COMPARISON OF VALUE AT RISK APPROACHES ON A STOCK PORTFOLIO

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Abstract

Value at risk is risk management tool for measuring and controlling market risks. Through this paper reader will get to know what value at risk is, how it can be calculated, what are the main characteristics, advantages and disadvantages of value at risk. Author compares the main approaches of calculating VaR and implements Variance-Covariance, Historical and Bootstrapping approach on stock portfolio. Finally results of empirical part are compared and presented using histogram.

Key words: *Value at risk (VaR), Variance-Covariance approach, Historical simulation, Bootstrapping, comparing approaches, stock portfolio, Pros and Cons of VaR*

1. DEFINING VALUE AT RISK

1.1. What is value at risk?

What is most I can lose on this investment? This is the question that every investor who is investing in risky asset asks at some point of time. Value at Risk tries to provide an answer within a reasonable bound. Value at Risk (VaR) is statistical measure that estimates potential loss in value of risky asset or portfolio, over defined period of time, for a given confidence level. VaR always comes with confidence level, which shows the probability that losses will not exceed given value.

Wide accepted definition of VaR is the following: "Value at risk is the maximum amount of money that may be lost on a portfolio over a given period of time, with given level of confidence" (Best, 1998).

For an example, daily VaR of 100 Euros with confidence level of 95%, means that in normal market conditions- or in 95 out of 100 days loss will not exceed 100 Euros.

There are three main approaches of calculating VaR: Variance-Covariance approach, Historical simulation and Monte Carlo simulation. First two approaches will be used in the empirical part of this paper. Bootstrapping will be implemented as third method, which is more similar to Historical then Monte Carlo simulation.

Value at risk is typically used by securities houses or investment banks to measure the market risk of their asset portfolios, but is actually a very general concept that has broad application.

1.2. Parameters of VaR

With just one look at the definition of VaR it can be seen that there are two main parameters:

- Holding period
- Confidence level

In this paper holding period of one day is used. Choice of holding period depends on how the resultant VaR is used, and VaR can be calculated for any holding period. The longer the holding period the larger the VaR. Normally you would expect a larger change in price over a period of one month than over 24 hours (Best, 1998). There are few confidence levels that are often used: 95%, 97,5% and 99%. In the empirical part VaR is calculated for 95% and 98% confidence levels. 95% confidence means that for about 5% of time, firm could expect to lose more than the number given by the VaR.

2. APPROACHES OF CALCULATING VALUE AT RISK

2.1. Variance-Covariance method

Variance-Covariance method is also known as Linear VaR or Delta normal VaR. This approach is relatively simple and is widely used. This method includes parts of modern portfolio theory of Harry Markowitz, by taking account of correlation coefficients between assets.

Historical data is used to calculate main parameters: mean, standard deviation, correlation. This method calculates VaR by assuming some theoretical distribution of asset returns. Usually normal distribution is used. This assumption allows volatility to be described in terms of standard deviations (SD). Another advantage of normal distribution is that it can be described by its first two moments: mean, and standard deviation (Žiković, 2005). This distribution is symmetrical so skewness is 0 and kurtosis 3. If we want to find position of a random variable (X) in a normal distribution we use standard value of variable Z (Z-score). Every variable can be transformed to standard variable with formula:

$$X = \mu + z\,\sigma \qquad (1)$$

(where z is simply calculated as $Z = X - \mu / \sigma$) , μ - mean, σ - standard deviation (SD). In a similar way VaR can be calculated as multiple of standard deviation.

Figure 1 shows normal distribution with value of x -2,326 SD. Probability for losses greater then x is shown as an area under the normal curve, left of x. This area is 1% of total area under the curve, so there is confidence of 99% that losses won't exceed -2,326 SD.

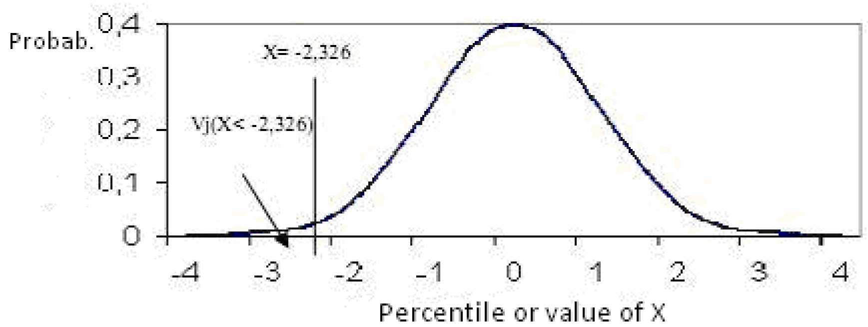

Figure 1: Percentile values and their probabilities with normal distribution (Žiković, 2005)

When measuring VaR only downward price changes are considered, or price changes that exceed some multiple of SD. Negative price change (in percentage) that corresponds to 1,65 SD gives confidence of 95% that loss wont exceed given value. And 2,33 SD give confidence of 99% (Best, 1998). Finally VaR for portfolio can be calculated using following formula:

$$VaR_p = (Z\ V\ P) \tag{2}$$

Where: Z is standard value (calculated from confidence level using formula "NORMSINV" in Excel), V- volatility or standard deviation of asset/portfolio, P- position (portfolio) value.

In practice portfolio VaR can be calculated using the following matrix formula:

$$VaR_p = (V\ C\ V^T)^{1/2} \tag{3}$$

Where V is row vector of VaRs for each individual position, C - matrix of correlations, V^T - transpose of matrix V.

Finally it is important to say that normal distribution also has a negative side. It can underestimate risk in tail of distribution (high levels of confidence). Returns don't always follow normal distribution, especially in crises, and variances and covariances can change over time.

2.2. Historical simulation

The correct method of calculating VaR using historical simulation is to use history of percentage price changes and apply these to today's portfolio, as follows (Best, 1998.):

1. Obtain price change series for every asset or risk factor needed to revalue the portfolio
2. Apply price changes to the portfolio to generate a "historical" series of portfolio values changes
3. Sort the series of portfolio value changes into percentiles

4. The VaR of the portfolio is the value change corresponding to the required level of confidence

This is unparametric method because it doesn't make assumptions about distribution of returns (or risk factors[1]). However it assumes history changes are going to repeat. Another limitation is that it values the same recent data and older data. That can cause bad estimates if there are recent trends, like higher volatility. For this simulation, like for all three approaches, is important to have sufficient data, and that is problem when dealing with new assets and risks. Typical trade-off for historical simulation is that we would like to have more data in order to observe the rare events, and on the other hand, we do not want to build our current risk estimates on very old market data.

2.3. Monte Carlo simulation and Bootstrapping

Monte Carlo simulation is similar to historical simulation. But instead of using historical changes, risk manager chooses distribution that adequately describes price changes. Then according to that distribution random values are simulated. Managers usually observe past changes to choose a distribution. After simulating price changes or changes in risk factors, hypothetical profits and losses are calculated. Finally VaR is calculated as a percentile corresponding to chosen confidence level. Bootstrapping is alternative to generating random numbers from hypothetical distribution. Instead bootstrap method samples from historical data with replacement (Jorion, 2001). This method includes fat tails (rare events) but it is important to have sufficient data (good sample). Advantage of bootstrapping is that any correlation between stock returns is saved, as we randomly pick a vector of original daily returns.

2.4. Comparison of methods

VaR methods differ in their ability to capture risks of options and option like-instruments, ease of implementation, ease of explanation to senior management, flexibility in analyzing the effect of changes in the assumptions, and reliability of the results (Linsmeier and Pearson,1996). The best choice will be determined by which dimension the risk manager finds most important. If you are assessing portfolios without options for short time period variance-covariance approach does a reasonably good job. If the VaR is being computed for a risk source that is stable and where there is substantial historical data, historical simulations provide good estimate.

In the most general case of computing VaR for nonlinear portfolios over long time periods, where the historical data is volatile and non-stationary and the normality assumption is questionable, Monte Carlo simulations do best (www.stern.nyu.edu/~adamodar/pdfiles/papers/VAR.pdf, 2010).

[1] A parameter whose value changes in the financial markets and whose change in value will cause change in portfolio value

	Historical Simulation	**Variance/Covariance**	**Monte Carlo Simulation**
Able to capture the risks of portfolios which include options?	Yes, regardless of the options content of the portfolio	No, except when computed using a short holding period for portfolios with limited or moderate options content	Yes, regardless of the options content of the portfolio
Easy to implement?	Yes, for portfolios for which data on the past values of the market factors are available.	Yes, for portfolios restricted to instruments and currencies covered by available "off-the-shelf" software. Otherwise reasonably easy to moderately difficult to implement, depending upon the complexity of the instruments and availability of data.	Yes, for portfolios restricted to instruments and currencies covered by available "off-the-shelf" software. Otherwise moderately to extremely difficult to implement.
Computations performed quickly?	Yes.	Yes.	No, except for relatively small portfolios.
Easy to explain to senior management?	Yes.	No.	No.
Produces misleading value at risk estimates when recent past is atypical?	Yes.	Yes, except that alternative correlations/standard deviations may be used.	Yes, except that alternative estimates of parameters may be used.
Easy to perform "what-if" analyses to examine effect of alternative assumptions?	No.	Easily able to examine alternative assumptions about correlations/standard deviations. Unable to examine alternative assumptions about the distribution of the market factors, i.e. distributions other than the Normal.	Yes.

Figure 2: Comparison of Value at Risk Methodologies (Linsmeier and Pearson,1996)

3. IMPLEMENTING VALUE AT RISK ON A STOCK PORTFOLIO

Empirical part of this paper refers to calculating VaR for portfolio using Variance-Covariance approach, Historical simulation, and Bootstrapping method. Portfolio consists of five stock listed on Zagreb stock exchange. Each stock has the same fixed proportion of 20% in portfolio and their total value is 20.000 kunas. A main criterion for choosing stocks into portfolio was their liquidity. The reason is obvious, it is important to have complete historical data of stock prices in order to calculate VaR. Portfolio consists of the following five stocks: Hrvatski telekom, Ina, Atlantska plovidba, Dalekovod and Ericsson Nikola Tesla.

Historical data is collected for period of 401 trading day (from 10.11.2008. till 18.06.2010.) in order to get 400 daily returns. Last two daily returns are shown in table 1. Portfolio returns are calculated as weighed sum of individual returns, where ponders are proportion of each stock in portfolio (20%).[2]

Table: 1 Daily returns

Stocks	HT-R-A	INA-R-A	ERNT-R-A	DLKV-R-A	ATPL-R-A	Portfolio
18. June	0,134%	-0,455%	-1,671%	-1,469%	0,133%	-0,666%
17. June	0,099%	-0,446%	-1,804%	0,231%	0,572%	-0,269%
…	…					

Source: research of author

3.1. Variance-Covariance approach

Variance-Covariance approach is started with assumption that stock returns are normally distributed. Daily prices are used to calculate daily returns, variances, standard deviations. Table 2. shows expected return, variance and standard deviation for individual stocks and portfolio. These parameters are calculated in Excel: expected return as mean of 400 daily returns (function "AVERAGE"), variance and standard deviation simply by using functions "VARP" and "STDEVP", where target array are 400 daily returns. VaR can be calculated with both formula 2 and 3. In this example we use formula 2, although both formulas give the same result.

Table 2: Basic parameters for each stock and whole portfolio

	HT	INA	ERCS	DLKV	ATPL	Portfolio
Expected return	0,036%	-0,021%	0,029%	-0,129%	0,014%	-0,014%
Variance	0,00019	0,000562	0,000367	0,000982287	0,00093115	0,000367
St. Deviation	0,013801	0,023706	0,019146	0,031341454	0,03051471	0,019159

Source: research of author

$VaR_{95\%}$ = SD x Z x Value of portfolio = 0,0191 x 1,65 x 20000 = 632,24 HRK

$VaR_{98\%}$ = 0,191 x 2,053 x 20000 = 786,95 HRK

3.2. Historical simulation

Historical simulation is implemented by taking 400 historical daily portfolio returns and multiplying them with value of today's actual portfolio - 20.000 HRK. Now there are 400 hypothetical profits and losses.

VaR with confidence of 95% is simply calculated as 5th percentile of hypothetical profits and losses or alternatively as 20th largest loss. Table 3. shows latest four historical (daily) returns as well as hypotetical profits/losses.

[2]Alternatively variance and standard deviation of portfolio returns can be calculated using variance-covariance matrix

Table 3: HS – applying historical price changes on today's portfolio

Date	Return on portfolio	Profit/ Loss (HRK)
18.06.´10	-0,666%	-133,12
17.06.´10	-0,269%	-53,8552
16.06.´10.	0,106%	21,22732
15.06.´10.	0,076%	15,1054

Source: research of author

VaR is calculated using formulas in Excel spreadsheet as following:

$VaR_{95\%}$= PERCENTILE(profit/loss array;5%) = -671,116 HRK
$VaR_{98\%}$= PERCENTILE(profit/loss array;2%) = -961,336 HRK

3.3. Bootstrapping

Bootstrapping method uses historical returns as well. In the first step each daily return is indexed with a number from 1 to 400 as shown in table 4. To simulate return for the next (n+1) day we generate random number 1- 400 using excel function "INT(rand())". This is repeated for hundred times in order to get 100 random numbers or indexes (hypothetical distribution). For each index corresponding daily return is multiplied with value of today's portfolio of 20.000 kunas. Again list of daily profits/ losses is formed, and VaR is calculated as a percentile matching to chosen confidence level.

Table 4: Bootstrapping method, indexing daily portfolio returns

Date	**Index**	Return on portfolio	Hypotetical profit/loss (HRK)
18.06.´10	**1**	-0,666%	-133,120
17.06.´10	**2**	-0,269%	-53,855
16.06.´10.	**3**	0,106%	21,227
15.06.´10	**4**	0,076%	15,105
…	…	…	…

Source: research of author

Table 5: Bootstrapping – generating random daily returns

Random Number 1-400	Correspond ing daily return	Hypothetical profit/loss (HRK)
159	-1,235%	-246,986
367	0,801%	160,299
185	-0,165%	-32,904
260	-6,077%	-1215,337
…	…	…

Source: research of author

$VaR_{95\%}$ = -592,524 HRK

$VaR_{98\%}$ = -903,381 HRK

3.4. Comparing the results

Table 6. shows values of VaR using three different approaches. Historical simulation resulted with higher values than the Var.-covar. approach, because actual price changes do not perfectly follow normal distribution, as often in practice. Kurtosis and skewness slightly differ from values characteristic for normal curve. Skewness is -0,44 what means that the distribution is asymmetrical (lean to the left) with higher probability of negative values.

Table 6: Values of VaR by three approaches (in kunas)

Variance-Covariance app.			**Historical simulation**			**Bootstrapping**	
VaR 95%	632,245		VaR 95%	671,116		VaR 95%	592,524
VaR 98%	786,953		VaR 98%	961,336		VaR 98%	903,381

Source: reaserch of author

Kurtosis is 2,03 because empirical distribution has higher peak than normal distribution and "bell" like shape is narrower then for theoretical normal distribution. Mean or expected return is slightly negative (-0,014%) and theoretical normal distribution has mean 0. Bootstrapping method samples from historical price changes so should result with similar value as historical simulation. Of course if sampling was repeated for more than 100 times resulting value would be closer to the value of historical simulation.

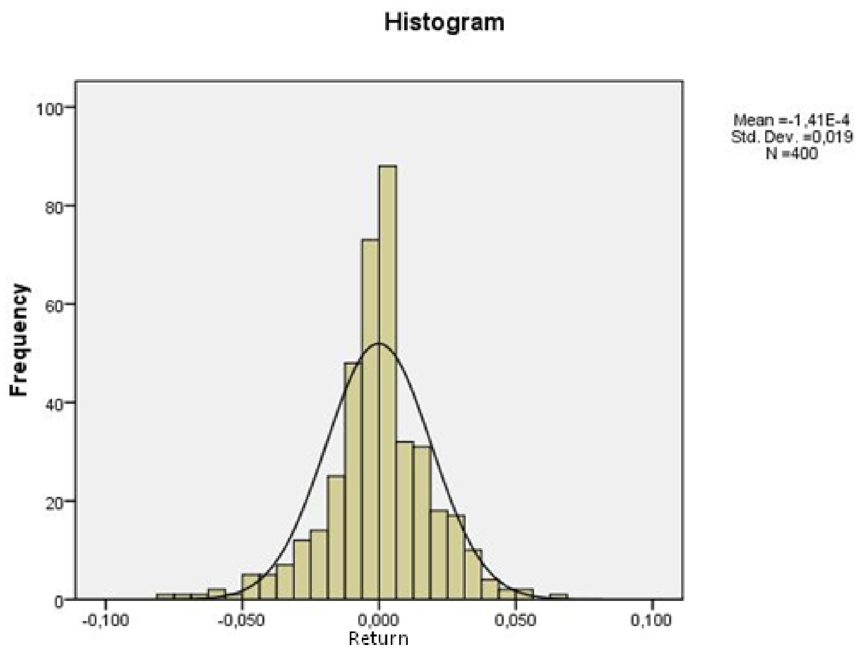

Figure 3: Histogram of empirical portfolio returns

New set of 100 samples could be easily created with pressing F9 in excel spreadsheet as numbers are automatically resampled. Then VaR could be calculated as average of VaRs for different sets of samples. If empirical distribution would follow normal distribution all three approaches should result with similar value. Naturally all three approaches have higher VaR when using higher confidence level.

4. PROS AND CONS FOR VAR

It may be the best choice not to use VaR at all. Small firms with exposure to just few market risks will find sensitivity analysis more appropriate and easy to implement. On other hand many non financial firms use alternative measure, cash-flow at risk.

When using VaR one should be aware of its limitations. Simply said VaR can be wrong. First of all history can be wrong predictor. All approaches take assumptions about return distribution based on historical data or rely directly on historical data. Choosing a historical period is crucial for good estimate. There is no rule for length of time period, but regulators often demand that banks calculate 10day VaR observing at least last 250 working days (1 year). Investor should know if observed period contains some unusual volatility that could lead to misleading results. Another problem is that we assume constant volatility and correlations, but they can change over time. Critics often highlight that different approaches result with different values and that is better "fly" without instruments then with bad ones. Finally VaR can have narrow focus because it looks at market risk, and other risks can also cause losses (political risk, liquidity risk and regulatory risk).

In defence to VaR there is no perfect risk measure or prediction of future. Company's risk mana-gement doesn't stop when VaR is calculated. If VaR gives 95% confidence, risk managers should take care at least for other 5 %. Further more VaR has its modifications that improve this risk measure. One of them is weighting the recent past more. This model uses decay factor, as time weighting mechanism. Some models use volatility updating to deal with non-stationary variables. Finally Conditional VaR deals with "fat tails" as it finds average value of losses exceeding VaR.

VaR is supplemented with stress test to deal with large price shocks. Regulators (Basel) demand that firms implement back testing in order to evaluate their risk models. If losses exceed VaR more then 4 times model is in yellow zone and certain improvements need to be taken.

VaR itself is not "the true way to measure risk" in the absolute sense. There are many other risk measurements. The major advantage of VaR is that it has become a widely accepted standard. Once the industry and the regulators agreed to use it as a primary risk management tool, it plays an important unifying role, and enables a good basis for comparison of risk between portfolios, institutions and financial intermediaries (Wiener, 1997).

6. CONCLUSION

There is no easy answer which method is the best. Investor or risk manager must look at composition of its portfolio and then choose appropriate method. It is useful to analyze historical data to see distribution of returns and see which approach can or can't be implemented. As said before, approaches differ in several categories, and one should decide which one he finds most important. Simple stock portfolio is convenient for all three approaches. It's a good idea to avoid more complex methods since we have linear portfolio without any options to evaluate. As seen, distribution of returns is not completely normal, so historical simulation and bootstrapping should give better estimate.

Investors probably invest on longer period and beside VaR use fundamental and technical analysis to predict possible return. To get more reliable results, VaR should be backtested and supplemented with stress testing to see will VaR be exceeded in case of extreme (unusual) price changes.

REFERENCES

Aljinović Zdravka, Marasović Branka, Šego Boško, *Financijsko modeliranje*, Zgombić & Partneri, 2008.

Best Philip, *Implementing Value at Risk*, John Wiley and Sons, 1998.

Burkhard Raunig, Gabriela de Raaji, *A comparison of Value at Risk approaches and their implications for regulators* , available at www.gloriamundi.org

Jorion Phillippe, *Value at Risk: The new benchmarking for managing financial risk*, 2nd edition, McGraw-Hill Trade, 2001..

Linsmeier Thomas J., Pearson Neil D., *Risk measurment: An introduction to Value at Risk*, University of Illinois at Urbana-Champaign, July 1996.

Mikulčić Dražen, *Value at Risk (rizičnost vrijednosti), Teorija i primjena na međunarodni portfelj instrumenata s fiksnim prihodom*, HNB, P-7, Kolovoz 2001.

Šverko Ivan, *Moguća primjena povijesne metode rizične vrijednosti pri upravljanju rizicima financijskih institucija u Republici Hrvatskoj*, stručni članak, 2001.

Šverko Ivan, *Rizična vrijednost (Value at risk) kao metoda upravljanja rizicima u financijskim institucijama*, 2002.

Wiener Zvi, *Introduction to VaR (Value-at-Risk)*, Risk Management and Regulation in Banking, Jerusalem 18 May 1997.

Žiković Saša, *Formiranje optimalnog portfolija hrvatskih dionica i mjerenje tržišnog rizika primjenom VaR metode*, magistarski rad, Ljubljana, veljača 2005.

http:// Rizici.blog.hr [Accessed 09/05/10]

http://www.stern.nyu.edu/~adamodar/pdfiles/papers/VAR.pdf [Accessed 09/05/10]

http://www.zse.hr [Accessed 18/06/10]

http://www.bionicturtle.com/learn/article/bootstrap_method_quantitative/ [Accessed 18/06/10]