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RELATIVISTIC EFFECTS ON SATELLITE NAVIGATION

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Subject review

The base of knowledge of relativistic effects on satellite navigation is presented through comparison of the main characteristics of the Newtonian and the relativistic space time and by a short introduction of metric of a gravity field. Post-Newtonian theory of relativity is presented as a background in numerical treating of satellite navigation relativistic effects. Time as a crucial parameter in relativistic satellite navigation is introduced through coordinate and proper time as well as terrestrial time and clocks synchronization problem. Described are relativistic effects: on time dilation, on time differences because of the gravity field, on frequency, on path range effects, caused by the Earth rotation, due to the orbit eccentricity and because of the acceleration of the satellite in the theory of relativity. Overviews of treatment of relativistic effects on the GPS, GLONASS, Galileo and BeiDou satellite systems are given.

Keywords: *satellite navigation, GNSS, post-Newton theory of relativity, relativistic effects, GPS, GLONASS, Galileo, BeiDou*

Relativistički utjecaji na satelitsku navigaciju

Pregledni rad

Osnovna znanja o relativističkim utjecajima na satelitsku navigaciju objašnjena su usporedbom glavnih karakteristika Newtonovog i relativističkog prostora vremena te kratkim uvodom u metriku gravitacijskog polja. Post-Newtonova teorija relativnosti objašnjena je kao osnova pri numeričkoj obradi relativističkih utjecaja u području satelitske navigacije. Vrijeme, kao vrlo važan parametar u relativističkoj satelitskoj navigaciji, objašnjeno je kroz koordinatno i vlastito vrijeme te kroz terestičko vrijeme i problem sinkronizacije satova. Opisani su relativistički utjecaji: na kašnjenje vremena, na razliku vremena zbog različitosti gravitacijskog polja, na frekvenciju, na širenje satelitskog signala te zbog rotacije Zemlje, zbog ekscentričnosti putanje satelita i zbog ubrzanja satelita prema teoriji relativnosti. Dan je pregled relativističkih utjecaja na GPS, GLONASS, Galileo i BeiDou satelitske sustave.

Ključne riječi*: satelitska navigacija, GNSS, post-Newtonova teorija relativnosti, relativistički utjecaji, GPS, GLONASS, Galileo, BeiDou*

1 Introduction

Italian mathematician, physicist and astronomer Galileo Galilei was the first to introduce the principle of relativity. Galileo argued that the motion has meaning only in relation to another object [20], and that was different in comparison to Aristotle's absolute space and time [1]. Isaac Newton introduced gravitational law, the concept of force, potential and kinetic energy and other basic laws of nature. But, he kept the framework of absolute space and time [35]. Newton's philosophy of nature served as the basis for describing the laws of nature. In 1905 Albert Einstein published the special theory of relativity (STR) [11], and ten years later, the general theory of relativity (GTR) [12]. STR and GTR have fundamentally changed our understanding of nature. Tab. 1 compares the main features of Newtonian and relativistic space time.

Development of satellite measurement techniques and increase of the sensitivity of the sensor technologies made relativistic effects significant. To use atomic clocks on artificial satellites and to test Einstein's theory of relativity was proposed already in 1955 by Friedwardt Winterberg [50].

In one of the general navigation principles, a receiver measures arrival times of the pulses from signal transmitters and calculates the position. When the Earth's satellites are used as the signal transmission sources, relativistic effects can be up to 12 kilometers after one day [15].

There are more Global Navigation Satellite Systems (GNSS): Global Positioning System (GPS), GLONASS, Galileo and BeiDou. The last two are under development. There are some regional GNSS augmentation systems: Wide-Area Augmentation System (WAAS), Local Area Augmentation System (LAAS), European Geostationary Navigation Overlay System (EGNOS), India Regional Navigation Satellite System (IRNSS), and Japanese Quasi-Zenith Satellite System (QZSS).

Table 1 The main characteristics of Newtonian and relativistic spacetime

Newtonian space time	Relativistic space time
Time is absolute.	Space time is relative and depends on the observer frame of reference.
Simultaneity of events is independent of the observer frame of reference.	Simultaneity of events is relative.
Space is absolute and does not dependent on the observer frame of reference.	Space time is relative and it depends on the observer frame of reference.
Euclidian metric is used.	Space time continuum metric is used.
Gravitational force is acting.	Space time is curved under the influence of gravity.

The main GNSS goal is precise navigation and time transfer. Stable and very accurate atomic clocks and frequency sources are enabling that.

2 Metric of the gravity field considering GNSS

There are more metric theories that can be used in the interpretation of a gravity field [36]: the Schwarzschild metric is a standard approach for a non rotating frame, the Langevin metric is used for a rotated body (Earth) with respect to a fixed-axis reference frame [5], the Schwarzschild-de Sitter metric for an isotropic, co-rotating frame of reference and the Kerr metric for complex geometrical relations of the space time in the vicinity of the rotating body [28].

German physicist Karl Schwarzschild defined in 1915 the metric of the symmetrical, conservative field outside of the spherical, non rotating body. The Schwarzschild metric is one of the classical mathematical physics used in the classical Newtonian interpretation of a gravity field. But, it is also used in the general theory of relativity and other theories of the gravity field. It defines complex mathematical relations.

The GPS uses the approximated linearized Schwarzschild metric in the Earth Centered Inertial frame (ECI) for the weak external gravitational field of the Earth. It has linear element, [5]:

$$\mathrm{d}s^2 = \left(1+\frac{2V}{c^2}\right)c^2\,\mathrm{d}t^2 - \left(1-\frac{2V}{c^2}\right),$$

$$(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\ \mathrm{d}\phi^2)\ . \tag{1}$$

Here c is speed of the light, t coordinate time, $(r,\ \theta,\ \phi)$ spherical coordinates and V the Newtonian gravitational potential of the Earth.

GNSS measurements are mostly done in the Earth-Centered Earth-Fixed (ECEF) referent frame; e.g. receiver on the Earth surface. The Langevin metric for rotating frame could be used for rotating Earth, [5]:

$$\mathrm{d}s^2 = \left(1-\frac{\omega_E^2\,r'^2}{c^2}\right)(c\ \mathrm{d}t')^2$$

$$-2\,\omega_E\,r'^2\ \mathrm{d}\phi'\ \mathrm{d}t' - \ (\mathrm{d}\sigma')^2\ . \tag{2}$$

Where c is speed of light, ω rotation of the Earth, $(t',\ r',\ \phi',\ z')$ spherical coordinates in the rotating frame and $(\mathrm{d}\sigma')^2 = (\mathrm{d}r')^2 + (r'\mathrm{d}\phi')^2 + (\mathrm{d}z')^2$ the square of the coordinate distance.

To experimentally improve practical differences between metrics of the space around the rotating and not rotating gravity source (Earth) two satellite missions are used: Gravity Probe-B and WeberSat-LARES [44, 19].

3 Post-Newton theory of relativity

Einstein's equations in the general theory of relativity are complex partial differential equations. For numerical solutions they could be parameterized through post-Newtonian (PN) formalism.

If the measurements are done between the transmitter and receiver reference frames that are relatively moving at low speeds, with weak gravitational field differences, usually a strict approach to Einstein's theory of relativity is not needed. This is regularly the case when we look at measurements in the vicinity of the Earth. The PN theory of relativity is the first approximation of Einstein's theory of relativity. The PN is the approximation of a complex mathematical relativistic operator in the way to keep only the significant order of development of the equation. The level of accuracy is usually associated with the quality of measurement and the PN approximation is sufficient in most practical cases.

Unlike the post-Newtonian theory of relativity, the post-post-Newtonian theory of relativity takes into account the relativistic effects of a finer order. It considers the effects of distant planets of the Solar system on the events on or near the Earth.

Corrections of Newtonian approach using PN are a regular way in treating relativistic effects in satellite navigation but there are approaches that treat the GNSS navigation problem in a strictly relativistic way [9, 7].

4 Coordinate and proper time

In relativity, space time is mapped out in four coordinates $(x_1,\ x_2,\ x_3,\ x_t) = (x,\ y,\ z,\ c_t)$; e.g. three dimensions for position and one for time; $x_t = c_t$. where c is speed of light and t is coordinate time. Coordinate time is the time defined for a coordinate frame. Coordinate time is defined for each of GNSS (e.g. GPS, GLONASS, Galileo and BeiDou). Coordinate time is theoretical time and proper time is actual clock reading of the observer [31]. Proper time was introduced by the German mathematician Hermanmn Minkowski [32].

If measurements with more moving clocks on separate locations are made, the observed proper times should be made consistent; e.g. transformed in coordinate times. To reach that, description of the clock movement path and metrical structure of the space time is required.

Satellite navigation techniques are very sensitive to time errors. An error of 1 nanosecond will result in a positional error of about 30 centimeters [31].

5 Terrestrial Time (TT)

Terrestrial Time (TT) is the Geocentric Reference System (GRS) coordinate time. It should provide consistent definitions for the coordinates and the metric tensor of the reference systems at the PN level [45]. Transformation between the observer proper time and coordinate time in the vicinity of the Earth could be made by [38]

$$\frac{\mathrm{d}\tau_\mathrm{A}}{\mathrm{d}t} = 1 - \frac{1}{c^2}\left[\frac{v_\mathrm{A}^2}{2} + U_\mathrm{E}(\overrightarrow{x_\mathrm{A}})\right]. \tag{3}$$

Where $U_\mathrm{E}(\overrightarrow{x_\mathrm{A}})$ is Newtonian potential of the Earth at the position $\overrightarrow{x_\mathrm{A}}$ of the clock in the geocentric frame, t coordinate time, c speed of the light, v_A^2 velocity of the clock, τ_A proper time of a clock A. This equation is valid up to the geosynchronous orbit or slightly above. This equation evaluates the contributions of the metric of the geocentric reference system, and it is sufficient for time and frequency applications in the Geocentric Celestial Reference System (GCRS). In the equation above are neglected tidal effects that have a contribution of 1×10^{-15} in frequency and a few pico seconds in time.

6 Time synchronization

According to the theory of relativity, the simultaneity of spatially separated events is not absolute. It is relative, and it depends on the observer. No observer has a privileged status. Because of relative movements, two events simultaneous in one inertial frame may not be simultaneous in another inertial frame. The clocks used in satellite navigation are Earth-orbiting (satellite), Earth-fixed (receiver) or near Earth surface (receiver).

If the clocks at different locations on a meridian, viewed from the ECI frame are observed, the clock near the equator, because of the Earth's rotation, moves more rapidly than one at the pole. But, if this effect is compared with the effects from the gravity field differences due to the Earth's oblate shape, it is cancelled. The reason is that the geoid is very close to hydrostatic equilibrium of the equipotential surface in the rotating frame and the clocks on the geoid beat at the same rate. Definition of the System International (SI) second is based on the time scale on the geoid. Clocks at different altitudes rapidly lose synchronization and they should be synchronized [21]. To have SI consistent time, clocks not on the geoid should be corrected for gravitational frequency shifts with respect to the geoid. For example, GPS time is defined by the reference (master) clock at Colorado Springs. It is at about 1,83 m above the geoid. It would be shifted with respect to the clocks on the geoid by about 16 nanoseconds per day.

Because of the daily Earth rotation, the Earth-fixed frames are rotation frames and they are not inertial frames. During the propagation time of a GNSS satellite signal an Earth-fixed observer moves because of the Earth rotation. GPS signal needs about 0,07 s to come to the observer on the Earth surface and the Earth rotation is 15"/s. The Earth rotates for about 1" during the GPS signal transmitting time. It depends on the observer's latitude, how much the observer on the Earth surface changed position because of 1" Earth rotation. On the equator the observer changed position for about 30 m.

According to the principles of the theory of relativity, time synchronization is not possible in a rotation frame. An acceptable way to avoid this problem is to synchronize the clocks in the nonrotating frame. An experimental synchronization procedure, so called Einstein synchronization, is used by GPS. The GPS system uses hypothetical time of the Earth Centered Inertial (ECI) frame with the origin attached to the Earth, but not spinning. Using a set of systematic corrections, clocks are synchronized with the referent clock.

Clock accuracy is improved every 10 years by about 10 times [40]. Clocks in Block I GPS satellites have fractional stabilities of about one part in 10^{12} over a period of one day, while clocks performance in Block II GPS satellites is 10 times better. The best ground clocks have accuracy of 3×10^{-17} and are improving [41].

ESA project Atomic Clock Ensemble in Space (ACES) [16] has an objective to achieve worldwide time and frequency transfer stability better than 10^{-16} [29]. ACES will enable absolute synchronization of ground clock time scales and intercontinental geodetic observatories with an uncertainty of 100 ps. It is using the knowledge of the two latest clock technologies [16]:

- PHARAO (Projet d'Horloge Atomique par Refroidissement d'Atomes en Orbite), giving frequency instability better than $1\times10^{-13}\cdot t^{-1/2}$ for 1 s ‹ *t* ‹ 10 days, where t is the integration time expressed in seconds and frequency inaccuracy at the 10^{-16} level;
- SHM (Space H-Maser), giving frequency instability better than $1{,}5\times10^{-13}$ at 1 s, $2{,}1\times10^{-14}$ at 10 s, $5{,}1\times10^{-15}$ at 100 s, $2{,}1\times10^{-15}$ at 1000 s, and $1{,}5\times10^{-15}$ at 10 000 s of integration time.

New clock generation will increase satellite navigation accuracy [39].

7 Sources of relativistic effects on GNSS

The main sources of relativistic effects on GNSS are relative motion between the satellite and the receiver, potential differences between the satellite and the receiver, and rotation of the Earth. The main relativistic effects on satellite navigation are [49, 51]:

- time dilation
- time differences because of differences of the gravity field
- relativistic effects on frequency
- relativistic path range effects
- relativistic Earth rotation effects
- relativistic effects due to the orbit eccentricity
- acceleration of the satellite in the theory of relativity.

There are more relativistic effects, but most of them are too small to be significant in satellite navigation [39].

7.1 Time dilation

Time dilation is a relativistic effect because of the relative movement of clocks. It is sometimes also called the second-order or transverse Doppler effect. Life of the processes in the reference frame that moves with speed *v* can be obtained using the Lorentz transformation:

$$t_\mathrm{s} = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}\,. \tag{4}$$

Here *c* is the speed of light and t_0 time in referent frame fixed with observed process. For small values of v/c binomial expansion could be used. Although the speed of the satellites is significantly smaller than the speed of light, time daily has significant effect on satellite navigation.

7.2 Time differences because of the differences of the gravity field

According to the general theory of relativity the time rate is slower in a more intensive gravity field; e.g. it is slower on the Earth's surface than in a satellite orbit. If we are measuring the time in two vertically shifted clocks A and B on the height difference *h* and if the potential differences between clocks are ΔU and the clock in the higher point is measuring time interval Δt, then the lower clock will measure:

$$\Delta\tau = \left(1-\frac{\Delta U}{c^2}\right)\Delta t. \tag{5}$$

For the potential can be used equation

$$U = -\frac{GM}{r}. \tag{6}$$

Here *M* is mass of the body, *G* gravitational constant, *c* speed of the light and *r* distance between masses.

If the two clocks are set at two points in Croatia with the biggest height differences e.g. one clock is on the highest peak of the Biokovo mountain and the other clock at the sea level, the height difference is about 2000 m and the normal gravity field of the Geodetic Reference System 1980 (GRS80) is used. If the time measured on the clock in the highest point is one second, the clock at the sea level will measure, because of the difference in gravity field, time interval of 0,999 999 999 999 792 6 s, e.g. the time rate of the clock at the sea level is $-2{,}1\times10^{-13}$ seconds slower because of the more intensive gravity field. Atomic clocks can measure time with the precision of 10^{-14} or better in the period of a few months.

7.3 Relativistic effects on frequency

The main relativistic effect on frequency is caused by the relative movement and difference in gravity field between the satellite and the receiver. The first is treated by the special theory of relativity and the other by the general theory of relativity. Relativistic effects on the satellite frequency can be computed by [51]

$$\delta_{\mathrm{rel}} = -\frac{f_0 - f'}{f'} = \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{\Delta U}{c^2} \ . \tag{7}$$

where ΔU is the Earth potential differences between satellite and receiver, c speed of the light, v velocity of the satellite relative to the receiver, f_0 frequency without relativistic effect and f' frequency of the moving transmitter. The first part of the equation is caused by relative movement and the second by the Earth potential differences. The Earth potential differences can be calculated using equation $\Delta U = GM/(R_{\mathrm{E}} + H) - GM/R_{\mathrm{E}}$. Where GM is geocentric gravitational constant of the Earth, R_{E} is the Earth radius and H is the height of the satellite above the Earth.

7.4 Relativistic path range effects

Because of anomaly of the gravity field the path of the satellite signal is not the Euclidian straight line but a curve. This causes the timing and ranging relativistic effects. Relativistic signal propagation model is considered in geocentric coordinate frame and only the influence of the Earth gravity field is taken into consideration. Gravitational effects of the Moon, Sun and other planets are reduced to tidal forces with very small relativistic correction [2, 3].

Coordinate time of the propagation of the electromagnetic signal from satellite in the position $\vec{x}_1$ at coordinate time t_1 to a receiver at the position $\vec{x}_2$ at the coordinate time t_2 is given by [38]:

$$t_2 - t_1 = \frac{|\vec{x}_2(t_2) - \vec{x}_1(t_1)|}{c} +$$

$$+ \sum_{\mathrm{J}} \frac{2GM_{\mathrm{J}}}{c^3} \ ln\left(\frac{r_{\mathrm{J}1} + r_{\mathrm{J}2} + \rho}{r_{\mathrm{J}1} + r_{\mathrm{J}2} - \rho}\right) \ . \tag{8}$$

Where the sum is including all the body J with mass M_J, $r_{\mathrm{J}1} = |\vec{x_1} - \vec{x_{\mathrm{J}}}|$, $r_{\mathrm{J}2} = |\vec{x_2} - \vec{x_{\mathrm{J}}}|$ and $\rho = |\vec{x_2} - \vec{x_1}|$. Amount of the effect can be up to a few seconds and uncertainty of this model is 3 ps. The second part of the upper equation is the Holdridge model [26] of the effects of general relativity on the path range.

To judge the amount of this effect the GPS orbit parameters are used for radius R = 6370 km, mean satellite height 20 200 km, maximum distance between satellite and receiver 25 800 km. Using these values, the effect has an amount of 18,7 mm. But, the effect on relative positioning, using two or more receivers is significantly smaller, and it amounts to 0,001 pm [53].

The same effect on GLONASS satellite is 20,0 mm when for the height of the satellite is used 19 100 km.

7.5 Relativistic Earth rotation effects

All corrections related to the rotation of the Earth are called the Sagnac effect. It arises because of the motion of the observer at the Earth's surface, due to the Earth's daily rotation. During the propagation time of the transmitted signal from the satellite, the observers are fixed on the Earth's move as the Earth rotates. Referent frame of the GNSS antenna on the Earth's surface is a rotating frame of reference, and it is not an inertial frame.

Observing from the non-rotating frame, the distance of the emitted signal can be calculated by the expression, [51]:

$$c\,\Delta t = |\vec{r}_{\mathrm{r}} + \vec{v}_{\mathrm{r}}\,\Delta t - \vec{r}_{\mathrm{s}}|. \tag{9}$$

Where c is speed of the light, $\vec{r}_{\mathrm{s}}$ geocentric vector of the satellites, $\vec{r}_{\mathrm{r}}$ geocentric vector of the receiver, $\vec{v}_{\mathrm{r}}$ receiver vector velocity, Δt time of signal travelling from satellite to receiver. During signal transmission time receiver has been moved because Earth rotation for amount $\vec{r}_{\mathrm{r}} + \vec{v}_{\mathrm{r}}\,\Delta t$. Correction of the transmitted satellite signal due to the rotation of the Earth is

$$\Delta\rho = |\vec{r}_{\mathrm{r}} + \vec{v}_{\mathrm{r}}\,\Delta t - \vec{r}_{\mathrm{s}}| - |\vec{r}_{\mathrm{r}} - \vec{r}_{\mathrm{s}}|. \tag{10}$$

This correction can be up to 30 m. If the measurements are done using a moving receiver, not fixed on the Earth surface, velocity vector should be calculated using

$$\vec{v}_{\mathrm{r}} = \vec{\omega}_{\mathrm{E}} \times \vec{r}_{\mathrm{r}} + \vec{v}_k \ . \tag{11}$$

Where the first part of the equation is the velocity vector of the receiver due to the Earth rotation $\vec{\omega}_{\mathrm{E}}$, $\vec{v}_k$ is kinematic velocity vector. Kinematic movement of the receiver of 100 km/h related to the Earth's surface can cause relativistic effect up to 2 m.

The Sagnac effect has a significant influence on time. One of the experimental improvements of the Sagnac effect on time was done in 1984. Three GPS satellites were used in simultaneous common view between three pairs of earth timing centers. The amount of the Sagnac correction varied from 240 to 350 ns [5].

According to Nelson [34] the Sagnac effect between points A and B is

$$\Delta t_{\mathrm{Sagnac}} = \frac{1}{c^2}\int_{\mathrm{A}}^{\mathrm{B}} (\vec{\omega} \times \vec{r}) \cdot \mathrm{d}\vec{r} = \frac{2\,\omega\,P}{c^2} =$$

$$= \frac{\omega}{c^2}(x_{\mathrm{A}}\, y_{\mathrm{B}} - y_{\mathrm{A}}\, x_{\mathrm{B}}). \tag{12}$$

Where ω is the Earth's rotational angular velocity, c the speed of light, P the perpendicular projection of the area formed by the center of rotation and end points (x_{A}, y_{A}) and (x_{B}, y_{B}) of the light path.

If the observations are made in inertial reference frame, the Sagnac effect should not be considered. But, the Sagnac effect should still be considered on the moving ground-based receivers.

The Sagnac correction is also used for the low-Earth orbit satellites that are using the GNSS satellite-to-satellite tracking (SST) systems.

7.6 Relativistic effects due to the orbit eccentricity

Precise orbit determination requires relativistic effects to be used. Relativistic effects in satellite orbit modeling using PN approach have an amount of about 30 cm daily [33]. Satellite clock correction caused by the eccentricity of the satellite orbit can be represented by [4, 34]:

$$\Delta t_{\mathrm{r}} = \frac{2}{c^2}\sqrt{\mu a}\; e \sin E + \; const. \tag{13}$$

Where a is semi major axis of the satellite orbit, e the eccentricity of the orbit, E is eccentric anomaly of the orbit and μ gravitational constant of the Earth. The constant part of the equation cannot be separated from the clock offset.

The relativistic effect caused by non circular orbit is in the GPS system distributed by the GPS satellite navigation message as polynomial coefficients corrections of the clocks. In relative positioning this effect is canceled [53].

7.7 Acceleration of the satellite in the theory of relativity

Relativistic correction of the GNSS satellites orbit determination includes corrections of the equation of motion, time transformations and measurement model. The main relativistic effects on the satellites are described by the Schwarzschild metric of the Earth. Relativistic correction to the acceleration of an artificial Earth satellite in GCRS is, [38]:

$$\Delta\vec{a} = \frac{GM_{\mathrm{E}}}{c^2 r^3}\left\{\left[2(\beta+\gamma)\frac{GM_{\mathrm{E}}}{r} - \gamma\vec{v}\cdot\vec{v}\right]\vec{r}\; 2\,(1+\gamma)(\vec{r}\cdot\vec{v})\vec{v}\right\} +$$
$$+ (1+\gamma)\frac{GM_{\mathrm{E}}}{c^2 r^3}\left[\frac{3}{r^2}(\vec{r}\times\vec{v})(\vec{r}\cdot\vec{J}) + (\vec{v}\times\vec{J})\right] +$$
$$+ \left\{(1+2\gamma)\left[\vec{v}_{\mathrm{R}} \times \left(\frac{-GM_{\mathrm{S}}\vec{R}}{c^2 R^3}\right)\right]\times\vec{v}\right\}. \tag{14}$$

Where c is speed of the light, GM_{E} and GM_{S} gravitational coefficient of the Earth and Sun, $\vec{r}$ geocentric satellite vector, $\vec{R}$ position of the Earth with respect to the Sun, $\vec{v}_R$ Earth velocity with respect to the Sun, $\vec{J}$ Earth's angular momentum per unit mass, β and γ are parameterized PN parameters and $\vec{v}$ satellite velocity vector.

The first lines in the equation above are the Schwarzschild terms. They have magnitude of about 10^{-10} m/s^2 for high orbits and 10^{-9} m/s^2 for low orbits. The second line is the Lense-Thirring precession (e.g. frame dragging), and it is of magnitude 10^{-11} m/s^2. The third line is geodesic (de Sitter) precession, and it is of magnitude 10^{-12} m/s^2.

The signal from the low orbit GOCE satellite (altitude of approx. 250 km) travels about 0,0008 s, and the signal from GNSS satellites (at a height of approx. 20 000 km) needs about 0,07 s to reach the observer on the Earth surface. Austrian physicists Josef Lens and Hans Thirring [46, 47, 30] predicted that the rotating bodies (Earth) drag space time for small amounts. This effect is called Lens-Thirring or frame dragging effect.

The curvature of the space time in the general theory of relativity is influenced by a vector carried along with an orbiting body. This effect is called geodetic precession or de Sitter precession. This effect was first predicted by a Dutch mathematician, physicist and astronomer Willem de Sitter [43]. He defined this relativistic correction for the Earth-Moon system. De Sitter effect is independent of the orbit altitude, and it has an amount of about 19 milliard seconds/year. Relativistic satellite mission Gravity Probe B (GP-B) confirmed theoretical amounts of frame dragging and geodetic precession effects. GP-B measured geodetic precession of –6,6018 milliard-seconds/year (theoretical value is –6,6061 milliard-seconds/year), and frame dragging of –37,272 milliard-seconds/year (theoretical value is –39,3 milliard-seconds/year).

After Gravity probe B, Einstein Gravity Explorer satellite mission is proposed. It would provide the GP-B tests at a higher accuracy level [42].

8 Relativistic effects on GPS

The GPS system introduced relativistic corrections. According to the specification of the GPS authorities [25] [22] GPS satellites have the same nominal frequency of the carrier frequencies of L1 and L2 signals. The nominal frequency received by the observer on the Earth is 10,23 MHz. The equation for relativistic effect on frequency (7) has the first part because of the relative velocity between the GPS satellite and the receiver on the Earth surface (special relativity) and the second part caused by gravity differences between the GPS satellite and the receiver on the Earth surface (general relativity). The effects have opposite sign and the total amount is $\Delta f/f$ = –4,4647×10^{-10}. GPS frequency, which would be measured at the satellite, is modified for this amount in the way (1 – 4,4647×10^{-10}) × 10,23 = 10,229 999 995 432 6 MHz. The observer on the Earth surface would receive nominal frequency of 10,23 MHz. Nominal frequencies (f_0) for L1 and L1 carriers are 1575,42 MHz and 1227,6 MHz respectively. Making this frequency correction, the GPS satellite transmitted time slowed down by 4,4647×10^{-10} × 60 × 60 × 24 = 38 575 ns/day.

In this way a major part of the relativistic effects is removed. But the temporal variations of the relativistic effects remain. They are removed during the receiver signal processing or during post processing.

The time received from the GPS satellites should be corrected by the user using the equation, [22]:

$$t = t_{\mathrm{sv}} - \Delta t_{\mathrm{sv}} \ . \quad (15)$$

Where t is GPS time, t_{sv} satellite codes phase time at the moment of the transmission and Δt_{sv} satellite code phase offset. Satellite code phase offset is given with polynomial expression

$$\Delta t_{\mathrm{sv}} = a_{\mathrm{f}_0} + a_{\mathrm{f}_1}(t - t_{0\mathrm{c}}) + a_{\mathrm{f}_2}(t - t_{0\mathrm{c}})^2 + \Delta t_{\mathrm{r}}. \quad (16)$$

Where a_{f0}, a_{f1} and a_{f2} are coefficients of the polynomial distributed in GPS message, $t_{0\mathrm{c}}$ clock referent time and Δt_{r} is relativistic correction calculated after modified equation (13)

$$\Delta t_{\mathrm{r}} = F\, e \sqrt{A} \sin E_k \ . \quad (17)$$

Where (e, $\sqrt{A}$, E_k) are orbit parameters of the satellite distributed in the navigational message, F is constant calculated by

$$F = \frac{-2\sqrt{\mu}}{c^2} = -4{,}442\,807\,633 \times 10^{-10},\ \frac{\mathrm{s}}{\sqrt{\mathrm{m}}}. \quad (18)$$

Where μ = 3,986 005×10^{-14} m^3/s^2 is Earth's universal gravitational parameter and c speed of light.

GPS navigational control segment is using an alternative form of the equation (17), but it is equivalent mathematical expression, [22]:

$$\Delta t_{\mathrm{r}} = -\frac{2\,\vec{R}\cdot\vec{V}}{c^2},\ \frac{\mathrm{m}^3}{\mathrm{s}^2}\ . \quad (19)$$

Where $\vec{R}$ is vector of the position of the GPS satellite, $\vec{V}$ vector of the velocity of the GPS satellite and c speed of the light.

GPS satellites distribute satellite ephemeris in an almanac. Almanac time is not corrected for relativistic effects because the periodic relativistic effect is less than 25 meters and accuracy of the almanac time is less than 135 meters [22].

There are also finer GPS clock corrections that are not considered by the GPS system, but can be modeled during processing of the GPS measurements. The GPS code, phase and Doppler measurements are influenced by the relativistic effects. Mathematical models of the GPS measurements for the code pseudo range R, carrier phase measurements Φ and Doppler measurements D can be written as, [51]:

$$R_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}}) = \rho_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}}) - (\delta t_{\mathrm{r}} - \delta t_{\mathrm{e}})c + \delta_{\mathrm{ion}} +$$

$$+ \delta_{\mathrm{trop}} + \delta_{\mathrm{tide}} + \delta_{\mathrm{rel}} + \varepsilon_{\mathrm{c}}\ , \quad (20)$$

$$\lambda\Phi_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}}) = \rho_{\mathrm{i}}^{\mathrm{k}}(t_{r}, t_{\mathrm{e}}) - (\delta t_{\mathrm{r}} - \delta t_{\mathrm{e}})c +$$

$$\lambda N_{\mathrm{i}}^{\mathrm{k}} - \delta_{\mathrm{ion}} + \delta_{\mathrm{trop}} + \delta_{\mathrm{tide}} + \delta_{\mathrm{rel}} + \varepsilon_{\mathrm{p}}\ , \quad (21)$$

$$D = \frac{\mathrm{d}\rho_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}})}{\lambda \mathrm{d}t} - f\,\frac{\mathrm{d}(\delta t_{\mathrm{r}}, \delta t_{\mathrm{e}})}{\mathrm{d}t} + \delta_{\mathrm{rel}_f} + \varepsilon_{\mathrm{d}}. \quad (22)$$

Where $R_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}})$ is pseudo range considering the time of the receiver t_{r} and time of the satellite t_{e}, $\rho_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}})$ geometric distance between satellite antenna and receiver antenna, δt_{r} receiver clock error, δt_{e} satellite clock error, c speed of the light, δ_{ion} ionospheric effect, δ_{trop} tropospheric effect, δ_{tide} Earth tides effect and ocean loading effect, δ_{rel} relativistic effects, ε_{c} remaining errors, $\lambda\Phi_{\mathrm{i}}^{\mathrm{k}}(t_{\mathrm{r}}, t_{\mathrm{e}})$ observed phase, λ wavelength, $N_{\mathrm{i}}^{\mathrm{k}}$ phase ambiguity between satellite k and receiver i, ε_p remaining errors, D Doppler measurements, f frequency, δ_{rel_f} relativistic effects and ε_{d} remaining errors.

Relativistic corrections for code, phase and Doppler measurements can be modeled in different ways and the modeling depends on the used software and the required level of accuracy.

9 Relativistic effects on GLONASS

GLONASS system has a more complex time/frequency standard than GPS [23, 24]. Each GLONASS satellite defines its own time/frequency standard and each satellite uses a different frequency for the L1 and L2 carrier waves. GLONASS satellites have a nominal frequency, measured at the Earth's physical surface, of 5,0 MHz. Like the GPS system the GLONASS system reduces nominal frequency for relativistic effects. Nominal frequency of the GLONASS satellites is reduced by –2,18×10^{-3} Hz and is 4,999 999 997 82 MHz. This value is given for a nominal height of 19100 km orbit. When the time/frequency standards are implemented in the GLONASS satellite, relative frequency differences of the carrier wave of amount ±2 ×10^{-11} are used.

10 Relativistic effects on Galileo

In the earlier Galileo documents it was stated that the Galileo satellite time does not include correction for relativistic effects, and the relativistic effects should be the responsibility of the user [13, 14, 5, 37].

But, in the latest Galileo Open Service Signal in the Space Interface Control document [17] is published correction for satellite time with relativistic correction. It has the same polynomial form as the GPS satellite time correction with included relativistic correction. The relativistic correction is the same as for GPS (17), but it uses the Galileo orbit parameters, and because of other Earth parameters it uses different constant value of F = –4,442 807 309×10^{-10} (s/$\sqrt{\mathrm{m}}$).

Using this relativistic correction the Galileo satellite time will not run rapidly away from International Atomic Time (TAI) and will not require large correction terms to be transmitted to users as it was previously planned.

Galileo has high clock standards. It will use the Rubidium Atomic Frequency Standard (RAFS) and the Passive Hydrogen Maser (PHM) as the baseline for clock technologies. A ground test indicates daily clock stability of 2×10^{-15} [10]. Not using other relativistic corrections will reduce effects of high Galileo clock performances.

11 BeiDou Navigation Satellite System

BeiDou is the Chinese GNSS [6, 8, 52]. Regional service is planned to be operational until 2012 and global service until 2020. BeiDou will consist of 5 geostationary satellites and 30 non geostationary satellites. Open and authorized services are planned. Open service will provide positioning accuracy better than 10 m, timing accuracy better than 20 ns and velocity accuracy better than 0,2 m/s. The difference between BeiDou time and UTC is less than 50 ns. Coordinate framework is the China Geodetic Coordinate System 2000. It is consistent with ITRF at 5 cm level.

The literature about relativistic effects is not widely available, but to reach these accuracy relativistic effects should be taken in the accounts. The Sagnac effect can amount to several hundred nanoseconds [5].

12 Numerical relativistic effects on GNSS

The observation of the relativistic effects depends on the characteristics of each satellite orbit. To illustrate aproximative magnitudes of relativistic effects, in Tab. 3 are compared relativistic effects of the GPS, GLONASS and Galileo satellites [34]. Following constants are used: speed of the light 299 792 458 m/s, gravitational constant of the Earth 398 600,44 km^3/s^2, Earth radius 6378,137 km, Earth's oblateness coefficient (J_2) 0,001 082 6, angular velocity of Earth rotation 7,292×10^{-5} rad/s, geopotential on the geoid 6,264×10^7 m^2/s^2 and $U_0/c^2 = -6{,}969\times10^{-10}$.

Table 2 Satellite orbits elements [34]

Orbits elements /Satellite	GPS	GLONASS	Galileo
Semi-major axis / km	26 561,8	25 510	29 994
Eccentricity	0,02	0,02	0,02
Inclination / °	55,0	64,8	56,0
Argument of perigee / °	0	0	0
Apogee altitude / km	20 715	19 642	24 216
Perigee altitude / km	19 652	18 622	23 016
Ascending node altitude / km	19 652	18 622	23 016
Period of revolution / s	43 082	40 549	51 697
Mean motion / revolution/day	2,0	2,1	1,7
Mean velocity / km/s	3,874	3,953	3,645

Table 3 Relativistic effects on the GPS, GLONASS and Galileo [34]

Relativistic effect / Satellite	GPS	GLONASS	Galileo
Time dilation / μs/d	–7,1	–7,4	–6,3
Red shift / μs/d	45,7	45,1	47,3
Effect due to orbit eccentricity / ns	46	45	49
Maximum Sagnac effect / ns	136	131	155
Amplitude of periodic tidal effect of the Moon / ps	1,2	1,0	1,8
Amplitude of periodic tidal effect of the Sun / ps	0,5	0,5	0,8

13 GNSS Augmentation Systems

GNSS can fail in more ways. The GNSS uses weak, easily disturbed, signal. Also the technology failure of even few seconds, as in the aviation transport, can be critical. These GNSS disadvantages are compensated by developing the GNSS augmentations. They are transmitting additional satellite signals that increase reliability of the navigation.

There are the U.S. Wide-Area Augmentation System (WAAS), European Geostationary Navigation Overlay System (EGNOS), Japanese Quasi-Zenith Satellite System (QZSS) and Indian Regional Navigation Satellite System (IRNSS). Relativistic corrections of each of the augmentations are sporadically documented.

WAAS improves navigation over the U.S.A. The WAAS is using 24 ground receivers that continuously monitor signals from GPS satellites and compute corrections that are uploaded to the Geostationary Earth Orbit (GEO) satellites for retransmission to the WAAS users. Time of GEO satellites is corrected for relativistic effects by the ground stations and should not be corrected by users [48].

LAAS is a ground-based augmentation system on a local area (e.g. an airport area). It is using pseudolites; emitters of satellite like local signals. Because of the local area no relativistic corrections are used.

Until Galileo operability, EGNOS is developing on GPS. EGNOS has developed similarly to WAAS to improve navigation over Europe [18].

QZSS is a satellite-based system consisting of three satellites in geosynchronous orbits. An orbit has semi-major axes of 42 164 km and large eccentricity 0,1. For atomic clocks in such satellites, relativistic effects would cause a fractional frequency shift of about –5,39×10^{-10}. The eccentricity effect is much larger than in GPS because the semi major axis and the eccentricity are larger than in GPS. The eccentricity effect has amplitude of about 290 ns [5].

IRNSS will use satellite constellation made of a geostationary Earth orbit (GEO) and the geosynchronous orbit (GSO) satellites over the Indian region [27]. Considering current standards of the GNSS accuracy for this kind of satellites constellation, relativistic effects should be taken into consideration.

Russia has developed a Wide Area Augmentation System (SDCM). It is at an early stage of development [18].

The problem of connecting augmentation systems worldwide is the problem of interoperability of the Space-Based Augmentation Systems (SBAS) [18]. The SBAS interoperability is not at the level to consider global, standardized use of relativistic corrections.

14 Conclusions

Relativistic effects in the satellite navigation are significant and no accurate positioning is possible if relativistic effects are not used. The GNSS relativity problem is treated as a correction of Newtonian approach and the post-Newtonian parameterization is used.

Relativistic effects depend on each orbit constellation and the GPS, GLONASS and Galileo have similar values of relativistic effects. The GNSS augmentation systems should consider relativistic corrections to reach today's navigation standards.

In the future development in the field of keeping accurate time scales, time transfers, theoretical development and that of the models, satellite missions in the field of relativity and clock synchronizations are only some of the state-of-the-art activities that are influencing relativistic approach of satellite navigation.

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