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New Approaches to Computer Assisted Instruction of Probability Topic with the Help of Buffon's Problem

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Abstract

In Russia, the topic of 'probability' is not included in the elementary schools' syllabi. However, as Russia entered the process of Bologna Education, the Russian Ministry of Education has recently started planning the introduction of some changes into the system of education. The principal aim of these changes is to teach the topic of 'probability' in the elementary schools in Russia. The topic is included in the 8th grade syllabi in the Turkish educational system. In this context, the subject, which is included into some schools' syllabi, has been taught for one year in Russia. The aim of this research is to show that in the Russian and Turkish educational systems this subject will be more efficiently taught with computer assisted systems when compared to the instruction carried out by means of traditional methods. In order to analyze the topic, a solution of Buffon's needle problem is attempted in the 'Reno Simulation' program. This simulation program is chosen because it includes the subject of probability. The program also eases the analysis of the subject of probability. Additionally, it may be suggested that Reno program is, in a way, a simulation tool for geometric probability events and presents a strong and flexible platform for simulation procedures. The program has example concepts; these concepts can be used as the bases, different scenarios can be built on these bases and these models' simulations can easily be presented in combinations with their algorithms. For the experiment, Buffon's needle problem is chosen from the set of examples on the probability topic available in the "Reno Simulation" program. At the end of the experiment, it is determined that the instruction of computer assisted probability topic with the help of Buffon's needle problem is more efficient than the instruction by means of traditional methods; additionally both Russian and Turkish pupils' behaviors are positively developed with computer assisted instruction.

Key words: *computer-assisted instruction; elementary education; mathematics education; probability.*

Introduction

The probability topic was not included in the elementary schools' syllabi in Russia. However, there have been some recent attempts to incorporate the topic into the elementary schools' second stage syllabi. In this context, people who do their master's and doctorate studies should choose and study the instruction of the probability topic in elementary education and new instruction developing techniques in order to ensure comprehensibility. Russian mathematics professors such as Kolyagin Y.M., Merlina N.İ., Savvina O.A., Terentyeva L.P., Avteyeva T.K., Merlin A.V., who enlightens Russian education, and Lukankin G.L., who has recently passed away, emphasize that this topic should be analyzed carefully (Kolyagin, Lukankin, Merlina, 2009, pp. 3-4):

"Our schools are going through a tough period; a difficult process has begun for mathematics teachers. The innovations in math teaching do not involve any changes in the education content. These innovations are surely important, but the most important point is to see the deeper changes that lie in the background of the issue. In order to achieve these important changes, we should widen our horizons, carefully compare the past and present situations, remember our national schools' traditions and apply innovations in the past to the modern schools. In addition to these, special attention should be given to the question of: "Are the innovations organically applied to traditions or are they contrary to the traditions?" On the other hand, everybody should also answer this important question: "What is good and what is bad?".

While the new generation is open-minded, the generation of 1940-1950 is trying to protect the traditional methods (Kolyagin, Savvina, Tarasova, 2007).

The decision of including the topics of statistics and the calculus of probabilities into school mathematics syllabi in Russia has increased the hope of creating a new instruction method content. This decision will enable creation, development and systematization of thoughts on the stochastic structure of the events around us. The teaching of mathematics in particular, with its limited stochastic structure (not separate but isolated blocks and sections), will be successfully enhanced in harmony with all the different departments and sections (Kolyagin, Lukankin, Merlina, 2009).

The first year experience of the teaching process with the new instruction material shows that, although there are not enough teaching methodology textbooks, the probability topic has not been effectively harmonized with other lessons and a traditional mathematics lesson still has no connection with the new method. The lesson is added to the education program in the last quarter of the academic year (Russian academic year is divided into four semesters). But the important point is not only the insertion of the topic into syllabus, but the effective repetition of the topic in the lessons. In order to do this, the new syllabus content should be harmonized with the older one and the potential of probability, which used to be ignored, should be used more. All the positive opportunities should be used in order to connect the new topic with the other lessons in order to compose the basis for the traditional instruction materials. As can be seen in Buffon's needle problem to some extent, the probability topic will reveal the accustomed mathematics' hidden secrets and widen its boundaries.

The first thing that comes to mind when computer assisted instruction is mentioned is that computers are assumed to be used as teacher's tools in a lesson, similar to the blackboard, chalk, ruler, overhead projector. In such an approach, computers' technical qualifications are used only as literals. However, computer assisted instruction is the method of benefiting from computers during the education process in order to enable pupils to acknowledge their own needs and performance and to control their own learning process by using feedback as well as to improve their interest in lessons with the aid of graphics, sound, animations and figures (Baki, 2002).

Computer assisted instruction is the method that aims at educating pupils by presenting the contents of the lessons directly, repeating the learnt information by means of other techniques, solving problems and practicing. Additionally, the method is used as an assisting tool in the teaching-learning processes of similar activities (Odabaşı, 2006).

The advantages of computer assisted instruction are presented below.

- It ensures an appropriate in-class teaching process by giving learners extra time and giving them the opportunity to control the amount and results of the information they learn.
- Pupils' morale increases when they learn quickly whether their answers are correct.
- Such programs provide a more positive educational environment especially to the children who learn slowly. As their mistakes are not seen by the whole class, they do not feel ashamed.
- Computer assisted education is effective for the pupils with learning difficulties, pupils belonging to different ethnical groups as well as the disabled ones.
- Colorful, musical and sprite graphics introduce authenticity and particularity to the subject.
- Data storage capacity of computers enables individual learning, and pupil development can be inspected if they prepare personal directions.
- Computers have databases appropriate for developing information.
- Computers can use all information about the graphics, texts, sounds and images.
- Computers can keep a lot of information that can be used by teachers; pupils gain the ability to learn on their own and various methods can be used through computers. Various educational methods are used in these learning experiences.
- Computers ensure reliable and proper teaching strategies, independent of the teacher, time and place.
- Computer assisted education increases teaching efficiency which allows for the increase in pupil success. This education also ensures sufficiency which enables achievement of goals in a shorter period of time at a reduced expense.
- Sufficiency is very important in business life and industry. In addition to this, its importance in education is increasing with each passing day.

– The emergence of easy-to-use systems has enabled some educators to create and develop their own educational programs (Earged, 2002).

In recent years, different researchers have emphasized that modern education methods make the teaching easier and consequently teaching can be made more appealing as well. Recently some important steps have been taken with this aim in mind.

Computers that have become an important part of our lives are important constituents of educational systems and the learning-teaching education environment as a part of the fact that information and communication technologies affect human life (Çiftçi, 2006).

The basic constituent of the conventional teaching is the teacher, followed by the classroom and blackboard which complete the process. In today's educational system, blackboards are being replaced by whiteboards, while chalks are being replaced by markers. The lesson is the unit in the classical teaching which brings all the relevant topics together. The instructor presents these topics in class following a particular structure. Presentation of the topics is synchronized; definite timeframes are separated for the education, for instance 3 hours on Thursday. One other main issue in the presentation is the speech of the instructor, namely his act of transferring the topics to the listeners by speaking. The process of transferring information through speech can be supported either slowly, using the "blackboard" environment in which topics or concepts can be observed step by step, or faster by overhead projector, videos, projections, computers or electronic "blackboards". Students take notes while listening and each student notes down the information in the way s/he comprehends and personalizes it. Thus, the pace of the provided support during the lectures is important (Çağlayan, Ufuk 2001).

Decrease in the cost of computers and increase in the aspects and abilities of computers have increased their usage for different purposes by all members of society. Accordingly, this has caused an increase in computer usage in education (Gürbüz, Eryılmaz, Yıldırım, Soner, 2001).

The most important factor that enables pupils to achieve success in mathematics by using computers is software. While most of these programs can cause impassivity of pupils as they sit in front of computers; they can also be affected by means of audible and visual effects. Users can answer multiple choice questions and get quick feedback on their results and success. Pupils also have the chance to go back to the previous explanations (Jinich, 1986).

During the non-standard problems teaching and solving processes the instructor-pupil-computer trio should be well organized. If the problems in computer support do appear, they should be studied beforehand and computer assisted teaching algorithm should be prepared. There should be a computer package program in computer laboratories and, most importantly, pupils should be motivated for the class.

It is known that the topic of "probability" is taught in the 8th grade mathematics class. In order to ensure the comprehensibility of the lesson and objectify the information, different teaching methods are used. For example it is mentioned that if the Graf theory is taught in classes, the pupils will understand and learn the topic of probability more easily (Seyhanlı, 2007).

The number Π has attracted mathematicians' as well as other people's attentions for centuries. This attention is the result of the change in the decimal places of Π. On the other hand, the number is seen in various fields of mathematics such as geometry, the calculus of probabilities, differential and integral calculations. All mathematicians are familiar with the mysterious aspect of this number and all its details. We will find the number Π with the simple experiment called Buffon's needle problem in the literature, tell the aspects of this mysterious number and show that the probability topic can be taught by means of computer assisted education.

Graphics, diagrams, various geometric shapes or models serve as tools for visualizations, thoughts, ideas and abstract concepts. Human thought connects the outer world and abstract concepts with these tools. Namely, presenting algebraic structures with geometric expressions helps to show pupils how a reasonable theory is built from a physical model (Konyalıoğlu, 2003).

The abstract concepts of mathematics can be made concrete by using some mathematical programs appropriated to the content of the problem. In this respect, the Buffon's needle problem which is a geometric probability problem is solved with graphic drawings in the shape of iteration and geometric shapes made of random straight lines or intersections.

Buffon's theory is based on whether the needle cuts the straight lines or not. If a needle thrown N times cuts the line at least once in R bounce, the probability is P=R/N. If a needle is thrown N times and at least one intersection occurs, then the probability is P=R/N. There is a non-combined evaluation for P's P/N (1-P) distribution. This is because there is a possibility of carrying out these experiments which give the approximate Π result. Many experiments are carried out in this way. Data are counted, calculated and discussed in the interesting study presented by Gridgeman (1960).

The Objective of the Research

The aim of this paper was to show that the topic *probability*, which is taught in the second stage of primary education, in contrast to conventional methods, may yield different solutions for Pi if presented by new approaches to computer aided education with the assistance of Buffon needle problem.

Method

Buffon's needle problem - the emergence of Π at the end of solving a probability problem is very interesting. This experiment can be performed by everybody and an estimation can be made for Π. Buffon's needle problem is mentioned in almost all of the books on probability and statistics (Morgan, Kennedy, 1992).

The Manner of Carrying out Buffon's Needle Problem Experiment

A medium sized needle (30 mm nail or ball pen) is taken. Parallel straight lines that are two times bigger than the needle are drawn on a sheet of paper. The needle is thrown at the paper randomly and the resulting situation is noted. The frequency table of the recurring situation is created as 'Needle intersects the line', and 'Needle doesn't intersect the line'. 'Needle intersects the line' situation's frequency probability is calculated and the reverse of the result is noted. After these, results are compared with the results of the experiments carried out by pupils.

After this process, a combined result table is organized that shows the results of a few classes (if possible) that are obtained by all the pupils in the class. Pupils will realize that the frequency of the situation 'Needle intersects the line', is two times lesser than the frequency of the situation 'Needle doesn't intersect the line'. Namely, the results of these situations are either equal or have approximate values.

By throwing a needle with the help of computers, 10,000, 20,000 and even 1,000,000 times in a similar way, children can see if the needle intersects the parallel lines. Pupils will finally come to realize the following rule: at the end of many throws, the reverse of the frequency of the situation 'Needle intersects the line' is almost 3.14.

Let us look at some of the details of the experiments that are carried out as part of the classical teaching method. In these experiments, the lengths of the needles are given as the distances between the lines.

Table 1. *Data on Buffon's needle problem experiments carried out by various scientists*

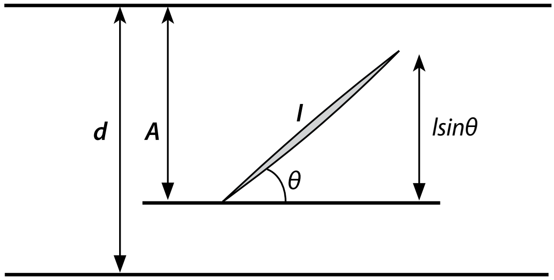
Experimenter	Length of the needle	Number of misses	Number of hits	Result
Volf, 1850	0.8	5000	2532	3.1596
Smit, 1855	0.6	3204	1218.5	3.1553
De Morgan, 1860	1.0	600	382.5	3.137
Fox, 1884	0.75	1030	489	3.1595
Lazzerini, 1901	0.83	3408	1808	3.14159
Reina, 1925	0.5419	2520	859	3.1795
Gridgeman, 1960	0.7857	2	1	3.143

The number Π becomes more easily explained when pupils meet geometric probabilities, sinusoid and integral in the coming years.

Data Collection Tools

Method 1

Different package programs are used in order to carry out the experiment with the applied method (MathCad, MatLab, Mathematica, SMath Studio, Maple, Advanced Grapher, MasterGraph, etc.). "Reno Simulation" computer program is used in this experiment.

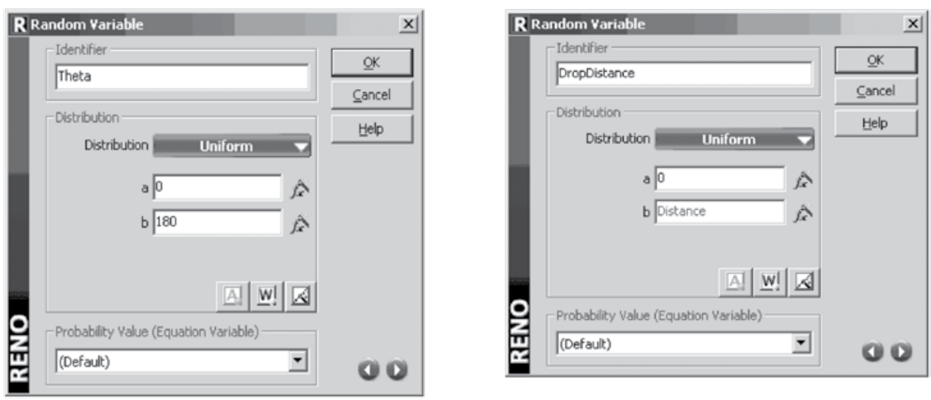


Picture 1. *Form of Buffon needle problem experiment*

Firstly, (d) distance between the parallel lines is determined. Then, the length of the needle is determined as (l). Thus, two fixed points necessary for the experiment are achieved.

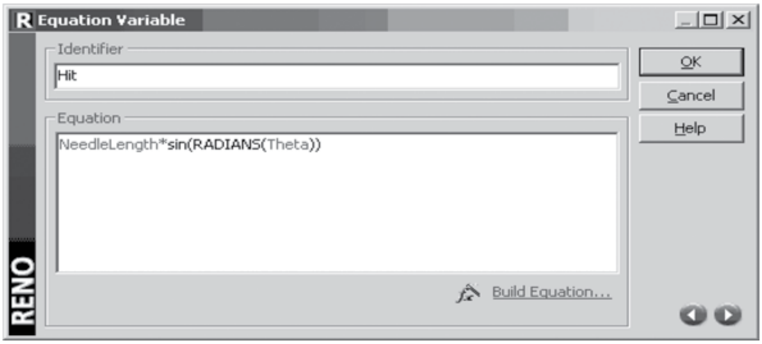
Picture 2. *First screenshot from Reno Simulation program*

Two variables are determined in order to define the distance of the perpendicular line on the left side of the needle (A) and the drop angle of the needle.

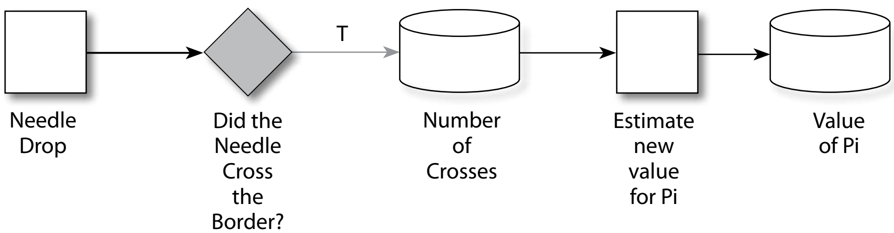


Picture 3. *Second screenshot from Reno Simulation program*

In order to determine the drop angle of I length needle, an equation is defined. If this definition is greater than A, this means that the needle intersects the horizontal line. Sinus and radian functions are determined in the program beforehand and can be noted as an equation. These functions can be used by clicking on the function icon in the menu.



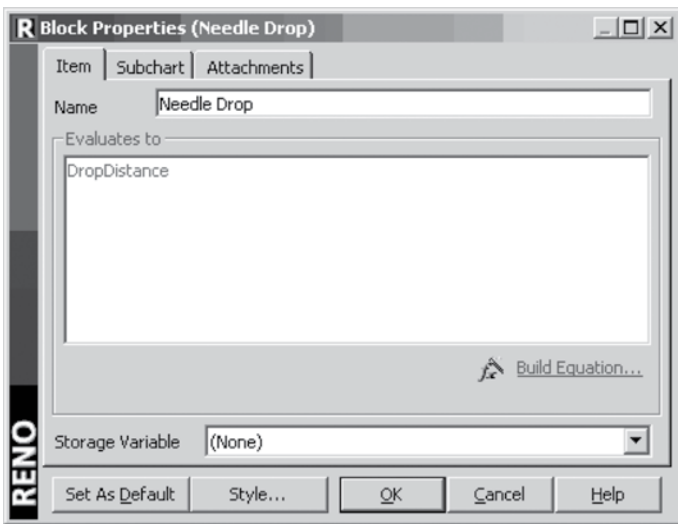
Picture 4. *Third screenshot from Reno Simulation program*



Picture 5. *The application algorithm of the experiment in the program*

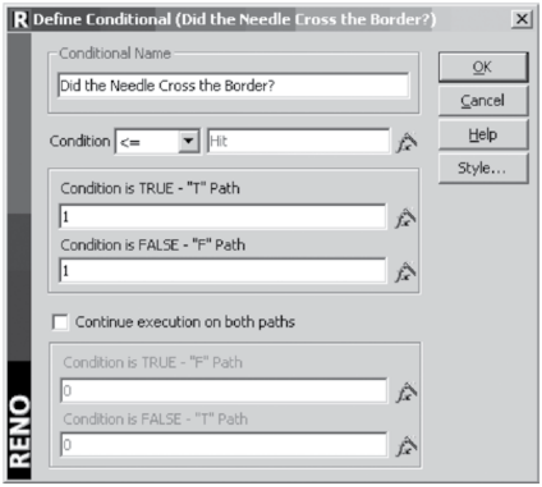
Inputting the Buffon's Needle Problem Parameters into the Reno Program

A term is used for the vertical distance from the position of the pinpoint based on a randomly produced variable to the point A (DropDistance).



Picture 6. *Fourth screenshot from Reno Simulation program*

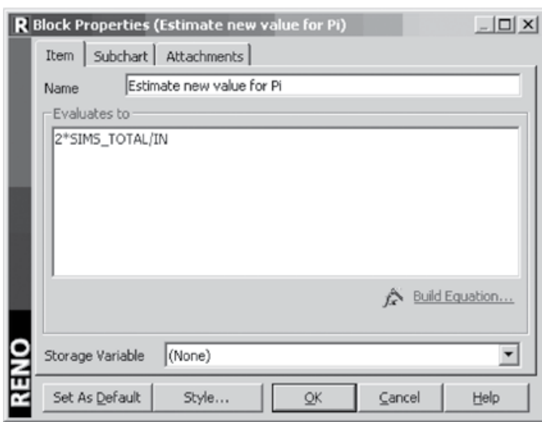
Conditional expression choices are used in order to identify if the needle intersects the parallel line. When needle-distance (determined by the equation variable named “Hit” **lsinθ**) is equal to or bigger than A distance, it intersects the vertical line. Number “1” the situation accepted as “Correct” in the program. The section “False” is chosen with the Tab key. As this situation is not used in the experiment, number “1” is marked in this space.



Picture 7. *Fifth screenshot from Reno Simulation program*

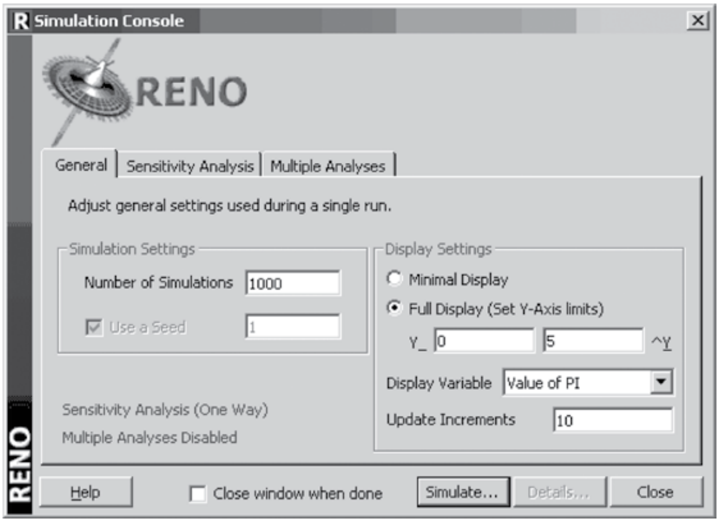
In order to save the total number of the repetitions of the needle’s intersection with the horizontal line, a result saving choice is used (M for approximately Π in Laplace equation).

Laplace equation is used in order to get closer to Π. The word IN in the equation stands for the M value in the previous situation and SIMS-TOTAL stands for the total number of simulations.

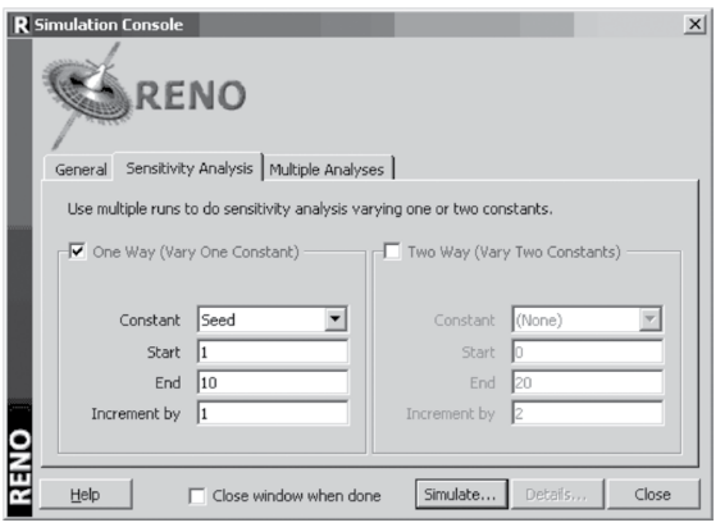


Picture 8. *Sixth screenshot from Reno Simulation program*

In order to save final results, another result saving should be used. Values are written according to the desired simulation numbers. In this example, it is mentioned that 1000 simulations will be made, so one apiece increment from 1 to 10 is made and diversified as is shown in the menu below. This is the result of the simulation of 1000 in 10 times. Each one is made by using randomly chosen numbers and every time, different results are obtained.

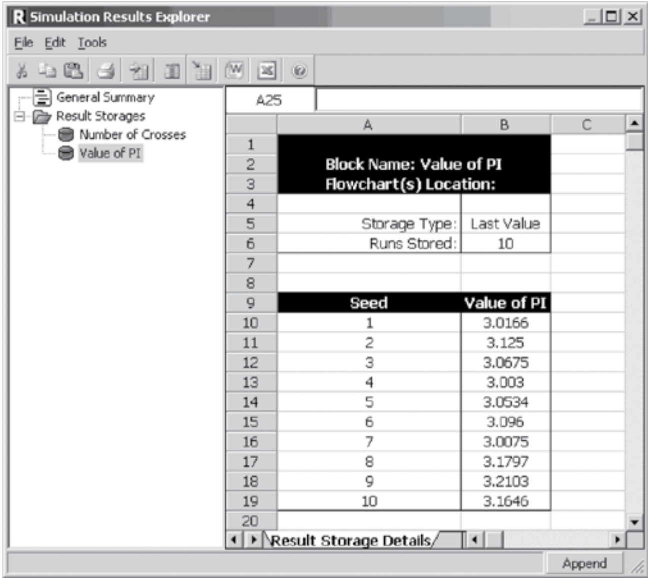


Picture 9. *Seventh screenshot from Reno Simulation program*

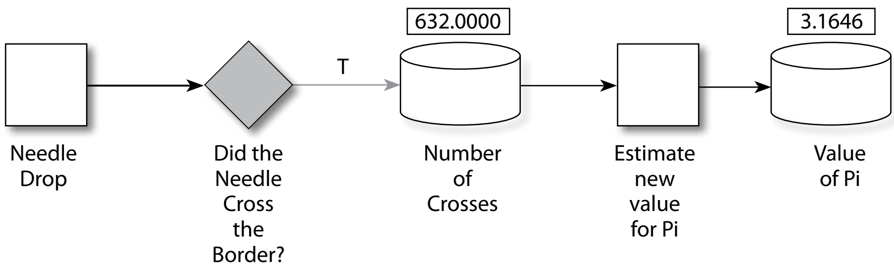


Picture 10. *Eighth screenshot from Reno Simulation program*

The results in the simulation result menu are shown below.



Picture 11. *Ninth screenshot from Reno Simulation program*



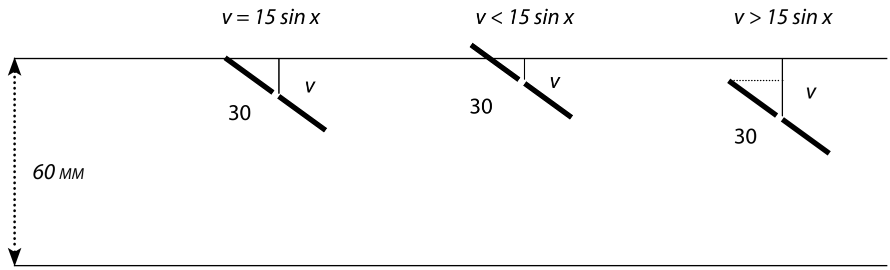
Picture 12. *Experiment's algorithm in the process of giving Π value*

The last one of the values in Picture 11 gives the appropriate Π value.

Method 2

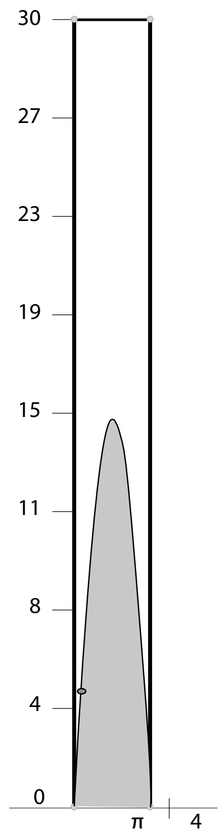
Number Π can also be defined by the ratio of the circumference and its diameter. However, second stage elementary education pupils can solve this problem using a different method. The middle point of the needle's distance to the close line is defined by y. The angle between the needle and the line is defined by x.

$$\begin{cases} Y \leq 15 \sin x; \\ 0 \leq x \leq \pi \end{cases}$$



Picture 13. *The inequalities for the problem*

The inequality system at the end of the "Needle intersects the straight line" positive situation constitutes $\mathbf{S_1}$ part.



Picture 14 . *The parts of* S_1, S_2

All the situation results under the same conditions create S_2 part defined by the inequality system.

$$\begin{cases} 0 \le y \le 30; \\ 0 \le x \le \pi. \end{cases}$$

$$S_1 = \int_0^{\pi} 15\sin x \, dx = 30$$

$S_2 = 30\pi$ is obtained.

The probability of the analyzed situation's occurrence is equal to

$$\frac{S_1}{S_2} = \frac{30}{30\pi} = \frac{1}{\pi}$$

Results and Suggestions

At the end of the experiment, it is found out that the instruction carried out by means of Buffon's needle problem is more effective than the instruction performed by means of the traditional method. Moreover, it develops pupils' attitude positively. As it has been two years since the probability topic was introduced into the school curricula in Russia, the perceptible projection of number Π on the screen after being solved in "Reno Solution" program has attracted pupils' attention more.

Secondary stage elementary education pupils' interest towards computer usage was significant, irrespective of the characteristic and difficulty of the problem. They used the computers consciously while solving the problems from their course books. They found reasonable evidence while trying to solve the problems. The program has attracted, not only the attention of pupils who were interested in computer assisted instruction, but also the pupils who were interested in the process itself.

Computer assisted problem-solving method increased the quality of information, helped and eased the understanding and perception of mathematics, and decreased teachers' class preparation and teaching time.

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Novi pristupi računalno potpomognutom podučavanju teme vjerojatnosti s pomoću Buffonova problema

Sažetak

Tema 'vjerojatnosti' nije uključena u osnovnoškolske silabe u Rusiji. No, kako se u Rusiji nedavno započeo provoditi bolonjski proces, rusko Ministarstvo znanosti planira uvesti određene promjene u obrazovni sustav. Temeljni cilj takvih promjena jest podučavanje teme 'vjerojatnosti' u osnovnim školama u Rusiji. Ta je tema uključena u silabe osmih razreda u turskom edukacijskom sustavu. U Rusiji se taj predmet, koji je uključen u silabe nekih škola, podučava jednu godinu. Cilj je ovoga istraživanja pokazati da će se taj predmet učinkovitije podučavati u ruskom i turskom obrazovnom sustavu ako se koristi računalno potpomognut sustav za razliku od podučavanja tradicionalnim metodama. Kako bi se ta tema analizirala, pokušalo se riješiti problem Buffonove igle u 'Reno Simulation' programu. Taj je simulacijski program izabran zbog toga što sadrži predmet vjerojatnosti. Program također olakšava analizu predmeta vjerojatnosti. Osim toga, može se reći da je Reno program, na neki način, simulacijski alat za događaje geometrijske vjerojatnosti te da predstavlja čvrstu i fleksibilnu platformu za simulacijske procedure. Program sadrži koncepte oprimjeravanja koji se mogu upotrijebiti kao temelji na kojima se mogu izgraditi različiti scenariji, a simulacije tih modela mogu se lako prikazati u kombinaciji s njihovim algoritmima. Za eksperiment je iz skupine primjera predmeta vjerojatnosti u „Reno Simulation" programu izabran problem Buffonove igle. Po završetku eksperimenta utvrđeno je da je računalno potpomognuto podučavanje teme vjerojatnosti s pomoću problema Buffonove igle učinkovitije od podučavanja tradicionalnim metodama. Uz pomoć računalno potpomognutog podučavanja ponašanje ruskih i turskih učenika bilo je pozitivno razvijeno.

Ključne riječi: *računalno potpomognuto podučavanje; osnovnoškolsko obrazovanje; matematika u obrazovanju; vjerojatnost*

Uvod

Tema vjerojatnosti nije bila uključena u osnovnoškolske silabe u Rusiji. No nedavno su se pojavili pokušaji da se tu temu uključi u silabe viših razreda osnovne škole. U tom bi kontekstu polaznici magistarskih i doktorskih programa trebali birati i učiti o podučavanju teme vjerojatnosti u osnovnoškolskom obrazovanju te razvijati nove pristupe podučavanju kako bi se osiguralo učeničko razumijevanje teme. Ruski profesori matematike Kolyagin Y. M., Merlina N. İ., Savvina O. A., Terentyeva L. P., Avteyeva T. K., Merlin A. V. koji unose prosvjetljenje u rusku edukaciju i nedavno preminuli Lukankin G. L. naglašavaju da se ta tema mora pomno analizirati (Kolyagin, Lukankin, Merlina, 2009, str. 3-4):

"Naše škole prolaze teško razdoblje; težak proces započinje za učitelje matematike. Inovacije u podučavanju matematike ne obuhvaćaju razlike u sadržaju podučavanja. Te su novine svakako važne, no najvažnije je vidjeti dublje promjene u pozadini te teme. Kako bi se postigle te važne promjene, trebamo proširiti svoje horizonte, detaljno usporediti prošle i sadašnje situacije, sjetiti se tradicija svojih nacionalnih škola i primijeniti novine u starim i modernim školama. Osim toga, posebnu pažnju treba posvetiti sljedećem pitanju: »Treba li inovacije organski primijeniti na tradicije ili su one u suprotnosti s tradicijama?« S druge strane, svatko bi također trebao odgovoriti na sljedeće važno pitanje: »Što je dobro i što je loše?«".

Dok je nova generacija otvorena prema promjenama, generacija iz vremena 1940.-1950. pokušava zaštititi tradicionalne metode (Kolyagin, Savvina, Tarasova, 2007).

Odluka o uvođenju tema statistike i kalkulusa vjerojatnosti u školske silabe iz matematike u Rusiji povećala je nadu za stvaranjem novog sadržaja metode podučavanja. Ta će odluka omogućiti stvaranje, razvoj i usustavljivanje misli o stohastičkoj strukturi događaja u našoj okolini. Podučavanje matematike, sa svojom ograničenom stohastičkom strukturom (ne odvojeni, već izolirani blokovi i sekcije), bit će pospješeni usklađenošću sa svim različitim odjelima i sekcijama (Kolyagin, Lukankin, Merlina, 2009).

Prva godina iskustva s procesom podučavanja s pomoću novog nastavnog materijala pokazuje da, usprkos nedovoljnom broju metodičkih udžbenika, tema vjerojatnosti nije bila učinkovito usklađena s ostalim satovima te tradicionalni sat matematike još uvijek nije povezan s novom metodom. Nastavna jedinica dodana je u program posljednje četvrtine nastavne godine (ruska nastavna godina podijeljena je u četvrtine). No, važno je ne samo uključiti temu u silab već također uspješno ponoviti temu na nastavnim satovima. Kako bi se to postiglo, novi sadržaj silaba treba biti usklađen sa starijim, pa bi se više trebao koristiti prethodno ignorirani potencijal teme vjerojatnosti. Sve pozitivne prilike trebale bi se iskoristiti kako bi se nova tema povezala s ostalim nastavnim satima i tako činila temelj tradicionalnim materijalima koji se u poduci koriste. Kao što se djelomično može vidjeti u problemu Buffonove igle, tema vjerojatnosti otkrit će uobičajene skrivene tajne matematike i proširiti njezine granice.

Prvo na što se pomisli kad se spomene računalno potpomognuto podučavanje jest da računala učitelju koriste kao sredstva podučavanja u nastavi, poput školske ploče, krede, ravnala, grafoskopa. U takvom se pristupu tehničke kvalifikacije računala koriste samo kao literali. No, računalno potpomognuto podučavanje je metoda u kojoj se računala upotrebljavaju u obrazovnom procesu kako bi se učenicima omogućilo da osvijeste svoje potrebe i rezultate svojega rada, da kontroliraju svoj osobni proces učenja s pomoću povratne informacije i da im se poveća zanimanje za nastavu s pomoću grafičkih prikaza, zvuka, animacija i crteža (Baki, 2002).

Računalno potpomognuto podučavanje je metoda kojoj je cilj obrazovanje učenika putem izravne prezentacije nastavnih sadržaja, ponavljanjem naučene informacije uz pomoć ostalih tehnika, rješavanjem problema i praktičnim zadacima. Osim toga, metoda se koristi kao pomoćno sredstvo u procesima učenja i podučavanja sličnih aktivnosti (Odabaşı, 2006).

U sljedećem popisu navedene su prednosti računalno potpomognutog učenja.

Osigurava odvijanje primjerena nastavna procesa u razredu tako što učenicima omogućava dodatno vrijeme i pruža im mogućnost kontroliranja broja i rezultata informacija koje uče.

Moral učenika raste kad im se brzo pruži uvid u točnost njihovih odgovora.

Programi osiguravaju pozitivniju obrazovnu okolinu, a osobito djeci koja uče sporo. Kako njihove greške ne vidi cijeli razred, ne osjećaju se postiđenima.

Računalno potpomognuto učenje učinkovito je u pomoći učenicima koji imaju poteškoće u učenju, učenicima iz različitih etničkih skupina i učenicima s teškoćama u razvoju.

Raznobojna, uglazbljena i animirana grafika unose autentičnost i posebnost u predmet.

Mogućnost računalnog spremanja podataka omogućuje individualno učenje, a učenički se razvoj može pregledati ako učenici pripreme osobne upute.

Računala imaju baze podataka primjerene razvijanju informacija.

Računala mogu koristiti sve informacije o grafici, tekstovima, zvukovima i slikama.

U računalima se može pohraniti puno informacija koje učitelji mogu koristiti; učenici stječu sposobnost samostalnog učenja. Osim toga, s pomoću računala mogu se koristiti različite metode. U takvim iskustvima učenja koriste se razne obrazovne metode.

Računala osiguravaju pouzdane i primjerene strategije podučavanja koje su neovisne o učitelju, vremenu i mjestu.

Računalno potpomognuto obrazovanje povećava učinkovitost podučavanja i tako pospješuje učenički uspjeh. Takva vrsta obrazovanja također osigurava dostatnost koja donosi postizanje ciljeva u kraćem vremenu, uz manje ulaganja.

Dostatnost je vrlo važna u poslovnom životu i privredi, a njezina važnost u obrazovanju raste iz dana u dan.

Pojava sustava koji su jednostavni za korištenje omogućila je nekim edukatorima da stvore i razviju svoje osobne edukacijske programe (Earged, 2002).

Posljednjih su godina različiti autori naglašavali da moderne obrazovne metode olakšavaju poučavanje, a time se i poučavanje može učiniti privlačnijim. Nedavno su poduzete važne mjere imajući na umu navedeni cilj.

Računala koja su postala važan dio naših života važni su dijelovi obrazovnih sustava i edukacijske okoline učenja i podučavanja, kao dio činjenice o tome koliko informacijsko-komunikacijske tehnologije utječu na ljudski život (Çiftçi, 2006).

Učitelji, uz razred i školsku ploču, čine važan dio uobičajenog nastavnog procesa. U današnjem obrazovnom sustavu tradicionalna se školska ploča zamjenjuje interaktivnom pločom; kreda se zamjenjuje flomasterom. Nastavni je sat u klasičnom podučavanju jedinice u kojoj se izlažu svi važni pojmovi. Predavač predstavlja dane teme u razredu u okviru određene strukture. Prezentacija, odnosno izlaganje tema je sinkronizirano; definitivni vremenski okviri u planu izlaganja su podijeljeni, primjerice u 3 nastavna sata u četvrtak. Druga važna karakteristika u predstavljanju gradiva jest govor predavača, odnosno njegov čin prenošenja informacija o temama slušateljima govorenjem. Proces prenošenja informacije govorenjem može imati sporu podršku u okolini koju predstavlja tradicionalna školska ploča te se tako teme i koncepti mogu gledati korak po korak, ili se može ostvariti brža podrška s pomoću grafoskopa, video-priloga, projekcija, računala ili elektroničkih „školskih ploča". Studenti vode bilješke tijekom slušanja i svaki student bilježi informacije na način koji mu/joj olakšava razumijevanje i personalizaciju. Stoga je dinamika podrške za vrijeme izlaganja važna (Çağlayan, Ufuk 2001).

Smanjenje cijena računala i povećanje u aspektima i mogućnostima računala populariziralo je njihovu uporabu u društvu pa se danas računala koriste za različite svrhe. To je uzrokovalo češću uporabu računala u obrazovanju (Gürbüz, Eryılmaz, Yıldırım, Soner, 2001).

Najvažniji čimbenik koji omogućuje učenicima postizanje uspjeha u matematici uz pomoć računala je korišteni softver. Većina programa uzrokuje pasivno sjedenje ispred računala, ali utječe na učenike i putem slušnih i vizualnih efekata. Korisnici mogu odgovarati na pitanja s višestrukim odgovorima i dobiti brzu povratnu informaciju o svojim rezultatima i uspjehu. Učenici također imaju priliku vratiti se na prethodna objašnjenja (Jinich, 1986).

Tijekom procesa podučavanja nestandardnih problema predavač-učenik-računalo trebaju biti dobro organizirani. Ako postoje problemi u računalnoj podršci, oni se trebaju unaprijed proučiti zbog čega je potrebno pripremiti računalno potpomognut algoritam podučavanja. U računalnim je učionicama potrebno imati instalaciju računalnog programa i, najvažnije od svega, učenici trebaju biti motivirani za rad.

Poznato je da se tema „vjerojatnosti" podučava na satu matematike u 8. razredu. Kako bi se osiguralo razumijevanje nastavne lekcije i nepristranost informacije, upotrijebljene su različite nastavne metode. Primjerice, spominje se da će učenici bolje razumjeti i naučiti temu vjerojatnosti ako se podučava Grafova teorija (Seyhanlı, 2007).

Broj Π stoljećima je privlačio pažnju matematičara, ali i nematematičara. To je bila posljedica promjene broja Π u decimali. S druge strane, taj se broj nalazi u različitim poljima matematike kao što su geometrija, izračunavanje vjerojatnosti, diferencijalni i integralni izračuni. Svim je matematičarima poznat tajnovit karakter toga broja u svim njegovim detaljima. Pronaći ćemo broj Π uz pomoć jednostavnog eksperimenta koji je u literaturi poznat kao problem Buffonove igle. Potom ćemo pokazati aspekte tog tajnovitog broja i da se tema vjerojatnosti može podučavati putem računalno potpomognutog obrazovanja.

Grafika, dijagrami, različita geometrijska tijela i modeli predstavljaju sredstva za vizualizaciju misli, ideja i apstraktnih koncepata. S pomoću tih sredstava ljudski um povezuje vanjski svijet i apstraktne koncepte. Naime, prezentacija algebarskih struktura s pomoću geometrije pomaže u tome da se prikaže učenicima kako se razumna teorija izgrađuje iz fizičkog modela (Konyalıoğlu, 2003).

Apstraktni matematički koncepti mogu postati konkretni korištenjem nekih matematičkih programa koji se primjenjuju u skladu sa sadržajem problema. Tako se problem Buffonove igle, kao problem geometrijske vjerojatnosti, rješava grafičkim crtežima u obliku iteracije i geometrijskim tijelima koja se sastoje od nasumičnih ravnih linija ili sjecišta.

Buffonova teorija temelji se na tome sijeku li igle ravne crte ili ne sijeku. Ako igla koja je bačena N puta presiječe crtu barem jednom u R odskoku, vjerojatnost je P=R/N. Ako je igla bačena N puta i ostvari se barem jedan presjek, vjerojatnost je P=R/N. Postoji i nekombinirana evaluacija distribucije P, P/N (1-P), zbog toga što postoji mogućnost provođenja eksperimenata koji otprilike daje rezultat Π. Na taj se način provode mnogi eksperimenti. Rezultati se broje, računaju i raspravljaju u zanimljivoj Gridgemanovoj studiji (1960).

Cilj istraživanja

Cilj je istraživanja bio pokazati da će podučavanje teme vjerojatnosti, koja se podučava u drugom stupnju osnovnoškolskog obrazovanja,uporabom računalno potpomognutog sustava i problema Buffonove igle, za razliku od podučavanja tradicionalnim metodama, rezultirati drugačijim rješenjima za Pi.

Metoda

Problem Buffonove igle; pojavljivanje broja Π na kraju rješavanja problema vjerojatnosti vrlo je zanimljivo. Bilo tko može provesti taj eksperiment i može se dati procjena za Π. Problem Buffonove igle spominje se u gotovo svim knjigama o vjerojatnosti i statistici (Morgan, Kennedy, 1992).

Način provođenja eksperimenta problema Buffonove igle

Odabire se igla prosječne veličine (30 mm čavao ili kemijska olovka) i na papir se ucrtavaju paralelne ravne crte dva puta veće od igle. Igla se nasumično baca na papir te se bilježi situacija koja je rezultat bacanja igle. Izrađuje se tablica čestotnosti dvaju mogućih situacija: „Igla siječe crtu“ i „Igla ne siječe crtu“. Potom se računa čestotnost vjerojatnosti situacije „Igla siječe crtu“ pa se bilježi suprotna vrijednost od vrijednosti dobivenoga rezultata. Zatim se rezultati uspoređuju s rezultatima eksperimenata koje provode učenici.

Po završetku tog procesa izrađuje se tablica kombiniranih rezultata u kojoj se prikazuju rezultati nekoliko razreda (ako je to moguće) koji se prikupljaju od svih učenika u razredu. Učenici će primijetiti da je čestotnost situacije „Igla siječe crtu" dva puta veća od čestotnosti situacije „Igla ne siječe crtu". Naime, rezultati tih situacija su ili jednaki ili su im vrijednosti približne veličine.

Bacajući igle uz pomoć računala, 10.000, 20.000, pa čak i 1.000.000 puta na sličan način, učenici mogu vidjeti hoće li igla sijeći paralelne crte. Na kraju će učenici uočiti sljedeće pravilo: po završetku mnogih bacanja, obratna vrijednost od čestotnosti situacije „Igla siječe crtu" je gotovo 3,14.

Pogledajmo neke detalje eksperimenata koji se provode u okviru klasične nastavne metode. U tim su eksperimentima duljine igle izražene kao razmaci između crta.

Tablica 1.

Broj Π postaje lakše objašnjiv kad se učenici susretnu s geometrijskim vjerojatnostima, sinusoidom i integralima u narednim godinama.

Sredstva prikupljanja podataka

Metoda 1

Različiti računalni programi koriste se s ciljem provođenja eksperimenta primijenjenom metodom (MathCad, MatLab, Mathematica, SMath Studio, Maple, Advanced Grapher, MasterGraph itd.). U ovome je eksperimentu upotrijebljen „Reno Simulation" računalni program.

Slika 1.

Najprije se određuje udaljenost (d) između linija. Zatim se određuje duljina igle (I). Na taj su način određene dvije točke potrebne za eksperiment.

Slika 2.

Dvije su varijable utvrđene s ciljem definiranja udaljenosti okomite crte s lijeve strane igle (A) i kuta pada igle.

Slika 3.

Kako bi se izračunao kut pada i duljine igle, definirana je jednadžba. Ako je ova definicija veća od A, to znači da igla siječe vodoravnu crtu. Funkcije Sinus i Radijan prethodno su određene u programu i mogu se ispisati kao jednadžba. Te funkcije mogu se upotrijebiti klikom na funkcijsku ikonu u izborniku.

Slika 4.i 5.

Unošenje parametara problema Buffonove igle u Reno program:

A označava okomitu udaljenost od položaja vrška igle na temelju nasumično proizvedene varijable do točke A (UdaljenostPada).

Slika 6.

Kako bi se utvrdilo siječe li igla paralelnu crtu, upotrijebljene su mogućnosti izraza kondicionala. Kad je udaljenost igle (koja je utvrđena varijablom jednadžbe označene s „Hit“ **lsinθ**) jednaka ili veća od udaljenosti A, tad ona siječe okomitu crtu. Broj „1“ označava situaciju koja je u programu prihvaćena kao „točna“. Sekcija „netočno“ odabire se tipkom Tab. S obzirom na to da ta situacija nije upotrijebljena u eksperimentu, u prazninu je upisan broj „1“.

Slika 7.

Kako bi se sačuvao ukupan broj ponavljanja pri kojima je igla presjekla vodoravnu crtu, upotrijebljena je mogućnost pohrane rezultata (M za približno Π u Laplace jednadžbi).

Laplace jednadžba upotrijebljena je kako bi se približilo vrijednosti Π. Riječ IN u jednadžbi koristi se za M vrijednost u prethodnoj situaciji, a SIMS-TOTAL označava ukupan broj simulacija.

Slika 8.

Kako bi se sačuvali konačni rezultati, potrebno je pribjeći drugačijem spremanju rezultata. Vrijednosti su ispisane kao željeni brojevi simulacije. U ovom primjeru spomenuto je da će se ostvariti 1000 simulacija, tako da je načinjeno jedno po jedno povećanje od 1-10 pa je preinačeno kao što je prikazano u izborniku. To je rezultat simulacije 1000 u 10 puta. Svaka je provedena uporabom nasumično odabranih brojeva pa su svaki put dobiveni različiti rezultati.

Slika 9. i 10.

Niže su prikazani rezultati u izborniku rezultata simulacije.

Slika 11. i 12.

Posljednja vrijednost na slici 11. daje primjerenu vrijednost Π.

Metoda 2

Broj *Π* se također može definirati omjerom obujma i njegova promjera. No učenici viših razreda taj problem mogu riješiti uporabom drugačije metode. Srednja točka udaljenosti igle od crte definirana je kao y, a kut između igle i crte definiran je kao x.

Slika 13.

Sustav nejednakosti na kraju pozitivne situacije „Igla siječe ravnu crtu” sadrži dio $\mathbf{S}_1$.

Slika 14.

Rezultati svih situacija u istim uvjetima čine dio S_2, definiran sustavom nejednakosti.

$$\begin{cases} 0 \le y \le 30; \\ 0 \le x \le \pi. \end{cases}$$

Tako je dobiveno

$$S_1 = \int_0^{\pi} 15 \sin x \, dx = 30$$

Vjerojatnost pojave analizirane situacije jednak je

$$\frac{S_1}{S_2} = \frac{30}{30\pi} = \frac{1}{\pi}$$

Rezultati i prijedlozi

Na kraju eksperimenta utvrđeno je da je podučavanje s pomoću problema Buffonove igle učinkovitije od podučavanja tradicionalnom metodom i da se tako razvijaju pozitivni stavovi učenika. Kako je prošlo dvije godine otkad je tema vjerojatnosti uključena u školske kurikule u Rusiji, pažnju učenika više je privukla zamjetna projekcija broja Π na ekranu nakon što je problem riješen „Reno Solution" programom.

Učenici drugog stupnja primarnog obrazovanja bili su vrlo zainteresirani za uporabu računala bez obzira na osobine i kompleksnost problema. Svjesno su se koristili računalima tijekom rješavanja problema zadanih u udžbenicima. Pronašli su razumne dokaze dok su pokušavali riješiti problem. Program je privukao ne samo pažnju učenika koji su zainteresirani za računalno potpomognuto podučavanje već i učenika koji su bili zainteresirani za opisani proces.

Računalno potpomognuta metoda rješavanja problema povećala je kvalitetu informacije, potpomogla i olakšala učeničko razumijevanje i viđenje matematike i skratila vrijeme koje su učitelji proveli na pripremu za sat i podučavanje.