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# DAMAGE TOLERANCE ANALYSIS OF STRUCTURAL COMPONENTS UNDER GENERAL LOAD SPECTRUM

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In this investigation fatigue behaviour of damaged structural components under cyclic loads constant amplitude and load spectrum is considered. Primary attention of this investigation is the establishment of computation procedure for the evaluation of the residual life of aircraft attachment lug type structural elements in the presence of initial cracks. In this investigation for residual life estimation and crack growth analysis Strain Energy Density (SED) method is used. The SED method uses the low-cycle fatigue (LCF) properties of the material in crack growth analysis. In this approach experimentally obtained dynamic properties of the material such as Forman's constants are not required when this method is concerned. The complete computation procedure for the crack propagation analysis using low-cycle fatigue material properties is illustrated with the damaged structural elements. To determine analytic expressions for stress intensity factors (SIF) singular finite elements are used. Results of numerical simulation for crack propagation based on strain density method have been compared with own experimental results.

***Keywords:*** *fatigue, residual life, cracked structural elements, aircraft attachment lugs, low-cycle fatigue properties, finite elements*

## Analiza elemenata konstrukcija s motrišta dopustivih oštećenja pod općim spektrom opterećenja

Izvorni znanstveni članak

U ovom se istraživanju razmatra zamor elemenata konstrukcije s inicijalnim oštećenjima pri cikličnim opterećenjima konstantne amplitude i spektrom opterećenja. Primarna pozornost ovog istraživanja usmjerena je na uspostavljanje postupka proračuna za ocjenu preostalog vijeka elemenata konstrukcije tipa uške avionske konstrukcije uz postojanje inicijalnih pukotina. U ovom istraživanju za procjenu preostalog vijeka i rasta pukotine, koristi se metoda gustoće energije deformacija (SED). SED metoda koristi niskociklične zamorne karakteristike materijala (LCF) za analizu širenja pukotine. U ovom pristupu eksperimentalno dobivene dinamičke karakteristike materijala poput Formanovih konstanti, nisu potrebne u analizi širenja pukotine. Kompletan postupak proračuna za analizu širenja pukotine na bazi korištenja niskociklčnih zamornih karakteristika materijala, ilustriran je na elementima konstrukcije s inicijalnim oštećenjima. Za definiranje analitičkih izraza za faktore intenziteta naprezanja (FIN), koriste se singularni konačni elementi. Rezultati numeričkih simulacija za širenje pukotine utemeljenih na metodi gustoće deformacije, uspoređeni su s vlastitim eksperimentalnim rezultatima.

***Ključne riječi:*** *zamor, preostali vijek, elementi konstrukcije s pukotinom, uške s inicijalnom pukotinom, niskociklične zamorne karakteristike materijala, konačni elementi*

## 1 Introduction

In assembling complex structures like military aircraft, riveted or bolted joints, attachment lugs are primarily used as they offer many options to the designer. To satisfy fatigue requirements, the designer can either keep the stress levels below the endurance limit or ensure the slow crack growth life of the component is greater than the design service goal plus some factor of safety. The latter approach is most commonly used and relies on the ability to predict fatigue crack growth at fatigue critical locations. Methods for design against fatigue failure are under constant improvement. In order to optimize constructions the designer is often forced to use the properties of the materials as efficiently as possible. One way to improve the fatigue life predictions may be to use relations between crack growth rate and the stress intensity factor range. These are fairly well established for constant amplitude loading, at least for common specimen geometries. Loading histories in engineering structures do however often exhibit varying amplitudes. For such cases the prediction capacity is markedly lower. Ideally, the crack advance under varying amplitude should be possible to predict using experimental data from constant amplitude testing. Numerous investigations address this problem but so far without reaching any total success. Design based on damage tolerance criteria often deals with notched components giving rise to localized stress concentrations which, in brittle materials, may generate a crack leading to catastrophic failure or to a shortening of the assessed structural life. For a successful implementation of the damage tolerance philosophy [1] to the design and in-service operation of structures subjected to fatigue loading it is crucial to have reliable crack growth prediction tools. Damage tolerance application to the aircraft structural components is limited to critical parts. A part, if it fails, alone may cause that the loss of an aircraft is classified as a critical part. This definition means that aircraft wing-fuselage attachments must comply with the damage tolerance requirements [1, 2]. The main goal is a safe life design, i.e. a slow crack growth structure not requiring any inspection during its full life. The Damage Tolerance approach assumes the components have an existing flaw from which a crack will grow under dynamic loads. This assumption makes it possible to account for in-service or manufacturing defects in determining the dynamic life. The Damage Tolerance Methodology uses fracture mechanics to predict the fatigue crack growth in a structure. In the design analysis of a slow crack growth structure it is most important to make correct estimates for the early portion of the crack growth process, because it is there the life is. In most cases this implies that maximum accuracy is needed for small corner cracks.

The ability to successfully maintain aircraft airworthiness and structural integrity is critically dependent on the application of appropriate fatigue crack growth (FCG) prediction tools. The prediction tools are required to accurately predict FCG in aircraft structures and components under flight spectrum loading, and thus reliably provide total economic lives or inspection intervals as part of a stringent aircraft structural integrity management plan.

Fatigue crack growth in aircraft structures and components under flight spectrum loading is traditionally predicted based on FCG rates obtained from constant-amplitude (CA) crack growth testing using the cycle-by-cycle approaches [14, 15].

Attachment lugs are particularly critical components in crack initiation and growth because of their inherently high stress concentration levels near the lug hole. For these reasons, it is important to develop analytical/ numerical as well as experimental procedures for assessing and designing damage tolerant attachment lugs to ensure the operational safety of aircraft. Over the years, several extensive studies [3 ÷ 5] have been made on lug fatigue performance, involving both experimental and numerical means.

In the work of fatigue crack growth and fracture behaviour of attachment lugs [6, 7], an accurate calculation of the stress intensity factor is essential. Over the years several methods have evolved to compute the stress intensity factors for structural components containing cracks. These methods include analytical as well as experimental approach. The experimental backtracking approach was used to derive empirically the stress intensity factors for structural components using the growth rate data of through-the-thickness cracks for simple geometry subjected to constant-amplitude loading. The finite element method is used to precisely determine SIFs using singular finite elements. Accurate stress-intensity factor (SIF) solutions are required to conduct thorough damage tolerance analyses of structures containing cracks. Exact closed form SIF solutions for cracks in three-dimensional solids are often lacking for complex configurations; therefore, approximate solutions must be used. Over the past two decades, considerable effort has been placed on developing computationally efficient methods which provide highly accurate SIF solutions for cracks in three-dimensional bodies.

The purpose of this investigation was to test the accuracy of the crack growth models. All necessary parameters, such as material property data, stress intensity solutions, and the load spectrum, were defined. To determine residual life of damaged structural components here are used two crack growth methods: conventional Forman`s crack growth method and crack growth model based on the strain energy density method. The last method uses the low cycle fatigue properties in the crack growth model.

## 2 Crack growth method based on strain energy density method

In this work fatigue crack growth method based on energy concept is considered and then it is necessary to determine the energy absorbed till failure. This energy can be calculated by using cyclic stress-strain curve. Function between stress and strain, using Massing's hypothesis provides good description of elastic-plastic behaviour of material, and the general hysteresis curve equation may be expressed as:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{k'}\right)^{\frac{1}{n'}}, \tag{1}$$

where $E$ is the modulus of elasticity, $k'$ – cyclic strength coefficient, $n'$ – cyclic strain hardening exponent, $\Delta\varepsilon/2$ is strain amplitude and $\Delta\sigma/2$ is stress amplitude. Equation (1) enables the calculation of the stress-strain distribution by knowing low cyclic fatigue properties. As a result the energy absorbed till failure becomes [10, 11]:

$$W_\mathrm{c} = \frac{4}{1+n'}\sigma'_\mathrm{f}\,\varepsilon'_\mathrm{f}, \tag{2}$$

where $\sigma'_\mathrm{f}$ is fatigue strength coefficient and $\varepsilon'_\mathrm{f}$ – fatigue ductility coefficient. Given the fact that strain energy density method is considered, the energy absorbed till failure must be determined after the energy concept is based on the following fact: The energy absorbed per unit growth of crack is equal to the plastic energy dissipated within the process zone per cycle. This energy concept is expressed by:

$$W_\mathrm{c}\cdot\delta a = \omega_\mathrm{p}, \tag{3}$$

where $W_\mathrm{c}$ is energy absorbed till failure, $\omega_\mathrm{p}$ – the plastic energy and $\delta a$ – is virtual crack length. In equation (3) it is necessary just to determine the plastic energy dissipated in the process zone $\omega_\mathrm{p}$. By integration of equation for the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [10] from zero to the length of the process zone ahead of crack tip $d^*$ it is possible to determine the plastic energy dissipated in the process zone $\omega_\mathrm{p}$. After integration relation of the plastic energy dissipated in the process zone becomes:

$$\omega_\mathrm{p} = \left(\frac{1-n'}{1+n'}\right)\frac{\Delta K_\mathrm{I}^2\,\psi}{E\,I_{n'}}, \tag{4}$$

where $\Delta K_\mathrm{I}$ is the range of stress intensity factor, $\psi$ – constant depending on the strain hardening exponent $n'$, $I_{n'}$ – the non-dimensional parameter depending on $n'$.

Fatigue crack growth rate can be obtained by substituting Eq. (2) and Eq. (4) in Eq. (3):

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{(1-n')\psi}{4\,E\,I_{n'}\,\sigma'_\mathrm{f}\,\varepsilon'_\mathrm{f}}\left(K_\mathrm{Imax} - K_\mathrm{Ith}\right)^2, \tag{5}$$

where $\Delta K_\mathrm{Ith}$ is the range of threshold stress intensity factor and is function of stress ratio i.e.

$$\Delta K_\mathrm{Ith} = \Delta K_\mathrm{Ith0}(1-R)^\gamma, \tag{6}$$

$\Delta K_\mathrm{Ith0}$ is the range of threshold stress intensity factor for the stress ratio $R = 0$ and $\gamma$ is coefficient (usually, $\gamma$ = 0,71). Finally number of cycles till failure can be determined by integration of relation for fatigue crack growth rate:

$$N_{\rm p} = 4\,E\,I_{n'}\sigma'_{\rm f}\,\varepsilon'_{\rm f}\int\limits_{a_0}^{a_k}\frac{{\rm d}a}{(1+n')\psi\left(\Delta K_{\rm I}-\Delta K_{\rm Ith}\right)^2} \tag{7}$$

and

$$\Delta K_{\rm I} = Y\cdot\sigma\cdot\sqrt{\pi\cdot a} \tag{8}$$

Eq. (7) enables us to determine crack growth life of different structural component. Very important fact is that Eq. (7) is easy for application since low cyclic material properties ($n'$, $\sigma'_{\rm f}$, $\varepsilon'_{\rm f}$) available in literature are used as parameters. The only important point is stress intensity factor which, depending on the geometry complexity and the type of loading, could be determined by using analytical and/or numerical approaches.

## 3 Crack growth analysis using conventional approach

Various conventional crack growth models have been used for crack growth analysis and fatigue life estimations. Many of these models achieve correct solutions of crack growth analyses for cracked structural elements under cyclic loads of constant amplitude. However, for construction under cyclic loads of variable amplitude in form of load spectrum such as in aircraft cases it is necessary to include the effects of shape of load spectra and its effects on estimation life of structural elements [9].

Forman, Newman and others [8] developed the equation, which is an equation often used to describe crack growth. This equation describes the crack growth curve in terms of the crack length $a$ , the number of cycles $N$, the stress ratio $R$, the stress intensity factor range $\Delta K$, and material constants, $C, n, p, q$ through best fits of the da/dN – $\Delta K_{\rm I}$ data

$$\frac{{\rm d}a}{{\rm d}N} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n\frac{\left(1-\dfrac{\Delta K_{\rm Ith}}{\Delta K}\right)^p}{\left(1-\dfrac{K_{\rm Imax}}{K_{\rm c}}\right)^q}, \tag{9}$$

where: $a$ – crack length, $N$-number of cycles, $C, n, p, q$ – are experimentally derived material parameters, $K_{\rm I}$ is the stress intensity factor (SIF), $K_{\rm Ith}$ is the threshold stress intensity factor, $R$ is the stress ratio, $K_{\rm Ic}$ – is the critical stress intensity factor. The Newman closure function is one of these terms and is defined as $f$:

$$f = \frac{K_{\rm Iop}}{K_{\rm Imax}} = \begin{cases}\max\left(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right); R \geq 0\\ A_0 + A_1 R; \quad -2 \leq R < 0\end{cases} \tag{10}$$

and the coefficients are given by:

$$A_0 = \left(0{,}825 - 0{,}34\,\alpha + 0{,}05\,\alpha^2\right)\left[\cos\left(\frac{\pi}{2}\cdot\frac{\sigma_{\max}}{\sigma_0}\right)\right]^{\frac{1}{\alpha}},$$

$$A_2 = 1 - A_0 - A_1 - A_3,$$

$$A_3 = 2A_0 + A_1 - 1,$$

where: $\alpha$ – is the plane stress/strain constraint factor, ($\sigma_{\max}/\sigma_0$) is the ratio of maximum stress to the flow stress. The threshold stress intensity factor range is calculated by the following empirical equation:

$$\Delta K_{\rm Ith} = \frac{\Delta K_{\rm I0}\cdot\left(\dfrac{a}{a+a_0}\right)^{0{,}5}}{\left(\dfrac{1-f}{(1-A_0)\cdot(1-R)}\right)^{(1-C_{\rm th}\cdot R)}}. \tag{11}$$

Relation (9) represents one general crack growth model based on conventional approach. This relation can be transformed to conventional Forman's crack growth model [8]. In region III rapid and unstable crack growth occurs, so Forman at all.

Proposed equation for region III as well as for region II [9]

$$\frac{{\rm d}a}{{\rm d}N} = \frac{C\cdot(\Delta K_{\rm I})^n}{(1-R)\cdot K_{\rm Ic} - \Delta K_{\rm I}}, \tag{12}$$

where $K_{\rm Ic}$ is the fracture toughness. Forman's equation has been developed to model of unstable crack growth domain (III).

## 4 Stress intensity factor solutions of cracked lugs

### 4.1 Damaged attachment lug with initial crack through the thickness

In this section are given relations for stress intensity factor with crack through the thickness.

In general geometry of notched structural components and loading is too complex for the stress intensity factor (SIF) to be solved analytically. The SIF calculation is further complicated because it is a function of the position along the crack front, crack size and shape, type of loading and geometry of the structure. In this work analytic results and FEM were used to perform linear fracture mechanics analysis of the pin-lug assembly. Analytic results are obtained using relations derived in this paper. Good agreement between finite element and analytic results is obtained. It is very important because we can use analytic derived expressions in crack growth analyses. Lugs are essential components of an aircraft for which proof of damage tolerance has to be undertaken. Since the literature does not contain the stress intensity solution for lugs which are required for proof of damage tolerance, the problems posed in the following investigation are: selection of a suitable method of determining of the SIF, determination of SIF as a function of crack length for various forms of lug and setting up a complete formula for calculation of the SIF for lug, allowing essential parameters. The stress intensity factors are the key parameters to estimate the characteristic of the cracked structure. Based on the stress intensity factors, fatigue crack growth and structural life predictions have been investigated. The lug dimensions are defined in Fig. 1.

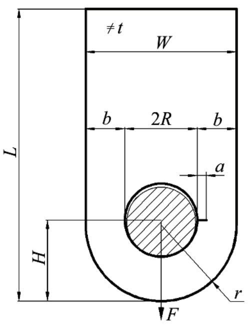



**Figure 1** Geometry and loading of lugs with crack through the thickness

To obtain stress intensity factor for the lugs it is possible to start with general expression for the SIF in the next form

$$K_{\mathrm{IT}} = Y_{\mathrm{SUM}} \cdot \sigma \cdot \sqrt{\pi \cdot a}, \tag{13}$$

where: $Y$ – correction function, $a$ – the crack length. This function is essential in determining of the stress intensity factor. Primarily, this function depends on stress concentration factor $k_{\mathrm{t}}$ and geometric ratio $a/b$. The correction function is defined using experimental and numerical investigations. This function can be defined in the next form [13]:

$$Y_{\mathrm{SUM}} = \frac{1{,}12 \cdot k_{\mathrm{t}} \cdot A}{A + \dfrac{a}{b}} \cdot k \cdot Q, \tag{14}$$

$$k = \mathrm{e}^{r \cdot \sqrt{\frac{a}{b}}}, \tag{15}$$

$$b = \frac{w - 2R}{2}, \tag{16}$$

$$r = -3{,}22 + 10{,}39 \cdot \left(\frac{2R}{w}\right) - 7{,}67 \cdot \left(\frac{2R}{w}\right)^2, \tag{17}$$

$$Q = \frac{U \cdot \dfrac{a}{b} + 10^{-3}}{\dfrac{a}{b} + 10^{-3}}, \tag{18}$$

$$U = 0{,}72 + 0{,}52 \cdot \left(\frac{2R}{H}\right) - 0{,}23 \cdot \left(\frac{2R}{H}\right)^2, \tag{19}$$

$$A = 0{,}026 \cdot \mathrm{e}^{1{,}895 \cdot \left(1 + \frac{a}{b}\right)}. \tag{20}$$

The stress concentration factor $k_{\mathrm{t}}$ is very important in calculation of correction function, Eq. (14). In this investigation a contact finite element stress analysis was used to analyze the load transfer between the pin and lug, Fig. 2.

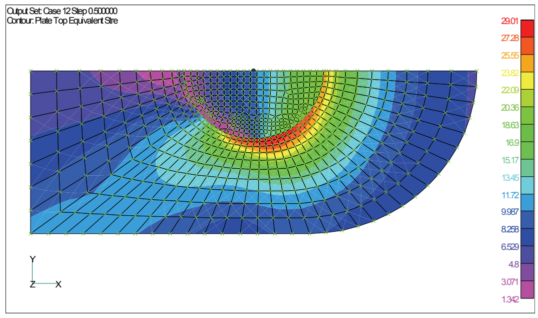


**Figure 2** Contact pin/load FE model of cracked lug

## 4.2 Damaged attachment lug with semi-elliptic surface crack

Here are considered the stress intensity factors for cracked lug with the semi-elliptic surface crack as shown in Fig. 3.

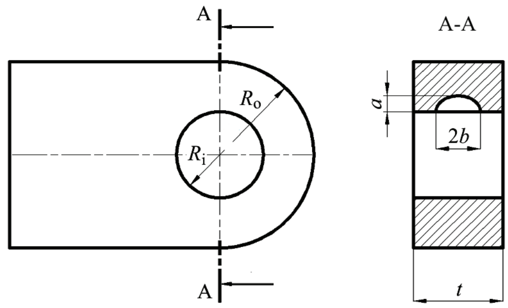



**Figure 3** Lug with semi-elliptic surface crack

As a start point for determination of stress intensity factor of attachment lug with semi-elliptic surface crack will be used Lukaš's model [16 ÷ 19]. Lukaš considered the problem of determination of SIF to plate with surface crack within zone of stress concentration. This approach is extended to the attachment lugs with semi-elliptic surface crack. By using this approach here are defined analytic expressions for determination of SIFs at the points A and B to lug with semi-elliptic surface crack as shown in Fig. 3, in the next form:

$$K_{\mathrm{IA}} = F_{\mathrm{A}} \cdot \sigma \cdot k_{\mathrm{t}} \cdot \sqrt{\pi \cdot a}, \tag{21}$$

$$F_{\mathrm{A}} = F_{\mathrm{A,0}} \cdot \sqrt{\frac{D_{\mathrm{A}}}{a}}, \tag{22}$$

$$F_{\mathrm{A,0}} = \frac{1{,}13 + 0{,}09 \cdot \left(\dfrac{a}{b}\right)}{E_2 \cdot \left(\dfrac{a}{b}\right)}, \quad 0 \le \frac{a}{b} \le 1, \tag{23}$$

$$D_{\mathrm{A}} = a + d \cdot \left[1 - e^{\left(-\frac{a}{a'}\right)}\right], \tag{24}$$

$$a' = \frac{d}{k_{\mathrm{t}}^2 - 1}, \tag{25}$$

$$K_{\mathrm{IB}} = F_{\mathrm{B}} \cdot \sigma \cdot k_{\mathrm{t}} \cdot \sqrt{\pi \cdot a}, \tag{26}$$

$$F_{\mathrm{B}} = F_{\mathrm{B},0} \cdot \sqrt{\frac{D_{\mathrm{B}}}{a}}, \tag{27}$$

$$D_{\mathrm{B}} = a + 4d \cdot \left[ 1 - e^{\left( -\frac{a}{4a'} \right)} \right], \tag{28}$$

$$F_{\mathrm{B},0} = \frac{1{,}243 + 0{,}099 \cdot \left( \frac{a}{b} \right)}{E_2 \cdot \left( \frac{a}{b} \right)} \cdot \sqrt{\frac{a}{b}}. \tag{29}$$

Previous analytic expressions of SIF for lugs with semi-elliptic surface cracks can be used for "static" fracture mechanics and residual life estimations using crack growth models.

## 5 Numerical validation

To illustrate computation procedures in damage tolerance analysis and residual life estimations of damaged structural components numerical examples are included.

### 5.1 Life estimation of damaged structural elements

Subject of this analysis are cracked aircraft lugs under cyclic load of constant amplitude and spectra. For that purpose conventional Forman crack growth model and crack growth model based on strain energy density method are used. Material of lugs is Aluminium alloy 7075 T7351 with the next material properties:
$R_{\mathrm{m}}$ = 432 MPa ⇔ Tensile strength of material,
$R_{\mathrm{p0,2}}$ = 334 MPa, $K_{\mathrm{Ic}}$ = 70,36 MPa·m$^{0,5}$.

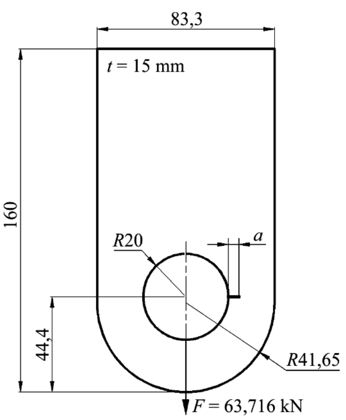



**Figure 4** Geometry of cracked lug 2

Dynamic material properties (Forman's constants): $C_{\mathrm{F}} = 3\times10^{-7}$, $n_{\mathrm{F}}$ = 2,39. Cyclic material properties: $\sigma'_{\mathrm{f}}$ = 613 MPa, $\varepsilon'_{\mathrm{f}}$ = 0,35, $n'$ = 0,121. The stress intensity factors (SIFs) of cracked lugs are determined for nominal stress levels: $\sigma_{\mathrm{g}} = \sigma_{\max}$ = 98,1 MPa and $\sigma_{\min}$ = 9,81 MPa. These stresses are determined in net cross-section of lug. The corresponding forces of lugs are defined as $F_{\max} = \sigma_{\mathrm{g}}$ (w-2R) t = 63716 N and $F_{\min}$= 6371,16 N, that are loaded of lugs. For stress analyses contact pin/lug finite element model is used. For cracked lugs defined in Tab. 1, Fig. 4, with initial cracks $a_0$, SIFs are determined using finite elements, Tab. 2. To obtain high-quality results of SIFs cracked lugs are modelled by singular finite elements around crack tip.

**Table 1** Geometric parameters of lugs [13]

| Lug No. | Dimensions / mm | | | | |
|---|---|---|---|---|---|
| | *2R* | *W* | *H* | *L* | *t* |
| 2 | 40 | 83,3 | 44,4 | 160 | 15 |
| 6 | 40 | 83,3 | 57,1 | 160 | 15 |
| 7 | 40 | 83,3 | 33,3 | 160 | 15 |

The stress intensity factors of cracked lugs are calculated under stress level: $\sigma_{\mathrm{g}} = \sigma_{\max}$ = 98,1 MPa, or corresponding axial force, $F_{\max} = \sigma_{\mathrm{g}} \cdot (w - 2R) \cdot t$ = 63 716 N. In present finite element analysis of cracked lug it is modelled with special singular quarter-point six-node finite elements around crack tip, Fig. 5.

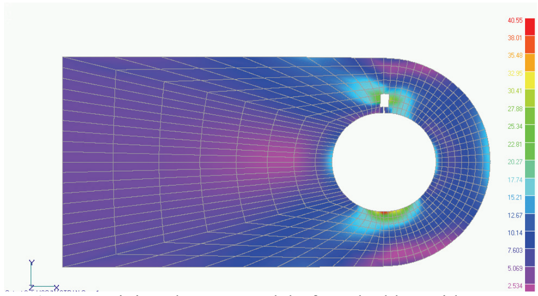


**Figure 5** Finite Element Model of cracked lug with stress distribution

To load the model, a concentrated force, $F_{\max}$, was applied at the centre of the pin and reacted at the other and of the lug. Spring elements were used to connect the pin and lug at each pairs of nodes having identical nodal coordinates all around the periphery. The area of contact was determined iteratively by assigning a very high stiffness to spring elements which were in compression and very low stiffness (essentially zero) to spring elements which were in tension. The stress intensity factors of lugs, analytic and finite elements, for through-the-thickness cracks are shown in Tab. 2. Analytic results are obtained using relations from previous sections, Eq. (13).

**Table 2** Comparisons of analytic with FEM results of SIF

| Lug No. | *a* / mm | $K_{\mathrm{Imax}}^{\mathrm{MKE}}$ / MPa·m$^{0,5}$ | $K_{\mathrm{Imax}}^{\mathrm{ANAL.}}$ / MPa·m$^{0,5}$ |
|---|---|---|---|
| 2 | 5,00 | 68,784 | 65,621 |
| 6 | 5,33 | 68,124 | 70,246 |
| 7 | 4,16 | 94,72 | 93,64 |

From the above Tab. 2 is evident good agreement between analytic and finite element results for determination of stress intensity factors. Accuracy of SIFs is very important in precise crack growth analyses and life estimation of cracked lugs. That means that the proposed analytic model for determination of SIFs is adequate in crack growth analyses. In design process it is very important to know how any geometric parameters of lug have the effects on fracture mechanics parameters.

In Fig. 6 are shown computational and experimental results of cracked lug No. 2 as defined in Fig. 4 and Tab. 1. In this computation analysis the Forman crack growth model is used. Good agreement between computation and experimental results is obtained. It is evident that the computational Forman's crack growth model is to a small extent conservative for longer crack, Fig. 6

In Fig. 7 are shown dependence SIF, $K_{\mathrm{Imax}}$, and height of head of lug $H$. In this analysis geometric properties of lugs are given in Tab. 1. From Fig. 7 is evident increasing of SIFs with increasing of crack length and reducing with increasing height of lug's head.

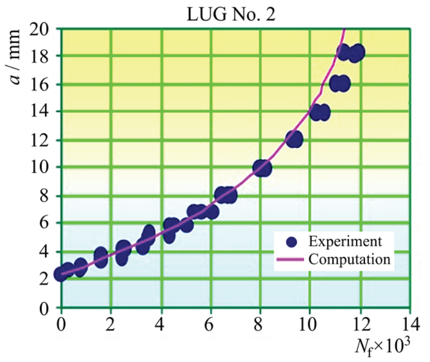



**Figure 6** Comparisons of computational with experimental crack growth results for lug No. 2 ($H$=44,4 mm); $k_t$=2,8

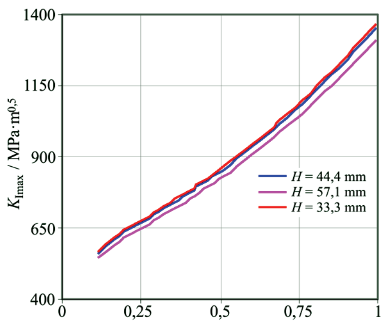



**Figure 7** The effects of head of length ($H$ in mm) in function of crack length to SIF of Lug. No: 7 ($H$=33,3), 2 ($H$=44,4), 6 ($H$=57,1)

In Fig. 8 are shown the results of crack growth for cracked lug using two methods: (1) conventional Forman's method and (2) strain energy density method (GED) [7].

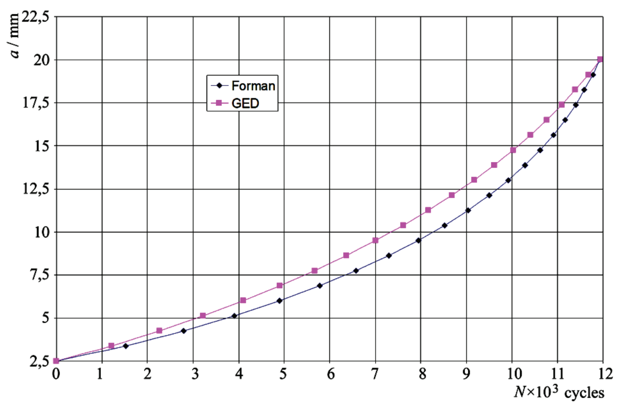


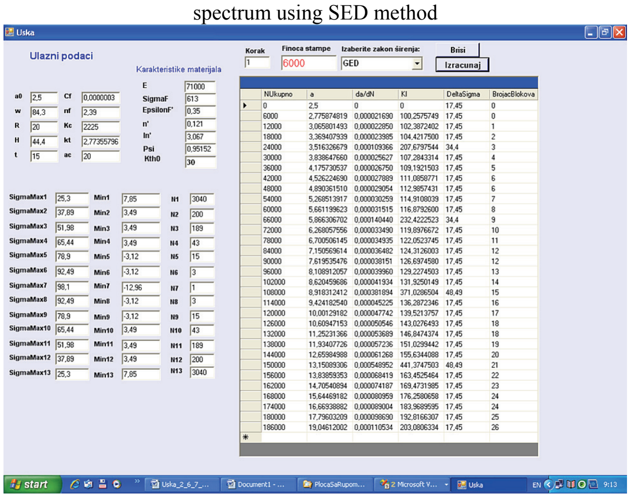

**Figure 8** Comparisons of crack growth results of cracked lug using two crack growth models: (1) Forman's and (2) Strain Energy Density (GED)

## 5.2 Comparison of SED with conventional crack growth method for two-level load spectrum

Here is considered residual life of cracked aircraft attachment lugs using in-house software "LUG". Aircraft lug No. 2, Fig. 4, defined in previous section 5.1, is under general load spectrum. The complete load spectrum and results of residual life of cracked lug No. 2 are given in Tab. 3 and Fig. 9. Residual life of cracked lug, expressed in the form of blocks, is $N_{\mathrm{bl}}$=26. One block of load spectrum corresponds to 100 flight hours of aircraft G-2.

In Fig. 9 is given graphical illustration of residual life of cracked lug No. 2 based on crack growth analysis under general load spectrum.

**Table 3** Results of residual life of cracked lug under general load spectrum using SED method



Uska

Ulazni podaci

| | | | |
|---|---|---|---|
| a0 | 2,5 | Cf | 0,0000003 |
| w | 84,3 | nf | 2,39 |
| R | 20 | Kc | 2225 |
| H | 44,4 | kt | 2,77355796 |
| t | 15 | ac | 20 |

Karakteristike materijala

| | |
|---|---|
| E | 71000 |
| SigmaF | 613 |
| EpsilonF' | 0,35 |
| n' | 0,121 |
| In' | 3,067 |
| Psi | 0,95152 |
| Kth0 | 30 |

| | | | | | |
|---|---|---|---|---|---|
| SigmaMax1 | 25,3 | Min1 | 7,85 | N1 | 3040 |
| SigmaMax2 | 37,89 | Min2 | 3,49 | N2 | 200 |
| SigmaMax3 | 51,98 | Min3 | 3,49 | N3 | 189 |
| SigmaMax4 | 65,44 | Min4 | 3,49 | N4 | 43 |
| SigmaMax5 | 78,9 | Min5 | -3,12 | N5 | 15 |
| SigmaMax6 | 92,49 | Min6 | -3,12 | N6 | 3 |
| SigmaMax7 | 98,1 | Min7 | -12,96 | N7 | 1 |
| SigmaMax8 | 92,49 | Min8 | -3,12 | N8 | 3 |
| SigmaMax9 | 78,9 | Min9 | -3,12 | N9 | 15 |
| SigmaMax10 | 65,44 | Min10 | 3,49 | N10 | 43 |
| SigmaMax11 | 51,98 | Min11 | 3,49 | N11 | 189 |
| SigmaMax12 | 37,89 | Min12 | 3,49 | N12 | 200 |
| SigmaMax13 | 25,3 | Min13 | 7,85 | N13 | 3040 |

Korak: 1 | Finoca stampe: 6000 | Izaberite zakon širenja: GED | Brisi | Izracunaj

| NUkupno | a | da/dN | KI | DeltaSigma | BrojacBlokova |
|---|---|---|---|---|---|
| 0 | 2,5 | 0 | 0 | 17,45 | 0 |
| 6000 | 2,775874819 | 0,000021690 | 100,2575749 | 17,45 | 0 |
| 12000 | 3,065801493 | 0,000022850 | 102,3872402 | 17,45 | 1 |
| 18000 | 3,369407939 | 0,000023985 | 104,4217500 | 17,45 | 2 |
| 24000 | 3,516326679 | 0,000109366 | 207,6797544 | 34,4 | 3 |
| 30000 | 3,838647660 | 0,000025627 | 107,2843314 | 17,45 | 4 |
| 36000 | 4,175730537 | 0,000026750 | 109,1921503 | 17,45 | 5 |
| 42000 | 4,526224690 | 0,000027889 | 111,0858771 | 17,45 | 6 |
| 48000 | 4,890361510 | 0,000029054 | 112,9857431 | 17,45 | 6 |
| 54000 | 5,268513917 | 0,000030259 | 114,9108039 | 17,45 | 7 |
| 60000 | 5,661199623 | 0,000031515 | 116,8792600 | 17,45 | 8 |
| 66000 | 5,866306702 | 0,000140440 | 232,4222523 | 34,4 | 9 |
| 72000 | 6,268057556 | 0,000033490 | 119,8976672 | 17,45 | 10 |
| 78000 | 6,700506145 | 0,000034935 | 122,0523745 | 17,45 | 11 |
| 84000 | 7,150569614 | 0,000036482 | 124,3126003 | 17,45 | 12 |
| 90000 | 7,619535476 | 0,000038151 | 126,6974580 | 17,45 | 12 |
| 96000 | 8,108912057 | 0,000039960 | 129,2274503 | 17,45 | 13 |
| 102000 | 8,620459686 | 0,000041934 | 131,9250149 | 17,45 | 14 |
| 108000 | 8,918312412 | 0,000381894 | 371,0286504 | 48,49 | 15 |
| 114000 | 9,424182540 | 0,000045225 | 136,2872346 | 17,45 | 16 |
| 120000 | 10,00129182 | 0,000047742 | 139,5213757 | 17,45 | 17 |
| 126000 | 10,60947153 | 0,000050546 | 143,0276493 | 17,45 | 18 |
| 132000 | 11,25231366 | 0,000053689 | 146,8474374 | 17,45 | 18 |
| 138000 | 11,93407726 | 0,000057236 | 151,0299442 | 17,45 | 19 |
| 144000 | 12,65984988 | 0,000061268 | 155,6344088 | 17,45 | 20 |
| 150000 | 13,15089306 | 0,000548952 | 441,3747503 | 48,49 | 21 |
| 156000 | 13,83859353 | 0,000068419 | 163,4525464 | 17,45 | 22 |
| 162000 | 14,70540894 | 0,000074187 | 169,4731985 | 17,45 | 23 |
| 168000 | 15,64469182 | 0,000080959 | 176,2580658 | 17,45 | 24 |
| 174000 | 16,66938882 | 0,000089004 | 183,9689595 | 17,45 | 24 |
| 180000 | 17,79603209 | 0,000098690 | 192,8166307 | 17,45 | 25 |
| 186000 | 19,04612002 | 0,000110534 | 203,0806334 | 17,45 | 26 |

In the previous Tab. 3 are: Sigma $F \equiv \sigma'_{\mathrm{f}}$, Epsilon $F' \equiv \varepsilon'_{\mathrm{f}}$, DeltaKth0$\equiv \Delta K_{\mathrm{Ith0}}$, Psi$\equiv \psi$, SigmaMax$\equiv \sigma_{\max}$, SigmaMin$\equiv \sigma_{\min}$, Y$\equiv Y_{\mathrm{SUM}}$ – is the corrective function defined by Eq. (14), GED≡SED and C,m – Paris's constants. Numerical simulation results for crack propagation of cracked lug, based on strain energy density method (SED), point out that the presented approach could be efficiently used in the analyses of residual life.

As previously mentioned, the presented strain energy density method uses cyclic characteristics of the material from low-cycle fatigue (LCF) domain instead of dynamic parameters from more conventional laws for crack propagation by Paris and others.

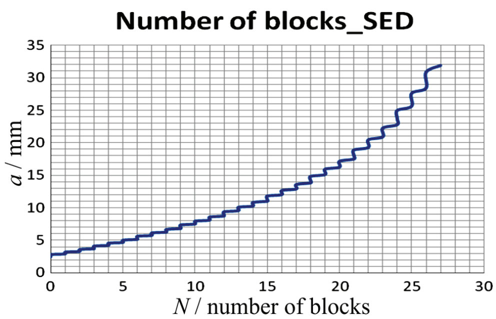



**Figure 9** Graphical illustration of residual life of cracked lug No. 2 under general load spectrum using SED method

In Fig. 9 is shown the effect of the lug's shape of surface crack on residual fatigue life.

## 5.3 The effects of lug's surface crack shape on fatigue life

Analytic expressions for determination of SIF for a lug with semi-elliptic surface crack, are given in section 4.2 and can be used for residual life estimation.

In Fig. 10 are shown the results of crack growth of cracked lugs No. 2 whose geometric properties are given in Figs. 1 and 3.

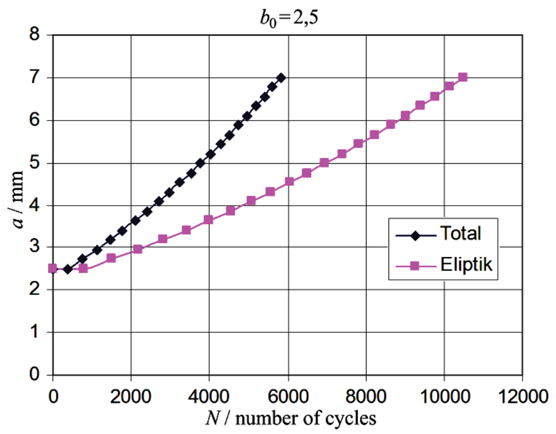



**Figure 10** Comparisons of crack growth results of attachment lug with crack through the thickness and semi-elliptic surface crack

As expected the lug with crack through the thickness ("total") has a more reduced life than the lug with semi-elliptic surface crack ("eliptic").

## 6 Conclusions

In this paper residual life estimations of cracked attachment lugs are considered. Special attention in this investigation has been focused on the effects of shape of lug`s surface crack of cracked lugs on residual fatigue life.

Predictions and experimental investigations for fatigue life of an attachment lug with through the thickness crack under load spectrum were performed. From this investigation the following is concluded:

- The stress intensity factor of cracked lug is defined in analytic form using finite elements.
- A model for the fatigue crack growth, based on SED, is included in this investigation, which incorporates the low cycle fatigue properties of the material.
- Comparison of the predicted crack growth rate using strain energy method with experimental data and conventional Forman's model points out the fact that this model could be effectively used for residual life estimations.
- The stress intensity factor of cracked lug is well defined by analytical method since there is really minor difference when compared with the results obtained by singular finite elements.
- The effects of surface cracks shape of the attachment lugs on residual fatigue life is evident.

In general the presented computation method based on SED can be effectively used in crack growth analyses and for residual life estimations.

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