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Horopters – Definition and Construction

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ABSTRACT

The feature of Horopter was studied allready since the arabic and persian school, where Aguilonius defined it in 1613 for the first time. From those times til now, horopter was investigated as a geometrical feature, but also as a physiological feature of single vision. In general, there is the geometrical or theoretical horopter (Vieth, G. 1818, Muller, J. 1823) and the empirical horopter (Wheatstone, C. 1838, Panum, P. L. 1858). Helmholtz includes cyclo-rotation of the eye and though geometrically defines the horopter as a »twisted cubic« fenomena, which accept also Schreiber, K.M. (2006). Our approach is geometrically and includes trigonometrical analysis of the visual lines and fixation points in space, but including the eye accommodation because the horopter plane in space is determined with the convergence angle of the bulbus and the accommodation sharpness of the eye near the fixation point and the whole presenting retina in the horopter space. We get the horopter with the presentation of both retinas in space, shaped as two spherical planes (calots), two semi-spheres with a common center of fixation. The width of their spacing which is the Panum's fusional area known as confusion of accommodation corresponds to the convergence angle of both bulbuae. If the fixation point is nearer, the Panum's fusional area is wider and hence the larger the disparation of imagies on the retina. The authors have mathematically estimated the radius of the horopter planes as: $R = PD/2\cos\alpha$.

***Key words**: horopter circle, binocular accommodative space curve, confusion of disparity, confusion of accommodation*

Introduction

In studies of binocular vision the horopter is the locus of points in space that yield single vision. This can be defined theoretically as the points in space which are imaged on corresponding points in the two retinas, that is, on anatomically identical points. An alternative definition is that it is the locus of points in space which make the same angles at the two eyes with the fixation lines. The horopter was first discovered in the eleventh century by the Arabian or Persian scholar Ibn al-Haytham, known to the west as »Alhazen«[1]. He built on the binocular vision work of Ptolemy and discovered that objects lying on a horizontal line passing through the fixation point resulted in single images, while objects a reasonable distance from this line resulted in double images.

The term horopter was introduced by Franciscus Aguilonius in the second of his six books in optics in 1613. In 1818, Gerhard Vieth argued from geometry that the horopter must be a circle passing through the fixation-point and the centers of the lenses of the two eyes. A few years later Johannes Müller made a similar conclusion for the horizontal plane containing the fixation point, although he did expect the horopter to be a surface in space (i.e., not restricted to the horizontal plane). The theoretical/geometrical horopter in the horizontal plane became known as the Vieth-Müller circle. Howarth[2] later clarified that the geometrical horopter is not a complete circle, but only its larger arc ranging from one nodal point (center of the eye lens) to the other.

In 1838, Charles Wheatstone invented the stereoscope, allowing him to explore the empirical horopter.[3] He found that there were many points in space that yielded single vision; this is very different from the theoretical horopter, and subsequent authors have similarly found that the empiral horopter deviates from the form expected on the basis of simple geometry.

As Wheatstone (1838) observed, the empirical horopter, defined by singleness of vision, is much larger than the theoretical horopter. This was studied by P. L. Panum in 1858. He proposed that any point in one retina might yield singleness of vision with a circular region centred around the corresponding point in the other retina. This has become known as Panum's fusional area, although recently that has been taken to mean the area in the horizontal plane, around the Vieth-Müller circle, where any point appears single.

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These empirical investigations used the criterion of singleness of vision, or absence of diplopia to determine the horopter. Other criteria used over the years include the drop-test horopter, the plumb-line horopter, and identical-visual-directions horopter, and the equidistance horopter. Most of this work has been confined to the horizontal plane or to the vertical plane.

The discrepancy of the theoretical and empirical approach has its origin in the fact that the empirical approach is based on the physiology of vision, hence, physiology optics, where the theoretical approach is based upon geometrical optics. In the attempt to fuse both approaches, especially indicating the dynamical change of the horopter plane in space, under influence of accommodation, in every new fixation plane in space (from punctum proximum to punctum remotum) among the convergence we have the presence of the accommodation.

This is the point, where each coresponding point of the horopter plane must have the same accommodation strength[4]. Our horopter study is based upon the geometrical horopter, hebce the trigonometrical approach, similar to Schreiber and Helmholz[6], but also the only theory which includes the presence of accommodation of the eyes, and which must be simultaneously in accordance with the convergence of both bulbuae.

Method

Drawing the horopter plane to present the retina in space, we decided to use the trigonometric method of analysis, hence in guidance of the geometrical horopter of previouse authors. But, the difference in respect to previouse authors is the fact that we included the accommodation of the eye, physiologically embeded to the convergence.

Results and Discussion

No matter whether we use the geometrical or the empirical theory to interpret the horopter, which differently define the horopter retina in space, (cyllinder or a hyperbolic paraboloid), one thing is for shure, if eye accommodation is included, things are completaly different.

If we put the horopter plane as the presentation of the retina, in shape of a centered optical spherical plane in space. The whole retina is being projected in space as an objective picture which distance from the eye is determined by accommodation. In Figure 1 we specify trigonometically in detail the projection of the retina in space and it is very clear that each eye determines its own horopter plane of the retina. The distance from nodal points of such a plane is determined with the expression:

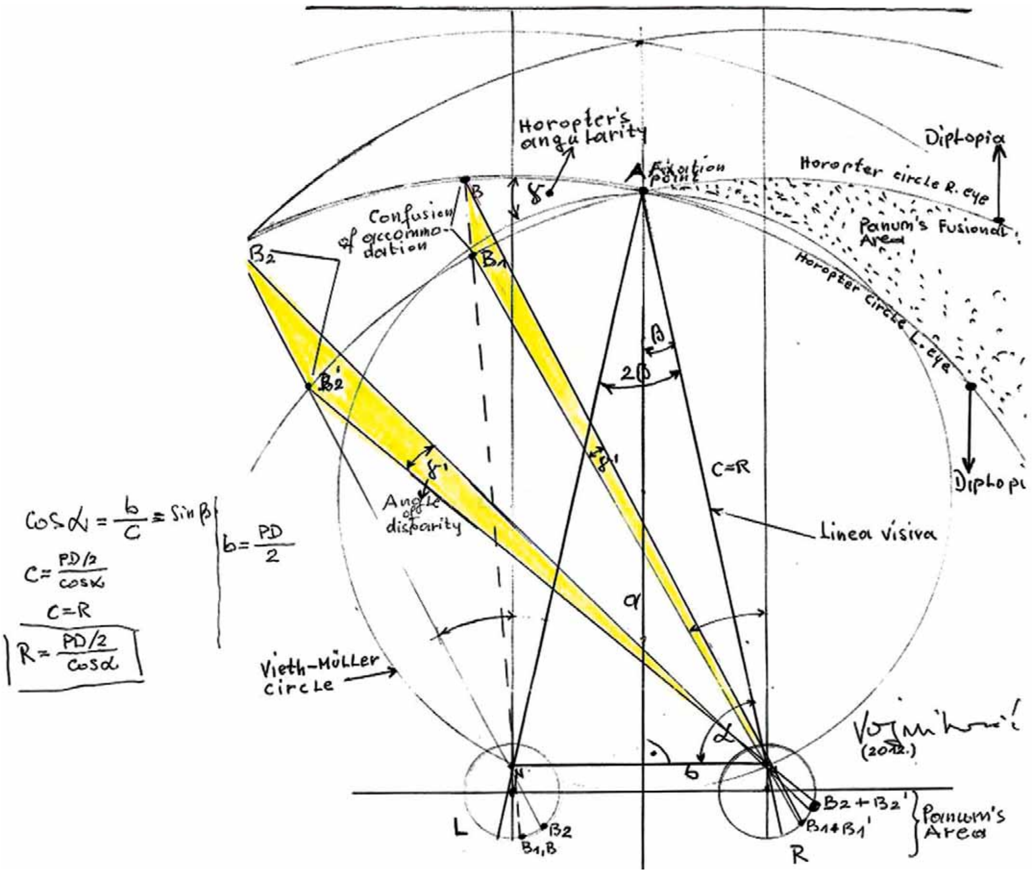

Fig. 1. The definition and trigonometry analysis of the horopters.

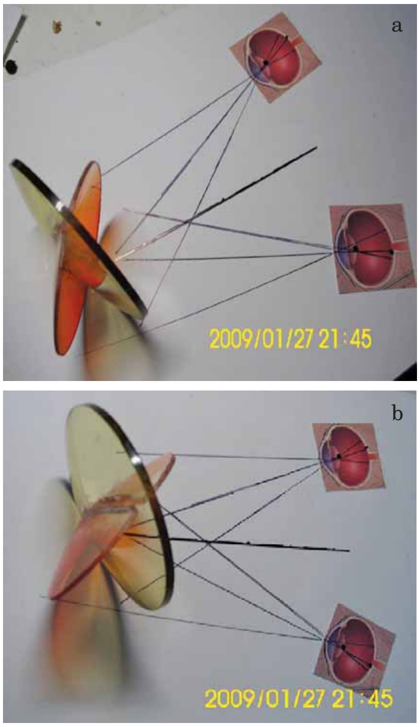

Fig. 2a-b. We have shown a horopter plane scheme, where each represents one of the retinal presentations. Gama angle between two horopter planes gets larger as the convergence gets larger, the observed object closer, and consquentially the accommodation width gets larger with the fixation point of the horopter plane.

$R = PD/2\cos\alpha$, where R is the vision line of the visive, in function of the convergence angle of the bulbus and the proper accommodation which determines the fixation point on the horopter. The radii R, are perpendicular to the spherical plane of the horopter, hence the convergence angle of both bulbuae determines the angle of both spherical horopter planes.

Is the angle of sight or fixation bigger (i.e. punctum proximum smaller), the bigger is the angle between the horopter planes. Consequently, the Panum's fusion areas are bigger. The more we get to the edge of the horopter and retina, the disparations are greater.

Let's follow the projection of point B1: point B1 on the horopter projection of the retina of the right eye, is being projected on the corresponding place of the retina of the left eye. But, the projection of the corresponding point of the retina of the left eye has its own projection, determined with accommodation, which is equal for both eyes, but indicates a different space projection of the horopter planes. In addition, we have the fact that the picture of point B1 on the left eye is fuzzy, or vice versa, the picture of point B1 in the left eye can be sharp, but then it is fuzzy in the right eye. This is a phenomenon called »accommodational confusion« (Vojniković). The angle gamma indicates the angulation of the horopter planes in space, and it is practical the same angle beta of the convergence of visual fixation directions (which in infinity becomes zero, and both horopter planes of both eyes concur in a single horopter plane, and mapps the far horizon which asimtotically approaches the flat horizonof Euclidean geometry).

Conclusion

Using the construction of geometrical optics to project a picture on the retina, one of the basic elements in constructing pictures, is the angle of convergence of the bulbus. But, fact is, that the eye represents a »dynamic optical system«, with respect to physiological optics, we cannot neglect the accommodation of the eye. In manner to analyse the projection of the picture on the retina, in terms of objective mapping of the outer items on the retina, and vice versa, the presentation of the retina in space on the spherical horopter plane, bottom line is to sharp the picture by accommodation. Our trigonometrical analysis constructs the horopter in terms of spherical planes on which is being projected the spherical retina through nodal points of both eyes. Our analysis has shown that each eye projects its own spherical horopter plane, perpendicular to the convergence angle, and sharped by the accommodation of the dioptric apparatus of the eye. We determined the law by which the holopter is being constructed, defined with the radius in terms of:

$$R = PD/2\cos\alpha$$

We conclude out of our trigonometry of spherical horopter planes, that the Panum´s areas are the bigger as we go to the edge of the retina and horopter, which is allready known. But with respect to our analysis, to construct a proper horopter plane, we must use the accommodation of the eye. The accommodation of the eye implicates the rotation of the spherial horopter plane in space, with a common fixation center, but forming a specific angle gamma between the horopters which determines the dimension of the Panum's area. It is not only matter of the disparity in the Panum's areas, but also the matter of accommodation of the eye which in the geometrical disparity of Panum's area generates alternate pictures of corresponding Panum's area and consequently the rivalry of the retina, having in mind the dominance of one eye and the basic to create the binocular stereoscopic vision. To conclude, unlike to previouse authors, the introduction of accommodational dynamics, the horopter is beiing defined as a spherical presentation of the retina in space, which radius is defined in respect with the convergence angle of the bulbus and the accommodational sharpness.

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HOROPTER – DEFINICIJA I KONSTRUKCIJA

SAŽETAK

Problemom horoptera bavili su se još od vremena perzijske filozofije i matematike (Ibn al-Haythan), pa sve do vremena autora Veth i Mullera, kao najčešće definiran horopter. Ono što je novina u našem pristupu definiranja horopterske krivine, jeste da je uključena akomodacija oka, bez koje nema pojma fiksacije u svijetlu geometrijske optike. Od pozicije fiksacije punctum proximum do punctum remotum, koliko god je distinktna akomodacija i prostorna fiksacija i prezentacija retine u prostoru, toliko možemo konstruirati horopterskih ploha u prostoru fiksacije i akomodacije. Od sferne početne punctum proximum sferne plohe, u beskonačnoj fiksacijskoj akomodacijskoj projekciji retine, prelazi od sherne plohe u asimptotski ravnu liniju. Autori definiraju horoptersku plohu kao relaciju R = PD / 2 cos alfa. Gdje je radius R horopterske akomodacijske plohe definiran kao umnožak polovične vrijednosti razmaka zjenica PD, kroz cos kuta alfa, kao kuta projekcije i fiksacije.