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A New Quantum Game Based on CHSH Game

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Abstract

The Quantum game theory is an extension of the classical game theory which describes games in which players share quantum information during the game. The use of the quantum mechanical effects enables players better results than they would achieve in the classical game without sharing any information at all. In this paper I would like to describe a new two player cooperative game which is based on the well known CHSH game. A quantum strategy for a new game will be introduced and it will be compared to the classical strategy and CHSH game strategy.
Keywords: quantum game, quantum strategy, quantum entanglement, CHSH game

1. Introduction

In 1935 Einstein, Podolsky and Rosen described a paradox with which they wanted to show that the theory of quantum mechanics is incomplete. The paradox is called "EPR paradox" and it is described in Einstein's, Podolsky's and Roesen's famous paper "Can quantum-mechanical description of physical reality be considered complete?" [9]. In 1964, John Bell succeeded to perform an experiment which showed that EPR paradox is not a paradox at all, that it is the way of nature functioning. Bell described his results and conclusions in his famous paper "On the Einstein-Podolsky-Rosen paradox" [4].

Quantum entanglement is the physical resource which is associated with correlations between separated quantum systems [19]. Quantum entanglement can be measured and the Bell states represent four special states of two-qubit quantum system which are maximally entangled [15]. One of the most known cooperative games which describes profit from quantum entanglement is the CHSH game which is named by its authors F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt who described it in their famous paper "Proposed experiment to test local hidden-variable theories" [8] in 1969. In theory, situation in which the predicted result by quantum mechanics differs from classical theories is called Bell's inequality. S. Popescu and D. Rohrlich shown that CHSH inequality can be violated even more than quantum mechanics predicts and at the same time preserve relativistic causality [17]. S.Cirel'son made some generalizations of Bell's inequality and shown that quantum correlations which violate Bell's inequality satisfy weaker inequalities of similar type [7]. B. Tsirelson stated some interesting problems regarding Bell's inequalities and shown correlations between sets of behaviors (sets of probabilities of outcome pair) which are related to particular physical theory [20]. A. Aspect, P. Grangier and G. Roger done some experimental research of CHSH inequality. They shown that experimental results are in agreement with the quantum mechanical predictions [3].

Until today, many quantum strategies for various games have been introduced. For instance, Luca Marinatto and Tullio Weber introduced the quantum strategy for Battle of the Sexes [13]. Jens Eisert and MartinWilkens introduced the quantum strategy for Prisoner's Dilemma [10]. David A. Meyer introduced the quantum strategy for coin flipping game [14]. Stefano Mancini and Lorenzo Maccone introduced the quantum strategy for Ulam's number game [12]. E.W.Piotrowski and J. Sladkowski explained the Newcomb's paradox using the quantum strategy [16]. Jiangfeng Du, Xiaodong Xu, Hui Li, Mingjun Shi, Xianyi Zhou, Rongdian Han turned classical unfair Card game into fair and zero-sum game using quantum strategy [11]. Gilles Brassard, Anne Broadbent and Alain Tapp recast Mermin's multi-player game in terms of pseudo-telepathy and introduced the quantum strategy for it [6]. Common to all mentioned quantum strategies is that players that use them can expect greater payoff than they would get if they were using classical strategies. Although, there are some games in which quantum strategies don't produce greater payoff than classical strategies. One of such game is GYNI (Guess your neighbor's input) introduced by M. L. Almeida, J.D Bancal, N. Brunner, A. Acín, N. Gisin, S. Pironio [1][21].

Today, quantum devices (i.e. quantum computer) are not available, but many governments around the world invest resources in developing them [5][18]. The power of quantum systems is recognized not only in the game theory, it brings improvements in other fields like computation and information theory as well. In the current state of the field it is very important to find situations where quantum systems outperform classical systems, because maybe one day classical systems will be indeed replaced by quantum systems.

In this paper I would like to analyze strategies for the game which is similar to the CHSH game and it has not been covered by previous research papers. The goal is to introduce a new way of using the quantum system that outperforms the classical system, even more than it was the case in the original CHSH game.

An original CHSH game is described in section 2, a new game based CHSH game is introduced in section 3, the benefits of the original CHSH quantum strategy for the new game are described in section 4, the proposed quantum strategy for the new game is described in section 5 and section 6 provides the conclusion and reference to future research.

2. CHSH Game

The game is played by two players, Alice and Bob which are far away from each other and they are not able to communicate in the classical manner at all. The game judge gives one random binary digit x to Alice and one random binary digit y to Bob. Alice must correspond to a game judge with a binary digit a and Bob must correspond with a binary digit b. Game Judge takes a look at all binary digits *x, y, a, b* and declares Alice and Bob winners if $a \oplus b = x \wedge y$, otherwise Alice and Bob lose the game [2]. Symbol $\oplus$ denotes the XOR operation (addition modulo 2).

If Alice and Bob play the classical version of the CHSH game without exchanging any information in the classical manner then the maximum probability for Alice and Bob to win is $\frac{3}{4}$. They can achieve such probability by using the following strategy [2]:

1. Bob will always correspond with $b = 0$.
2. If Alice receives $x = 0$, she will correspond with $a = 0$. The game is won by Alice and Bob with the probability 1.
3. If Alice receives $x = 1$ then she has to gamble because she does not know binary digit y which Bob received. Game is won by Alice and Bob with the probability $\frac{1}{2}$.

Overall probability for Alice and Bob to win the game if they use the described strategy is $\frac{1}{2} * 1 + \frac{1}{2} * \frac{1}{2} = \frac{3}{4}$.

But, if Alice and Bob share two-qubit system which is initialized in the Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, they can gain a probability of 0.8 if they use the following strategy [2]:

1. Alice takes the first qubit and Bob takes the second qubit from the quantum system.
2. If Alice receives $x = 1$ then she applies a rotation by $\frac{\pi}{8}$ to her qubit.

$$R_A = \begin{bmatrix} \cos\left(\frac{\pi}{8}\right) & -\sin\left(\frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{8}\right) & \cos\left(\frac{\pi}{8}\right) \end{bmatrix}$$

If Alice receives $x = 0$ she does not apply any operation on her qubit.

3. If Bob receives $y = 1$ then he applies a rotation by $-\frac{\pi}{8}$ to his qubit.

$$R_B = \begin{bmatrix} \cos\left(\frac{\pi}{8}\right) & \sin\left(\frac{\pi}{8}\right) \\ -\sin\left(\frac{\pi}{8}\right) & \cos\left(\frac{\pi}{8}\right) \end{bmatrix}$$

If Bob receives $y = 0$ he does not apply any operation on his qubit.

4. Alice and Bob measure their qubits and output the values obtained as their answers a and b.

If Alice receives $x = 0$ and Bob receives $y = 0$, they do not apply any operations to their qubits, so the quantum system remains in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice and Bob win the game with the probability 1.

If Alice receives $x = 0$ and Bob receives $y = 1$, Alice applies any operations to her qubit and Bob applies R_B operation to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(|0\rangle\left(\cos\left(\frac{\pi}{8}\right)|0\rangle - \sin\left(\frac{\pi}{8}\right)|1\rangle\right)\right) + \left(|1\rangle\left(\sin\left(\frac{\pi}{8}\right)|0\rangle + \cos\left(\frac{\pi}{8}\right)|1\rangle\right)\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right)$$

Alice and Bob win the game with the probability $\cos^2\left(\frac{\pi}{8}\right)$.

If Alice receives $x = 1$ and Bob receives $y = 0$, Alice applies R_A operation to her qubit and Bob does not apply any operations to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(|0\rangle\left(\cos\left(\frac{\pi}{8}\right)|0\rangle + \sin\left(\frac{\pi}{8}\right)|1\rangle\right)\right) + \left(|1\rangle\left(-\sin\left(\frac{\pi}{8}\right)|0\rangle + \cos\left(\frac{\pi}{8}\right)|1\rangle\right)\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{8}\right)|00\rangle + \sin\left(\frac{\pi}{8}\right)|01\rangle - \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right)$$

Alice and Bob win the game with the probability $\cos^2\left(\frac{\pi}{8}\right)$.

If Alice receives $x = 1$ and Bob receives $y = 1$, Alice applies R_A operation to her qubit and Bob applies R_B operation to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(\left(\cos\left(\frac{\pi}{8}\right)|0\rangle + \sin\left(\frac{\pi}{8}\right)|1\rangle\right)\left(\cos\left(\frac{\pi}{8}\right)|0\rangle - \sin\left(\frac{\pi}{8}\right)|1\rangle\right)\right)\right.$$
$$\left. + \left(\left(\left(-\sin\left(\frac{\pi}{8}\right)|0\rangle + \cos\left(\frac{\pi}{8}\right)|1\rangle\right)\right)\left(\sin\left(\frac{\pi}{8}\right)|0\rangle + \cos\left(\frac{\pi}{8}\right)|1\rangle\right)\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right)|00\rangle - 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)|01\rangle + 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)|10\rangle\right.$$
$$\left. + \left(\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right)|11\rangle\right)$$

Because of $\left(\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right) = \cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$, all amplitudes of the base states have the same absolute value. In this case Alice and Bob will win the game with the probability $\frac{1}{2}$.

The overall probability for Alice and Bob to win the game if they use the described strategy is $\frac{1}{4} * 1 + \frac{1}{4} * \cos^2\left(\frac{\pi}{8}\right) + \frac{1}{4} * \cos^2\left(\frac{\pi}{8}\right) + \frac{1}{4} * \frac{1}{2} \approx 0.8$.

3. New Game

The game is played by two players, Alice and Bob which are far away from each other, and they cannot communicate at all in the classical manner. The game judge gives one random binary digit x to Alice and one random binary digit y to Bob. Alice must correspond to the game judge with a binary digit a and Bob must correspond with a binary digit b. Game Judge takes a look at all binary digits *x, y, a, b* and declares Alice and Bob winners if a ∧ b = x ⊕ y (operator on the left side is swapped with the operator on the right side compared to the equation in the original CHSH game), otherwise Alice and Bob lose. Symbol ⊕ denotes the XOR operation (addition modulo 2).

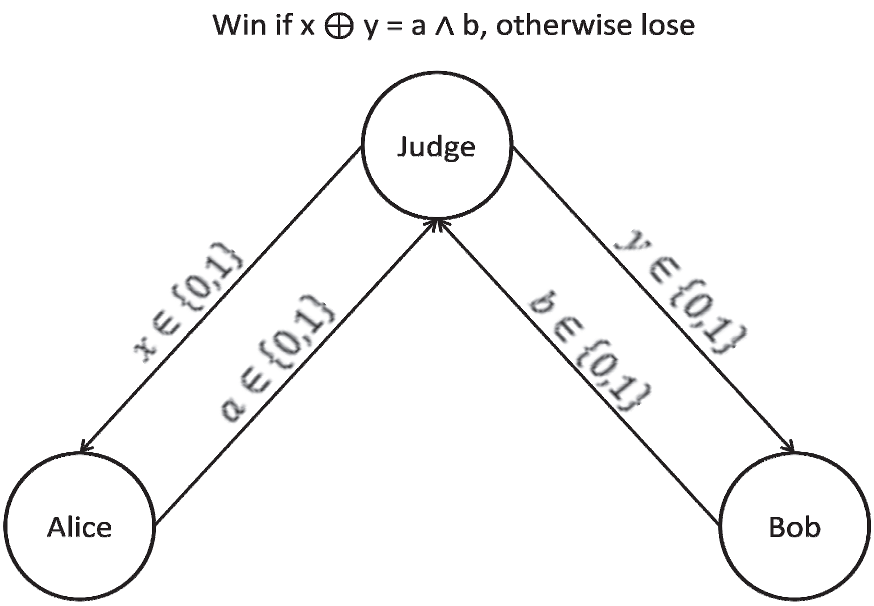

Figure 1. New game based on CHSH game

If Alice and Bob play the classical version of the game without exchanging any information in the classical manner, then they have a maximum probability of $\frac{1}{2}$ to win the game because no matter what binary digits Alice and Bob received from the Judge they have to gamble with their answers.

4. CHSH Quantum Game Strategy for the New Game

If Alice and Bob share a two-qubit quantum system which is initialized in Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and if they use the same strategy as they do in the CHSH game they will win the game with the probability of 0.526.

If Alice receives $x = 0$ and Bob receives $y = 0$, they do not apply any operations to their qubits so the quantum system remains in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice and Bob win the game with the probability $\frac{1}{2}$.

If Alice receives $x = 0$ and Bob receives $y = 1$, Alice does not apply any operations to her qubit and Bob applies R_B operation to his qubit. The quantum system is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right)$. Alice and Bob win the game with the probability $\frac{1}{2}\cos^2\left(\frac{\pi}{8}\right)$.

If Alice receives $x = 1$ and Bob receives $y = 0$, Alice applies R_A operation to her qubit and Bob does not apply any operation to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right)$$

Alice and Bob win the game with the probability $\frac{1}{2}\cos^2\left(\frac{\pi}{8}\right)$.

If Alice receives $x = 1$ and Bob receives $y = 1$, Alice applies R_A operation to her qubit and Bob applies R_B operation to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Bigg(\left(\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right)|00\rangle - 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)|01\rangle + 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)|10\rangle + \left(\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right)|11\rangle\Bigg)$$

Alice and Bob win the game with the probability $\frac{3}{4}$.

Overall probability for Alice and Bob to win the game if they use the described strategy is $\frac{1}{4} * \frac{1}{2} + \frac{1}{4} * \frac{1}{2}\cos^2\left(\frac{\pi}{8}\right) + \frac{1}{4} * \frac{1}{2}\cos^2\left(\frac{\pi}{8}\right) + \frac{1}{4} * \frac{3}{4} \approx 0.526$.

5. Proposed Game Strategy for the New Game

The proposed game strategy for the new game is the following:

1. Alice and Bob share the two-qubit quantum system which is initialized in Bell state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Alice takes the first qubit and Bob takes the second qubit.
2. If Alice receives $x = 1$, she applies the Hadamard transform $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ to her qubit. If Alice receives $x = 0$, she does not perform any operation on her qubit.
3. If Bob receives $y = 1$, he applies the Hadamard transform $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ to his qubit. If Bob receives $y = 0$, he does not perform any operation on his qubit.
4. Alice and Bob measure their qubits and output the values obtained as their answers *a* and *b*.

If Alice receives $x = 0$ and Bob receives $y = 0$, they do not apply any operations to their qubits so the quantum system remains in the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Alice and Bob win the game with the probability 1.

If Alice receives $x = 0$ and Bob receives $y = 1$, Alice does not apply any operations to her qubit and Bob applies the Hadamard transform to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(|0\rangle\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\right) - \left(|1\rangle\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|01\rangle\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$$

$$|\psi\rangle = \frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$$

Alice and Bob win the game with the probability $\frac{1}{4}$.

If Alice receives $x = 1$ and Bob receives $y = 0$, Alice applies the Hadamard transform to her qubit and Bob does not apply any operations to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)|1\rangle\right) - \left(\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)|0\rangle\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|10\rangle\right)$$

$$|\psi\rangle = \frac{1}{2}|01\rangle + \frac{1}{2}|11\rangle - \frac{1}{2}|00\rangle + \frac{1}{2}|10\rangle$$

Alice and Bob win the game with the probability $\frac{1}{4}$.

If Alice receives x= 1 and Bob receives y=1, Alice applies the Hadmard transform to her qubit and Bob applies the Hadamard transform to his qubit. The quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left(\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\right) - \left(\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\right)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle\right)$$

$$|\psi\rangle = -\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

Alice and Bob win the game with the probability 1.

Overall probability for Alice and Bob to win the game if they use the described strategy is $\frac{1}{4} * 1 + \frac{1}{4} * \frac{1}{4} + \frac{1}{4} * \frac{1}{4} + \frac{1}{4} * 1 = 0.625$.

6. Conclusion and Future Work

A new cooperative game for two players has been introduced. A new game is based on the CHSH game and the only difference is the equation which is used by the game judge to declare whether players are winners or losers. It has been shown that the proposed quantum strategy for the new game outperforms the best classical strategy for the introduced game. If the players use the best classical strategy, then they have a maximum probability of $\frac{1}{2}$ to win the game, but if the players use the proposed quantum strategy, then the winning probability is much higher, even 0.625. The topic of the future work is to describe and analyze new and better quantum strategies for the game introduced in this work.

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