USEFULNESS OF BOOTSTRAPPING IN PORTFOLIO MANAGEMENT

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Abstract

This paper contains a comparison of in-sample and out-of-sample performances between the resampled efficiency technique, patented by Richard Michaud and Robert Michaud (1999), and traditional Mean-Variance portfolio selection, presented by Harry Markowitz (1952). Based on the Monte Carlo simulation, data (samples) generation process determines the algorithms by using both, parametric and nonparametric bootstrap techniques. Resampled efficiency provides the solution to use uncertain information without the need for constrains in portfolio optimization.

Parametric bootstrap process starts with a parametric model specification, where we apply Capital Asset Pricing Model. After the estimation of specified model, the series of residuals are used for resampling process. On the other hand, nonparametric bootstrap divides series of price returns into the new series of blocks containing previous determined number of consecutive price returns. This procedure enables smooth resampling process and preserves the original structure of data series.

Key words: *Resampled efficiency, Mean-Variance portfolio, parametric bootstrap, nonparametric bootstrap, Capital asset pricing model*

1. INTRODUCTION

The purpose of this paper is to develop a more robust methodology for asset allocation for the stock investment markets which takes more seriously into account inherent valuation and data issues. This includes the integration of mean-variance optimizer using resampled data inputs, passive investment management, the selection of appropriate asset classes and time rebalancing technique to ensure that the portfolio remains aligned with the dynamic nature of stock markets.

The proposed methodology will prove to be useful for making asset allocation decisions, especially in highly volatile financial markets. The chosen bootstrap procedure selectively resamples the return time series by maintaining the economic cycle. After constructing resampled efficient portfolios, the research process resumes with comparison made on the traditional Mean-Variance portfolio optimization problem.

2. MARKOWITZ EFFICIENCY

Sixty years ago Harry Markowitz (1952) developed the portfolio selection theory that became a foundation of financial economics for asset management and revolutionized investment practice. Markowitz noticed well a basic premise for his theory that all economic decisions are made upon trade-offs. In situation of investment selection, a trade-off, risk versus expected return, is observed. The theory extends the techniques of linear programming to develop the critical line algorithm. Mentioned algorithm identifies all feasible portfolios that minimize risk for a given level of expected return and maximize expected return for a given level of risk in order to form a set of portfolios graphically presented as the efficient frontier. Showing the level of diversification in portfolio selection, the efficient frontier indicates the importance of achieving risk reduction.

However, the portfolio selection is based on assumption that investment decision only depends on expected value $E(R_p)$ and variance σ_p^2 of the total portfolio return. Fallowing this background, the portfolio optimization procedure requires the knowledge of $E(R_i)$ as the expected return of the asset i, σ_i as the standard deviation of the return of the asset i, ρ_{ij} as the correlation between the returns of the assets i and j for $i, j = 1,..., n$, and σ_{ij} as the covariance between two asset or security returns. Consequently, the classical Mean-Variance optimization model is presented in following form:

$$\begin{aligned} Min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \\ st \quad & \\ & \sum_{i=1}^{n} E(R_i)x_i \geq M \\ & \sum_{i=1}^{n} x_i = 1 \\ & x_i \geq 0 \qquad i = 1,..., n \end{aligned} \tag{1}$$

The formulation (1) indicates a convex quadratic programming problem where is denote by M the required level of return for the portfolio and by x_i the fraction of a given capital to be invested in each asset i.

3. RESAMPLED EFFICIENCY

With often use of Markowitz portfolio selection procedure it became obvious that problems in creating adequate composition are occurring in extreme portfolio weights, an unbalanced asset allocation and a lack of diversification. In fact, the composition of optimal portfolios is very sensitive to changes in expected returns, variances and covariances. Tending to pick those assets with most attractive features and to short or deselect those with worst features are exactly the cases in which estimation error is likely to be highest. Hence, the process maximizes the impact of estimation error on portfolio weights and decreasing out-of-sample performances. There are several attempts made to reduce estimation errors and improve portfolio performance. This paper embraces the resampled efficiency technique patented by Richard Michaud and Robert Michaud (1999), which is based on resampling of portfolio returns to reflect the uncertainty in return process.

In order to analyze the performance of the resampled efficiency, some studies made the comparison between mean-variance optimization and mentioned resampled efficiency. In many of those studies Michaud’s procedure outperforms the approach of Markowitz. In simulation studies of Michaud and Michaud (2008) and also Markowitz and Usmen (2003) was found a strong evidence for better performance of resampled efficiency. However, there are also completely different results. For instance, study of Harvey and other authors (2008), with more sophisticated prior distribution and more appropriate algorithm, obtain rather balanced results between the resampled efficiency and the optimization of Markowitz or even better results using their Bayesian estimator.

Nevertheless, even if there are some studies comparing these two techniques, each of them concentrates on a specific setting, which rarely leads to general recommendations. Based on the analysis of the mentioned papers the results of Markowitz versus Michaud are rather balanced and very sensitive to the length of the estimation horizon with capability to give advice for different initial situations of investment.

Why resampled efficiency? The starting point to explain the purpose of the resampling efficiency is a well known set of rigid assumptions used in Markowitz optimization framework. Thus the utility function often became more complex involving preferences beside mean and variance. Instead of fallowing dynamic nature of market Markowitz selection model mainly offers static optimization (one-period optimization). With such a rigid set of constrains small changes in input assumptions could imply large changes in the optimized portfolio. All previously observed facts magnify estimation errors and in the end lead to decrease of utility value of the portfolio selection model.

Therefore, the resampling efficiency technique has been presented to overcome shortcomings in the portfolio selection procedure. Michaud (1999) patented the resampled efficiency[TM], but keeping some underlying assumptions from the portfolio selection procedure. Scherer (2002) summarized this procedure as fallows:

1) Estimate variance-covariance matrix and mean vector of historical inputs.
2) Resample from inputs by taking B draws from input distribution θ (this paper includes both, parametric and nonparametric bootstrap procedure). The number of draws reflects the degree of uncertainty in the inputs. Calculate new variance-covariance matrix from sampled series.
3) Calculate the efficient frontier from inputs derived in second iteration and save optimal portfolio weights for m equally distributed returns along the frontier.
4) After repeating step 2 and 3 B times, calculate average portfolio weights for each return point.

Recreating the history of time series bootstrap procedures imply different output results then standard portfolio selection procedure. On the other hand, with consideration of the ability to present a variety of different investment solutions, resampled portfolios have desirable characteristics for investors. Delcourt and Petitjean (2011) were elaborated the opinion that low degree of diversification and the sudden shifts in allocation along portfolios are undesirable characteristics of mean-variance portfolio.

4. BOOTSTRAPPING TIME SERIES

Bootstrapping is related with simulation, but with one crucial difference. With simulation, the data are constructed completely artificially, while bootstrapping obtains a description of the properties of estimators by using the sample data points themselves, and involves sampling repeatedly with replacement from the actual data. There are two obvious advantages of bootstrap procedure over analytical results of traditional statistical methods. First, bootstrapping allows the researcher to make inferences without making strong distributional assumptions. The bootstrap involves empirically estimating the sampling distribution by looking at the variation of the statistic within sample. Hence, this procedure treating the sample as a population from which samples can be drawn. Second, the bootstrap are more robust then the classical statistical methods. Therefore, it could be used effectively

[TM] U.S. Patent #6,003,018 by Michaud et al., December 19, 1999 with relatively small samples and preserved the estimator stability during the periods of unexpected volatility shifts.

The bootstrap, originally created by Efron (1979), begins with a set of *n* independent and identically distributed (iid) observations with distribution function *F* and unknown parameter θ as a function of *F*. The bootstrap methodology allows an approximation of the distribution of θ under very general conditions and it is based on obtaining a bootstrap replicate of the available data set by drawing with replacement random samples from *F*.

A described method is the simplest version and is only valid in the case of independent and identically distributed observations. If the iid bootstrap is applied directly to dependent observations, the resampled data will not preserve the properties of the original data set, providing inconsistent statistical results. Including dynamic correlation and conditional heteroscedasticity, Ruiz and Pasqual (2002) offered two, parametric and nonparametric, bootstrap procedures recently developed for time series data. There are several versions of parametric and nonparametric bootstrap method, but this paper contains two most popular, the residual bootstrap and the moving block bootstrap method.

4.1. Residual bootstrap

The parametric bootstrap procedure is based on assumption that there is always a specific model suitable enough for time series data. In this case, it is usually not recommended to bootstrap from the row data but from the residuals of a given model. However, it is necessary to decide which form of model to be used and which residuals to be bootstrapped. This paper uses the Capital Asset Pricing Model (CAPM) for estimating returns of observed time series as the fallowing regression equation:

$$E(R_{i,t}) - R_f = \alpha_i + \beta_i(E(R_{M,t}) - R_f) + u_t \quad (2)$$

where,

$E(R_{i,t})$ –security expected return based on a concept of the random variable shows the weighted-average return of i-th security in observed time t

R_f –risk free rate

$E(R_{M,t})$ – market expected return, calculated from time series of the BELEX15 stock exchange index returns

α_i – slope coefficient

β_i – measure of sensitivity to a movement in the overall market

After defining the form of estimation model, the residual bootstrap procedure contains following four steps:

1) Estimate the model on the actual data, obtain the fitted values of dependent variable and calculate the residuals
2) Take the sample of size n with replacement from these residuals and generate a bootstrapped dependent variable by adding the fitted values to the bootstrapped residuals $(E(R_i)-R_f)^* = (E(R_i)-R_f)+u_i^*$
3) Regress this dependent variable on the original data $(E(R_M)-R_f)$ to get a bootstrapped coefficient β^*
4) Go back to stage 2, and repeat a total of B times.

Nevertheless, it is important to emphasize the other forms of regression models, particularly in situations where some adjustments are needed. The parametric model has to present a good approximation of true model. Thus the utility value of the residual bootstrap procedure predominantly depends on appropriate model selection process.

4.2. Moving block bootstrap

If the serial dependence of the date is misspecified, the parametric bootstrap could be inconsistent. Consequently, alternative approaches that not require fitting a parametric model have been developed to deal with dependent time series data. Kunsch (1989) proposed the moving block bootstrap method that divides the data into overlapping blocks of fixed length and resample with replacement from these blocks. Mentioned method preserves the original structure of time series by doing the resample process within defined blocks.

However, the accuracy of the moving block bootstrap procedure mainly depends on optimal block length selection. Otherwise, the optimal block length selection depends on sample size, applied data generating process and chosen statistics of interest. When sample size increases, the block length must follow the changes in order to secure the bootstrap consistency and empirical distribution function. By choosing the optimal block length it is possible to minimize the mean squared error.

The moving block bootstrap method contains four inevitable iterations in order to assemble an efficient resample algorithm:

1) Divide time series data into the equal size blocks with overlapping, where first block contains the set of $X_1,\ldots,X_l$ elements, second $X_2,\ldots,X_{l+1}$ etc.
2) Do the resampling process with overlapping within defined blocks and align resampled block in one bootstrap sample $X_1^*,\ldots,X_n^*$
3) Estimate the statistics of interest by using the constructed bootstrap sample $T_n^* = T_n(X_1^*,\ldots,X_n^*)$
4) Repeat steps 2 and 3 B times to get bootstrap distribution and probability of obtaining a test statistic $\hat{G}_n(t,F_n) = P^*(T_n^* \le t) = \frac{1}{B}\sum_{b=1}^{B} I(T_{N,b}^* \le t)$

5. EMPIRICAL EVIDENCE ON THE BELGRADE STOCK EXCHANGE

Considering the effects of the financial crises this paper involves 45 monthly stock prices data from the beginning of the year 2009 in order to preserve relative investment stability violated during the year 2008. With relatively small number of available data, the resampling algorithms find their place in making an appropriate data set. Six stocks from the Belgrade stock exchange with a high turnover rate comparing with other trading stocks are included in portfolio analysis. The six company stocks denoted by the stock symbols are: IMLK – Imlek, BMBI – Bambi Banat, MTLC – Metalac, AIKB – AIK banka, FITO – Galenika Fitofarmacija and GMON – Goša montaža.

5.1. In-sample portfolio analysis

At the beginning of the mentioned analysis we compute mean-variance efficient frontier from the original set of inputs and emphasize that only the weights computed with the Markowitz equations are optimal regarding to original set of inputs. Following the first iteration, the analysis resumes with the resampled efficiency procedure using two separate bootstrap algorithms. In combination with the original set of inputs, all resampled portfolio weights will form frontiers below the mean-variance efficient frontier, indicated in Figure 1.

Figure 1. gives us a starting point in fulfilling the idea on how sampling errors can effect the determination of an efficient frontier. This figure demonstrates that even small changes in the sample data can cause significant changes in mean-variance efficient portfolio decision. We noticed that the moving block bootstrap efficient frontier has the higher slope coefficient, while the mean-variance efficient frontier and its residual bootstrap analogue are approximately parallel.

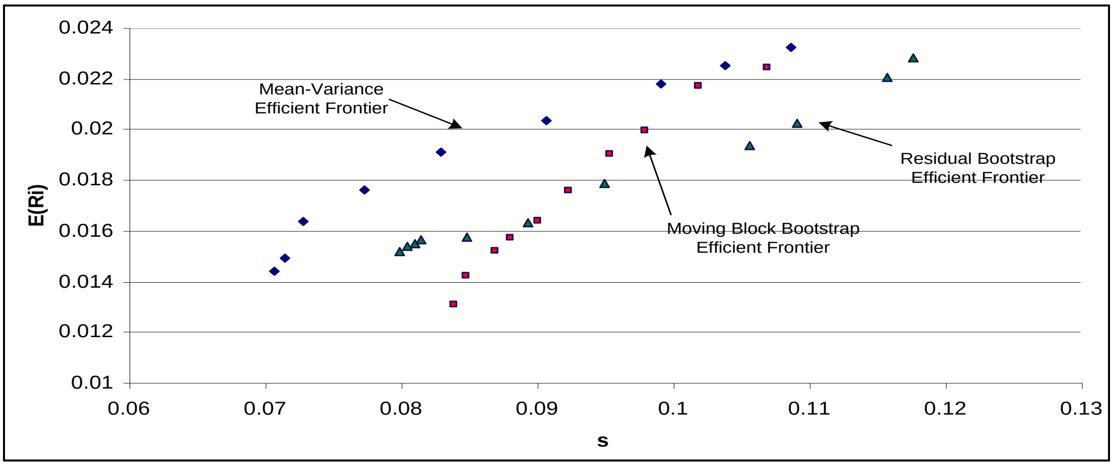

Figure 1: Mean-variance and resampled efficient frontiers

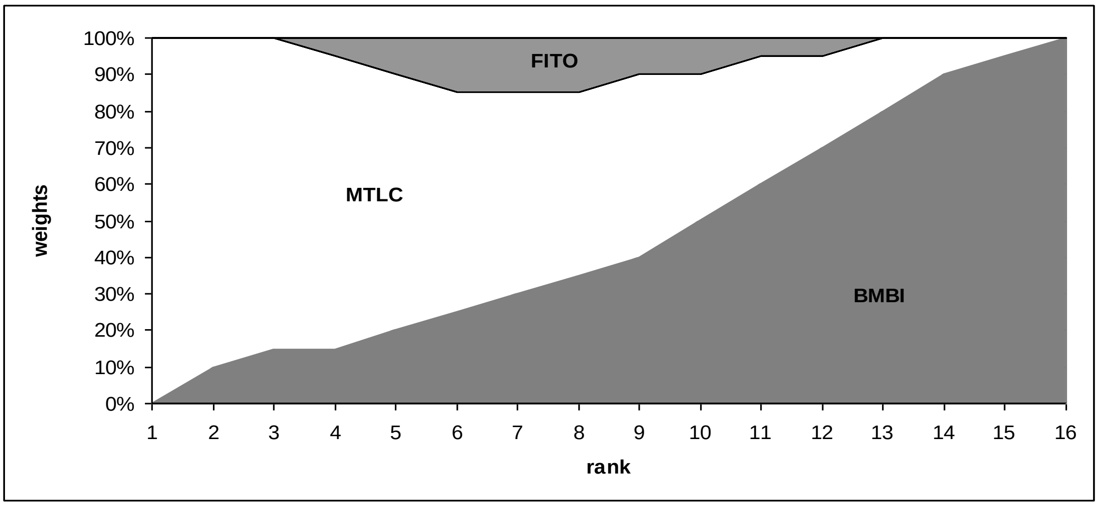

Figure 2: Mean-variance portfolio allocation

During the optimization process the allocation problem has become more important feature in investment strategy. Figure 2. shows that mean-variance optimization applying quadratic programming with no additional constrains emphasize weak diversification amongst six selected stocks. According to the mean-variance procedure, only few intermediate ranks (ranks define the expected value of stock return rate) include three common stocks, whereas the diversification of only two common stocks in smaller or bigger ranks (lowest or highest expected returns) are shown. At the other hand, two resampled versions, presented in Figures 3. and 4., involve all six analyzed stocks.

Two resampled portfolio solutions show smoothed transition in allocation along the resampled frontier. There are no sudden shifts in weights according to changes in expected return, particularly in the moving block bootstrap example. Shown in Figure 4., the residual bootstrap smoothes the original set of data, but keeping the level of average return per stock it reduces weights of the small return stocks in the highest ranks of portfolio choice. The portfolio with characteristics of mean-variance portfolio model is likely to maximize sampling errors and exhibit poor out-of-sample performance. In contrast to mean-variance allocation, diversification is preserved in the resampling procedures where resampled portfolios show a tendency towards the BMBI common stock.

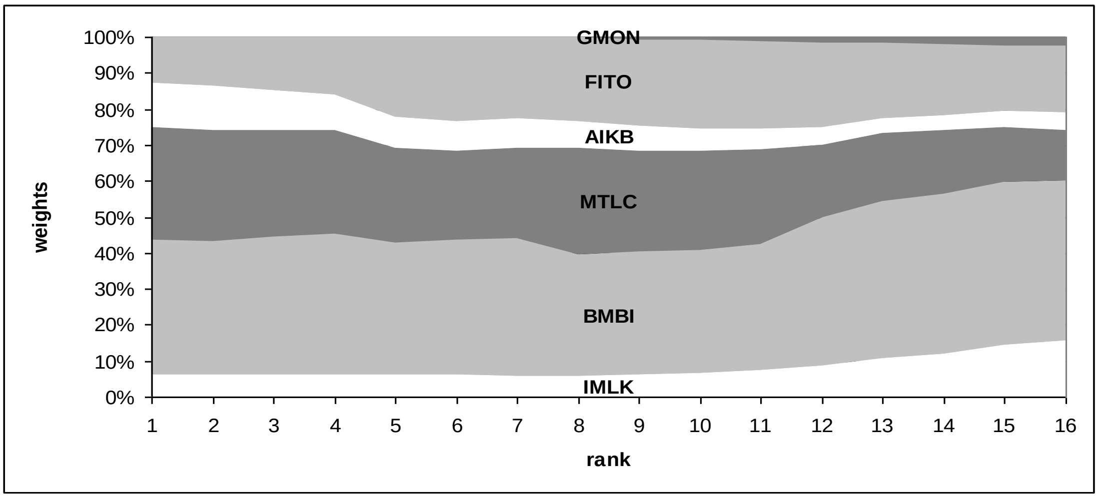

Figure 3: Moving block bootstrap portfolio allocation

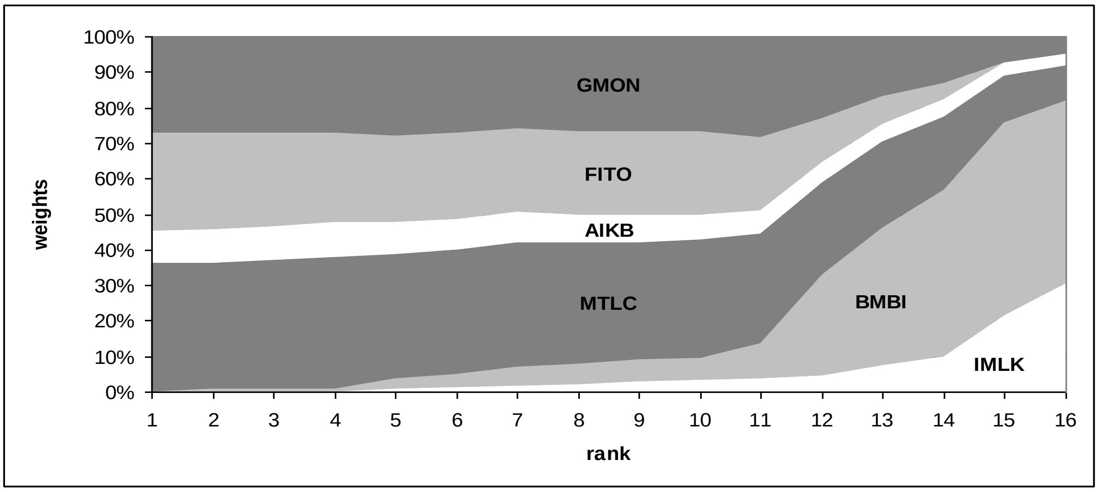

Figure 4: Residual bootstrap portfolio allocation

Presented differences between mean-variance portfolio selection and resampled allocation are implication of historical variance. They are more likely to be far from their historical value among all different scenarios in the simulation process. Hence, the riskier is the portfolio, the higher is the estimation risk.

5.2. Out-of-sample portfolio analysis

Out-of sample simulation study is performing to compare the performance between mean-variance and two resampled portfolio allocation strategies. Analogous with simulation study elaborated by Delcourt and Petitjean (2011) we consider 12 monthly periods ahead for three different expected return (minimum, intermediate and maximum return). Optimal portfolios are computed for each period, then the sample period is moved foreword a month and optimization process is repeated. At the beginning, we computed the realized returns generated by the optimal portfolios. Afterwards, we estimate the average realized returns and the average risk of the portfolios during the out of sample period. Therefore, we are capable to compute an average Sharpe ratio as the appropriate out of sample performance measure.

Table 1: Out-of-sample performance measured by Sharpe ratio

method	rank	return	risk	Sharpe
Mean-variance	0.0010	0.0101	0.0879	0.1143
	0.0140	0.0164	0.0728	0.2247
	0.0230	0.02324	0.1086	0.2138
Moving block bootstrap	0.0010	0.0105	0.0968	0.1076
	0.0140	0.0157	0.0901	0.1742
	0.0230	0.0224	0.0919	0.2437
Residual bootstrap	0.0010	0.0151	0.0878	0.1701
	0.0140	0.0157	0.0729	0.2159
	0.0230	0.0228	0.1087	0.2101

Source: Authors

Following the out of sample methodology we could summarize measured performances in Table 1. Selected strategy varies due to the investor’s required return. The moving block bootstrap process shows dominance towards higher expected returns, whereas the residual bootstrap procedure advantage towards lower expected returns are noticed. In this case, the estimation period is quite short and it is therefore reasonable to expect that the effect estimation risk will be more significant, favoring the resampled methods. Hence, the larger the sample size, the better the performance of the mean-variance portfolios with respect to the resampling procedure.

6. CONCLUSION

Keep in mind that the difference between the resampled and the traditional efficient frontier arises because resampling provides portfolios that are too diversified. Concerning Scherer (2002), instances can occur in resampling in which diversification becomes smaller as the maximum-return solution is approached. However, all resamplings are derived from the same vector and covariance matrix, where true distribution is unknown. Hence, all resampled portfolios will suffer from the deviation of the parameters. Averaging will not help greatly in this case because the averaged weights are the results of the input vector, which is itself very uncertain.

Portfolio resampling offers an intuitive way to develop tests for statistical difference between portfolios. Simulated return and risk help to quantify the effect on the optimization process of uncertainty inherent in the investment decision. The comparison between mean-variance optimization and resampled optimization shows that resampled strategies lead to more stabile and more diversified portfolios regardless to transaction costs. Moreover, it is important to notice that there is significant difference between two resampling optimization procedures, but their common feature is greater portfolio stability and diversification over the mean-variance optimization. The end result may be useful for controlling risk and structuring the allocation so that is consistent with investor objectives.

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