QUANTITY DISCOUNTS IN SUPPLIER SELECTION PROBLEM BY USE OF FUZZY MULTI-CRITERIA PROGRAMMING

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Abstract
Supplier selection in supply chain is a multi-criteria problem that involves a number of quantitative and qualitative factors. This paper deals with a concrete problem of flour purchase by a company that manufactures bakery products and the purchasing price of flour depends on the quantity ordered. The criteria for supplier selection and quantities supplied by individual suppliers are: purchase costs, product quality and reliability of suppliers. The problem is solved using a model that combines revised weighting method and fuzzy multi-criteria linear programming (FMCLP). The paper highlights the efficiency of the proposed methodology in conditions when purchasing prices depend on order quantities.

Keywords: vendor selection, fuzzy linear programming, revised weighting method, price breaks.

1. INTRODUCTION

The problem of vendor selection and determination of material quantities supplied is the key element in the purchasing process in manufacturing which is one of the most important activities in supply chain. If all the selected vendors are able to meet the buyer's requirements completely, then the selection process becomes easier and is based only on the selection of the most suitable vendor in terms of purchasing costs, product quality, and vendor reliability. Nevertheless, practice shows that it is not good to rely on one vendor only. Therefore the management of the purchasing company generally enters into contracts with several vendors. Their number usually ranges from two to five for each sort of material. Also, there are cases when no vendor can meet the buyer's demand, or will not do it in order to protect his own business interests.

In this paper we will discuss the supplier selection problem when there are some limitations on suppliers’ capacity, quality and so on, where no supplier can satisfy the buyer’s total requirements and the buyer needs to purchase some of the needed material from one supplier and some from another to compensate for the shortage of capacity or low quality of the first supplier. The model combines two methods used in operational researches. The first of them, revised weighting method is used to determine the coefficient weights of complex criteria functions (quality and reliability). Coefficients determined in this way present the coefficients of the objective functions in the fuzzy multi-criteria programming model providing the final selection and the quantity supplied from a particular vendor. The constraints in the multiple objective programming model are the total demand and the limitations of supplier capacities. Vendor selection is an important issue dealt with by numerous researchers. Great efforts are made to define appropriate models for vendor selection and determination of supply quotas from the selected vendors and to apply the adequate methods to solve such models.

The problem of supplier selection and determination of supply quotas from selected vendors becomes more complex if it takes into account the quantity discounts granted. This problem was dealt with by a number of

1 This article is dedicated to Professor Sanjo Zlobec on occasion of his 70th birthday
This study focuses on the proposed methodology and the specific problem of vendor selection and determination of supply quotas in a bakery.

The rest of the paper is organized as follows: We will first present the methodology of vendor selection and determination of supply quotas. Then the proposed methodology will be tested on the concrete example of vendor selection by a bakery.

2. METHODOLOGY OF VENDOR SELECTION WITH PRICE BREAKS

For vendor selection and determination of supplied quantity we will use the revised weighting method and fuzzy multi-criteria linear programming. The main steps in the proposed model are:

- Determining criteria for vendor selection.
- Applying revised weighting method to determine the variable's coefficients in criteria functions.
- Building and solving the FMCLP model to determine supply quotas from selected vendors.

2.1. Determining criteria for vendor selection

The first step in the proposed methodology is selection of criteria for vendor selection. Numerous criteria are stated in literature and their selection depends on the concrete problem (Weber Current and Benton (1991)). The most important criteria may certainly be: the total purchasing costs in a particular period, product quality offered by particular vendors, and vendor reliability. Each of these criteria is expressed through a number of sub-criteria, which can further be expressed through a number of sub-sub-criteria, etc. This reveals the hierarchical structure of criteria for vendor selection, which directs us to apply the revised weighting method to solve this problem.

2.2. The revised weighting method

We will give a brief outline of the basic propositions of this multi-criteria method used in a large number of factual cases.

The main idea of the weight coefficient method as presented by Gass and Satty (1955) and Zadeh (1963) is to relate each criteria function with the weight coefficient and to maximize/minimize the weighted sum of the objectives. In that way the model containing several criteria functions is transformed into the model with one criteria function. It is assumed that the weight coefficients \( w_j \) are real numbers so that \( w_j \geq 0 \) for all \( j = 1, \ldots, k \). It is also assumed that the weights are normalized, so that \( \sum_{j=1}^{k} w_j = 1 \). Analytically presented, the multi-criteria model is modified into a mono-criterion model and is called the weighting model:

\[
\text{max/ min } \sum_{j=1}^{k} w_j f_j(\chi) = \sum_{j=1}^{k} \sum_{i=1}^{n} w_j c_{ji} x_i
\]

s.t.

\( \chi \in X \).  

(1)

To make the weight coefficients \( w_j \) express the relative importance of criteria functions \( f_j \) we propose linear transformation of criteria functions coefficients. To allow addition of weighted criteria functions we have to transform all of them either into functions that have to be maximized or into functions to be minimized. For more details see Perić and Babić (2010).

In this paper we use the revised weight coefficients method to reduce the number of complex criteria functions. This idea originates from Koski and Silvennoinen (1987). According to it, the normalized original criteria functions are divided into groups so that the linear combination of criteria functions in each group forms a new criteria function while the linear combination of new criteria functions form a further criteria function, etc. In this way we obtain a model with a reduced number of criteria functions. According to this
each Pareto optimal solution of the new model is also Pareto optimal solution of the original model, but the reverse result is not generally true.

To determine the set of compromise solutions and the preferred solution, we will here use the fuzzy linear programming method. For that purpose we cannot use the revised weight coefficients method as the coefficients of cost function (purchasing price) are not fixed but depend on the order quantity.

2.3. Fuzzy multi-criteria programming (FMCP)

The FMCP model for solving the problem of determining the supply quotas by selected vendors with fuzzy goals and fuzzy constraints can be presented as:

\[ f_r^* = \sum_{i=1}^{n} c_r x_i \leq f_r^0, \quad r = 1,2,\ldots,p \]  
\[ f_s^* = \sum_{i=1}^{n} c_s x_i \geq f_s^0, \quad s = p+1, p+2,\ldots,q \]  
\[ g_l(x) = \sum_{i=1}^{n} a_l x_i \geq b_l, \quad l = 1,\ldots,m \]  
\[ g_p(x) = \sum_{i=1}^{n} x_i = D, \quad 0 \leq x_i \leq u_i, \quad i = 1,\ldots,n. \]

In this model the sign \( \approx \) indicates fuzzy environment. The symbol \( \leq \approx \) denotes the fuzzy version \( \leq \), and is interpreted as “essentially smaller than or equal to”, the symbol \( \geq \approx \) is interpreted as “essentially greater than or equal to”, while the symbol \( \approx \approx \) is interpreted as “essentially equal to”. \( f_r^0 \) and \( f_s^0 \) represent the aspiration levels of criteria functions by the decision maker.

Assuming that the membership functions based on preference or satisfaction are linear, we can present the linear membership functions for criteria functions and constraints as follows:

\[ \mu_{f_r}(x) = \begin{cases} 
1 & \text{for } f_r^0 \leq f_r(x) \\
\frac{(f_r^0 - f_r(x))/(f_r^+ - f_r^0)}{f_r^+ - f_r^0} & \text{for } f_r^0 \leq f_r(x) \leq f_r^+ \\
0 & \text{for } f_r^+ \leq f_r(x) 
\end{cases}, \quad \forall r = 1,2,\ldots,p \]  
\[ \mu_{f_s}(x) = \begin{cases} 
1 & \text{for } f_s(x) \leq f_s^0 \\
\frac{(f_s(x) - f_s^0)/(f_s^+ - f_s^0)}{f_s^+ - f_s^0} & \text{for } f_s^0 \leq f_s(x) \leq f_s^+ \\
0 & \text{for } f_s^+ \leq f_s(x) 
\end{cases}, \quad \forall s = p+1, p+2,\ldots,q \]  
\[ \mu_{g_l}(x) = \begin{cases} 
1 & \text{for } g_l(x) \geq b_l \\
\frac{(g_l(x) - b_l)/(b_l^+ - b_l)}{b_l^+ - b_l} & \text{for } b_l \leq g_l(x) \leq b_l^+ \\
0 & \text{for } g_l(x) \leq b_l \]  
\[ \mu_{g_p}(x) = \begin{cases} 
1 & \text{for } g_p(x) = D \\
\frac{(g_p(x) - D^-)/(D^- - D})/(D^+ - D^-) & \text{for } D^- \leq g_p(x) \leq D \\
\frac{(D^+ - g_p(x))/(D^+ - D)}{D^+ - D} & \text{for } D \leq g_p(x) \leq D^+ \\
0 & \text{in other cases,} \]  

where \( b_l = b_l^+ - d_l \) and \( b_l^+ = b_l + d_l \), and \( D = D^+ - p_1 \), \( D^+ = D + p_2 \). \( d_l \) are subjectively determined constants expressing the limits of allowed deviations of \( l \) inequation (tolerance interval) and \( p_1 \), \( p_2 \) are subjectively determined constants expressing the limits of allowed deviations of equation \( g_p(x) \).

The optimal solution \( \hat{x} \) of the above model can be obtained by solving the following linear programming model (Zimmermann (1978)):
\begin{align}
\text{(max)} \lambda \\
\text{s.t.} \\
\lambda & \leq \mu_{j_1}(x), \quad j = 1, 2, \ldots, k \\
\lambda & \leq \mu_{j_2}(x), \quad l = 1, 2, \ldots, m \\
\lambda & \leq \mu_{l_1}(x) \\
0 & \leq x_i \leq u_i, \quad i = 1, \ldots, n; \quad \lambda \in [0, 1],
\end{align}

where \( \mu_{j_1}(x) \) is the membership function for the optimal solution, \( \mu_{j_2}(x) \) represents membership functions for criteria functions, \( \mu_{l_1}(x) \) represents membership functions for constraints of type \( \geq \), and \( \mu_{l_2}(x) \) represents a membership function for constraint of type \( = \). In this model the relation between constraints and criteria functions is totally symmetrical (Zimmermann (1978)), and here the decision maker cannot express the relative importance of criteria functions and constraints.

In order to express the relative importance of criteria functions and constraints we have to solve the so-called additive weighting model in which weights present utility functions of criteria functions and constraints (Bellman and, Zadeh (1970), Sakawa (1993), Tiwari, Dharmahr and Rao (1987), Amid, Ghodsypour and O’Brien (2006)).

To solve the additive weighting FMCP model we will use the following linear programming model:

\begin{align}
\text{(max)} f = & \sum_{j=1}^{k} w_j \lambda_j + \sum_{l=1}^{m} \beta_l \lambda_{l_1} + \lambda_2 \\
\text{s.t.} \\
\lambda_j & \leq \mu_{j_1}(x), \quad j = 1, 2, \ldots, k, \\
\lambda_{l_1} & \leq \mu_{l_1}(x), \quad l = 1, 2, \ldots, m, \\
\lambda_2 & \leq \mu_{l_2}(x) \\
0 & \leq x_i \leq u_i, \quad i = 1, \ldots, n; \\
\lambda_1, \lambda_2, \lambda_3 & \in [0, 1], \quad j = 1, 2, \ldots, k; \quad l = 1, 2, \ldots, m,
\end{align}

2.4. Vendor selection model with price breaks

In order to formulate the vendor selection model with price breaks, the following notations are defined:

- \( x_{ij} \) the number of units purchased from the \( i \)th supplier at price level \( j \)
- \( P_{ij} \) price of the \( i \)th supplier at level \( j \)
- \( V_{ij} \) maximum purchased volume from the \( i \)th supplier at \( j \)th price level
- \( D \) demand over the period
- \( V_{ij}^* \) slightly less than \( V_{ij} \)
- \( m_i \) number of price level of the \( i \)th supplier
- \( Y_{ij} \) integer variable for the \( i \)th supplier at \( j \)th price level
- \( C_i \) capacity of the \( i \)th supplier
- \( F_i \) quality coefficient of the \( i \)th supplier
- \( S_i \) reliability coefficient of the \( i \)th supplier
- \( n \) number of suppliers
Let $P_{X_i}$ be the price that has to be paid to the vendor $i$ for the delivered quantity $x_i$. Then the net price to be paid for $D$ equals $\sum_{i=1}^{n} P_{X_i}$.

If vendors offer different prices for different order quantities in terms of intervals set by vendors, order cost of $x_i$ units from the vendor $i$, $P_{X_i}$, are defined as $P_{X_i} = P_{y_i} x_i, 1 \leq j < m_i$, where $P_{y_i}$ is the unit price for price level $j$, and $m_i$ is the number of quantity ranges in supplier $i$’s price schedule.

The cost objective function can be stated as

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} P_{y_i} x_j.$$  \hspace{1cm} (22)

The Eq. (22) is not a linear function because the $P_{y_i}$ depends on the purchased amount. In order to overcome this non-linearity, some binary (0, 1) variables are employed. As $x_j$ is non-zero only if it lies between $V_{i,j-1}$ and $V_{i,j}$, the following integer variables are defined: $Y_{ij} = 0$ if $x_j = 0$, and $Y_{ij} = 1$ if $x_j > 0$. These integer variables are taken into account by using the following constraints: $V_{i,j-1} Y_{ij} \leq x_j$, and $V_{i,j} Y_{ij} \geq x_j$, $i = 1, \ldots, n$, $j = 1, \ldots, m_j$. Therefore the buyer can purchase from one price level of each supplier.

The aggregate performance measure for a quality objective function is given by

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} S_{X_j},$$  \hspace{1cm} (23)

which maximizes quality indicators.

The equation of reliability objective function

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} F_{X_j}$$  \hspace{1cm} (24)

should maximize supplier’s reliability indicators.

In accordance to that MCP model of supplier selection and determination of material quantities supplied in the case of price breaks can be presented as:

$$\text{Min } f_1 = \sum_{i=1}^{n} \sum_{j=1}^{m_i} P_{y_i} x_j \quad \text{(cost)}$$  \hspace{1cm} (25)

$$\text{Max } f_2 = \sum_{i=1}^{n} \sum_{j=1}^{m_i} S_{X_j} \quad \text{(quality)}$$  \hspace{1cm} (26)

$$\text{Max } \sum_{i=1}^{n} \sum_{j=1}^{m_i} F_{X_j} \quad \text{(reliability)}$$  \hspace{1cm} (27)

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} x_j \geq D \quad \text{(demand constraint)}$$  \hspace{1cm} (28)

$$V_{i,j-1} Y_{ij} \leq x_j, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_j$$  \hspace{1cm} (29)

$$V_{i,j} Y_{ij} \geq x_j, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_j$$  \hspace{1cm} (30)

$$\sum_{j=1}^{m_i} Y_{ij} \leq 1, \quad i = 1, \ldots, n \quad \text{(at most one price level can be chosen)}$$  \hspace{1cm} (31)

$$Y_{ij} = 0, 1, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_i$$  \hspace{1cm} (32)

$$x_j \geq 0, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_i$$  \hspace{1cm} (33)

3. CASE STUDY

3.1. Data required for vendor selection and determination of supply quotas

We will show the example of vendor selection for a bakery. It is to be noted that in production of bread and bakery products the purchase of flour is contracted for the period of one year, from harvest to harvest. After the harvest flour producers have the information on the available wheat quantity, price and quality which allows them to define the price, quality and quantity of flour they can supply in the subsequent one-year period.
Table 1. Purchasing costs for flour Type 550

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Quantity level</th>
<th>Price (E/t)</th>
<th>Vendor</th>
<th>Quantity level</th>
<th>Price (E/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Q&lt;500</td>
<td>220.10</td>
<td>3</td>
<td>Q&lt;700</td>
<td>207.10</td>
</tr>
<tr>
<td>500 ≤ Q&lt;1000</td>
<td>211.40</td>
<td></td>
<td>700 ≤ Q&lt;1000</td>
<td>186.60</td>
<td></td>
</tr>
<tr>
<td>1000 ≤ Q</td>
<td>201.90</td>
<td></td>
<td>1000 ≤ Q</td>
<td>180.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Q&lt;300</td>
<td>207.60</td>
<td>4</td>
<td>Q&lt;500</td>
<td>230.80</td>
</tr>
<tr>
<td>300 ≤ Q&lt;800</td>
<td>200.20</td>
<td></td>
<td>500 ≤ Q&lt;1000</td>
<td>220.90</td>
<td></td>
</tr>
<tr>
<td>800≤Q</td>
<td>189.90</td>
<td></td>
<td>1000 ≤ Q</td>
<td>210.70</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Quality indicators for flour Type 550

<table>
<thead>
<tr>
<th>Quality indicators</th>
<th>Criteria weights</th>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Vendor 3</th>
<th>Vendor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>General characteristics of flour (A1) (0.20)</td>
<td></td>
<td>13.53</td>
<td>13.27</td>
<td>13.49</td>
<td>13.33</td>
</tr>
<tr>
<td>Moisture in % (B3) min (0.30)</td>
<td></td>
<td>0.57</td>
<td>0.549</td>
<td>0.53</td>
<td>0.486</td>
</tr>
<tr>
<td>Ash in % (B4) min (0.20)</td>
<td></td>
<td>1.5</td>
<td>1.5</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Acidity level in ml/100 grams (B5) min (0.10)</td>
<td></td>
<td>26.7</td>
<td>25.8</td>
<td>25.1</td>
<td>24.0</td>
</tr>
<tr>
<td>Wet gluten in % (B6) max (0.40)</td>
<td></td>
<td>60.8</td>
<td>59.8</td>
<td>58.5</td>
<td>61.1</td>
</tr>
<tr>
<td>Farinograph (A2) (0.30)</td>
<td></td>
<td>70</td>
<td>65</td>
<td>85</td>
<td>60</td>
</tr>
<tr>
<td>Water absorption in % (B7) max (0.40)</td>
<td></td>
<td>81</td>
<td>104</td>
<td>87.2</td>
<td>107.3</td>
</tr>
<tr>
<td>Degree of mellowness in FJ (B8) min (0.60)</td>
<td></td>
<td>137</td>
<td>162</td>
<td>180</td>
<td>165</td>
</tr>
<tr>
<td>Extensigraph (A3) (0.30)</td>
<td></td>
<td>7.17</td>
<td>1.19</td>
<td>1.07</td>
<td>0.75</td>
</tr>
<tr>
<td>Energy u cm² (B9) max (0.40)</td>
<td></td>
<td>395</td>
<td>280</td>
<td>235</td>
<td>350</td>
</tr>
<tr>
<td>Elasticity in mm (B10) max&lt;190 (0.30)</td>
<td></td>
<td>1054</td>
<td>860</td>
<td>1275</td>
<td>1325</td>
</tr>
<tr>
<td>Resistance (B11) max (0.30)</td>
<td></td>
<td>0.65</td>
<td>0.49</td>
<td>0.52</td>
<td>0.35</td>
</tr>
<tr>
<td>Amylograph (A4) (0.20)</td>
<td></td>
<td>7.12</td>
<td>0.88</td>
<td>0.87</td>
<td>0.92</td>
</tr>
<tr>
<td>Peak viscosity in BU (B12) max (1.00)</td>
<td></td>
<td>49.36</td>
<td>23.6</td>
<td>48.92</td>
<td>49.69</td>
</tr>
</tbody>
</table>

Table 3. Vendor reliability indicators

<table>
<thead>
<tr>
<th>Reliability indicators</th>
<th>Criterion</th>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Vendor 3</th>
<th>Vendor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial stability, indebtedness and liquidity (A5) (0.60)</td>
<td></td>
<td>1.12</td>
<td>0.88</td>
<td>0.87</td>
<td>0.92</td>
</tr>
<tr>
<td>Coverage of fixed assets and stocks by capital and long term resources, (B13) max (0.20)</td>
<td></td>
<td>49.36</td>
<td>23.6</td>
<td>48.92</td>
<td>49.69</td>
</tr>
<tr>
<td>Share of capital in source of funds in %, (B14) max (0.10)</td>
<td></td>
<td>7</td>
<td>19</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>Indebtedness factor, number of years (B15) min (0.10)</td>
<td></td>
<td>0.65</td>
<td>0.49</td>
<td>0.52</td>
<td>0.35</td>
</tr>
<tr>
<td>Total assets turnover coefficient (B16) max (0.10)</td>
<td></td>
<td>7.17</td>
<td>1.19</td>
<td>1.07</td>
<td>0.75</td>
</tr>
<tr>
<td>General liquidity coefficient (B17) max (0.30)</td>
<td></td>
<td>86</td>
<td>101</td>
<td>102</td>
<td>58</td>
</tr>
<tr>
<td>Short term receivables collection period, in days (B18) min (0.20)</td>
<td></td>
<td>1.06</td>
<td>1.03</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>Performance indicators (A6) (0.40)</td>
<td></td>
<td>4.81</td>
<td>1.85</td>
<td>2.66</td>
<td>1.02</td>
</tr>
<tr>
<td>Coefficient of total revenue and expenditure ratio (B19) max (0.20)</td>
<td></td>
<td>3.14</td>
<td>0.91</td>
<td>1.39</td>
<td>1.01</td>
</tr>
<tr>
<td>Share of profit in total income in % (B20) max (0.30)</td>
<td></td>
<td>60538</td>
<td>21189</td>
<td>12370</td>
<td>15446</td>
</tr>
</tbody>
</table>

In the one-year period the bakery plans to consume 4000 tons of flour Type 550. The company contacts 4 potential flour suppliers and defines the upper limit of flour supplied by a single vendor in the amount of 2000 tons. The proposed prices of flour and price levels (Criterion C1) are shown in the Table 1. The potential vendors supply the data on flour quality that they have to maintain throughout the contract period (Criterion C2). It is to be noted that the quality of flour depends on the wheat sort and quality and on technology used in flour production. The vendors also should supply data on their reliability by the forms SOL-2 and BON-1 (Criterion C3).
The Tables 2 and 3 show the flour quality indicators and vendor reliability. The weights expressing the relative importance of criteria and sub-criteria are given in brackets, and they are determined by the decision maker where in every group of sub-criteria the sum of weights is 1.

3.2. Application of revised weighting method

Considering the data from the Tables 1, 2 and 3 we form a hierarchical structure of goals and criteria for vendor selection.

The hierarchical structure in our example consists of five levels. Level 1 represents the vendor general efficiency (or total value of purchasing - TVP), Level 2 represents criteria for vendor selection, Level 3 represents criterion sub-criteria, Level 4 represents sub-criterion sub-criteria, and Level 5 represents the available alternatives (vendors).

After decomposition of the problem and formation of the hierarchical structure of goals and criteria, we have applied the revised weighting method to calculate the coefficients quality and reliability functions.

Using the data on coefficient weights with variables of grouped sub-criteria and weight coefficients with sub-criteria A1, A2, A3 and A4, and by applying the relation (1) we calculate the coefficients with criterion C2 variables. Reliability criterion coefficients are calculated in a similar way:

Table 4: Normalized coefficient weights with quality and reliability criterion variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff. ( c_{2i} )</th>
<th>Coeff. ( c_{3i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.244824</td>
<td>0.397097</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.241625</td>
<td>0.191739</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.241354</td>
<td>0.208131</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>0.272198</td>
<td>0.203032</td>
</tr>
</tbody>
</table>

3.3. FMCP model building and solving

As there are constraints in terms of capacity or limited quantity supplied by a single vendor, we have to form a MCLP model to determine the quantities to be supplied by selected vendors.

Let \( x_j \) is the number of units purchased from the \( i \)th supplier at price level \( j \). Considering the data on purchasing costs from Table 1, normalized coefficient weights with variables of quality, and reliability functions, the total demand for flour in the given period (4000 t) and limited quantities supplied from single vendors (2000 t each), and the theoretical results for forming the vendor selection model with price breaks, we form the MCLP model with three criteria functions and 26 constraints. The obtained model is first solved by mixed linear programming method optimizing separately each of the three criteria function on the given set of constraints. The results are given in the Payoff table:

Table 5. Payoff values

<table>
<thead>
<tr>
<th>Solution</th>
<th>( (\min)f_1(x) )</th>
<th>( (\max)f_2(x) )</th>
<th>( (\max)f_3(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1^* )</td>
<td>739800</td>
<td>965.96</td>
<td>799.74</td>
</tr>
<tr>
<td>( x_2^* )</td>
<td>825200</td>
<td>1034.04</td>
<td>1200.26</td>
</tr>
<tr>
<td>( x_3^* )</td>
<td>763800</td>
<td>972.36</td>
<td>1210.46</td>
</tr>
</tbody>
</table>
It can be seen that the obtained solutions differ and that we have to choose a compromise solution. This work proposes methodology for vendor selection and determination of supply quotas by application of fuzzy linear programming on the MCLP model. The application of FMCP requires determination of the highest and lowest value for each criteria function. These values represent the aspiration levels in FMCP. The lowest and highest values for criteria functions are shown in the following table:

Table 6. Fuzzy goals

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Value-I</th>
<th>Value-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>739800*</td>
<td>825200</td>
</tr>
<tr>
<td>$f_2$</td>
<td>965.96</td>
<td>1034.04*</td>
</tr>
<tr>
<td>$f_3$</td>
<td>799.74</td>
<td>1210.46*</td>
</tr>
</tbody>
</table>

Based on the above data we calculate the linear membership functions:

Based on the calculated membership functions the MCLP model can be transformed into the following linear programming model:

$$\begin{align*}
\text{(max)} & \lambda \\
\text{s. t.} & \lambda \leq \mu_{f_1}(x) \\
& \lambda \leq \mu_{f_2}(y) \\
& \lambda \leq \mu_{f_3}(z) \\
& 0 \leq \lambda \leq 1 \\
& x_{11} - 499 y_{11} \leq 0 \\
& 500 y_{12} - x_{12} \leq 0 \\
& x_{12} - 999 y_{12} \leq 0 \\
& 1000 y_{13} - x_{13} \leq 0 \\
& x_{21} - 299 y_{21} \leq 0 \\
& 300 y_{22} - x_{22} \leq 0 \\
& x_{22} - 799 y_{22} \leq 0 \\
& 800 y_{23} - x_{23} \leq 0 \\
& x_{31} - 699 y_{31} \leq 0 \\
& 700 y_{32} - x_{32} \leq 0 \\
& x_{32} - 999 y_{32} \leq 0 \\
& 1000 y_{33} - x_{33} \leq 0 \\
& x_{41} - 499 y_{41} \leq 0 \\
& 500 y_{42} - x_{42} \leq 0 \\
& x_{42} - 999 y_{42} \leq 0 \\
& 1000 y_{43} - x_{43} \leq 0 \\
& y_{11} + y_{12} + y_{13} \leq 0 \\
& y_{21} + y_{22} + y_{23} \leq 0 \\
& y_{31} + y_{32} + y_{33} \leq 1
\end{align*}$$
\[ y_{41} + y_{42} + y_{43} + y_{44} \leq 1 \]  \hspace{1cm} (58) \\
\[ y_j = 0,1; \ i = 1,2,3,4; \ j = 1,2,3. \]  \hspace{1cm} (59)

where \( \mu_{f_i}(x), \mu_{f_j}(x) \) and \( \mu_{f_k}(x) \) are linear membership functions. By solving it we obtain the following optimal solution:

\[ \lambda_{\max} = 0.5554, \ x_{33} = 1065, \ x_{33} = 1813, \ x_{43} = 1122, \]
\[ f_1 = 777769, \ f_2 = 1003.719, \ f_3 = 1028.052. \]

The fuzzy technique applied in the model solving does not take into account the subjective importance of criteria functions. In order to include the subjective importance of criteria functions for the decision maker we solve the theoretical model (16-21) by using data from the model (34-59), where we determine the criteria weights: \( w_1 = 0.40, \ w_2 = 0.40 \) and \( w_3 = 0.20. \) We obtain the following solution: \( \lambda_1 = 0.4000, \)
\( \lambda_2 = 0.9215, \lambda_3 = 0.2572, \ x_{11} = 440, \ x_{33} = 1560, \ x_{43} = 2000, \ f_1 = 799044, \ f_2 = 1028.631, \ f_3 = 905.471. \)

The decision maker has accepted the obtained solution.

Application of the additive weighting FMCP model also allows sensitivity analysis of the obtained solution depending on the weights assigned to criteria functions and constraints.

4. CONCLUSION

Solving the concrete example by application of the proposed methodology we can make a number of conclusions presenting the advantages of using the proposed methodology in solving the problem of vendor selection and determination of order quotas at price breaks.

The revised weighting method allows efficient reducing of complex criteria functions into simple criteria functions. For DM, it is easier to determine weighting coefficients if he/she deals with few criteria functions than if he/she deals with a large number of them. If there are a large number of criteria and sub-criteria, there is a high probability of error in determining of weighting coefficients.

The revised weighting method applied alone needs determination a set of Pareto optimal solutions, which is not possible, because the coefficients by cost criteria function are not determined in a unique way. Alongside this method has some shortcomings so that it is not the most appropriate one to create a set of Pareto optimal solutions. The shortcomings are: (1) varying weight coefficients do not guarantee that we will determine all Pareto optimal solutions, and (2) the determined Pareto optimal solutions are those that are situated in the extreme points of the convex polyhedron but not those that connect the two extreme points. To determine the set of compromise solutions and the preferred solution it is better to use the fuzzy linear programming model.

When solving the MCP model the use of fuzzy technique proves to be very efficient. The efficiency of the fuzzy technique in solving the model can be seen in the possibility to define weights for criteria functions that express the decision maker's preferences. However, if you deal with complex criteria functions it is complicated to use the FMCP method alone because of arising problems by determination of weighting coefficients.

Application of revised weighting method and FMCP to solve the problem of vendor selection and determination of supply quotas allows a simple sensitivity analysis of the obtained solutions. The proposed methodology can be used in solving similar business problems.

REFERENCES


