of the investment project’s life span. We calculate them in the following way:

$$CCF = \sum_{n=0}^{t} NNT_n$$

(9)

If by $CCF_i$ we mark the cumulative cash flows of the alternative $i$, we get the following objective function:

$$f_5(x) = \sum_{i=1}^{m} CCF_i x_i$$

(10)

6. **Return on investment (ROI)** represents the ratio between net gain/losses of the investor $ND_t$ realised in the reporting period $t$ and the total value of the investment $I$, which is calculated in the following way:

$$ROI = \frac{ND_t}{I}$$

(11)

If by $ROI_i$ we mark the return on investment of the alternative $i$, we get the following objective function:

$$f_6(x) = \sum_{i=1}^{m} ROI_i x_i$$

(12)

7. **Net profit margin (NPM)** represents the ratio of net gain/losses $ND_t$ and the total business revenue $PP_t$ of the investor in the reporting period $t$, which is calculated in the following way:

$$NPM = \frac{ND_t}{PP_t}$$

(13)
If by $NPM_i$ we mark the net profit margin of the alternative $i$, we get the following objective function:

$$f_7(x) = \sum_{i=1}^{m} NPM_i x_i$$  \hspace{1cm} (14)

8. **Interest coverage ratio (ICR)** represents the ratio of the operational gain $OD_t$ and the expenses of the financing i.e. interests of the investor, $TF_t$, in the reporting period $t$, which is calculated in the following way:

$$ICR = \frac{OD_t}{TF_t}$$  \hspace{1cm} (15)

If by $ICR_i$ we mark the interest coverage ratio of the alternative $i$, we get the following objective function:

$$f_8(x) = \sum_{i=1}^{m} ICR_i x_i$$  \hspace{1cm} (16)

9. **Current ratio (CR)** represents the ratio of the current assets $TI_t$ and current obligations $TO_t$ of the investor in the reporting period $t$, which can be shown in the following relation:

$$CR = \frac{TI_t}{TO_t}$$  \hspace{1cm} (17)

If by $CR_i$ we mark the current ratio of the alternative $i$, we get the following objective function:

$$f_9(x) = \sum_{i=1}^{m} CR_i x_i$$  \hspace{1cm} (18)

The problem we are solving is as follows ($P$):

$$\max( f_1(x), ..., f_9(x))$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \in \{0,1\}, i = 1, ..., m.$$  \hspace{1cm} (19)

3. **UTILITY FUNCTIONS**

In the second chapter we introduced nine attributes by which we measure the performance or value of alternatives. Since the multi-criteria decision-making problem is multi-dimensional, we need to reduce it to a one-dimensional problem. In decision-making the utility functions for different criteria, in general, are not explicitly known. For every individual objective function (attribute) we introduce decision-maker specific utility function. Construction is done in a way that a set of ordered pairs called utility points $(y_i, u_i), i=1,...,n$ is introduced for every objective function, whereby the $y_i$ is the value of the objective function (attribute,
indicator) and \( u_i \) is the corresponding utility. The utility \( u_i \) of the performance \( y_i \) is determined by the decision-maker (DM). By means of utility points individual utility functions are constructed. Different techniques are used for such constructions. Ehrgott etc... (2004) recommend linear, quadratic or cubic interpolation and smoothing techniques around the chosen utility points.

The investor suggests several different performances for each of the projects. The decision-maker will choose the optimal performance for each of the projects.

Based on different performances of investment projects local utility functions are constructed, of the value of every attribute for every alternative. The generated performances of alternatives are generally not dominated by a certain performance. The obtained utility points are approximated by function via least squares method.

For a global utility function we take the score:

\[
S_i = \sum_{j=1}^{9} w_j u_j(f_j(x))
\]

where \( S_i \) is the score of the performance \( i \) of the alternative. The variable \( x_i=1 \) if the chosen performance is \( i \), \( x_i=0 \) if performance \( i \) is not chosen. If it is so that \( 0 < x_i < 1 \) for any \( i \), a new alternative performance is constructed and it is suggested to the investor to determine whether that alternative performance is possible.

Let's assume that \( n \) performances of the project are given.

We solve the following optimization problem (\( R \)):

\[
\begin{align*}
\max & \quad \sum_{j=1}^{9} w_j u_j(f_j(x)) \\
\text{s.t.} & \quad \sum_{i=1}^{q} x_i = 1 \\
& \quad x_i \geq 0, \ i = 1, \ldots, n
\end{align*}
\]

Notice that the binary condition has been substituted by the non-negativity condition. Doing this we also allow, beside the created performance offers, that the optimal project could be subsequently created in agreement with the investor.

For project \( i \) the optimal solutions give optimal performance. With that performance we enter the negotiations with the investor of project \( i \).

### 4. IMPLEMENTATION

On the basis of the proposed criteria the decision-maker has chosen three – Net Present Value (\( NPV \)), Payback Period (\( PBP \)), and Return on Investment (\( ROI \)). Based on sources (Burns and Walker, 2001), Net Present Value is considered to be the most reliable indicator for assessment of investment projects which use
discounted cash flows of the project (primary indicator). Payback Period is a dynamic criterion that does not use discounted cash flows, but has a long term application in global practice, mostly as a secondary indicator (Adler, 2000). Lastly, the decision-maker chose Return on Investment, as a static criterion.

The top management of the company was given a limited capital budget for the investment, on the basis of which it acquired five investment offers on behalf of the interested management of the investment centers in the company. All the values for all the attributes for the five alternatives (investments) have been listed in the decision-making matrix. For every project a single performance is indicated.

<table>
<thead>
<tr>
<th>CRITERION Cj:</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>INVESTMENT Ai:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>96.82</td>
<td>5.33</td>
<td>5.52%</td>
</tr>
<tr>
<td>A2</td>
<td>84.77</td>
<td>6.55</td>
<td>6.28%</td>
</tr>
<tr>
<td>A3</td>
<td>78.20</td>
<td>8.75</td>
<td>6.88%</td>
</tr>
<tr>
<td>A4</td>
<td>54.68</td>
<td>11.71</td>
<td>9.45%</td>
</tr>
<tr>
<td>A5</td>
<td>97.60</td>
<td>6.62</td>
<td>6.92%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>criterion name:</th>
<th>1. NPV</th>
<th>2. PBP</th>
<th>3. ROI</th>
</tr>
</thead>
<tbody>
<tr>
<td>criterion:</td>
<td>max</td>
<td>min</td>
<td>max</td>
</tr>
</tbody>
</table>

For the given information, the decision-maker has identified the value points. In Table 2 the column \( U_1 \) represents the utility of Net Present Value of corresponding alternatives. Column \( U_2 \) represents the utility of the corresponding Payback Period, and column \( U_3 \) represents the utility of Return on Investment. At the end we have the score with the weight values \( w_1=0.4 \), \( w_2=0.4 \) and \( w_3=0.2 \).

<table>
<thead>
<tr>
<th>INVESTMENT Ai:</th>
<th>NPV:</th>
<th>( U_1 ):</th>
<th>PBP:</th>
<th>( U_2 ):</th>
<th>ROI:</th>
<th>( U_3 ):</th>
<th>SCORE:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>96.62</td>
<td>45.00</td>
<td>5.33</td>
<td>83.00</td>
<td>5.52%</td>
<td>28.00</td>
<td>56.80</td>
</tr>
<tr>
<td>A2</td>
<td>84.77</td>
<td>41.00</td>
<td>6.55</td>
<td>65.00</td>
<td>6.28%</td>
<td>50.00</td>
<td>52.40</td>
</tr>
<tr>
<td>A3</td>
<td>78.20</td>
<td>37.00</td>
<td>8.75</td>
<td>43.00</td>
<td>6.88%</td>
<td>60.00</td>
<td>44.00</td>
</tr>
<tr>
<td>A4</td>
<td>54.68</td>
<td>16.00</td>
<td>11.71</td>
<td>30.00</td>
<td>9.45%</td>
<td>85.00</td>
<td>35.40</td>
</tr>
<tr>
<td>A5</td>
<td>97.60</td>
<td>46.00</td>
<td>6.62</td>
<td>64.00</td>
<td>6.92%</td>
<td>60.00</td>
<td>56.00</td>
</tr>
<tr>
<td>w</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the first alternative, the decision-maker constructs acceptable performances for each of the attributes, thus obtaining a sequence of performances for the first alternative.
For the first alternative (project) the investor has suggested ten performances. For every value of the first attribute (NPV) the decision-maker has assigned utility points. The approximation of the utility function is generated from the points obtained using the least squares method (Figure 1).

<table>
<thead>
<tr>
<th>Value of NPV</th>
<th>Utility Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.50</td>
<td>43.00</td>
</tr>
<tr>
<td>96.00</td>
<td>43.50</td>
</tr>
<tr>
<td>96.50</td>
<td>44.00</td>
</tr>
<tr>
<td>97.00</td>
<td>44.50</td>
</tr>
<tr>
<td>97.50</td>
<td>45.00</td>
</tr>
<tr>
<td>98.00</td>
<td>45.50</td>
</tr>
<tr>
<td>98.50</td>
<td>46.00</td>
</tr>
</tbody>
</table>

The same procedure is followed for the remaining two criteria (PBP and ROI). For the Payback Period we attained the following utility function:

$$u_2 = 10,472y_2^2 - 114,87y_2 + 397,42$$

Finally, for the Return on Investment the attained utility function is:

$$u_3 = -90098 y_3^2 + 10179y_3 - 258,9$$

For already chosen weight values ($w$), we formulate the following optimization problem ($R_i$):

$$\max \sum_{j=1}^{3} w_j u_j(f_j(x))$$

$$\sum_{i=1}^{10} x_i = 1$$

$$x_i \geq 0, i = 1,...,10$$

(22)

For the given optimization problem ($R_i$) we have attained the optimal score of $S_i=57,45$ which was attained for the first project in which $NPV=98,20$, $PBP=5,11$ and $ROI=5,40\%$.

The result is that the optimal performance of the first project is one of the performances offered by the investor.

For each other project we conduct the same procedure thus attaining the Table 3.

Figure 1. Approximation of the utility function for the first attribute of the first alternative.
Based on the results obtained, the decision-maker has the right to a preferential decision with which he influences the performance of the project. The previous table is the basis for negotiation between the decision-maker and the investor. The decision-maker enters the negotiations with the optimal performance. Apart from the listed project performances, it is possible to enter in further negotiations and new assessments.

**Table 3. Optimum utility functions scores for the five investments.**

<table>
<thead>
<tr>
<th>INVESTMENT $A_i$:</th>
<th>NPV</th>
<th>PBP</th>
<th>ROI:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>96.82</td>
<td>5.33</td>
<td>5.52%</td>
</tr>
<tr>
<td>$A_2$</td>
<td>84.77</td>
<td>6.55</td>
<td>6.28%</td>
</tr>
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<td>$A_3$</td>
<td>78.20</td>
<td>8.75</td>
<td>6.88%</td>
</tr>
<tr>
<td>$A_4$</td>
<td>54.68</td>
<td>11.71</td>
<td>9.45%</td>
</tr>
<tr>
<td>$A_5$</td>
<td>97.60</td>
<td>6.62</td>
<td>6.92%</td>
</tr>
</tbody>
</table>

The compromise between the decision-maker and the investor is reached with regard to concessions the negotiators are willing to make. For the first alternative the decision-maker can improve the values of the first two attributes, whereas he allows for a deviation of the third attribute.

5. CONCLUSION

The suggested procedure allows for the decision-maker to be involved in all the phases. His preferences and concessions he is willing to make are thus taken into account as the basis for negotiations. This kind of procedure is applicable to problems where negotiations are crucial to decision-making.

REFERENCES:


