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CONTRIBUTION TO RESEARCH OF MATHEMATICAL PROPERTIES OF PRE-CHRISTIAN SLAVIC SACRED LANDSCAPE STRUCTURES

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ABSTRACT

Considerable amount of interpreted data indicates that the ancient Slavs positioned their sacred sites in a way which refers to characteristic Sun angles. The article addresses the question whether distances among such sites are based on a common unit of length. In particular, this article tackles that question applying the mathematical formalism on the following two assumptions: (i) the absolute value of a distance between sacred sites was significant to the pre-Christian Slavic priests, along with the angles between lines connecting pairs of sites; (ii) the distances were prevalently measured utilising the projections of isosceles right triangle on the horizontal plane, with the exceptions of flat grounds for which the distances were measured by walk. That assumption follows from the frequent occurrence of ratio $1{:}\sqrt{2}$ in the analysed sacred sites. Based on the two stated assumptions the attempts are done to find the best possible length modules by using the probability distribution method of arithmetic sequences. The main property of length modules which are the least probable to appear by mere chance is that they account for as many as possible of distances from the analysed set of distances. The stated method is applied on numerous sacred systems described in literature. The result is that several common modules are extracted. The modules are subsequently correlated with the modules extracted in my recent article using the novel method which extracts the optimal common sub-module. Value of the length module thereby obtained is 30,9 m. It has 60 sub-units 0,515 m long (a cubit) and 100 sub-units 0.309 m long (a foot). Multiples of 100 or 365 sub-units, respectively, are regularly encountered in the analysed set of sacred sites in the form of sub-harmonics of the observed distances. One may argue that results of the analysis of the distances contributed to the fact that the ancient Slavs were giving a lot of attention to a solar calendar and accurate determination of the time of a year.

KEY WORDS

myth in space, metrology, archeoastronomy, spatial analysis, probability distribution

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INTRODUCTION

The pre-Christian Slavs positioned their sacred places in a tripartite structures [1]. Such structures were related to the central Slavic myth of a divine battle between Perun and Veles [2-4]. Measuring the angles which the Sun takes through the annual cycle and comparing them with the angles between sacred sites, the pre-Christian priests could accurately determine the days of religious festivals. The number of sacred triangles have already been described in Croatia, Slovenia, Austria and Germany [5-15]. These triangles probably give enough evidence that the ancient Slavs positioned their sacred places in spatial constellations which formed the sun angles. This paper as a start point takes the assumption that besides the angles also the distances between sacred sites were important to the pre-Christian Slavic priests and then questions was that distancing done by using some common units of length. The problem is approached by relying on the mathematical theory of probability.

In my recent article [15] it is described a mathematical method which helps to find out the possible common modules of length between some set of sacred sites. The method relies on the mathematical properties of arithmetic and geometric sequences. If for some of the initial parameters of the sequences the probability function for a given distribution of sacred sites in some area shows to be significantly smaller than the average, then it is an indication that the respective distribution is not random. The parameters in such case may point to a system of measures used during creation of the structure. Method tests have shown that any arithmetic or geometric sequence which is not a result of randomness and which is part of some intended system, will appear with its sub-harmonics and harmonics in the histogram of the probability function despite any reasonable number of added points which are not part of the system. But, on the other side, some moderately improbable arithmetic or geometric sequences can appear as a result of completely random distribution of points. It means that if a real distribution of sacred points in some area really is a result of intention, we surely will detect the used modules, but we will not be sure if they are a result of coincidence. But comparing the resulting modules from various Slavic regions it should be possible to determine if among them there is a correlation which can significantly minimize the accidental results. Results form my recent article were indicative, but made on insufficient number of sacred landscape structures. In this article the modified proposed method is applied to a several sacred systems discovered by the other recognized researchers.

This paper is divided into three main sections. In section one the mathematical methods used to calculate the common modules of distances between sacred sites are described along with the explanation how the pre-Christian priests were able to measure them. In section two the least probable modules of arithmetic sequences separately for Župa Dubrovačka and 10 sacred triangles from different regions are computed. Then the modules obtained modules from different regions are taken as input into the common module method with the aim of finding out the ultimate metric module common to the majority of analyzed sacred landscape systems. In section three I give the interpretation of the obtained ultimate metric module.

MATHEMATICAL METHODS

In this article the two main methods for calculating the common modules are used. Probability distribution method uses the properties of arithmetic sequences and is more appropriate for situations when every distance from some set of distances does not necessarily belong to the same system.

Common sub-module method calculates the best common sub-module among several distances and is suitable for situations when all of them are surely part of the same system.

PROBABILITY DISTRIBUTION OF ARITHMETIC SEQUENCE

Arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant. The probability distribution of points around some central position is calculated depending on the value of difference and the value of tolerance.

If for some of the difference values the probability function for a given distribution of sacred sites in some landscape shows to be significantly smaller than the average, then it is an indication that the respective distribution is not random.

The general case [15; pp.129-131] was defined in a two-dimensional plane as a sequence of distances (D_1, D_2, D_3, ... D_k) measured from the central point (S_0) with the associated limits of tolerance (t_1, t_2, t_3, ... t_k). But, practically the same results are given by the simplified formula in which all the input lengths are projected on one-dimension. Since in [15] the tolerance depended on the difference value D, the smaller differences had the absolute values of tolerance too small compared to the value of uncertainty with which the sacred sites were determined. Moreover, the larger differences had the absolute values of tolerance too large in comparison with that uncertainty. Here, the absolute value of tolerance for the arithmetic sequences is taken to be a constant.

In the case of an arithmetic sequence with the difference D and the absolute value of tolerance T (Fig. 1) the probability p, that a sacred site satisfies the conditions to be considered as a member of a sequence, equals the ratio of thickened segments length to the total length. On average, that can be expressed by the following expression:

$$p = \frac{2T}{D}. \tag{1}$$

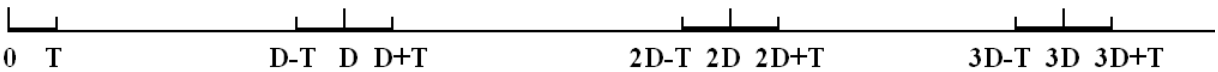


Figure 1. Arithmetic sequence with the difference D and the absolute value of tolerance T. Points satisfying conditions stated in the text are within the thickened segments.

The likelihood P of any distribution of sacred sites in respect to such sequence is expressed using the formula for the binomial distribution:

$$P = q^{N-n} p^{n} \cdot \frac{N!}{n!(N-n)!}, . \tag{2}$$

with N the total number of sacred points, n being the total number of sacred points which meet the conditions, p the probability that a sacred point satisfies the conditions of any member of sequence and $q = 1 - p$ the probability that a sacred point does not satisfy the conditions.

PROBABILITY HISTOGRAM

The results which gives the formula (2) for every arithmetic sequence from D = 100 m to D = 1500 m with the resolution of 1 m are presented in a form of histogram in which on the ordinate are plotted the logarithms of the reciprocal values of probability for every difference value D of the abscissa. The logarithms of the reciprocal values of probability are the most practical way for representing the probabilities, because the less probable sequences result with the higher values and the relative difference among any of them is not very big. The span from 100 m to 1500 m shows to be optimal for the usual size of tolerance with which the sacred sites can be located (approximately ±10 m to ±20 m) and how they are usually distanced (500 m to 30 000 m).

AGGREGATED PROBABILITY HISTOGRAM

This is a method in which the probability distribution of arithmetic sequences around every sacred point in some area is calculated separately. Subsequently the obtained results are aggregated in a single histogram. That means that the reciprocal values of probability for a certain value *D* for every point are summed and plotted on the histogram with the logarithmic scale. Such a histogram represents the overall probability distribution for all points. If there is any common module around any of the sacred points in the area, such graph will reveal it with a significantly large confidence.

COMMON SUB-MODULE METHOD

Among several distances (A_1, A_2, A_3, ... A_n) the method tries to find a common sub-module x in such a way that its harmonics (a_1, a_2, a_3, … a_n) give the lengths as close as possible to the measured distances:

$$x \approx A_1/a_1 \approx A_2/a_2 \approx A_3/a_3 \approx ... \approx A_n/a_n.$$

The harmonics are either integers or integers multiplied by √2.

The possible set of harmonics is searched dividing the minimal distance value A_{min} by every integer and integer multiplied by √2 within a range from 1 to 100 and from √2 to 100√2, respectively, giving the test sub-module values x_i. The values of harmonics are then calculated dividing every distance (A_1, A_2, A_3, ... A_n) with the test sub-modules x_i and rounding up the result to the nearest integer:

$$a_i = [A_i/x_i],\ i = 1, ..., n.$$

The candidate sub-module x which is a real number very close to the value of test sub-module x_i is determined using the function:

$$f(x) = \min_x \sum_{i=1}^{n} \left| a_i x - A_i \right|. \tag{3}$$

At its minimum, the first derivation of function $f(x)$ its sign, which occurs only at the one of the following, non-differentiable points:

$$a_i x - A_i = 0 \rightarrow x_i = A_i/a_i,\ i = 1, ..., n.$$

The candidate sub-module is determined by testing which of the values x_1, x_2, … x_n gives the minimal value for $f(\cdot)$ in (3). Within a given set of candidate sub-modules, the optimal module is the one which gives the minimal value of total error. The fact is that the sub-modules which have the smaller harmonic numbers will more probably appear by intention than the ones which have the larger harmonic numbers.

DISTANCE MEASUREMENT

This article is based on the assumption that the absolute values of distances between sacred sites were important to the pre-Christian Slavic priests, so here I conjecture how they measured these distances. The simplest way to measure some distance is with the help of a measuring rope and stakes for maintaining the direction.

In the plain areas the distance between sacred sites has been measured simply by walk, but in the mountainous and hilly areas or the areas filled with lakes or some other bodies of water it surely has not always been possible. There is also a dilemma as to whether the pre-Christian priests in hilly areas measured the walk distances or the horizontal plane distances.

The pre-Christian priest had the simple means to measure the horizontal plane distances using the rules of the isosceles right triangle (Fig. 2). In this case only one side of the triangle has to lie on the the plain area.

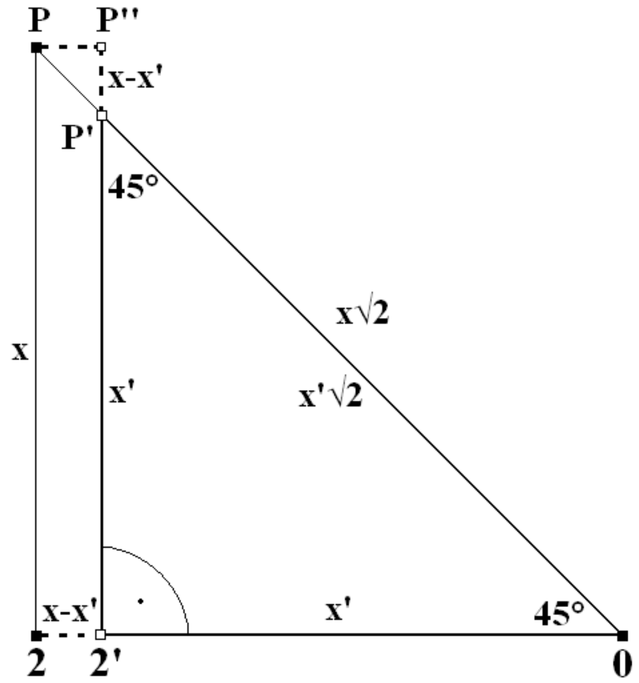


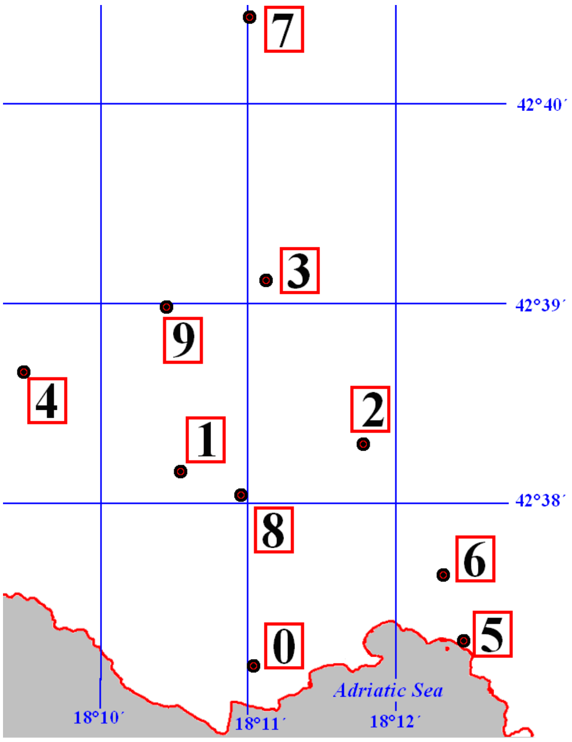
Figure 2. Locating the point 2 using the characteristics of the isosceles right triangle.

If the wish of priest was that the distance 0-2 has the length of *x*, he could have estimated the position of point 2 by locating it temporarily at the point 2'. Then, using the rules of the isosceles right triangle he could walk from point 2' in the one or the other direction perpendicular to the side 0-2' until the point 0 was seen at the angle of 45°, which he achieved at the point P'. If the distance *x*' which he traveled was smaller than *x*, he knew that the point 2' must be moved away from point 0 for the value of *x* –*x*'. If the distance *x*' which he traveled was bigger than x, then he knew that the point 2' must be moved towards point 0 for the value of *x* –*x*'. After that the distances 0-2 and 2-P were equal to *x*, and the distance 0-P equals *x*√2. This method naturally accounts for frequent appearance of ratio 1:√2 among distances.

RESULTS

ŽUPA DUBROVAČKA

The structure of sacred sites at Župa Dubrovačka[1] southeast of Dubrovnik in Croatia, described by I. Kipre [6; p.130], indicates that pre-Christian priests had really measured distances between sacred sites and that they had frequently used relation 1:√2. The system has a number of points so the probability distribution method of arithmetic sequences demonstrates its real strength and reveals what otherwise would not be easily detectable. The method brings about the aggregated probability histogram of arithmetic sequences with an exceptionally improbable length of 432 m and an additional slightly more probable length of 610 m (Fig. 4a and Table 1). These lengths are mutually related by √2: 610 = 432 × √2 which is rather indicative. Graphical representation of lengths which are multiples of 432 m (Fig. 3b) reveals that the lengths which are related by √2 prevalently appear at point 7 which is a mountain peak of height 880 m. This indicates that these lengths were measured using the isosceles right triangles.



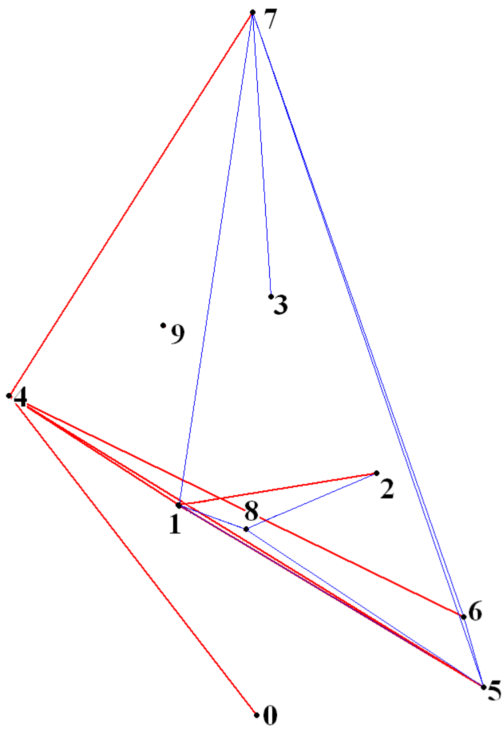


Figure 3. The spatial distribution of sacred sites at Župa Dubrovačka: a) sketch of the map of the area, b) presented distances which are multiples of 432 m (red lines) and 432√2 = 611 m (blue lines), with the tolerance 20 m. Sites are denoted as: 0 – St. Peter in Prenj, 1 – St. Vincent Ferrer in Rovanj, 2 – St. George in Suđurađ, 3 – The Church of the Assumption in Martinovići, 4 – St. Anne (St. Petka or St. Parascheva) in Brgat Gornji, 5 – St. Hilarion in Mlini, 6 – The Church of the Holy Salvation in Krstac, 7 – Peak of Elijah, 8 – St. Mary Magdalene in Mandaljena and 9 – Gradac.

Results indicate the lengths of 648 m and 775 m to be the least probable to appear by chance in a given set of 10 sacred triangles.

THE SET OF TEN SACRED TRIANGLES

In [15], in the area northwest of Zagreb in Croatia, the module of 620 m was detected in the probability diagrams of geometric sequences around Babožnica and St. Vitus in Javorje, and also in the probability diagram of arithmetic sequences around Gradna. The modules of 677 m, 873 m and 1234 m are detected among important archeological sites on the island of Rügen in Germany.

The literature about Slavic pre-Christian sacred tripartite landscape structures describes the number of other sacred triangles in Croatia, Slovenia and Austria. The most of these are shown[2] in Figure 5. It is not very practical to plot the probability histogram of the probability distribution method of arithmetic sequences for every sacred triangle separately, because in every one of them there are only three distances. Better results are obtained if all the distances[3] between the sacred sites which appear in the maps in Figure 5 are taken to be the input into the algorithm which calculates the probability histogram of arithmetic sequences upon such distribution of lengths. Even if some of the triangles are falsely identified as sacred, it will not prevent the algorithm to detect the common modules if they really exist among the remaining triangles or at least among several of them. Falsely identified triangles should not affect the algorithm if their distribution of lengths is random.

The results given by the probability distribution method of arithmetic sequences are presented in Figure 4b and Table 2.

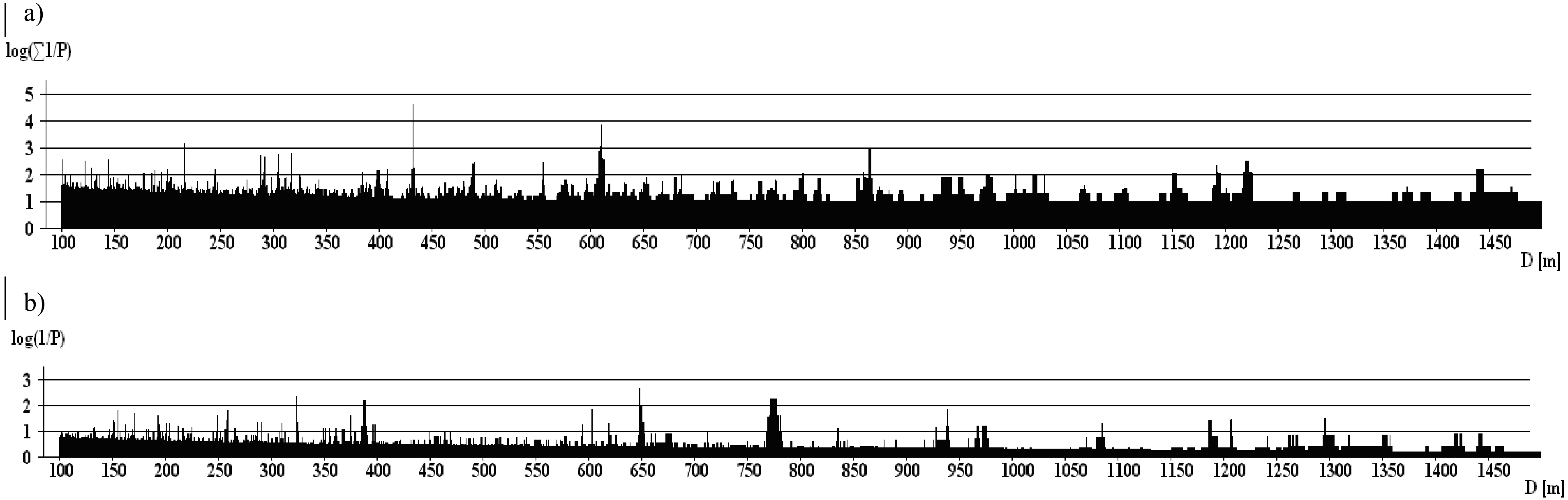


Figure 4. Aggregated histogram of the logarithm of the reciprocal values of probability of arithmetic sequences for: a) all the points in Figure 3, $T = 10$ m, b) all distances in Figure 5, $T = 15$ m.

Table 3. Computed candidate common sub-modules from the list of detected modules. The harmonics with error larger than 6 m are hatched because the calculated error is too large.

Modules, m / Sub-modules, m	432	610	620	648	677	775	873	877	1234	Total error, m
154,326	$2\sqrt{2}$ (–4,5)	~~4 (–7,3)~~	4 (2,7)	~~$3\sqrt{2}$ (–6,8)~~	~~$3\sqrt{2}$ (22,3)~~	5 (3,4)	$4\sqrt{2}$ (0,0)	$4\sqrt{2}$ (4,0)	8 (–0,6)	51,5
109,125	4 (–4,5)	~~$4\sqrt{2}$ (–7,3)~~	$4\sqrt{2}$ (2,7)	~~6 (–6,8)~~	~~6 (22,3)~~	$5\sqrt{2}$ (3,4)	8 (0,0)	8 (4,0)	$8\sqrt{2}$ (–0,6)	51,5
77,125	$4\sqrt{2}$ (–4,3)	~~8 (–7,0)~~	8 (3,0)	~~$6\sqrt{2}$ (–6,4)~~	9 (–17,1)	10 (3,8)	$8\sqrt{2}$ (0,4)	$8\sqrt{2}$ (4,4)	16 (0,0)	46,4
43,628	$7\sqrt{2}$ (0,1)	14 (–0,8)	$10\sqrt{2}$ (3,0)	~~15 (–6,4)~~	$11\sqrt{2}$ (–1,7)	~~18 (–10,3)~~	20 (0,4)	20 (4,4)	$20\sqrt{2}$ (0,0)	27,2
30,857	**14 (0,0)**	**$14\sqrt{2}$ (–0,9)**	**20 (2,9)**	**21 (0,0)**	**22 (–1,9)**	**25 (3,6)**	**$20\sqrt{2}$ (0,2)**	**$20\sqrt{2}$ (4,2)**	**40 (–0,3)**	**14,0**

Table 1. Minimal values of probability of arithmetic sequences aggregated for all points in Fig. 3.

Arithmetic module, m	$\log(\sum p^{-1})$	Relation
432	**4,654**	**432**
610	**3,935**	$\approx 432\sqrt{2}$
216	3,244	432/2
864	3,053	432×2

Table 2. Minimal values of probability of arithmetic sequences for all distances in Figure 5.

Arithmetic module, m	$\log(\sum p^{-1})$	Relation
648	**2,727**	**648**
324	**2,428**	$\approx 648/2$
775	2,336	775
388	2,277	775/2

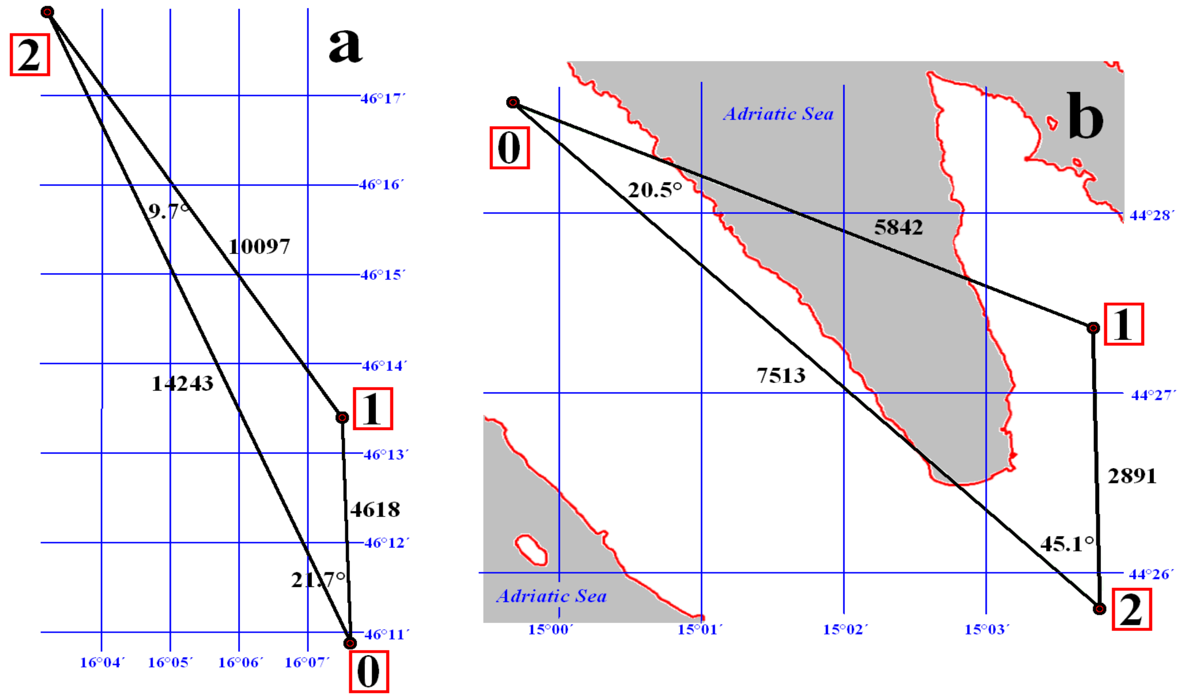
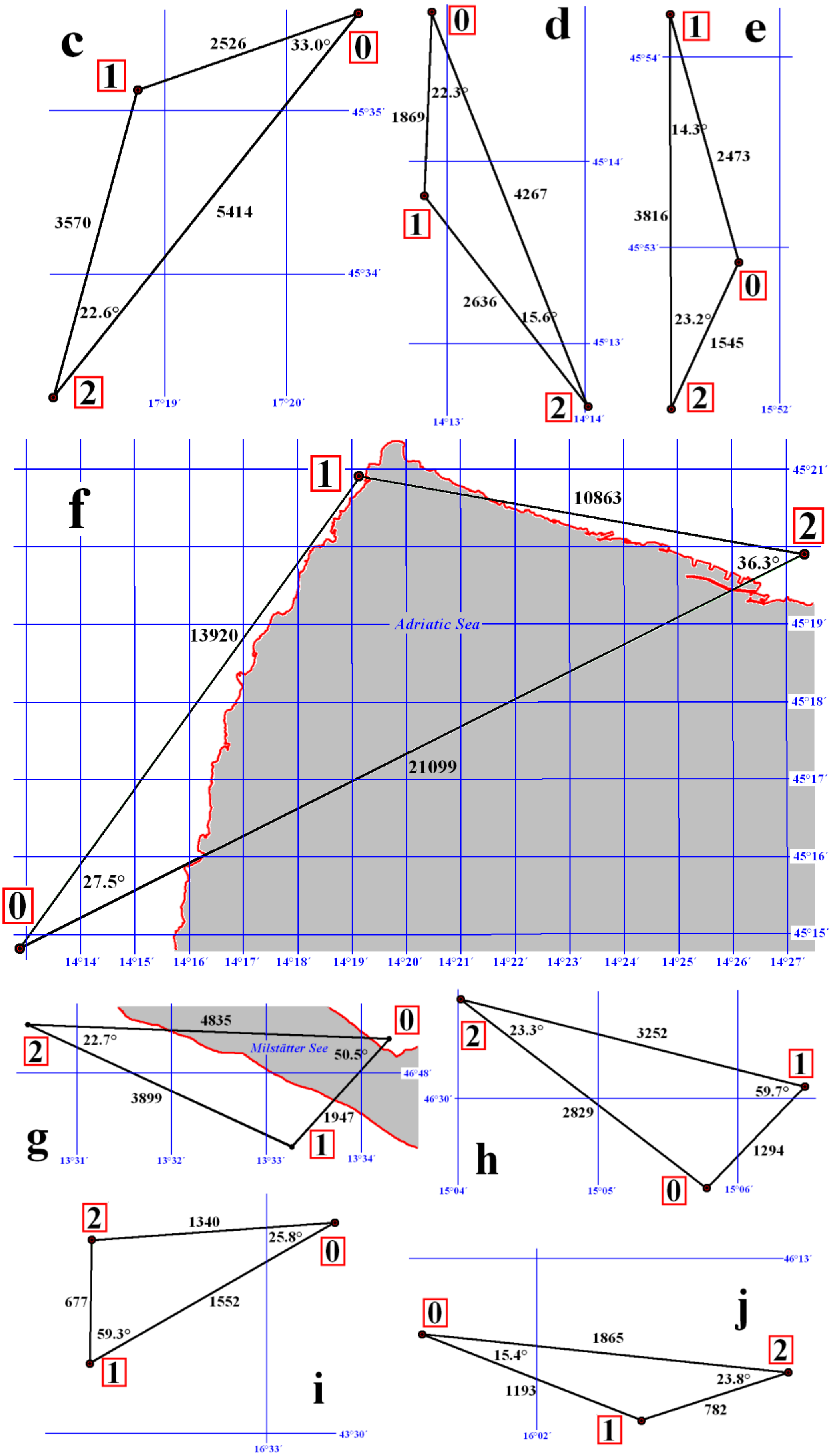


Figure 5. (This page and p.79) Ten sacred triangles, a) Ivanec in Croatia (0 – Ivančica, 1 – St. Mary in Ivanec, 2 – St. Wolfgang in Vukovoj[4]) [5; pp.426-433, 14; p.193], b) Pag in Croatia (0 – St. Vitus, 1 – St. George, 2 – St. Mary[5]) [5; pp.445-449, 10, 11, 14; p.193] and [15; pp.151-152], c) Papuk in Croatia [9; p.54, 14; p.193] (0 – Pogani vrh, 1 – Petrov vrh, 2 – Bijela (St. Margaret)[6]), d) Mošćenice in Croatia [5; pp.437-439, 14; p.194] (0 – Perun in Mošćenice, 1 – Gradac, 2 – St. Helen in Jelena[7]), e) Kameni svati in Croatia (0 – The former Puzjak's mill at Lončar in Novaki, 1 – St. Venedelin in Donja Bistra, 2 – Kameni svati[8]) [13; p.77], f) Rijeka in Croatia [5; p.439, 14; p.193] (0 – Perun in Mošćenice, 1 – At. Anne in Volosko, 2 – St. George Trsat9), g) Millstatt in Austria [1; pp.165-166] (0 – St. Salvator in Millstatt, 1 – Hochgosch, 2 – St. Wolfgang[10]), h) Slovenj Gradec in Slovenia [1; pp.167-168] (0 – St. Mary in Homec, 1 – St. George in Legen, 2 – St. Pancras in Puščava[11]), i) Žminjača in Croatia (0 – St. George in Perunsko, 1 – The Lady of Sita in Strožanac[12], 2 – The Snake's stone in Žminjača[13]) [5; pp.441-443, 7, 8, 14; p.194], j) Lepoglava in Croatia [5; pp.443-444, 14; p.194] (0 – St.George in Purga Lepoglavska, 1 – St. Mary in Lepoglava, 2 – St. John (Vitus) in Gorica Lepoglavska[14]).


c
0
1
2
2526
33.0°
3570
5414
22.6°
45°35′
45°34′
17°19′
17°20′
d
0
1
2
1869
22.3°
4267
2636
15.6°
45°14′
45°13′
14°13′
14°14′
e
1
0
2
14.3°
2473
3816
23.2°
1545
45°54′
45°53′
15°52′
f
1
2
0
10863
36.3°
13920
Adriatic Sea
21099
27.5°
45°21′
45°19′
45°18′
45°17′
45°16′
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Milstätter See
50.5°
3899
1947
46°48′
13°31′
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3252
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2829
1294
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15°04′
15°05′
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i
2
0
1
1340
25.8°
677
1552
59.3°
43°30′
16°33′
j
0
2
1
1865
15.4°
23.8°
1193
782
46°13′
16°02′

COMMON SUB-MODULE

So far we detected the next modules as the least probable to appear by mere chance:

- Babožnica, Gradna: 620 m; 877 m [15; p.142, p.154].
- Rügen: 677 m; 873 m; 1234 m [15; p.142, p.154].
- Župa Dubrovačka: 432 m; 610 m.
- set of 10 sacred triangles in Croatia, Slovenia and Austria: 648 m; 775 m.

If these modules are taken as the input values $A_1 = 432$, $A_2 = 610$, $A_3 = 620$, $A_4 = 648$, $A_5 = 677$, $A_6 = 775$, $A_7 = 873$, $A_8 = 877$ and $A_{)} = 1234$, for the common sub-module method, then it results with several candidate common sub-modules, listed in Table 3.

The optimal common sub-module with the minimal total error is the candidate common sub-module with length 30,857 m.

DISCUSSION

From Table 3 it can be seen that the module of 432 m has 14 sub-modules of (30,857 ± 0.15) m, 610 m has 14√2 of them, the module 620 m have 20 detected sub-modules, the module of 648 m 21 of them, the module of 677 m 22 of them, the module of 775 m has them 25, the modules of 873 m and 877 m have 20√2 of them and the module of 1234 m has 40 of them.

The other detected candidate modules in Table 3 are the value of 43,628 m which is equal to 30,857 × 1,414 and the members of a geometric sequence which is nearly identical to the geometric sequence detected at Babožnica: 77,2 ↔ 109,2 ↔ 154,4 ↔ 218,4 ↔ 308,8 ↔ 436,7 ↔ 617,6 ↔ 873,4 ↔ 1235,2 m [15; p.138; p.142].

In addition, the length of 0,517 m, a cubit was proposed to be a basic unit behind 620 m [15; p.154]. The detected sub-module of 30,865 m has about 60 such anthropomorphic sub-units.

It is possible to propose also the sub-unit of 0,3086 m (a foot) because in the detected sub-module there is a hundred of such sub-units. Behind both of these sub-units could be the length of 0,1029 m which is equal to 0,3086/3 m or 0,5143/5 m, which is their first common sub-harmonic. The above mentioned geometric sequence has a member with the length of 154,3 m which is the first common harmonic of 30,86 m and 51,43 m.

The largest number of sacred triangles in Figure 5 have sides which can be decomposed into the subharmonics equal to the sub-module values of approximately 30,9 m and 51,43 m.

In this way, few of them have sides which are some multiple of 7 detected sub-modules or 700 detected sub-units. These are the Papuk's triangle which can be factorized into[15] 0,5155 m × 700 × (7, 7√2, 15), the Slovenj Gradec's triangle which can be factorized into 0,3097 m × 700 × (6, 13, 15) and the Rijeka's triangle which can be factorized into 0.3107 m × 700 × (50, 64, 97).

The Kameni svati's triangle and Žminjača's triangle can be factorized into the multiples of 2 sub-modules or 200 sub-units: the first one has sides 0,5152 m × 200 × (15, 24, 37) and the other 0,3090 m × 200 × (11, 22, 25).

The Millstatt's triangle and the Pag's triangle can be factorized into multiples of 3 sub-modules or 300 feet: the first one into 0,3094m × 300 × (21, 42, 52) and the other into 0,3092 m × 300 × (22√2, 63, 81).

Other triangles deviate from the above mentioned rule. The two triangles from Ivanec's area have sides which are multiples of 120 feet: the Ivanec's triangle can be factorized into 0,3092 m × 8 × 120 × (11√2, 34, 48) and the Lepoglava's triangle into 0.3107 m × 120 × (21, 32, 50).

The Mošćenice's triangle cannot be simply factorized into any multiples of 100 or 120 sub-units. But the possible common module factorization of all 3 sides of this triangle is with a factor of 365 cubits: 0,5166 m × 365 × (7√2, 14, 16√2).

If the mentioned module of 1234 m from Rügen has 40 sub-modules of 30,85 m or 4000 feet of 0,3085 m, then the module of 677 m can be factorized into 0,3085 m × 6 × 365,75, which is very close to 0,3085 m × 2200 (678,7 m).

However, it is not clear whether the Rügen's north triangle (Arkona-Venzer Burgwall--Rugard) [12; p.242, 13; p.81, 15; pp.147-148] should be factorized into 0,3085 m × 365 × 6 × (18, 31, 42) or 0,3078 m × 2200 × (18, 31, 42). That dilemma may be resolved looking at the distance between Wanda mound and Krakus mound in Krakow which has 8629 m [16], as given in Fig. 6 and Table 5. That length is almost exactly for the factor of √2 less than the length between Rugard and Venzer Burgwall on the island of Rügen [15; p.149]: 12 188 m = 1,41245 × 8629 m.

The distance between Wanda mound and Krakus mound can be decomposed into 28 000 feet or 16 800 cubits: 0,3082 m × 28 000 = 0,3082 m × (40 × 700) = 0,3082 m × (14 × 2000) = 0,5136 m × 16 800 = 0,5136 m × (14 × 1200) = 0,5136 m × 46 × 365,217.

It means that this distance is at the same time the multiple of 700 and 1000 feet, and simultaneously the multiple of 1200 cubits and 365,22 cubits.

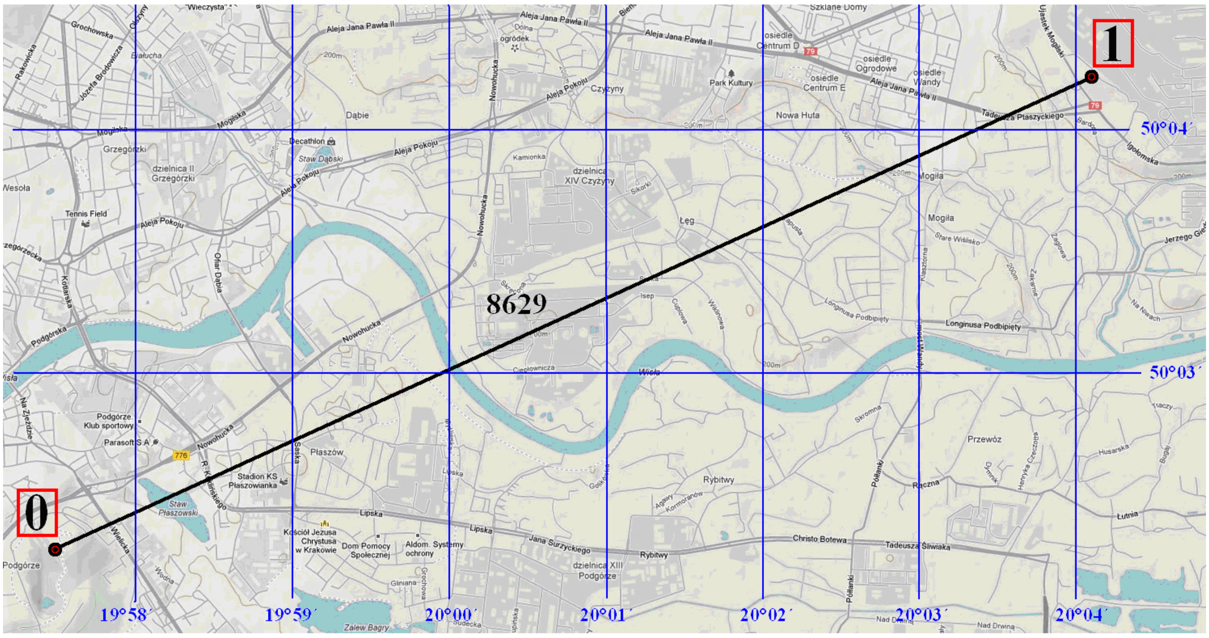

Figure 6. Geographic location of Krakus mound and Wanda mound in Krakow, Poland[16], 0 – Krakus mound and 1 – Wanda mound.

Table 5. Geographic location of Wanda mound and Krakus mound in Krakow Poland [16].

Point	**φ**		**λ**		***h* (*H*), m**		**Point name**	
KR	50°02'17"		19°57'30"		309,2 (269,4)		Krakus Mound	
KW	50°04'13"		20°04'05"		277,9 (238,5)		Wanda Mound	
Direction	**Distance *d*, m**	**A1-2**	**A2-1**	***δ***	**Date Sunrise**		**Date Sunset**	
KR-KW	8629	65°29' 38"	245°34' 41"	15°28'	2. V	10. VIII	6. II	4. XI

The fact that 16 800 days equal exactly 46 years, or that 8400 days or 1200 weeks equal exactly 23 solar years accounting also the leap-year days is a well known fact since ancient times. The number 8400 is the lowest common harmonic of 100 and 365,2. In reality one solar year duration is 365,242 days. The observed accuracy is rather indicative.

If one cinsiders that the length of 12 188 m between Rugard and Venzer Burgwall equals 16 800√2 cubits with a cubit of 0,513 m and a feet of 0,3078 m (0,513 × 3/5), then the module of 677 m actually equals 2200 feet which is only approximately equal to the number of days in 6 years, 2191,5 days.

After performing that correction, it appears that the Rügen's north triangle can be factorized into 0,513 m × 2400 × (7√2, 17, 23) and also into 0,3078 m × 2200 × (18, 31, 42). The Rügen's south triangle (Zudar-Venzer Burgwall-Rugard) [13; p.81, 15; pp.147-148] can be factorized into 0,3078 m × 4400 × (9, 14, 20).

It can also be observed that in the area of Župa Dubrovačka the distance between Ilijin vrh (Peak of Elijah) and the Church of St. Hilarion in Mlini amounts to 6105 m, which is 8629/1,4134 m. According to the perfect ratio observed in the case of Krakow this length has exactly 8400√2 cubits or 14 000√2 feet. The modules of 432 m and 611 m which we detected there amount to 1400 and 1400√2 feet. The triangle Ilijin vrh – St. Anne in Brgat Gornji – St. Hilarion in Mlini is almost right angled (Fig. 3). Its angles are 89,2°, 51,2° and 39,5° and sides 432 m × (9, 11, 10√2)), but it is not the isosceles triangle. One may argue whether the constructor used the approximation that the right angled triangle with the legs in a ratio 9:11 has the hypotenuse almost equal to 10√2.

In the area of Babožnica the length which could encode the number of days in 46 solar years is the distance between Kameni svati and the Church of St. Catharine in Hrebine [15; p.144]. This line crosses exactly above Babožnica and amounts to 8685 m giving a cubit of 8685/16800 m = 0,517 m. This is the same value which gives the module of 620 m when divided by 1200. Just to the south, the triangle St. Anthony in Gradna – St. Nicholas in Strmec – St. Anastasia in Samobor has the characteristic sun angles of 23,5° and 34,2° [15; p.145]. The length between St. Anthony in Gradna and St. Nicholas in Strmec amounts to 4342 m which is 8685/2 m, so it contains exactly 8400 cubits and codes the number of days in 23 solar years.

That analysis points to the fact that not only the angles, as previously shown, but also the distances between the sacred sites could refer to a some form of a solar calendar (the number of days in a year, 365). Indeed, the identification of the common sub-module relied upon the property that the distances concentrate around the values which are both the multiple of 100 sub-units and the multiple of 365 sub-units.

Rather important confirmation of the sub-unit which equals 0,309 m (a foot) is a fact that the length of 3,00 m to 3,13 m was as a basic measuring unit of burial sites and sacral objects detected by A. Pleterski in his key article about three-partite Slavic pre-Christian landscape structures [1; p.182].

If the detected modules are intentionally formed, which means that they did not appear by pure chance, then one can assume that the basic unit of length had sacred meanings. That brings about the further thought that it was probably represented in some form inside the Slavic pre-Christian temples[17].

CONCLUSIONS

Despite that it is not possible to prove beyond absolute doubt that the discovered length of approximately 30,9 m belongs to the common system of measures of pre-Christian Slavs, it is possible to assume that if distances between sacred sites were important to the pre-Christian Slavic priests and if they possessed some common system of units then the discovered common sub-module is the most probable candidate for such system. The fact that the units of 30,9/60 m and 30,9/100 m multiplied by 100 and 365 are often the common sub-harmonics of the observed distances is an additional confirmation of the correctness of the theory. The appearance of the distances which contain some multiple of 8400 cubits referencing in this way 8400 days, 1200 weeks and exactly 23 solar years accounting also the leap-year days is also indicative.

We can surmise that the performed analysis of the distances indicates the same which was already known from the observed angles between sacred places: the ancient Slavs were giving a lot of attention to solar calendar and accurate determination of the time of year.

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[1]Geographic coordinates of sacred sites are: 0 (423711,50 181101,93), 1 (423809,70 181032,70), 2 (423818,55 181146,86), 3 (423907,31 181107,34), 4 (423839,93 180929,06), 5 (423719,06 181226,94), 6 (423738,57 181219,60), 7 (424026,28 181100,20), 8 (423803,07 181057,78) and 9 (423859,47 181027,48). These, as well as geographic coordinates in remarks 2-9, 13 and 14, were determined using www.arkod.hr.

[2]Some triangles are excluded from the analysis because the available sources did not make possible to accurately locate the sacred sites. These are the triangles at Wechsel in Austria, Paški Kozjak in Slovenia, Dejlovce in Macedonia [1], Mrdakovica near Vodice in Croatia [18] and some others. The triangle northeast of Perun at Žrnovnica is excluded because it is not clear whether the third point of this triangle is the Church of St. Michael or the peak of Gračić [5, 6, 18] and for the Zagreb's triangle it is not clear where to precisely locate Županići.

[3]The lengths in meters are: 1340, 677, 1552, 3252, 2829, 1294, 2526, 3570, 5414, 4835, 3899, 1947, 1865, 1193, 782, 1869, 4267, 2636, 2473, 3816, 1545, 10 097, 14 243, 4618, 5842, 7513, 2891, 10 863, 13 920 and 21 099.

[4]Geographic coordinates of sacred sites are: 0 (461053,14 160737,91), 1 (461322,53 160730,40) and 2 (461755,75 160311,83).

[5]Geographic coordinates of sacred sites are: 0 (442836,27 145941,73), 1 (442721,70 150344,65) and 2 (442548,08 150347,34).

[6]Geographic coordinates of sacred sites are: 0 (453535,47 172035,59), 1 (453507,60 171846,02) and 2 (453315,60 171804,93).

[7]Geographic coordinates of sacred sites are: 0 (451449,32 141253,23), 1 (451348,82 141250,06) and 2 (451239,60 141400,83).

[8]Geographic coordinates of sacred sites are: 0 (455255,58 155142,77), 1 (455413,11 155113,98) and 2 (455209,53 155114,82).

[9]Geographic coordinates of sacred sites are: 0 (451449,32 141253,23), 1 (452054,59 141907,76) and 2 (451954,72 142719,39).

[10]The coordinates of sacred sites are determined using GoogleEarth: 0 (464814,58 133417,32), 1 (464727,63 133316,08) and 2 (464821,01 133029,49).

[11]Cartesian coordinates of sacred sites are determined using gis.arso.gov.si/atlasokolja: 0 (507744 149856), 1 (508651 150779) and 2 (505498 151576).

[12]The position of sacred site according to some researchers is not located at The Lady of Sita, but rather in the close vicinity (at 'Krug' in Strožanac). This article uses the older positioning.

[13]The coordinates of sacred sites are: 0 (433037,61 163316,51), 1 (433012,53 163216,62) and 2 (433034,47 163217,04).

[14]The coordinates of sacred sites are: 0 (461247,66 160132,91), 1 (461233,54 160224,71) and 2 (461241,27 160259,47).

[15]The sides of triangles are written in a form: a measuring unit *x* a number of units *x* (a factor for the first side, a factor for the second side, a factor for the third side).

[16]Map adapted from maps.google.com.

[17]This could be the height of some statue. For example, the idol of Zbruch is 2,57 m tall, which is equal to 0,514 m × 5. The height of the Plomin tablet is 0,52 m. It is symptomatic that this Late Antique Roman plastic with carved glagolitic letters has a figure with unnaturally shortened legs, perhaps to accommodate the length of one cubit.

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DOPRINOS ISTRAŽIVANJU MATEMATIČKIH SVOJSTAVA SVETIH KRAJOBRAZNIH STRUKTURA PREDKRŠĆANSKIH SLAVENA

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SAŽETAK

Znatna količina interpretiranih podataka upućuje na to kako su stari Slaveni postavljali svoja posvećena mjesta na način koji povezan s karakterističnim Sunčevim kutovima. Ovaj članak razmatra jesu li udaljenosti između takvih svetih mjesta višekratnik zajedničke mjerne jedinice za duljinu. Razmatranje se provodi sa striktno matematičkog stajališta polazeći od sljedećih pretpostavki: (i) pretkršćanskim slavenskim svećenicima bile su važne apsolutne vrijednosti udaljenosti među posvećenim mjestima, zajedno s kutovima između pravaca koji ih povezuju, (ii) predkršćanski svećenici često su mjerili udaljenosti metodom koja koristi svojstva istorkačnog pravokutnog trokuta, osim kad su mjerili udaljenosti po ravnome tlu. Ta pretpostavka posljedica je relativno česte pojave omjera 1:√2. Na temelju navedenih pretpostavki, primjenom metode raspodjele vjerojatnosti u aritmetičkim nizovima, tražen je optimalni modul duljine. Moduli duljine najmanje nasumične vjerojatnosti pojavljivanja predstavljaju subharmonike što većeg broja stvarnih udaljenosti između posvećenih mjesta. Navedena metoda primijenjena je na brojna posvećena mjesta opisana u literaturi. Kao rezultat, izdvojeno je nekoliko modula duljine koji su zatim korelirani s modulima izdvojenim u prethodnom radu novom metodom kojom se izdvaja optimalni zajednički submodul. Tako je dobiven iznos 30,9 m. Ta duljina sadrži 60 duljina 0,5143 m (lakat) i 100 duljina 0,309 m (stopa). Spomenute jedinice pomnožene sa 100 i 365 su često zajednički subharmonici promatranih udaljenosti. U radu se raspravlja o doprinosu analize duljine prethodnom stavu da su stari Slaveni mnogo pažnje poklanjali solarnom kalendaru i točnom određivanju doba godine.

KLJUČNE RIJEČI

mitovi u prostoru, mjeriteljstvo, arheoastronomija, prostorna analiza, raspodjela vjerojatnosti