Kiril Arnaudov
Dimitar Karaivanov

ISSN 1333-1124

THE TORQUE METHOD USED FOR STUDYING COUPLED TWO-CARRIER PLANETARY GEAR TRAINS

UDC 621.833.6

Summary

Using the torque method, one can determine not only the speed ratios in complex compound planetary gear trains, but also the magnitude and direction of internal power flows and thus the efficiency. A brief description of the torque method is given in this paper. As an example, a two-carrier compound planetary gear train connected in series was studied. Some more specific and difficult cases of application of this clear and simple method are reviewed – a planetary gear train with two degrees of freedom and a self-locking planetary gear train.

Key words: *torque method, planetary gear train, speed ratio, efficiency*

1. Introduction

Planetary gear trains are complex technical devices and comprise a wide technical field [1-4]. Common methods for simple planetary gear train analysis (e.g. Willis, Kutzbach, Swamp, etc.) [1, 2, 5]) are either difficult or impossible to apply to complex compound planetary trains. In such cases, the analysis can be performed by means of known formulae, tables and diagrams [2, 4]. Although they do accomplish given objectives, this method of work has some disadvantages:

1. The designer is dependent on a particular literary source and cannot act autonomously;
2. The designer acts mechanically, even blindly, which is not appropriate;
3. The method lacks any visibility. Clarity and visibility are dominant in the thoughts and endeavours of the engineer. And very important to him.

These issues create a clear need for another method, one that overcomes the aforementioned disadvantages and allows the designer to achieve more goals.

The torque method is **extremely clear, easy to understand and easy to apply.** Using it, more than one objective could be achieved:

1. Determination of the speed ratio as with the methods of Willis, Kutzbach, Swamp, but also
2. clarification of whether the internal power division or circulation initially exist. Generally, determination of the magnitude and direction of internal power flows
3. and thus determination of the efficiency.

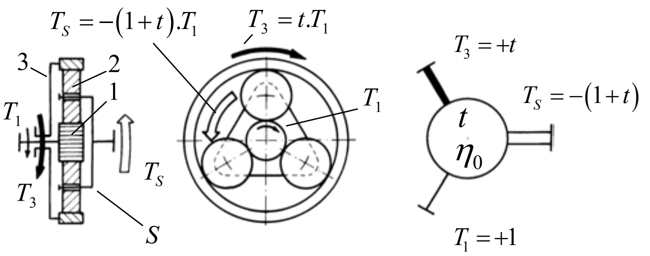
There are studies [6 and 7] where the torque is used for the power analysis using the fundamental circuits, but the kinematic analysis is performed by using the Willis method.

The torque method is developed in a number of studies [8 - 13], and considered as the two-carrier [8 - 10], as the three- and the four-carrier compound planetary gear trains [11 - 13]. It is also based on the lever analogy of three-shaft planetary gear trains – simple and compound [8]. The main objective is to review a more specific and difficult case not yet studied in the previous publications [8 - 13] which illustrate the application of this simple, clear and easy-to-apply method.

2. Torque method

2.1 Essence of the torque method

This method is based on the fact that the torques of the three external shafts of a simple single-carrier planetary gear train (Fig. 1) are in a constant ratio in stationary mode, regardless of the operating mode of the gear train (with one or two degrees of freedom, as a reducer or multiplier, as a standalone train or a part of a compound train). For the purpose of clarity, the single-carrier planetary train (Fig. 1) is represented with the symbol of Wolf [14] but modified according to [8 - 13], the three external shafts being denoted with lines of different width corresponding to the magnitude and direction of their torques (T_1 and T_3 are in the same direction, opposite to T_S). In Fig.1 the three ideal (with no account of losses) torques (T_1 of the sun gear, T_3 of the ring gear, and T_S of the carrier) are expressed with the torque ratio t [8 and 9] defined by the authors.



Prerequisite: $\eta_0 = \eta_{13(S)} = \eta_{31(S)} = 1$;

Torque ratio: $t = \dfrac{T_3}{T_1} = \left|\dfrac{z_3}{z_1}\right| > +1$

Torques: $T_1 : T_3 : T_S = T_1 : t{\cdot}T_1 : -(1+t)T_1 = +1 : +t : -(1+t)$

Fig. 1 The most common single-carrier planetary gear train and its torques

The ideal (T_1, T_3, and T_S) as well as the real (T_1', T_3', and T_S') torques are in equilibrium in stationary mode

$$\sum T_i = T_1 + T_3 + T_S = 0\,;\ \sum T_i' = T_1' + T_3' + T_S' = 0 \tag{1}$$

The ideal input torque T_A and output torque T_B are used [1 and 2] for the determination of the speed ratio i of the gear train

$$i = \frac{\omega_A}{\omega_B} = -\frac{T_B}{T_A} \tag{2}$$

where ω_A and ω_B are the input and output angular velocities. The real input torque T_A' and the output torque T_B' are used first for the determination of the torque transformation μ and then the efficiency η [1 and 5]

$$\mu = \frac{T_B'}{T_A'} \tag{3}$$

$$\eta = -\frac{\mu}{i} \tag{4}$$

In this paper, the method is illustrated on coupled two-carrier planetary gear trains (Figs 2, 3, and the rest), also referred to as elementary compound planetary gear trains [2, 3], but the same procedure can be applied to higher compound planetary trains which are multi-carrier planetary gear trains (three-, four-, etc.) [13].

2.2 Sequencing in the application of the method

The following are determined:

1) Torque ratios tI and tII of the component gear trains I and II

$$t_I = \frac{T_3}{T_1} = \left|\frac{z_3}{z_1}\right| > 1; \qquad t_{II} = \frac{T_6}{T_4} = \left|\frac{z_6}{z_4}\right| > 1 \tag{5}$$

where z is the teeth number of the gears.

2) Basic (internal) efficiencies η_{0I} and η_{0II} of the component gear trains I and II

The simplest formula according to [15] is as follows (ζ_0 - basic coefficient of losses)

$$\begin{aligned}\eta_{0I} &= 1-\zeta_{0I} = 1-\left[0.15\left(\frac{1}{z_1}+\frac{1}{z_2}\right)+0.2\left(\frac{1}{z_2}-\frac{1}{|z_3|}\right)\right]\\ \eta_{0II} &= 1-\zeta_{0II} = 1-\left[0.15\left(\frac{1}{z_4}+\frac{1}{z_5}\right)+0.2\left(\frac{1}{z_5}-\frac{1}{|z_6|}\right)\right]\end{aligned} \tag{6}$$

If a more accurate calculation is required, sources [16 through 18] should be used.

3) Ideal torques T_i

The ideal torques are calculated serially (following the relations in Fig.1) in a determined order where it is appropriate to begin with a sun gear shaft, preferably with a value of +1 (but not mandatory) [13]. The torques at the two ends of the internal compound shaft (Fig. 2) are equal by the absolute value, but with opposite signs. The torque of the external (outer) shaft is obtained as a sum of the torques of its component shafts. The sequence for the determination of ideal torques is shown in Fig.3 with the numbers in circles.

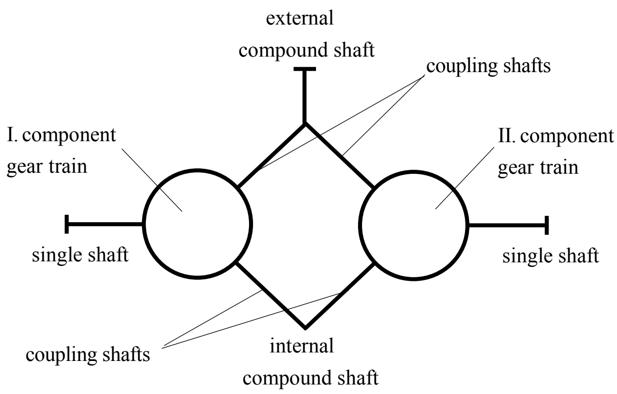


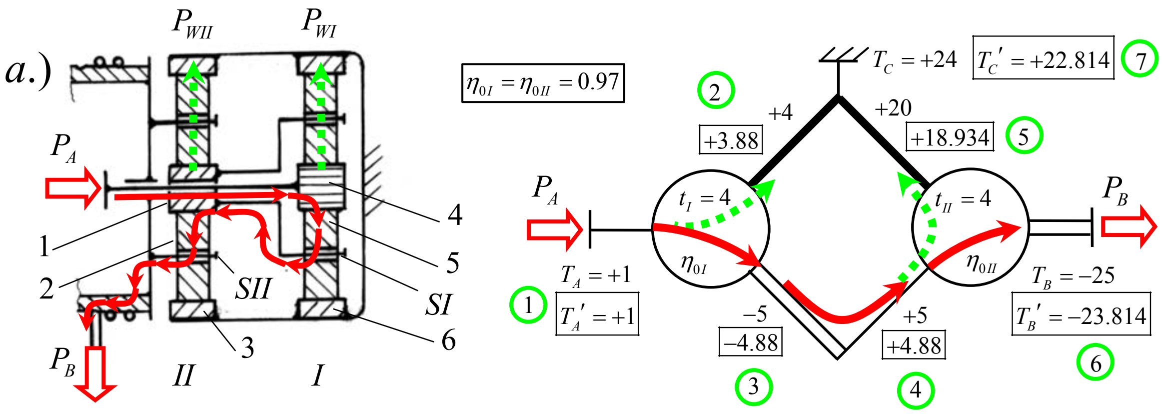
Fig. 2 Coupled two-carrier planetary gear train and the names of its shafts

4) Verification of the ideal torques T_i

If the calculations are correct, the three outer ideal torques, i.e. the input torque T_A, the output torque T_B, and the reaction torque T_C, should be in equilibrium in stationary mode, i.e.

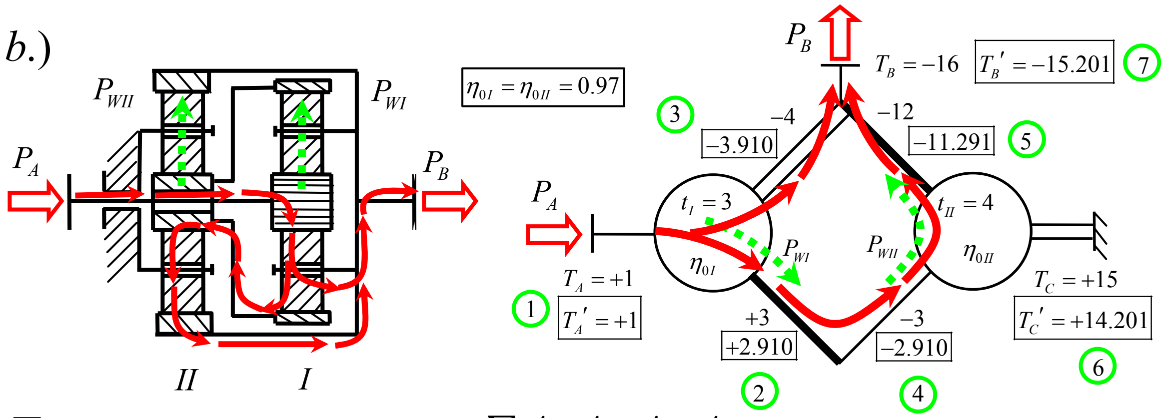
$$\sum T_i = 0\,;\; T_A + T_B + T_C = 0 \tag{7}$$

Otherwise, the calculations are incorrect.



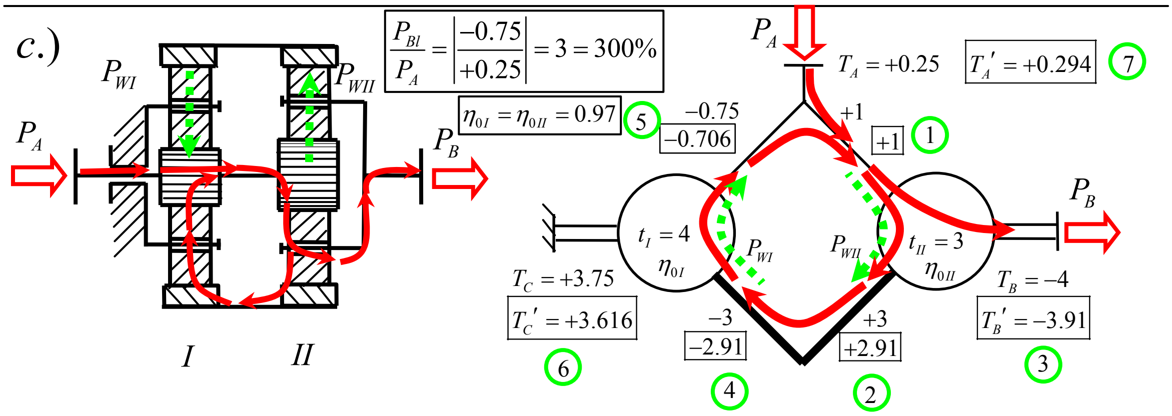
$$\sum T_i = T_A + T_B + T_C = +1 - 25 + 24 = 0; \qquad \sum T_i' = T_A' + T_B' + T_C' = +1 - 23.814 + 22.814 = 0$$

$$i = -\frac{T_B}{T_A} = -\frac{-25}{+1} = +25; \quad \mu = \frac{T_B'}{T_A'} = \frac{-23.814}{+1} = -23.814; \quad \eta = -\frac{\mu}{i} = -\frac{-23.814}{+25} = 0.9526 = 95.26\%$$



$$\sum T_i = T_A + T_B + T_C = +1 - 16 + 15 = 0; \qquad \sum T_i' = T_A' + T_B' + T_C' = +1 - 15.201 + 14.201 = 0$$

$$i = -\frac{T_B}{T_A} = -\frac{-16}{+1} = +16; \quad \mu = \frac{T_B'}{T_A'} = \frac{-15.201}{+1} = -15.201; \quad \eta = -\frac{\mu}{i} = -\frac{-15.201}{+16} = 0.950 = 95\%$$



$$\sum T_i = T_A + T_B + T_C = +0.25 - 4 + 3.75 = 0; \qquad \sum T_i' = T_A' + T_B' + T_C' = +0.294 - 3.91 + 3.616 = 0$$

$$i = -\frac{T_B}{T_A} = -\frac{-4}{+0.25} = +16; \quad \mu = \frac{T_B'}{T_A'} = \frac{-3.91}{+0.294} = -13.3; \quad \eta = -\frac{\mu}{i} = -\frac{-13.3}{+16} = 0.8313 = 83.13\%$$

Fig. 3 Coupled two-carrier planetary gear trains – torques, speed ratio, efficiency

5) Speed ratio i

Determined by (2)

6) Internal power flows, absolute and rolling (relative) power, internal power division and power circulation

This is a delicate problem and sometimes not easy to understand, especially in complex compound multi-carrier planetary trains. It is necessary to distinguish between the absolute and the relative (rolling) power. The latter is related to the losses due to the relative rotation of the gears to the corresponding carrier. Most often the meshing losses are dominating, with exceptions like the turbine-trains, which have sliding (plain) bearings. It is necessary to determine the direction of the rolling powers P_{WI} and P_{WII} in each of the component trains I and II in order to calculate the real torques T_1', T_3', and T_{SI}' (as well as T_4', T_6', and T_{SII}') and thus the efficiency η of the gear train. In some simple cases the directions are determined easily.

However, the **general approach** is as follows:

When the directions of the torque T_1 and the relative angular velocities $\omega_{1rel} = \omega_1 - \omega_{SI}$ of the sun gears 1 (relative to the carriers SI) are the same, i.e. when

$T_1 > 0$ and $\omega_{1rel} > 0$,

meaning that

$T_1 \cdot \omega_{1rel} > 0$,

the rolling power P_{WI} is transmitted by the sun gear 1 through the planets 2 of the ring gear 3.

Otherwise, when

$T_1 \cdot \omega_{1rel} < 0$,

the rolling power P_{WI} is transmitted in the opposite direction. The directions of the two rolling powers P_{WI} and P_{WII} in the two component trains I and II are illustrated in Fig. 3.

If the directions of the rolling powers are not that easy to determine, there is another possible method – the method of trial [19] demonstrated in section 3.2.

As for the absolute power, in gear trains with a fixed external compound shaft, its direction is obvious (Fig. 3a). When the external compound shaft is not fixed, there are two possible cases:

- internal power division (Fig. 3b);
- internal power circulation (Fig. 3c);

The particular case can be easily determined by the **rule of algebraic signs** [10] (Fig. 3):

- if the algebraic signs of the torques of the two coupling shafts are the same, there is an internal power division (Fig. 3b);
- if the signs are different, there is an internal power circulation (Fig. 3c).

The direction of the circulating power is the same as the direction of the input P_A or output P_B power (acting on the compound shaft) in the coupling shaft, the sign of which is the same as the sign of the corresponding input P_A or output P_B power. This is clearly demonstrated in Fig. 3c. It is also obvious that the circulating power P_{BI} is three times the input power P_A.

7) Real torques T_i'

Their determination is done by the same sequence used for the ideal torques with regard to the direction of transmission of rolling powers P_{WI} and P_{WII} in each of the component trains I and II as well as the corresponding basic efficiencies η_{0I} and η_{0II}. If the calculations (e.g. for the first component train I) are based on the real torque T_1' of the sun gear 1, there are two options for the torque T_3' of the ring gear 3:

- $T_3' = t_I \cdot T_1' \cdot \eta_{0I}$ when the rolling power P_{WI}' is transmitted from the sun gear 1 to the ring gear 3;
- $T_3' = t_I \cdot T_1' / \eta_{0I}$ when P_{WI}' is transmitted from the ring gear 3 to the sun gear 1;

This applies to the second component train II as well.
In the figures, the real torques are denoted in rectangles.

8) Verification of the real torques T_i'

The same as with ideal torques, the real torques must also be in equilibrium, i.e.

$$\sum T_i' = 0\,;\; T_A' + T_B' + T_C' = 0 \tag{8}$$

if the calculations are correct.

9) Transformation of the torques μ and efficiencies η

They are determined by the real input torque T_A' and the output torque T_B' according to (3) and (4).

Two more specific cases of torque method application are reviewed in this paper:

– a gear train with two degrees of freedom operating with power summation as a „summation train" in case of two-motor driving;
– a self-locking train consisting of two non-locking component trains;

3. Study of a coupled two-carrier planetary gear train with two degrees of freedom

3.1 Determination of angular velocities and power flows

Fig. 4 shows a gear train with two degrees of freedom which operates as a summation train with internal power division.

With given torque ratios $t_{\mathrm{I}} = 3.5$ and $t_{\mathrm{II}} = 2.818$ of the two component trains I and II and angular velocities of the input shafts $\omega_{\mathrm{AI}} = 25\,s^{-1}$ and $\omega_{\mathrm{AII}} = -12\,s^{-1}$, the ideal torques and angular velocities can be determined for all the shafts of the coupled train.

The sequence for the determination of torques is also shown with encircled numbers (Fig. 4).

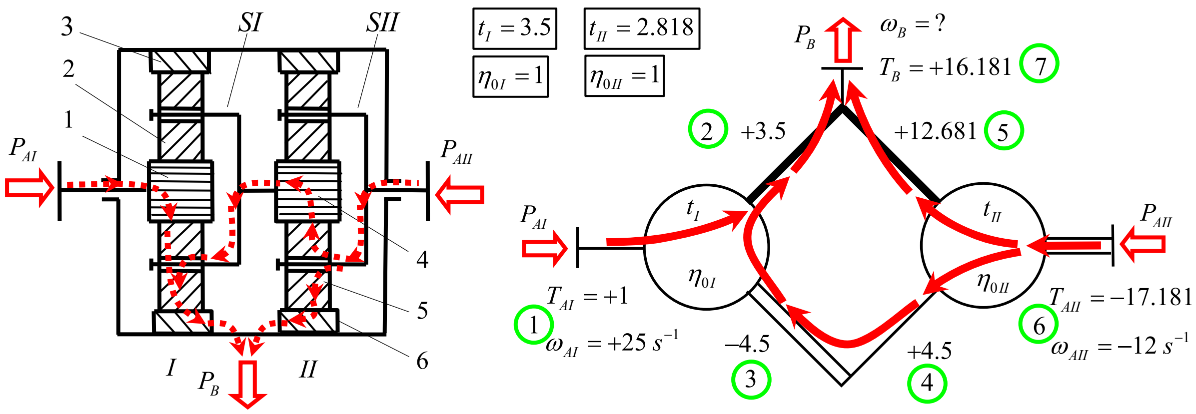


Fig. 4 Coupled two-carrier planetary gear train with two degrees of freedom and its ideal torques

After assuming that $T_{\mathrm{AI}} = +1$ and having in mind the equilibrium of power flows in the coupled train

$$\sum P_i = P_{AI} + P_{AII} + P_B = T_{AI} \cdot \omega_{AI} + T_{AII} \cdot \omega_{AII} + T_B \cdot \omega_B =$$
$$= 1 \cdot 25 + (-17.181)(-12) + 16.181\omega_B = 0$$

one can determine the output angular velocity

$$\omega_B = \omega_3 = \omega_6 = -\frac{25 + 206.172}{16.181} = -14.29\,s^{-1}.$$

From the equilibrium of power flow in the component train I:

$$\sum P_i = P_1 + P_3 + P_{SI} = T_1 \cdot \omega_1 + T_3 \cdot \omega_3 + T_{SI} \cdot \omega_{SI} = \\ = 1 \cdot 25 + 3.5(-14.29) + (-4.5)\,\omega_{SI} = 0$$

the angular velocity of the internal compound shaft $\omega_{SI} = \omega_4 = -\dfrac{25-50.015}{4.5} = -5.559\ s^{-1}$ can be determined.

This angular velocity is necessary for the determination of relative angular velocities of the gear wheels. The relative angular velocities are expressed as:

$$\omega_{1rel} = \omega_1 - \omega_{SI} = +25 - (-5.559) = +30.559\ s^{-1}$$

$$\omega_{2rel} = \omega_{1rel} \frac{z_1}{z_2}$$

$$\omega_{3rel} = \omega_3 - \omega_{SI} = -14.29 - (-5.559) = -8.731\ s^{-1}$$

$$\omega_{4rel} = \omega_4 - \omega_{SII} = -5.559 - (-12) = +6.441\ s^{-1}$$

$$\omega_{5rel} = \omega_{4rel} \frac{z_4}{z_5}$$

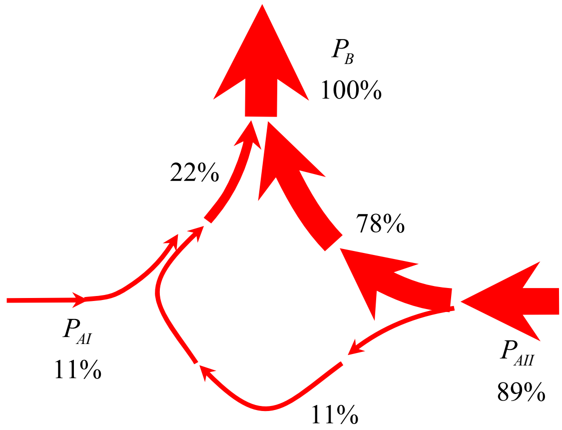


Fig. 5 Power flows in the gear train from Fig. 4

$$\omega_{6rel} = \omega_6 - \omega_{SII} = \omega_B - \omega_{AII} = -14.29 - (-12) = +2.29\ s^{-1}$$

The directions of the torques and angular velocities determined this way indicate that the train has internal power division. The magnitudes of the two power flows are shown in Fig. 5.

3.2 Efficiency determination

Proper determination of the real torques (and thus the efficiency (4) of the coupled train) requires the determination of the directions of the rolling powers P_{WI} and P_{WII} in the component trains I and II.

Fig. 6 shows this determination (for the train from Fig. 4) by means of the trial method (the aim is to illustrate a method different from the one described above).

When determining the direction of the rolling power P_{WI} of the first component train I, one must choose a basic efficiency with a lower value (for method effectiveness and result clarity), for example $\eta_{0I} = 0.8$, while the efficiency of the other train is chosen as $\eta_{0II} = 1$.

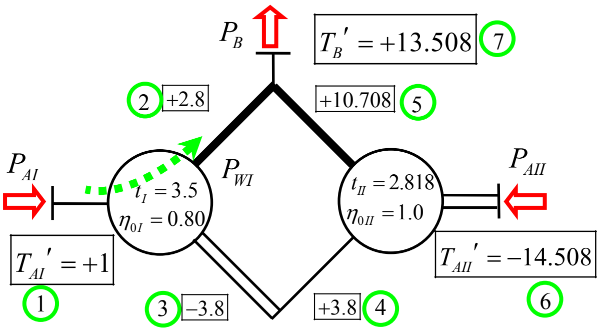
The total efficiency η of the coupled train is calculated with the two possible directions of the rolling power P_{WI} of the first component train I. The direction which gives a realistic value ($\eta < 1$) is chosen. The same method is applied to the second component train II.

Fig. 6 shows that in both component trains I and II, the rolling powers flow from the sun gears 1 and 4 to the respective ring gears 3 and 6.

The real torques at given basic efficiencies of the component trains $\eta_{0I} = \eta_{0II} = 0.97$ (a realistic value) are determined in Fig. 7. The efficiency of the coupled gear train is:

$$\eta = \frac{P_B'}{P_{AI}' + P_{AII}'} = \frac{T_B' \cdot \omega_B}{T_{AI}' \cdot \omega_{AI} + T_{AII}' \cdot \omega_{AII}} = \\ = \frac{15.409(-14.29)}{1 \cdot 25 + (-16.409)(-12)} = \frac{220.195}{221.908} = 0.9923 \tag{9}$$

Trial of the component train I

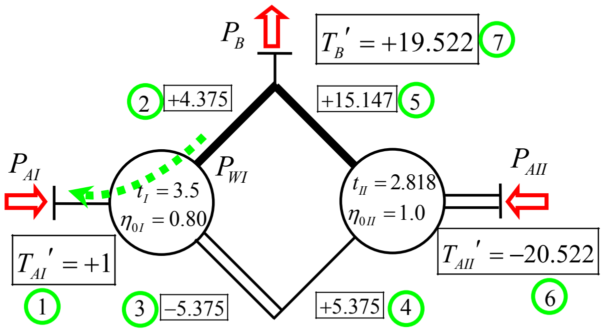


$$\sum T_i' = T_{AI}' + T_{AII}' + T_B' = +1 - 14.508 + 13.508 = 0$$

$$\eta = -\frac{P_B'}{P_{AI}' + P_{AII}'} = -\frac{T_B' \cdot \omega_B}{T_{AI}' \cdot \omega_{AI} + T_{AII}' \cdot \omega_{AII}} =$$

$$= -\frac{13.508(-14.29)}{1 \cdot 25 + (-14.508)(-12)} = +0.9695 < 1$$

Right direction of P_{WI} !



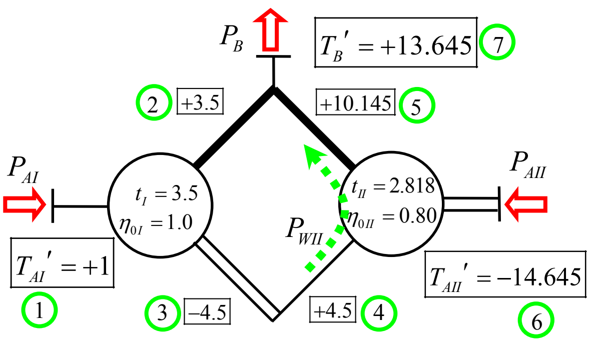
$$\sum T_i' = T_{AI}' + T_{AII}' + T_B' = +1 - 20.522 + 19.522 = 0$$

$$\eta = -\frac{P_B'}{P_{AI}' + P_{AII}'} = -\frac{T_B' \cdot \omega_B}{T_{AI}' \cdot \omega_{AI} + T_{AII}' \cdot \omega_{AII}} =$$

$$= -\frac{19.522(-14.29)}{1 \cdot 25 + (-20.522)(-12)} = +1.028 > 1$$

Wrong direction of P_{WI} !

Trial of the component train II

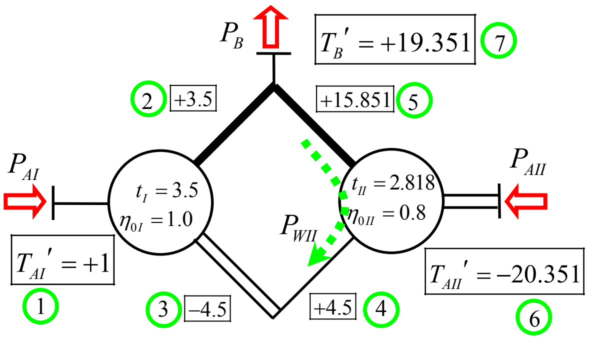


$$\sum T_i' = T_{AI}' + T_{AII}' + T_B' = +1 - 14.645 + 13.645 = 0$$

$$\eta = -\frac{P_B'}{P_{AI}' + P_{AII}'} = -\frac{T_B' \cdot \omega_B}{T_{AI}' \cdot \omega_{AI} + T_{AII}' \cdot \omega_{AII}} =$$

$$= -\frac{13.645(-14.29)}{1 \cdot 25 + (-14.645)(-12)} = +0.971 < 1$$

Right direction of P_{WII} !



$$\sum T_i' = T_{AI}' + T_{AII}' + T_B' = +1 - 20.351 + 19.351 = 0$$

$$\eta = -\frac{P_B'}{P_{AI}' + P_{AII}'} = -\frac{T_B' \cdot \omega_B}{T_{AI}' \cdot \omega_{AI} + T_{AII}' \cdot \omega_{AII}} =$$

$$= -\frac{19.351(-14.29)}{1 \cdot 25 + (-20.351)(-12)} = +1.027 > 1$$

Wrong direction of P_{WII} !

Fig. 6 Determination of the directions of the rolling powers P_{WI} and P_{WII} in the component trains of coupled planetary gear train from Fig. 4

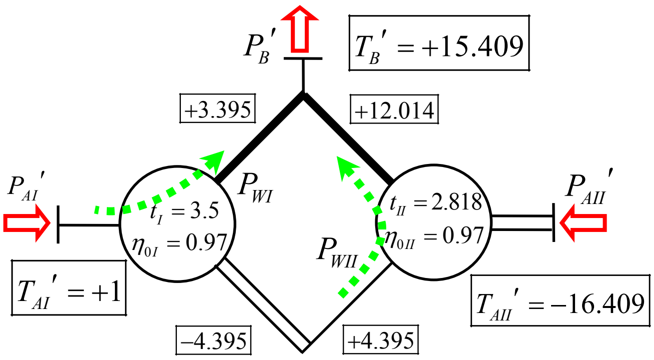


Fig. 7 Real torques in the gear train from Fig. 4

4. Study of a coupled two-carrier planetary gear train with self-locking

4.1 The train operates as a multiplier

Self-locking usually occurs in planetary trains with two external [20 and 21] or two internal [2] meshes, in the so-called "positive" [2 and 3] trains. Fig. 8 shows a coupled gear train with a possibility of self-locking, although it is composed of two simple planetary trains (with external and internal meshes, i.e. the so-called "negative" [2 and 3] trains) which are free from self-locking.

Self-locking occurs when the values of the torque ratios t_I and t_{II} of the component trains I and II are close. The train has internal power circulation and the direction of the circulating power (as well as the rolling powers) changes according to which of the torque ratios t_I and t_{II} is greater. The torque method facilitates the orientation in this complex situation.

Fig. 9 shows the ideal and real (in rectangles) torques and the directions of the power flows when $t_I < t_{II}$. The direction of the internal power for this case is shown in Fig. 8.

Check of the torques (7):

$$\sum T_i = T_A + T_B + T_C = 0\,;\; +4 + 0.2 - 4.2 = 0$$

$$\sum T_i' = T_A' + T_B' + T_C' = 0\,;\; +4.124 - 0.05 - 4.074 = 0$$

Speed ratio (2):

$$i = \frac{\omega_A}{\omega_B} = -\frac{T_B}{T_A} = -\frac{+0.2}{+4} = -0.05 = -\frac{1}{20}$$

shows that the train operates as a multiplier.

The torque transformation

$$\mu = \frac{T_B'}{T_A'} = \frac{-0.05}{+4.124} = -0.0121$$

for the efficiency (4) yields a negative value

$$\eta = -\frac{\mu}{i} = -\frac{-0.0121}{-0.05} = -0.242 = -24.2\% < 0\,,$$

i.e. self-locking occurs.

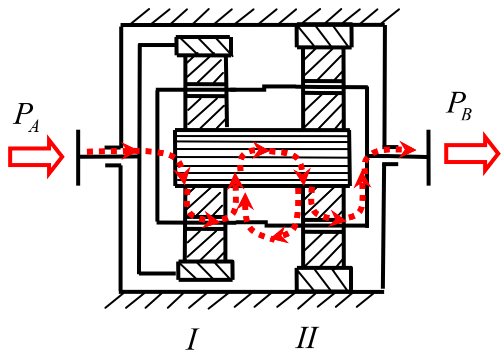


Fig. 8 Coupled two-carrier planetary gear train with self-locking

In this case there is a circulating power P_{BI}. The determination of its magnitude is based on the real torques $T_{\mathrm{S}}' = -5.124$ and $T_{\mathrm{A}}' = +4.124$. The first torque T_{S}' acts on the carrier $\mathrm{S_I}$ of the first component train, with only the circulating power P_{BI} flowing through it (Fig. 9). The second torque T_{A}' acts on the input shaft through which the input power P_{A}' flows. The calculation is also based on the ratio of the angular velocities ω_{SI} and ω_{A} of the corresponding elements. This ratio depends on the speed ratio $i = -\frac{1}{20}$. As the angular velocity ω_{SI} and the angular velocity ω_{B} of the output shaft are identical ($\omega_{\mathrm{SI}} \equiv \omega_{\mathrm{B}}$), one can write:

$$\frac{\omega_{SI}}{\omega_A} = \frac{\omega_B}{\omega_A} = \frac{1}{i} = \frac{1}{-\frac{1}{20}} = -20$$

The same is valid for the ratio of the powers:

$$\frac{P_{BI}}{P_A'} = \frac{T_{SI}' \cdot \omega_{SI}}{T_A' \cdot \omega_A} = \frac{T_{SI}'}{T_A'} \cdot \frac{1}{i} = \frac{-5.124}{+4.124}(-20) \approx 25$$

It is obvious that the circulating power P_{BI} is 25 times the input power P_{A}' (Fig. 9).

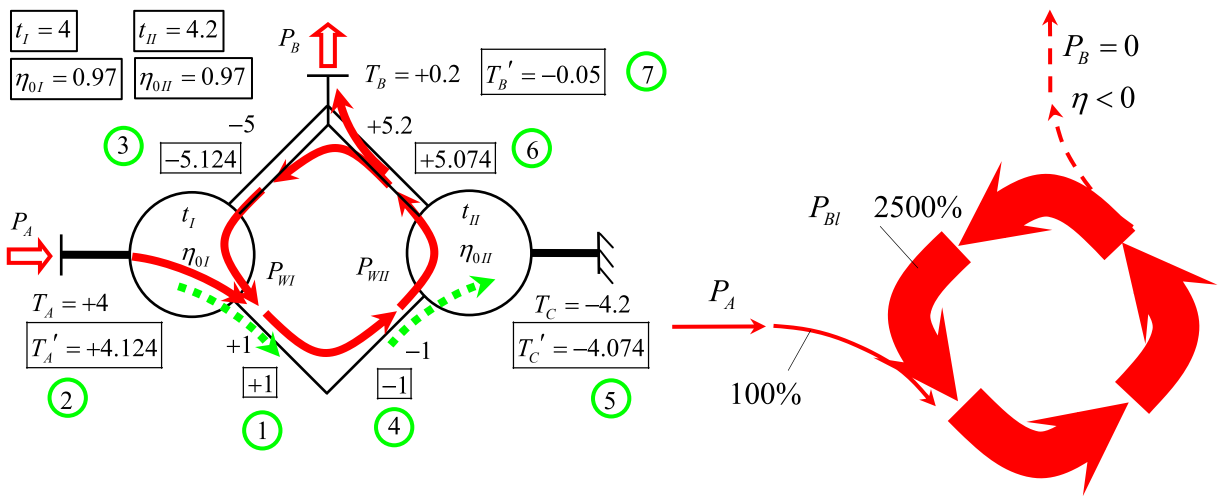


Fig. 9 Torques and power flows of the gear train from Fig. 8 operating as a multiplier

Of course, in the case of a self-locking train which cannot move (as in this case) the term “circulating power” is relative.

4.2 The train operates as a reducer

Fig. 10 shows the determined ideal and real torques as well as the directions of the power flows for the same gear train (Fig. 8) operating as a reducer.

The magnitude of the ideal torques is not changed. The change is in the directions of the power flows, both absolute and rolling, and thus in the magnitudes of the real torques.

Check of the torques (7):

$$\sum T_i = T_A + T_B + T_C = 0\,;\ 0.2 + 4 - 4.2 = 0$$

$$\sum T_i' = T_A' + T_B' + T_C' = 0\,;\ +0.45 + 3.88 - 4.33 = 0$$

Speed ratio (2):

$$i = \frac{\omega_A}{\omega_B} = -\frac{T_B}{T_A} = -\frac{+4}{+0.2} = -20$$

shows that the train operates as a reducer.

The torque transformation

$$\mu = \frac{T_B'}{T_A'} = \frac{+3.88}{+0.45} = +8.622$$

for the efficiency (4) yields a positive value

$$\eta = -\frac{\mu}{i} = -\frac{+8.622}{-20} = +0.431 = 43\% > 0$$

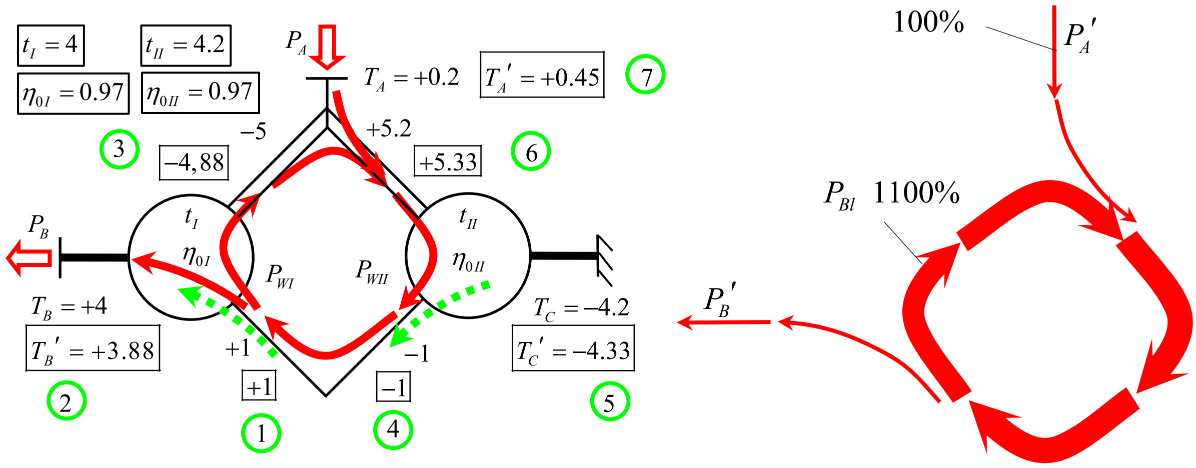


Fig. 10 Torques and power flows of the gear train from Fig. 8 operating as a reducer

Despite being rather low, this efficiency is not negative as in the case of the train operating as a multiplier. Here the ratio of the circulating power P_{BI} to the real input power P_A' is better and it is determined in a way similar to the previous case with the real torque $T_{SI}' = -4.88$ and the real input torque $T_A' = +0.45$ by the following relation (considering $\omega_A \equiv \omega_{SI}$):

$$\frac{P_{BI}}{P_A'} = \frac{T_{SI}' \cdot \omega_{SI}}{T_A' \cdot \omega_A} = \left|\frac{T_{SI}'}{T_A'}\right| = \left|\frac{-4.88}{+0.45}\right| \approx 11$$

It is obvious that when the train operates as a reducer (Fig. 10), the circulating power P_{BI} is much lower than when the train operates as a multiplier (Fig. 9) - about 2 times (25:11). Thus, the losses in the train are smaller, which explains the higher efficiency.

The relations obtained by the torque method allow the designer to achieve his desired variant (by altering the torque ratios t_I and t_{II} of the two component trains):

- high-efficiency reducer and multiplier with no self-locking or
- reducer with lower efficiency and self-locking multiplier (as in the example case).

5. Conclusion

Use of the torque method in the analysis of complex compound multi-carrier planetary gear trains allows the accomplishment of more objectives than the use of other methods:

1. Accomplishment of the initial objectives regarding the speed ratio, internal power flows (division and circulation) and efficiency;
2. The method combines accuracy and clarity which are separately presented in the methods of Willis and Kutzbach;
3. Conditions for equilibrium of the ideal and real torques enable an easy verification of the accuracy of calculations;
4. The method is convenient for the optimization analysis of compound planetary gear trains [22]. By varying t_I and t_{II}, one can look for their best combination for obtaining maximum efficiency, minimal overall dimensions, backlash, etc. Multi-factor optimization can also be pursued [23 and 24];
5. Due to its clarity and easy application, the method is appropriate for industrial application (i.e. for engineers) as well as for educational purposes (for students).

NOMENCLATURE

Symbols:	Subscripts:
i – speed ratio;	1 – sun gear of first component train
P – transmitted power;	2 – planet (satellite) of first component train
P_{Bl} – circulating (blind) power	3 – ring gear of first component train
P_W – rolling (relative) power in the simple planetary train;	4 – sun gear of second component train
T – ideal torque;	5 – planet (satellite) of second component train
T' – real torque;	6 – ring gear of second component train
t – torque ratio;	I – first component train
z – number of teeth;	II – second component train
η – efficiency;	A – input shaft
η_0 – basic (internal) efficiency of the component simple planetary gear train;	B – output shaft
μ – torque transformation;	C – fixed shaft
ς_0 – coefficient of internal losses of the component simple planetary gear train;	S – carrier
ω – angular velocity;	rel – relative

REFERENCES

[1] Looman, J. Zahnradgetriebe. 3. Auflage. Berlin: Springer Verlag, 1996.

[2] Müller, H. W. Die Umlaufgetriebe. 2. Auflage. Berlin: Springer Verlag, 1998.

[3] Müller, H. W. Epicyclic Drive Trains. Detroit: Wayne State University Press, 1982.

[4] Tkachenko, V. Planetary Mechanisms. Optimal Design, Harkov: HAI, 2003. (in Russian)

[5] VDI-Richtlinie 2157. Planetengetriebe – Begriffe, Symbole, Berechnungsgrundlagen.

[6] Kahraman, A., Ligata, H., Kienzle, K. and Zini, D.M. A Kinematics and Power Flow Analysis Methodology for Automatic Transmission Planetary Gear Trains. In: ASME J. of Mechanical Design, **126** (2004), pp. 1071-1081.

[7] Esmail, E. L. and Hassan, S. S. An Approach to Power-Flow and Static Force Analysis in Multi-Input Multi-Output Epicyclic-Type Transmission Trains. In: ASME J. of Mechanical Design, **132** (2010).

[8] Arnaudov, K., Karaivanov, D. Engineering Analysis of the Coupled Two-Carrier Planetary Gearing Through the Lever Analogy. In: Proceedings of the International Conference on Mechanical Transmissions 2001, Chongqing, China, pp. 44-49.

[9] Karaivanov, D., Arnaudow, K. Die Zusammengesetzen Mehrsteg-Planetengetriebe und ihre Gesetzmäßigkeiten. In: Proceedings of the IRMES'2002, 1/2, Srpsko Sarajevo - Jahorina, 19-20 September 2002, pp. 19-26.

[10] Arnaudow, K., Karaivanov, D. Systematik, Eigenschaften und Möglichkeiten von zusammengesetzten Mehrsteg-Planetengetrieben. In: Antriebstechnik (2005) 5, pp. 58-69.

[11] Arnaudov, K., Karaivanov, D. Higher Compound Planetary Gear Trains. IN: VDI-Berichte 1904 (2005) , pp. 327-344.

[12] Karaivanov, D. Theoretical and Experi-mental Studies of the Influence of the Structure of the Coupled Two-Carrier Planetary Gear Trains on Its Basic Parameters. Dissertation. Sofia, 2000. (in Bulgarian)

[13] Arnaudov, K., Karaivanov, D. The Complex Compound Multi-Carrier Planetary Gear Trains – a Simple Study. In: VDI-Berichte 2108 – 2 (2010), pp. 673-684.

[14] Wolf, A. Die Grundgesetze der Umlaufgetriebe. Braunschweig: Friedr. Vieweg Sohn, 1958.

[15] Förster, H. J. Zur Berechnung des Wirkungsgrades von Planetengetrieben, In: Konstruktion (1969) 5, pp. 165-178.

[16] Niemann, G., Winter, H., 1995, Maschinenele-mente. Bd. 2. Zahnradgetriebe. 2. Auflage. Berlin: Springer Verlag, 1995.

[17] Pedrero, J. I., Pleguezuelos, M., Muños, M. Simplified Calculation Method for the Efficiency of Involute Spur Gears. In: Proceedings of the ASME Int. Power Transmissions and Gearing Conf. (PTG), August 30 – September 2, 2009, San Diego, USA. (Paper DETC2009-87179)

[18] Schoo, A. Verzahnungsverlustleistungen in Planetenradgetrieben. Dissertation. Institut für Konstruktionstechnik der Ruhr-Universität, Bochum, 1985.

[19] Seeliger, K. Das einfache Planetengetriebe. In: Antriebstechnik (1964) pp. 216-221.

[20] Ikejo K., Nagamura K., Yada T, Kagary Y. Self-Locking of 2F-C Type Planetary Gear Train Composed of External Gears. In: Proceedings of the ASME Int. Power Transmissions and Gearing Conf. (PTG), August 30 – September 2, 2009, San Diego, USA. (Paper DETC2009-86291)

[21] Karaivanov, D., Sanjin, T., Pavlova, R. Investigation into Self-Looking Planetary Gear Trains through the Lever Analogy. In: Transactions of FAMENA, **36** (2012), No. 1, pp. 13-24.

[22] Karaivanov, D. Structural Analysis of Coupled Multi-Carrier Planetary Gear Trains – from Lever Analogy to Multi-Objective Optimization. In: Proceedings of the 3rd Int. Conf. on Manufacturing Engineering (ICMEN), 1-3 October 2008, Chalkidiki, Greece, pp. 579 – 588.

[23] Troha, S., Petrov, P., Karaivanov, D. Regarding the Optimization of Coupled Two-Carrier Planetary Gears With Two Coupled and Four External Shafts. In: Machinebuilding and Electrical Engineering (Sofia), 2009 (LVIII), № 1, pp. 49-55.

[24] Stefanović-Marinović, J., Milovančević, M. An Application of Multicriteria Optimization to Planetary Gear Transmissions. In: Proceedings of the International Conference "Mechanical Engineering in XXI Century", 25-28 November 2010, Niš, Serbia, pp. 133-136.

Submitted: 19.6.2012

Accepted: 22.02.2013

Kiril Arnaudov
Bulgarian Academy of Sciences, Sofia, Bulgaria
k_arnaudov@abv.bg

Dimitar Karaivanov
Univ. of Chem.. Techn. and Metallurgy, Sofia, Bulgaria
dipekabg@yahoo.com