GA-BASED OPTIMAL SIZING OF PV UNDER PAY AS BID AND UNIFORM POWER MARKET PRICING CONSIDERING UNCERTAINTY OF SOLAR RADIATION

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This paper presents a multi-objective formulation for optimal sizing of photovoltaic resources as grid connected mode in distribution systems with the aim of maximizing net present worth of system. The proposed system in this paper includes two sources such as upstream network i.e. 63/20 kV substation and photovoltaic arrays as private sector for meeting load. Total net present worth considered in this paper as objective function consists of net present worth of distribution company (DISCO) and of photovoltaic owners (PV-Owner). In order to obtain accurate results, in this paper the uncertainty of solar radiation is considered. The implemented technique is based on GA and weight method employed to obtain the best compromise between these costs. Simulation results on 33-bus distribution test system under pool market operation are presented to demonstrate the effectiveness of the proposed procedure.

Keywords: genetic algorithm, Monte Carlo, optimization, photovoltaic arrays (PV), pool electricity market

1 Introduction

There are a lot of models of power markets and transactions to achieve a competitive electricity environment. Three basic models based on types of transactions are: pool model, bilateral contract model and hybrid market model [1].

Pool market is a central market place that clarifies the market for both buyers and sellers. Electric power sellers/buyers propose bids to the pool for the value of power that they want to trade in the market.

Many studies have investigated the optimization of solar energy in a distribution system. In [2] Unit sizing and cost analysis of stand-alone hybrid wind/PV/fuel cell power generation systems is presented. The Size optimization of a PV/wind hybrid energy conversion system with battery storage using simulated annealing is analyzed in [3]. Stand alone mode is considered and the uncertainly of solar radiation and wind speed is not considered. Economic analysis of standalone and grid connected hybrid energy systems, is discussed in [4]. Evaluation of utilizing solar and wind energy with hydrogen as a storage device to cover the electricity demand is investigated. The method of analyzing in this paper is based on costs of the system and simulated using Homer software. Authors in [5] studied the Multi-objective design of PV-wind-diesel-hydrogen-battery systems. In the mentioned research, minimizing, simultaneously, the total cost throughout the useful life of the installation, pollutant emissions (CO₂) and unmet load are considered. Paper [6] investigates the multi-objective Particle Swarm Optimization for the optimal design of photovoltaic grid-connected systems. The objective function describing the economic benefit of the proposed optimization process is the lifetime system’s total net profit which is calculated according to the method of the Net Present Value (NPV). The second objective function, which corresponds to the environmental benefit, equals to the pollutant gas emissions avoided due to the use of the PV. The uncertainty of solar radiation is not considered in this study.

In [7] investigation of the dynamic modeling and sizing optimization of stand-alone photovoltaic power systems using hybrid energy storage technology is illustrated.

In [8] cost benefit analysis of a photovoltaic-energy storage electrification solution for remote islands is presented.

Optimal sizing of hybrid wind/photovoltaic/battery considering the uncertainty of wind and photovoltaic power using the Monte Carlo has been studied in [9]. In the mentioned research, a micro-grid consisting of hybrid wind/photovoltaic/battery in stand-alone mode based on cost optimization has been analyzed.

In restructured power market place, in rivalry and open access environment of power market, different transactions can directly happen among buyers and sellers; these transactions are bilateral, multilateral and ancillary services transactions [10].

Under hybrid market, several features of the previous two types of market are coalesced. Many transactions are expected between buyers and sellers in order to create a flexible and economic market operation in the hybrid power market model. These transactions need to be...
assessed ahead of their scheduling time to check their feasibility with respect to system operating conditions. Infeasible transactions can alter cause congestion and threaten system security and stability of the network. They can alter the economic schedule too. Hence the issue of DG optimal sizing in deregulated electricity market model needs to be addressed.

In this paper, optimal sizing of photovoltaic arrays and battery in interconnected mode is described. Two main power sources, considered in this study for covering demand load of system, are main grid and photovoltaic arrays. The objective function is maximizing the total net present worth (NPW) of the active power generation system i.e. sum of net present of photovoltaic arrays-owners and distribution company over its 10-year lifetime. With using the Monte Carlo method the uncertainty of solar radiation is considered. For probabilistic analysis of solar radiation, 15 years data of solar radiation are used. The data have been used as every four-hour step. For every four-hour data, the best probability density function (PDF) of solar radiation is determined. For a year, month and day the Weibull distribution is known as suitable distribution for solar radiation. This means that one year is separated to 2190 samples of solar radiation each four-hour sample has 15 samples of all 15 years, so the time step is four-hour. Using the Monte Carlo and Genetic Algorithm (GA), the best sizing of photovoltaic arrays is determined.

2 Pool-based power market using pay as bid and uniform pricing

In this paper, the photovoltaic owners are the names such as micro-grid, so in all equations under pool market, the term of "micro-grid" is used for photovoltaic owners that produce power and trade with the main grid or upstream grid. Based on this, it can be said that power producers offer suggestions for n-sequential rounds and stay in the game even if they do not manage to find a market portion for a long time. In other words it can be said that they are not acquainted with each other’s payoffs and offers and costs and at the end of each round are aware only of the market-clearing price and their own market portion and profits. Therefore power producers evaluate their upshot, in terms of their profit, with the one they obtained in the preceding round and if it is better they reward the last randomly chosen action using growing its possibility in the distribution of probability of potential actions. Or else, they decrease that probability. Hence new randomly preferred values from the adjusted probability distribution clarify and identify the next offer of power market. The adjustment sizes in the values of the offer curve coefficients and in the actions’ probability values are determined at the introduction of the game and they are called steps. Therefore, during the game each player progressively forms a probabilistic profile with reference to its potential moves, which is essentially a behavioral tool based on its recent experience, conducting him to react proportionately to different market's trends [11-13].

Two basic laws under pool market are as follows:

1: i - Power Producers (Players i.e. component of micro-grid and main grid) with a capacity range \([x_{min}, x_{max}]\) that defines: (i) the technical lowest output, below which the \(i\)-power producer i.e. micro-grid and main grid, cannot operate and (ii) the maximum output that they can produce. So, the total value of capacity exceeds the expected power demand load.

2: The main supervisor in power market environment i.e. ISO, in other words Independent System Operator, whose aim is to cover and meet the demand power load at the lowest total cost and therefore purchases electricity from the main grid or micro-grid evaluating their offers. In the first offer that the players submit to the ISO is their marginal cost and it is illustrated as the following equation:

\[
F_i(x) = A_i + B_i \cdot x, \tag{1}
\]

where are:

\[
A_i = a_i \quad \text{and} \quad B_i = 2b_i. \tag{2}
\]

The market-clearing price and the dispatched generation \((x_{m})\) for each generator are then calculated. The corresponding net revenue for each player in uniform pricing is:

\[
J_i = x_{m} \cdot SMP_i - TC(x_{m}). \tag{3}
\]

While in pay-as-bid is:

\[
J_i = \left(x_{min} \cdot F_i(x_{min})\right) + \int_{x_{min}}^{x_{m}} (F_i(x))dx - TC(x_{m}). \tag{4}
\]

Of course in this study, for applying these constraints of pool market operation, the sigma is used for implementation of Eqs. 3 and 4.

At the end of the round, producer \(i\) (PV as considered micro-grid and Utility Grid (Upstream Grid) knows only the market-clearing price and the quantity \(x_{m}\) the ISO purchased from it.

Fig. 1 represents the mode of payment in Uniform Pricing and Pay-As-Bid in a pool based electricity market.

![Figure 1 Mode of payment in uniform pricing and pay-as-bid](image)

Based on pool operation of power market, at the beginning of the next round, micro-grid or PV arrays and upstream network or main grid may modify their offer, before they submit it to the ISO by changing the values of the coefficients of their offer \(A_i, B_i\).
Based on pool operation of power market, Micro-grid and main utility grid can modify just one of the coefficients, at each round, and only the same coefficient for a predefined number of sequential rounds (modification period). It is notable that the duration of these periods may vary per player and per coefficient and based on pool operation of power market it is assigned at the beginning of the game. The modification of the in turn coefficient consists in the increment or decrement of the coefficient’s value.

3 OPF equations and constraints

The power flow equations corresponding to both real and reactive electrical power balance for all the buses of the system are described as follows [14]:

\[
P_i = \sum_{j=1}^{N_b} V_j V_j \left[ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right],
\]

\[
Q_i = \sum_{j=1}^{N_b} V_j V_j \left[ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right],
\]

Real and reactive power injections at bus-\(i\) are presented with the following equations:

\[
Q_i = Q_{gi} + \theta \cdot Q_{PV} - Q_{di},
\]

\[
P_i = P_{gi} + \theta \cdot P_{PV} - \rho \cdot P_{di}.
\]

That \(\rho\) is the demand variation factor for both pool and bilateral demand. It represents the change in the operating point of the system. Also \(\theta\) is decision variable with value \(\{0, 1\}\). With the existence of PV at each bus of the system, this value is equal to one and/or else, it is zero.

These constraints are power generation, voltage and angle limits, expressed by the following equations:

\[
P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max},
\]

\[
\delta_i^{max} \leq \delta_i \leq \delta_i^{max},
\]

\[
|P_i|^{max} \leq |P_i| \leq |P_i|^{max}.
\]

Other constraints that must be considered in this optimization are power flow direction and the capacity of feeders. Power flow direction is from upstream network i.e. 63/20 substation to distribution system. So based on this constraint, the answers obtained from optimization procedures that do not satisfy this condition is eliminated from answers. Also using power flow for each of optimized answers the power line is calculated and compared with this limitation. These constraints are presented by (11).

\[
P_{feeder-i}^{max} \leq P_i^{max},
\]

\[
0 \leq P_{Sub.63/20}.
\]

4 Photovoltaic arrays

PV technology is identified as most environment friendly technologies. It requires only sunlight and no other energy fuel. Being modular in design, the capacity can be increased to meet additional demand. It is easy to dismantle and configure these systems for other applications. PV systems require little maintenance. These components can be manufactured and assembled locally. PV system has several f as follows [15]:

- Without any fuel usage
- Not emission of gaseous and liquid pollutants
- They do not have any noise
- Generation of DC electricity due to its being easily stored in batteries
- Can be easily transported, assembled and installed in remote areas.

Another feature of PV is low operation and maintenance cost. Therefore the PV energy cost is still higher than the utility energy price. Therefore, the PV applications have been limited to remote locations. However in this paper it is supposed that the PV is connected to power system and it is previously installed into system due to every reason such as reduction power loss, improvement of voltage profile and improvement of power quality or reliability indices. But in this paper only optimal operation of PV as grid connected based on optimum economical purpose is presented and considered.

In this study the main characteristic of PV is considered and modeled, this feature is dependence of output power of PV on climatic conditions. The PV output depends on climatic conditions. In order to solve this problem, the idea of combination of PV with battery will, therefore, construct a very reliable distributed power system, because the battery acts as a back-up during low PV out-put.
In this equation, \( C_F \) is the total cost of upgrading the distribution system feeders and is expressed by the following Eq. [19]:

\[
C_F = \sum_{i=1}^{TN} \sum_{j=1}^{M} C_y \sigma_{ij}.
\]

The second cost is the cost of the operation and maintenance of substations and is expressed as follows [19]:

\[
C_{S-M&O} = \sum_{i=1}^{TN} \sum_{u=1}^{M} C_{i,u} \cdot \sigma_{i,u} + 8760 \cdot \sum_{i=1}^{TN} \sum_{u=1}^{M} \beta \left( \sum_{i=1}^{TN} \sum_{u=1}^{M} C_{i,u} \cdot P_{i,u} \right).
\]

Another cost, considered in cost function of DISCO is the total cost used for buying electricity from main grid or from photovoltaic owner in order to meet the demand load, and it is illustrated by the following Eq. [17]:

\[
C_{buy} = \sum_{k=1}^{T} P_{k, buy}^U \cdot \rho_{k, buy}^U + \sum_{k=1}^{T} P_{k, sell}^U \cdot \rho_{k, sell}^U.
\]

It is more important that \( \sigma_{i,u} \) and \( \sigma_{i,u} \) are decision variables for the repair. They are equal to one in case of repair, or else they are zero.

C. Income of photovoltaic owner
The total revenue of photovoltaic owner can be considered as the amount of the monetary income received using selling electricity to DISCO on peak. It is expressed as follows [17]:

\[
R_{DG} = \sum_{k=1}^{T} \sum_{i=1}^{N_{DG}} P_{k, sell}^e \cdot \rho_{k, sell}^e.
\]

D. Costs of photovoltaic owner
The total cost function for a photovoltaic owner can be expressed by the following equation:

\[
C_{PV} = C_{r&m} + C_{P(PV)}.
\]

The first term is the repair and maintenance cost and it is expressed as follows [19]:

\[
C_{r&m} = 8760 \cdot \sum_{i=1}^{M} \beta \left( \sum_{i=1}^{N_{PV}} C_{i} \cdot P_{i} \cdot S_{DG} \right).
\]

The next cost is the production cost. It is considered as a quadratic function of produced power, as follows [20]:

\[
C_{P(PV)} = \sum_{i=1}^{N_{PV}} C_{PV,i}.
\]

That \( C_{PV,i} \) describes the production cost of photovoltaic arrays and is expressed as follows [21]:

\[
P_{PV} = A_{PV} \cdot x^2 + B_{PV} \cdot x + C_{PV}.
\]
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Optimalno dimenzioniranje fotonapona uz ponudom dogovorenu cijenu i određivanje jednake tržišne cijene električne energije za

\[ C_{PV,i} = a' + b' \times P_{PV,i}. \]  

(24)

6 Proposed method

In this research, the numbers of photovoltaic array and battery are optimized for supplying the load. Maximization of Net Present Worth is considered as objective function and satisfies defined pool market indices as constraint. Uncertainty in power output of photovoltaic arrays is also considered. The algorithm of presented method is represented in Fig. 3.

![Figure 3](image)

**Figure 3** The algorithm of presented method

6.1 Monte Carlo Method

Monte Carlo method is a set of computational algorithms based on repeated random sampling to subtract their results. The Monte Carlo method is a broadly used apparatus in various courses, counting engineering, finance, biology, computer graphics, physics, chemistry, operations research and management science [22].

Monte Carlo approach works as a pattern described as follows:
1. Determination of the boundary of data.
2. Data generation from probability distribution of input.
3. Deterministic calculation on the data.

6.2 Best distribution of solar radiation

In order to clarify the best probability density functions (PDF) of solar radiation, 15 years data are used. The data have been used as every four-hour period. This means that one year is separated to 2190 samples of solar radiation and each four-hour sample has 15 samples of all 15 years, so the time step is four-hour. The purpose is getting the PDF model for every four-hour of solar radiation. In order to compute the PDF for solar radiation, various representations of solar radiation for a year, month and day are investigated. For a year, month and day the Weibull distribution is known as suitable distribution for solar radiation. In this research, a distribution style for every time step solar radiation data is considered, so that for each time step the parameters of distribution are different from other hours. The algorithm of getting the most suitable distribution for each time step of solar radiation is shown in Fig. 4.

![Figure 4](image)

**Figure 4** The algorithm of finding the suitable and best distribution for each time step of solar radiation

In this paper, GA is considered as optimization algorithm.

6.3 GA-based optimization method

The GA is basically an evolutionary algorithm, comparable to a part of the physical world. GA is a stochastic optimization approach introduced by [23] and further discussed by [24]. GA is a strong search method that relies on the natural selection and genetics to find the optimum answer. GA is computationally simple yet powerful and it is not restricted by assumptions about the search environment. The most important aim of optimization should be improvement. Even if GA cannot give assurance that the solution will converge to the optimum, it tries to achieve the optimum, that is, it works for the enhancement. Binary and floating-point representations are used to utilize GA, for the sake of comparison. In the binary implementation, each element of a chromosome vector was coded with the same number of bits and each occupied its own fixed position.

The implemented GA begins with randomly generating an initial population of probable solutions. For each solution a value of power generation units is chosen between 0 and a maximum limit, fixed by the planner on the ground of economical and technical constraints; then, a different size of DG units are randomly chosen until the
total amount of power installed reaches the DG penetration level assigned. At this point, the objective function is assessed verifying all the technical limitations. In Fig. 5 the block diagram of optimization problem is shown.

Parameters used for the GA with binary representation are listed in Tab. 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>80</td>
</tr>
<tr>
<td>Number of populations, generations</td>
<td>100</td>
</tr>
<tr>
<td>Probability of crossover</td>
<td>95 %</td>
</tr>
<tr>
<td>Probability of mutation</td>
<td>5 %</td>
</tr>
<tr>
<td>Solution precession</td>
<td>0,001</td>
</tr>
</tbody>
</table>

7 Simulation results

In this paper, photovoltaic and battery is considered for supplying the load. Pool market is considered as constrains in optimization procedure in net present value of total system including Distribution Company (DISCO) and photovoltaic owner. The power market considered in this research is shown in Fig. 6. In this market, multi participants including (1) Main Grid = sub. 63/20, (2) PV panels as DG, (3) Distribution company (DISCO) and (4) Customers are considered.

The histogram of power generated from a photovoltaic array for a time step of a year are shown in Fig. 7. Optimization algorithm run and the variables are updated in each time step. In this research Genetic Algorithm is used for optimization procedure.

The convergence of GA algorithm is shown in Fig. 8 and the best combinations of photovoltaic and battery units with optimized net present worth are listed in Tabs. 2 and 3.

<table>
<thead>
<tr>
<th>Case</th>
<th>NPWPV_Owner</th>
<th>NPW_DISCO</th>
<th>NPW_Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Pricing</td>
<td>35 498</td>
<td>212 426</td>
<td>315 403</td>
</tr>
<tr>
<td>Pay As Bid Pricing</td>
<td>34 878</td>
<td>223 426</td>
<td>306 803</td>
</tr>
</tbody>
</table>
Table 3 Optimal number for PV units and battery under pool electricity market

<table>
<thead>
<tr>
<th>Case</th>
<th>Costs / US$</th>
<th>PV units</th>
<th>Batt num.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Pricing</td>
<td>202 310</td>
<td>223</td>
<td>18</td>
</tr>
<tr>
<td>Pay As Bid Pricing</td>
<td>203 290</td>
<td>216</td>
<td>14</td>
</tr>
</tbody>
</table>

8 Conclusion

In this research a multi-objective formulation for optimal sizing of photo-voltaic resources in distribution systems for maximizing net present worth of system is analyzed. The presented system in this paper consists of upstream network i.e. 63/20 kV substation as main grid and photovoltaic cells as DG to supply load. The objective function considered in this paper consists of net present worth of distribution company (DISCO) and of photovoltaic owners (PV-Owner). In this paper the Monte Carlo technique is used for considering uncertainty of solar radiation. The implemented approach relies on GA and weight coefficient method (WCM). Simulation results on 33-bus distribution test system under pool-based power market operation are presented to show evidence of the proposed procedure effectiveness. In order to clarify the best probability density functions (PDF) of solar radiation, 15 years data are used. The data have been used as every four-hour period, i.e. one year is separated to 2190 samples of solar radiation and each four-hour sample has 15 samples of all 15 years, so the time step is four-hour. The purpose is getting the PDF model for every four-hour of solar radiation. In order to compute the PDF for solar radiation, various representations of solar radiation for a year, month and day are investigated. For a year, month and day the Weibull distribution is known as suitable distribution for solar radiation. In this research, a distribution style for every time step solar radiation data is considered, so that for each time step the parameters of distribution are different from other hours.

9 References


Symbols

\[ P_i, Q_i \] – Net power injected at each buses of system

BiH

517

511-518
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$P_{g_{ij}}, Q_{g_{ij}}$ – Power injected at each bus from sub. 63/20

$P_{PV_{ij}}, Q_{PV_{ij}}$ – Active and reactive power injected at each bus from PV panels

$P_{di}, Q_{di}$ – Demand active and reactive power at each buses of system

$P_k^{e, sell}$ – Amount of electricity buying from main grid at year $k$

$\rho_k^{e, sell}$ – Price of electricity buying from main grid at year $k$

$M$ – Total number of system load buses

$TN$ – Total number of system buses

$TU_s$ – Total number of substation transformers in distribution system

$\sigma_{iu}$ – Transformer $u$ in substation $i$ decision variable for repair

$\sigma_{ij}$ – Feeder $i$ to $j$ binary decision variable for repair

$C_{ij}$ – Total repair and maintenance cost of feeder $i$ to $j$ ($/km)$

$C_{lu}$ – Total repair and maintenance cost of transformer $u$ in substation $i$

$C_e$ – Electricity market price ($/(kW \cdot h)$)

$P_{lu}$ – Transformer $u$ in substation $i$ dispatched power (kW)

$P_{k_{buy}}^{e, UP}$ – Amount of electricity buying from main grid at year $k$

$\rho_{k_{buy}}^{e, UP}$ – Pricing of electricity buying from main grid at year $k$

$P_{k_{buy}}^{e, PV}$ – Amount of electricity buying from PV at year $k$

$\rho_{k_{buy}}^{e, PV}$ – Pricing of electricity buying from PV at year $k$

$\beta$ – Present worth factor

$T$ – Lifetime of project

$pf$ – System power factor

$P_{k_{sell}}^{e, PV}$ – Amount of electricity selling of PV to DISCO at interval $k$

$\rho_{k_{sell}}^{e, PV}$ – Pricing of electricity selling of PV to DISCO at interval $k$

$P_{PV}$ – Power of each PV array.

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