**MONOPOLY INNOVATION WITH EXHAUSTIBLE RESOURCE AND LABOR INPUT**

Pu-yan Nie*  
Peng Sun†

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**Abstract**

This paper focuses on the propensity to innovate for a monopolist with two inputs, an exhaustible resource and labor. When this exhaustible resource is used up, the monopolist quits this industry. This paper characterizes the relationship between the two types of elasticity of innovation. With this relationship, the equilibrium is captured. This study argues that the lower the marginal cost incurred by innovation, the longer it takes for the monopolist to quit the industry and the higher the profits.

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*Institute of Industrial Economics, Jinan University, Guangzhou, 510632, P.R. China. Tel. 86-20-85221069. E-mail: pynie2005@yahoo.com.cn
†Institute of Industrial Economics, Jinan University, Guangzhou, 510632, P.R. China. E-mail: newsp2008@126.com

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1 Introduction

By the new growth theory, innovation can be seen the driving force to improve the quantity (quality) of intermediate goods, and obtains the sustained growth through endogenous technical change. Romer (1990) regarded innovation as the engine of growth and investigated the determinants of technological progress, and there are many important papers about innovations in economics that discuss how innovation is achieved under various economic circumstances. Arrow (1962) examined the relationship between innovation and market power. Vives (2008) addressed the relationship between innovation and competitive pressure and derived some interesting conclusions about innovation and competition. Sacco and Schmuzler (2011) recently confirmed the U-shaped relationship between competition and investment in laboratory experiments.


Another group of scholars pay attention on the innovation in exhaustible resources. Grimaud and Rouge (2003) achieved the equilibrium paths in the model of the exhaustible resource with vertical innovations. Bretschger (2005) stated that effective innovations act as a remedy for the natural resource scarcity to realize economic development. Di Vita (2006) considered the technical substitution with exhaustible and renewable resources in an endogenous growth model.

Through the data in the past 100 years, Wils (2001) analyzed the different efforts among three types of technical innovation in the exhaustible resource exploitation. Nie (2012) addressed the emission of monopolist.

Recently, more researchers begin to pay close attention to the innovation both in human capital and exhaustible resource. Acemoglu (1998, 2002, 2003), and Corrado and Simone (2008) developed the innovation models with directed technical
change, where final output is obtained by means of two inputs, e.g. resource and labor. The technical progress may be either labor or resource augmenting, or both. Acemoglu et al. (2012) studied the effect of exhaustible resources and human capital inputs on innovation and derived some significant conclusions. Moreover, Acemoglu et al. (2012) introduced a growth model with environmental constraints.

The existing literature has focused on various types of innovations. In reality, a firm seeks to innovate in diverse ways. For example, Shell Cooperation is a global group of energy and petrochemical companies. On one hand, Shell supports a series of Training Programmes of human capital\(^1\). On the other hand, Shell launches the innovation to improve the efficiency\(^2\). How about the relationship about various innovations? This is the motivation for this study.

This paper further focuses on innovators that have the constraint of an exhaustible resource as a production input among industrial companies. As it is expressed in Acemoglu et al. (2012), exhaustible resource inputs are a variable considered in monopoly innovation models. To expand upon the existing research, this study focuses on two types of monopoly inputs: human capital and exhaustible resources. Additionally, this study focuses on two types of innovations, increasing the efficiency of the use of the exhaustible resource and human capital. The relationships between the two types of innovations are illustrated in this work. Based on a theory developed from social phenomena, this study develops a theory about innovations with multiple dimensions.

This work also employs dynamic models to analyze exhaustible resources. This is different from Acemoglu et al. (2012) and Acemoglu et al. (2012) focused on a growth model and just one type of innovation. This work considers exhaustible resources in industrial organizations and pays attention to two innovations, simultaneously.

This paper is organized as follows. The model of the monopoly innovation defined with variables representing exhaustible resources and human capital is

\^[1] Innovation in human capital, see the webpage http://www.shell.com/home/content/sdo/environment_society/shell_in_the_society/social_investment/training_programmes/training_for_employment.html

\^[2] http://www.shell.com/home/content/innovation/

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formally outlined in Section 2. Analysis and results are presented in Section 3. In Section 3, the strategies about investment of the innovations both in the exhaustible resource and in the human capital of the monopolist are all characterized. Some existed empirical evidence is outlined in Section 4. Some concluding remarks are given in the final section.

2 The Model

Consider an industry with a unique producer. For production in this industry, human capital and an exhaustible resource are required. The exhaustible resource is continuously reduced because it is consumed at each stage. When this resource is exhausted, this monopolist quits this industry. We further assume that there are no alternative inputs. We formally establish a dynamic model of monopoly innovation with variables for human capital and an exhaustible resource.

**Demand.** $p_t$ is the vector of price and the quantity of production is $D_t = q_t$ at time $t$. The utility function at time $t$ is

$$u_t(p_t, q_t) = Aq_t - \frac{1}{2}q_t^2 - p_tq_t,$$

where $A > 0$ and is a constant. The inverse demand function, which is the same as that in Sacco and Schmutzler (2011), is given as follows

$$p_t = A - q_t$$

Please note that the inverse demand function is directly induced by the above utility function.

**Producer variables:** We postulate that the production inputs are human capital and a type of exhaustible resource. The following notation is employed throughout.

$h_t$: human capital for production at time $t$.
$S_0$: stock of the exhaustible resource at time $t = 0$.
$S_t$: stock of the exhaustible resource at time $t$.
$er_t$: consuming exhaustible resource at time $t$.
$w_0$: reservation wages for each worker. 

$c(S_t)$: marginal cost incurred to consume the stock of exhaustible resource $S_t$, also described as the price to consume a unit of the exhaustible resource. $c(S_t)$ is decreased in $S_t$, or the higher the marginal cost, the less the consumption of the exhaustible resource. It is apparent that 

$$S_t = S_0 - \int_0^t e_{rt} dt = -\frac{dS_t}{dt}$$

(3) is also employed in Acemoglu et al. (2012). This paper employs a continuous model while Acemoglu et al. (2012) used a discrete time model. There is a unique final good using human capital and the exhaustible resource. The variable $I^h$ represents an innovative investment intended to promote efficiency in human capital and $I^{er}$ is an innovative investment intended to promote efficiency in the use of the exhaustible resource. The production function is a Cobb-Douglas production function 

$$q_t = (1 - e^{-I^h})^\alpha (1 - e^{-I^{er}})\beta h_t^\alpha (er_t)^\beta,$$

(4) 

where $1 \geq \alpha > 0$ and $1 \geq \beta > 0$ are two constants.

The cost function of the monopolist is mainly determined by two parts: one is incurred by the use of the exhaustible resource and the other comes from the cost of labor. The profit function of the monopolist is given as follows:

$$\pi_t = p_t q_t - c(S_t) er_t - w_0 h_t,$$

(5) 

$$\pi = \int_0^T e^{-\delta t} \pi_t dt - \tau_h I^h - \tau_{er} I^{er},$$

(6) 

where $S_T = 0$. At $S_T = 0$, the monopolist quits this industry because the resource is exhausted. In this model, the discounting factors are postulated to be $\delta$, $\tau_h$ and $\tau_{er}$, two positive constants, which represent the marginal cost of innovative investment. The term $\tau_h I^h$ represents the costs incurred by making innovative investments in human capital and $\tau_{er} I^{er}$ represents the costs incurred by making innovative investments to promote the efficiency in the use of the exhaustible resource.

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The timing of this pattern is outlined as follows. At the initial stage, the monopolist determines the amount of innovative investments in both human capital and the exhaustible resource. At the second stage, the monopolist determines the amount of labor to hire, the price and the quantity of the products for all $t < T$. At the last stage, the exhaustible resource is used up and the firm quits the market. Moreover, no potential entrants are considered in this work. No substitutes are introduced in this industry.

2.1 Analysis and Primary Results

Here the models (1)-(6) are discussed. The model above is a type of fixed endpoint optimal control problem, which is also a type of Euler equation. The problem is restated as follows.

$$\max_{I^h,I^{er},S_t,h_t} \pi = \int_0^T e^{-\delta t} \left\{ (1 - e^{-I^h})^\alpha (1 - e^{-I^{er}})^\beta h^\alpha_t (er_t)^\beta \right\} dt - \tau_h I^h - \tau_{er} I^{er}$$

S.t. \( (3) \) and \( S_T = 0. \)

The solution is determined by the following optimal conditions.

$$\frac{\partial \pi}{\partial I^h} = -2ae^{-I^h}(1 - e^{-I^h})^{2\alpha - 1}(1 - e^{-I^{er}})^{2\beta} \int_0^T e^{-\delta t} h_t^{2\alpha} (er_t)^{2\beta} dt + \alpha e^{-I^h}(1 - e^{-I^h})^{\alpha - 1}(1 - e^{-I^{er}})^{\beta} \int_0^T Ae^{-\delta t} h_t^{\alpha} (er_t)^{\beta} dt - \tau_h = 0 \quad (8)$$

$$\frac{\partial \pi}{\partial I^{er}} = -2\beta e^{-I^{er}}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta - 1} \int_0^T e^{-\delta t} h_t^{2\alpha} (er_t)^{2\beta - 1} dt + \beta e^{-I^{er}}(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta - 1} \int_0^T Ae^{-\delta t} h_t^{\alpha} (er_t)^{\beta} dt - \tau_{er} = 0 \quad (9)$$

$er_t$ and $S_t$ are determined by the following equation:

$$e^{-\delta t} \frac{\partial \pi}{\partial er_t} + \lambda_t = \lambda_t + e^{-\delta t} [A\beta(1 - e^{-I^h})^\alpha (1 - e^{-I^{er}})^{\beta} h^\alpha_t (er_t)^{\beta - 1} - 2\beta(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha} (er_t)^{2\beta - 1} - c(S_t)] = 0 \quad (10)$$

where $\lambda$ satisfies

$$\frac{d\lambda_t}{dt} = -e^{-\delta t} \frac{\partial \pi}{\partial S_t} = e^{-\delta t} er_t \frac{dc(S_t)}{dS_t}. \quad (11)$$
The human capital investment function is explained with the following relation
\[
e^{-\delta t} \frac{\partial \pi}{\partial h_t} = e^{-\delta t}[\alpha A(1 - e^{-I^h})^{\alpha-1}(1 - e^{-I^{er}})^{\beta} h_t^\alpha (er)^{\beta}
- 2\alpha(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha-1}(er)^{2\beta} - \varpi_0] = 0
\] (12)

The equilibrium state is determined by the above system of equations (8)-(12). The equilibrium solution is denoted as \((I^{h*}, I^{er*}, S_t, h_t^*, er_t^*)\) along with equilibrium value \(\pi^*\). After examining the second-order optimal conditions, the equilibrium is then characterized.

For the second-optimal conditions, we have reached the following conclusions.

**Proposition 1:** \(\pi\) is concave in both \(I^h\) and \(I^{er}\). Moreover, \(\pi\) is concave in both \(h_t\) and \(er_t\) for all \(t\).

**Proof.** See in Appendix. |

**Remarks:** Proposition 1 manifests that the equilibrium is a local maximum. The existence and uniqueness of the solution is, therefore, guaranteed.

The equilibrium is then characterized.

1. Innovative investment

The innovative investment is described by equations (8)-(9). Based on (8) and (9), by the implicit function theorem, we have \(\partial I^h / \partial \tau^h < 0\) and \(\partial I^{er} / \partial \tau^{er} < 0\). The higher marginal cost of an innovative investment yields lower levels of the corresponding innovative investment. For the parameter \(\delta\), by the comparative static analysis approach, we can see that \(\partial I^h / \partial \delta < 0\) and \(\partial I^{er} / \partial \delta < 0\). This means that more patient producers launch into more innovative investments when they are a monopoly.

For the parameters \(\alpha\) and \(\beta\), by the comparative static analysis method, under \(\ln h_t + \ln(1 - e^{-I^h}) > 0\) and \(\ln er_t + \ln(1 - e^{-I^{er}}) > 0\), we have \(\partial I^h / \partial \alpha > 0\), \(\partial I^h / \partial \beta > 0\), \(\partial I^{er} / \partial \alpha > 0\), and \(\partial I^{er} / \partial \beta > 0\). Under \(\ln h_t + \ln(1 - e^{-I^h}) > 0\) and \(\ln er_t + \ln(1 - e^{-I^{er}}) > 0\), the innovative investments are improved with a more elastic coefficient of inputs. We note that \(\ln h_t + \ln(1 - e^{-I^h}) > 0\) and \(\ln er_t + \ln(1 - e^{-I^{er}}) > 0\) manifest that the innovation increases the output. Therefore, this assumption of \(\ln h_t + \ln(1 - e^{-I^h}) > 0\) and \(\ln er_t + \ln(1 - e^{-I^{er}}) > 0\) is very moderate.
Moreover, equations (8) and (9) jointly indicate the interesting formulation

\[
\frac{(1 - e^{\mu_{h,t}}) \tau_h}{\alpha e^{\mu_{h,t}}} = \frac{(1 - e^{\mu_{er,t}}) \tau_{er}}{\beta e^{\mu_{er,t}}}.
\]  

Here we consider the elasticity of innovation. We denote the elasticity of labor innovation function at time \(t\) to be 

\[\varepsilon_{I_{h,t}} = \frac{\partial q_t}{\partial I_{h,t}} \cdot I_{h,t} / q_t.\]

Similarly, we define the elasticity of the exhaustible resource innovation function at time \(t\) to be 

\[\varepsilon_{I_{er,t}} = \frac{\partial q_t}{\partial I_{er,t}} \cdot I_{er,t} / q_t.\]

By definition, we find the following proposition:

**Proposition 2:** The ratio of the two types of elasticity of innovation is the corresponding ratio of two types of innovation cost at any time \(t\). Or 

\[\frac{\varepsilon_{I_{h,t}}}{\varepsilon_{I_{er,t}}} = \frac{\tau_h I_{h,t}}{\tau_{er} I_{er,t}}.\]

**Proof.** See in Appendix. 

**Remarks:** The relationship between the two types of innovative investments is captured and an interesting conclusion is achieved. The ratio of the two types of innovation elasticity is exactly the same as the ratio of the costs of the two types of innovation at any time \(t\).

We also note that the elasticity of innovation is closely related to time \(t\) in general, while 

\[\varepsilon_{I_{h,t}} = \alpha e^{\mu_{h,t}} / (1 - e^{\mu_{h,t}})\]

and 

\[\varepsilon_{I_{er,t}} = \beta e^{\mu_{er,t}} / (1 - e^{\mu_{er,t}}),\]

have no relation with time \(t\) in this work because of the special production function.

Moreover, (13) is rewritten as 

\[
(1 - e^{\mu_{h,t}}) \tau_h / (1 - e^{\mu_{er,t}}) \tau_{er} = \frac{\alpha}{\beta} \text{ or } (1 - e^{\mu_{h,t}}) \tau_h / (1 - e^{\mu_{er,t}}) \tau_{er} = \frac{\alpha e^{\mu_{h,t}}}{\beta e^{\mu_{er,t}}}.
\]

This means that the ratio of the elastic coefficient of inputs is equal to the ratio of marginal cost incurred by innovation multiplying a ratio related efficient to the innovation, which is a very interesting result. As we known, there is no relationship between multiple innovations in the existed papers and this conclusion firstly describes the relationship between two innovations.

1. Equilibrium exhaustible resource and human capital input

By comparative statistical analysis, equations (10), (11) and (12) are manifestly \(\partial I_{er} / \partial \tau_h > 0, \partial I_{er} / \partial \tau_{er} > 0, \partial h_t / \partial \tau_h > 0\) and \(\partial h_t / \partial \tau_{er} > 0\) for all \(t\).
Under the conditions of lower marginal costs incurred by innovation, the monopolist consumes less of the exhaustible resource and human capital at each stage.

Moreover, because we have $\frac{\partial S_t}{\partial h_t} < 0$ and $\frac{\partial S_t}{\partial \text{er}_t} < 0$ for all $t$, the stock of exhaustible resources decreases with the marginal cost incurred by innovations in both human capital and the use of the exhaustible resource for all $t$. Similarly, under $\ln h_t + \ln(1 - e^{-I^h}) > 0$ and $\ln \text{er}_t + \ln(1 - e^{-I^\text{er}}) > 0$, the inputs are correspondingly improved with a more elastic coefficient of inputs. Equations (11) and (12) indicate that $\frac{\partial \text{er}_t}{\partial \delta} > 0$ and $\frac{\partial h_t}{\partial \delta} > 0$ for all $t$. This means that more patient producers consume less exhaustible resource and hire less labor at each stage in monopoly conditions. $\frac{\partial \text{er}_t}{\partial \delta} > 0$ yields $\frac{\partial S_t}{\partial \delta} < 0$ for all $t$, which means that the more patient producer cares more about the stock of exhaustible resources.

For price and quantity, we have $\frac{\partial p_t}{\partial \tau_h} > 0$, $\frac{\partial p_t}{\partial \tau_{er}} > 0$, $\frac{\partial q_t}{\partial \tau_h} < 0$ and $\frac{\partial q_t}{\partial \tau_{er}} < 0$. These inequalities address the conditions that determine the quitting time of the monopoly producer. Obviously, $\frac{\partial T}{\partial \text{er}_t} < 0$, or $T$ is decreased in $\text{er}_t$, under the definition of quitting time. Based on the relation $\frac{\partial \text{er}_t}{\partial \tau_h} > 0$, $\frac{\partial \text{er}_t}{\partial \tau_{er}} > 0$, $\frac{\partial h_t}{\partial \tau_h} > 0$ and $\frac{\partial h_t}{\partial \tau_{er}} > 0$, we reach the following conclusion.

**Proposition 3:** Quitting time $T$ is decreased in marginal cost incurred by innovative investment $\tau_h$ and $\tau_{er}$.

**Proof.** Combined with $\frac{\partial T}{\partial \text{er}_t} < 0$, this is a direct conclusion from the above relationship $\frac{\partial \text{er}_t}{\partial \tau_h} > 0$, $\frac{\partial \text{er}_t}{\partial \tau_{er}} > 0$, $\frac{\partial h_t}{\partial \tau_h} > 0$ and $\frac{\partial h_t}{\partial \tau_{er}} > 0$. The proof in detail is deleted and the proof is complete. 

**Remarks:** Under the condition of lower marginal costs for innovation, the monopolist will increase its innovative investments and the exhaustible resources will take longer to be used up. With higher marginal costs for innovation, an exhaustible resource is consumed faster. This is consistent with the reality. The oil industry in Indonesia is a suitable example. Indonesia is the only member of OPEC organization in Asia & Pacific area and its oil output accounts for the 20th in the world. But because of the poor circumstance of technical innovation, the margin cost incurred by both crude oil refinement and human
capital investment are very high. The technical innovation lagging behind the resource exploitation leads the increasing consumption of oil and the outstandingly declining stock of oil. The stock is near to 50 millions ton until 2010, which only equivalently one third of 1980. In 2004, Indonesia had became the net importer.\(^3\)

The profit function is addressed here. The envelope theorem indicates the relation \(\frac{\partial \pi}{\partial \tau_h} < 0\) and \(\frac{\partial \pi}{\partial \tau_{er}} < 0\). Lower marginal costs are incurred as a result of innovation yielding higher profits for the monopolist. Under \(\ln h_t + \ln (1 - e^{-I_h}) > 0\) and \(\ln er_t + \ln (1 - e^{-I_{er}}) > 0\), the profits of the monopolist are improved with a more elastic coefficient of inputs. For the discounting factor, we have \(\frac{\partial \pi}{\partial \delta} < 0\).

The costs incurred through the use of the exhaustible resource are further discussed here. These costs are exogenously determined. The monopolist has no other choice but to accept this price. For example, in many mining industries, when the stock of the corresponding mine is reduced, the cost to develop mine is increased. Moreover, the reservation wages for each worker in production are exogenously given and the human capital is abundant enough that the producer can hire human capital with those reservation wages.

### 3 Existed Theoretic and Empirical Evidence

Some other theoretic conclusions are consistent with the above results. Bretschger (2005) confirmed that innovation is potential to compensate for natural resource scarcity. Under natural resource scarcity, innovation elasticity of exhaustible resource is small. By Proposition 2, the innovation investment in human capital becomes large while exhaustible resource innovation becomes small. This is a case of Proposition 2.

Because of page limitation and data restriction, we did not launch the empirical study. There exists some empirical research supporting the above theoretical conclusions. Subramanian and Nilakanta (1996) confirmed the existence of the

\(^3\)http://www.eia.gov/countries/
substantive relationship between two types of innovations based on the questionnaire about bank industry of United States. Battisti and Stoneman (2010) used the information contained in CIS4 to explore the pattern of use of innovations in UK industry and to test for the existence of complementarities among seven types of innovations, i.e. process, product, machinery, marketing, organization, management and strategic innovations. Binswanger (1974) measured the technical change biases of multiple types of innovations and the empirical evidence also supports the relationship between multiple innovations.

By Proposition 2, given the innovation elasticity, the optimal innovation in the human capital increases with the exhaustible resource innovation. Based on the patents of the German Federal Ministry of Education and Research, Ostertag, Sartorius and Espinoza (2010) examined that two types of patents keep pace with the general increase of patent applications. This highly supports our theoretic conclusions.

4 Concluding Remarks

In this paper, the innovation patterns of the monopolist are highlighted in relation to human capital competition and the limitations of an exhaustible resource. The equilibrium solution is achieved and characterized. The conditions for quitting the industry are also discussed. This study argues that the equilibrium innovative investment and inputs have a close relationship to the elasticity coefficient of inputs and marginal costs to innovate. When the properties of an exhaustible resource are introduced, the firm strategy changes correspondingly. This is the beginning of the research on the effects of an exhaustible resource in industrial organizations. In actuality, the exhaustible resource has deep effects on firms’ strategies. This work focuses on innovative investments. Other strategies of firms with exhaustible resource inputs will be the subject of our future research.

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6 Appendix

Proof of Proposition 1

Equation (8) indicates

\[
\frac{\partial \pi}{\partial I_h} = \frac{-2\alpha e^{-I_h}}{1-e^{-I_h}} \int_0^T e^{-\delta t}(1-e^{-I^{tr}})^{2\alpha} (1-e^{-I^{tr}})^{2\beta} h_t^{2\alpha} (er_t)^{2\beta} dt + \frac{\alpha e^{-I_h}}{1-e^{-I_h}} \int_0^T A(1-e^{-I^{tr}})^\alpha (1-e^{-I^{tr}})^\beta e^{-\delta t} h_t^\alpha (er_t)^\beta dt - \tau_h = 0
\]

or

\[
2 \int_0^T e^{-\delta t}(1-e^{-I^{tr}})^{2\alpha} (1-e^{-I^{tr}})^{2\beta} h_t^{2\alpha} (er_t)^{2\beta} dt - \frac{A}{\delta T} e^{-\delta t}(1-e^{-I^{tr}})^\alpha (1-e^{-I^{tr}})^\beta e^{-\delta t} h_t^\alpha (er_t)^\beta dt < 0
\]

This indicates that there exists a unique solution to equations (8) and (9). We further consider the second-order optimal conditions. According to equation (8), we have

\[
\frac{\partial^2 \pi}{\partial (I_h)^2} = -2\alpha (2\alpha - 1)(e^{-I_h})^2 (1-e^{-I^{tr}})^{2\alpha - 2}(1-e^{-I^{tr}})^{2\beta} \int_0^T e^{-\delta t} h_t^{2\alpha} (er_t)^{2\beta} dt + 2\alpha (e^{-I_h})(1-e^{-I^{tr}})^{2\alpha - 1}(1-e^{-I^{tr}})^{2\beta} \int_0^T e^{-\delta t} h_t^{2\alpha} (er_t)^{2\beta} dt + \alpha (\alpha - 1)(e^{-I_h})^2 (1-e^{-I^{tr}})^{\alpha - 2}(1-e^{-I^{tr}})^{\beta} \int_0^T A e^{-\delta t} h_t^\alpha (er_t)^\beta dt - \alpha (e^{-I_h})(1-e^{-I^{tr}})^{\alpha - 1}(1-e^{-I^{tr}})^{\beta} \int_0^T A e^{-\delta t} h_t^\alpha (er_t)^\beta dt.
\]

The first-order optimal conditions yield

\[
2\alpha (e^{-I_h})^2 (1-e^{-I^{tr}})^{2\alpha - 1}(1-e^{-I^{tr}})^{2\beta} \int_0^T e^{-\delta t} h_t^{2\alpha} (er_t)^{2\beta} dt - \alpha (e^{-I_h})^2 (1-e^{-I^{tr}})^{\alpha - 1}(1-e^{-I^{tr}})^{\beta} \int_0^T A e^{-\delta t} h_t^\alpha (er_t)^\beta dt < 0.
\]
Moreover, for \( \alpha \leq 1 \), we have

\[
-2\alpha(2\alpha - 1)(e^{-I^h})^2(1 - e^{-I^h})^{2\alpha - 2}(1 - e^{-I^{er}})^{2\beta} \int_0^T e^{-\delta t} h_t^{2\alpha}(er_t)^{2\beta} dt \\
+\alpha(\alpha - 1)e^{-I^h}(1 - e^{-I^h})^{\alpha - 2}(1 - e^{-I^{er}})^{\beta} \int_0^T A e^{-\delta t} h_t^{\alpha}(er_t)^{\beta} dt \\
= \alpha e^{-I^h}(1 - e^{-I^h})^2[(\alpha - 1)A \int_0^T e^{-\delta t}(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta} h_t^{\alpha}(er_t)^{\beta} dt \\
- (4\alpha - 2) \int_0^T e^{-\delta t}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha}(er_t)^{2\beta} dt] \\
= \alpha e^{-I^h}(1 - e^{-I^h})^2[(\alpha - 1)A \int_0^T e^{-\delta t}(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta} e^{-\delta t} h_t^{\alpha}(er_t)^{\beta} dt \\
- (2\alpha - 2) \int_0^T e^{-\delta t}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha}(er_t)^{2\beta} dt \\
- 2\alpha \int_0^T e^{-\delta t}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha}(er_t)^{2\beta} dt] \\
< 0.
\]

The last inequality comes from the relation

\[
-2\alpha \int_0^T e^{-\delta t}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha}(er_t)^{2\beta} dt < 0,
\]

and

\[
(\alpha - 1)A \int_0^T e^{-\delta t}(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta} e^{-\delta t} h_t^{\alpha}(er_t)^{\beta} dt \\
- (2\alpha - 2) \int_0^T e^{-\delta t}(1 - e^{-I^h})^{2\alpha}(1 - e^{-I^{er}})^{2\beta} h_t^{2\alpha}(er_t)^{2\beta} dt \\n\leq 0.
\]

For \( \alpha \leq 1 \), the first-order optimal conditions yield the above inequality.

We thus find that \( \pi \) is concave in \( I^h \). In a similar way, for \( 1 \geq \beta > 0 \), we find that \( \pi \) is concave in \( I^{er} \). Similarly, \( \pi \) is concave in both \( h_t \) and \( er_t \) for all \( t \). The method is highly similar to the above case.

Conclusions are obtained and the proof is complete.

Proof of Proposition 2

The equation (4) and the definitions of the two types of the elasticity of innovation functions jointly indicate the following relation

\[
\varepsilon_{I^h,t} = \frac{\partial q_t}{\partial I^h} \frac{I^h}{q_t} = \frac{\alpha e^{-I^h}(1 - e^{-I^h})^{\alpha - 1}(1 - e^{-I^{er}})^{\beta} h_t^{\alpha}(er_t)^{\beta} I^h}{(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta} h_t^{\alpha}(er_t)^{\beta}}
\]

and

\[
\varepsilon_{I^{er},t} = \frac{\partial q_t}{\partial I^{er}} \frac{I^{er}}{q_t} = \frac{\beta e^{-I^{er}}(1 - e^{-I^{er}})^{\beta}(1 - e^{-I^{er}})^{\alpha - 1} h_t^{\alpha}(er_t)^{\beta} I^{er}}{(1 - e^{-I^h})^{\alpha}(1 - e^{-I^{er}})^{\beta} h_t^{\alpha}(er_t)^{\beta}}.
\]

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Taking equation (13) into account, we immediately have the relationship $\frac{\varepsilon_{Ir,t}}{\varepsilon_{er,t}} = \frac{\tau_{hr}I_{hr}}{\tau_{er}I_{er}}$. The conclusion is achieved and the proof is therefore complete.

**INOVACIJA MONOPOLA S POTROŠNIM RESURSIMA I UTROŠKOM RADA**

**Sažetak:**

Ovaj rad je usredotočen na sklonost koju monopol s dva utroška, potrošnim resursom i radnom snagom, ima ka inovacijama. Kad se potrošni resursi istroše, monopolisti gase tu industriju. Ovaj rad analizira vezu između dva tipa elastičnosti inovacija. Ovom se vezom postiže ravnoteža. Članak postavlja tezu da što je manji marginalni trošak inovacije, monopolistu treba duže vremena da ugasi industriju a profit je isto tako veći.

**Ključne riječi:** inovacija, ljudski kapital, potrošni resursi, industrijska organizacija

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