Designing a Forest Road Network Using Mixed Integer Programming

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Abstract – Nacrtak

Forest roads are an essential yet costly part of forest management, and optimization methods are important tools for planning road systems to support harvesting. This paper presents a Mixed Integer Programming (MIP) optimization model to design a forest access system consisting of logging roads for trucking and access spurs for skidding. The network designed is hierarchical in the sense that the two transport systems require significantly different road standards, and timber may only be transferred from access spurs to forest roads. All timber must be transported from harvest sites to exit nodes that connect the forest road network to public roads. A dense network of potential connections is formed by overlaying a regular grid onto the forest, and then calculating costs of inter-node connections using GIS topographical data. Feasible arcs thus determined are input to the optimization model. The model minimizes total cost of road construction and maintenance, skidding and whole transportation in forest. It can be used to develop road system alternatives to support the process of planning the total access system. The model performance is explored on a study area in a mountainous region, where a persistent access network for partial harvesting is required. High quality solutions were achieved in reasonable computational time.

Keywords: optimization, forest harvesting cost, forest road network designing, access spur, harvest access planning

1. Introduction – *Uvod*

Harvesting is one of the most important forest activities, for which purpose thousands of kilometers of roads and access spurs are constructed and millions of dollars are spent annually. Access roads may be used for other forestry, development and construction purposes, such as fire protection, silviculture, or recreational use. Nonetheless, their main economic purpose is for the extraction of timber. These roads have a large construction cost as well as maintenance costs (Najafi et al. 2008). It is thus important to develop methods to design road systems that carry out this purpose at minimal cost and that also minimize negative impacts such as erosion and water sedimentation.

Network design models can help forest management planners choose appropriate placement, design standard, and construction period for road segments. Such models are characterized by construction and transport decisions chosen from amongst a finite number of feasible road projects and a finite number of landing points for timber. Road costs, capacities, requirements for timber extraction, linkages between the forest road network and public roads that link to customers, and defined potential transportation systems grouped by type and their road standard requirements are the inputs used to form problem constraints. Depending on the situation, the harvesting sites may be determined a priori, or the choice of where to harvest may be included in the optimization model along with the road network design decisions.

To create a forest road network design model, one must first systematically discretize the landscape to design a finite number of construction decisions, or potential road segments. These functions are typically accomplished with a Geographical Information System (GIS) by overlaying a grid of road intersection points, harvest location points, and extraction points on the landscape. From these, a network of potential arcs for road placement is created. Second, one must acquire information about the forest topography, such as slope, soil type and ground bearing strength, to determine feasibility of placing roads on the network arcs, and to estimate costs. Furthermore, volume of cut at each harvest site and skidding cost on each link must be estimated. Again, this information is usually supported by a GIS system. To explore feasible decisions thoroughly, the point grid and consequently the network of potential arcs should be dense so as not to preclude good opportunities. An optimization model will then be formulated to minimize total network costs, with constraints to ensure network connectivity, access to all harvesting sites, and optimal routes to exit points.

This problem is related to the integrated harvest and road construction problem, an important tactical planning problem that arises when cutting and road construction schedules are integrated over a medium-term planning horizon. If the road network has already been designed, the problem is to optimally schedule harvest and construction activities to create a plan that is cost-efficient and meets timber flow requirements. On the other hand, if a harvest schedule is first fixed, the problem is to design an optimal road network system to provide access to harvesting sites and routes to extract timber to main highways. In the case of single entry clearcutting, it is better to include both harvesting and road construction in an integrated model, since the capital cost of road construction is a significant factor in these problems. For example, Jones et al. (1986) showed that integrated models, which included both harvesting and road construction decisions in an integrated model, produced plans with costs reduced by as much as 30% over those that included only harvesting.

Solutions to the harvest-scheduling problem and road construction problem have been found using two strategies: exact mathematical optimization using Mixed Integer Programming (MIP) and heuristic solution methods. One of the earliest harvesting, road construction and transportation models is the Integrated Resources Planning Model (IRPM) presented by Kirby et al. (1986). This is an MIP optimization model to simultaneously select road construction and harvest projects, while minimizing costs of road construction and transportation of timber products. Guignard and Yan (1993) used Lagrangian substitution schemes to solve the IRPM. Subsequently, Guignard et al. (1998) improved the model by adding logical inequalities, lifting of inequalities and careful selection of Branch-and-Bound branching priorities based on »double-contracting« variables.

The appeal of exact methods such as integer programming is that their solutions are optimal, but the disadvantage is that problem size is limiting. In its original form, and using the 1980's computer hardware and branch-and-bound technology, the MIP approach was considered to be suitable for only small problem instances. Jones et al. (1986) and Kirby et al. (1986) tested the IRPM integer programming models and concluded that they were capable of solving only modest-sized problems. Even with the improvements of Guignard et al. (1998), time to solve realistic problem instances was prohibitive. Accordingly, many researchers have applied heuristic methods, which seek to approximate an optimal solution at a reasonable computational cost, to these problems. For example, Weintraub et al. (1995) created an iterative procedure with linear programming and heuristics to solve harvest and road construction problems. They relaxed the integrality constraints on road construction variables, and applied rounding heuristic rules to fractional variables in the solution. Clark et al. (2000) developed a three-stage heuristic to solve the harvest and road construction problem. Others have used various meta-heuristics such as genetic algorithms, evolutionary programs, tabu search, and simulated annealing to address similar forest management problems. Lu and Eriksson (2000) used a genetic algorithm to form harvest units, and Falcao and Borges (2001) used evolution programs to generate harvest schedules. Richards and Gunn (2003) used tabu search to solve a forest management problem that includes spatial constraints for maximum opening size, adjacency delay (green up), and road construction decisions. They used parameterized weighting of objective function penalty components to develop a trade-off frontier between lost forest productivity due to timing of harvests, and capital cost of road construction.

Some methods to design forest road networks using GIS and heuristic solution methods have been published. Anderson and Nelson (2004) developed a computer algorithm to generate road networks under a variety of assumptions related to road design standards. Their method mimicked the processes an engineer might use to design a road network to access a number of stands, and does not optimize harvest timing or construction decisions. In contrast, the NETWORK 2001 method of Chung and Sessions (2001) uses metaheuristics (genetic algorithms and simulated annealing) to optimize road networks that access a given set of stands and uses digital terrain data to estimate costs of constructing various road links. They also, as in this work, deal with two road standards, with spur (skidding) roads for each stand. Their method designs the main road routes (trucking standard roads) using a genetic algorithm (GA) first, then adds spur roads as branches using simulated annealing (SA). Heinimann et al. (2003) and Newnham (1995) are two other examples of methods to create a road network using digital terrain data and a dense network of potential arcs, using a heuristic method.

Despite the documented difficulty of solving these and other spatial forest planning models using integer programming, there has recently been a recurrence of interest in MIP. Crowe et al. (2003), Goycoolea et al. (2005), and Gunn and Richards (2005) have re-visited the adjacency problem in forest harvesting using integer programming models with new formulations. In addition to developing improved formulations for these models, these researchers have found that commercially available solvers have been much improved, and that problems which were previously intractable are now solvable using modern solvers with sophisticated branch and cut methodology. Their work has motivated us to develop an MIP model to design a transportation network of road and skidding trails for a forest to be harvested using multiple entry selection. This paper presents a rudimentary Mixed Integer Programming (MIP) model that is able to design an access system consisting of roads and access spurs for skidding, and which minimizes the total cost of road construction, transportation costs, and skidding costs.

2. Methods – *Metode*

The forest management problem we address is to design a least cost access network for forests to support continuous selective harvesting system. Cutting sites are pre-determined, and they are selection harvested on a fixed return cycle (10 years). Timber from each harvesting site is extracted using skidding systems to the banks of a spur road, and then transported to associated highway road systems using trucks. Since selective harvesting in a number of sites will persist over time, the access network must be entirely constructed during first 10 years and remain in place.

Skidding trails are constructed using cut and fill without paving. Their maximum allowable gradient is 25% and trail width is 4 meters. Roads are constructed to a much higher standard, where cut and fill beds are appropriately drained and paved with three compacted layers. Their maximum allowable gradient is 10%, and road width is 5.5 meters. The higher quality roads have a significant construction cost but lower unit transportation cost when compared to skidding trails.

The two transportation systems are hierarchical. By this, we mean that skidding trails are not suitable for trucking, and that timber is never shifted from the higher level trucking system to the lower level skidding system. The input network of potential skidding trails and road segments is a very dense system of directional arcs. We also note that performance and applicability of this network design model may not be generalized to situations where access requirements are time dependent. For example, when single-entry

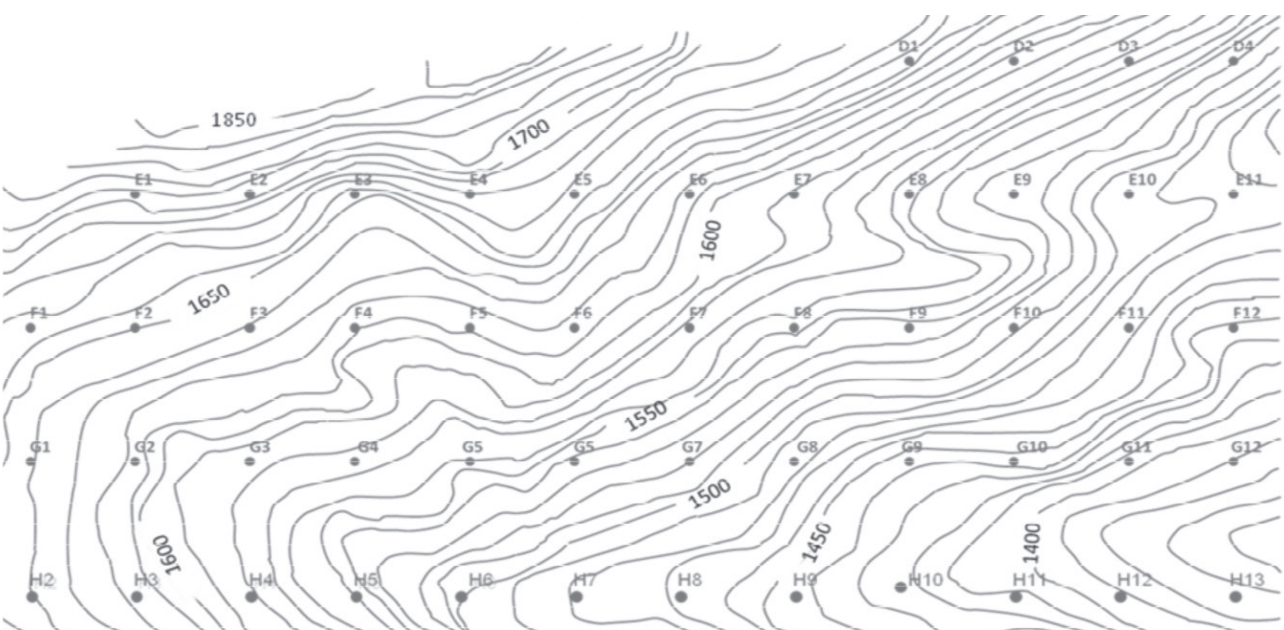

Fig. 1 Systematic nodes placed on the forest map
Slika 1. *Prostorni razmještaj mreže mjernih točaka*

clearcut harvesting is used, the road network need not be permanent, and it need not be constructed at once. The network design problem then becomes a multi-period decision situation, which requires much larger models and is likely to be less »MIP-friendly« than the single period case.

The optimization problem is formulated, using an MIP model, to choose a least cost transportation network that supports all harvesting activities and the two transportation systems. Its objective is to minimize total cost of road construction, maintenance costs, transportation costs, and skidding costs.

2.1 Mathematical Model Definition – *Definicija matematičkoga modela*

To define the network design model, 298 nodes are placed in the center of each real harvesting area and located on the forest map (Fig. 1).

Each node has a known volume of timber cut that is to be extracted in each period. These real world data were already collected by forest inventory group. In this model, each node is defined with two arguments, *i* and *j*, indicating the horizontal axis, or columns, and the vertical axis, or rows, respectively (Fig. 2). Additional nodes that are potential road intersections are then added. The distribution of the nodes over the forest can be done in a systematic or a random way. The examples presented in this paper are based on a systematic placement of nodes on the forest map. Then, potential road segments are defined. For each node, a fixed number of arcs to near nodes are mapped, and their construction costs calculated using digital terrain data. Thus, all potential transportation decisions are based on directed arcs that connect nodes. Subject to terrain restrictions, each arc may be used as a skidding trail, or constructed to a road standard, or omitted from the solution altogether. In this work, there are potentially 20 adjacent nodes to each node, and hence as many as 20 outgoing and 20 incoming arcs at each node, creating a high density design network (Fig. 2). Finally, exit nodes are chosen, or added if necessary, to identify the connection(s) between the forest road system and the public highway systems.

The objective is to minimize the sum of road network, timber transportation, and skidding costs. Road network costs consist of road construction costs based on the segment length, terrain conditions, and road standard, as well as road maintenance costs which are calculated based on traffic volume over each segment, its length, and its characteristics. Transport costs along each road segment are based on its unit cost of transportation and volume transported. Unit costs vary according to the characteristics of individual road segments. Similarly, cost of skidding timber on access

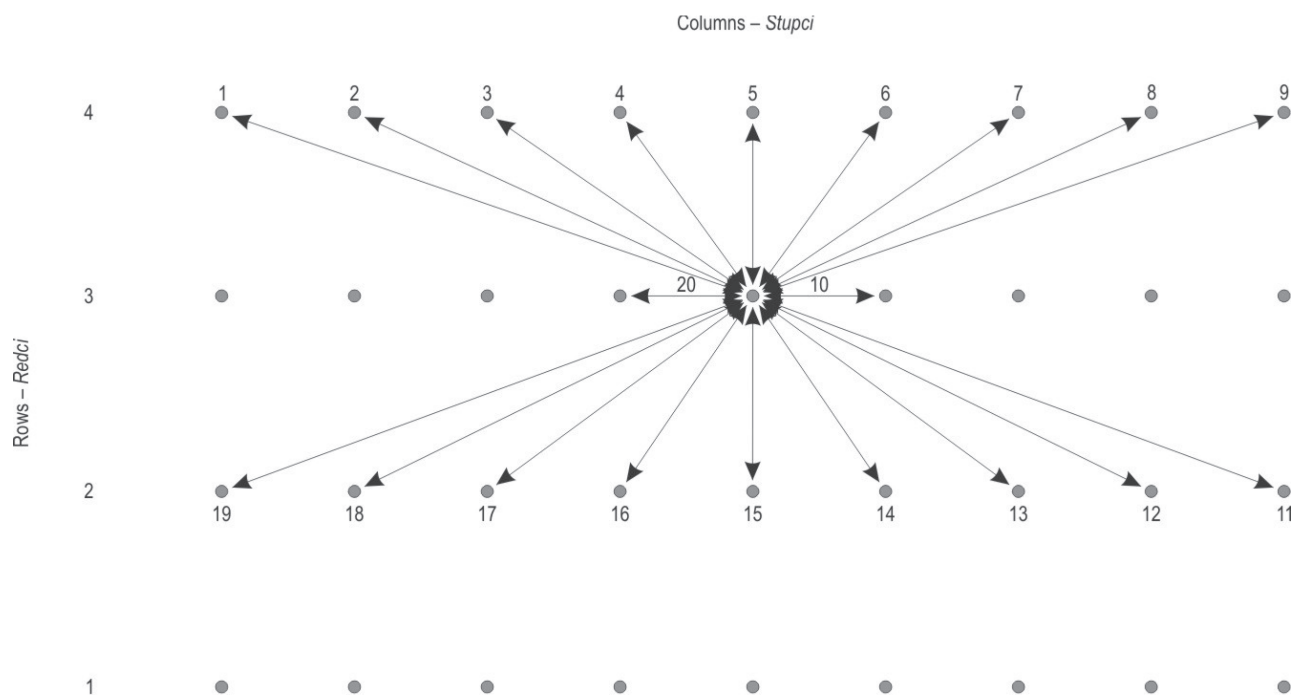

Fig. 2 Outgoing links at a systematic point
Slika 2. *Veze među mjernim točkama*

spurs is based on cost and hourly production of skidders, volume skidded and unit cost on each segment.

The objective function is to minimize total costs:

$$\min \sum_{roads(i,j)} BC_{ij} \times B_{ij} + \sum_{arcs(i,j)} (TC_{ij} \times TV_{ij} + SC_{ij} \times SV_{ij} + MC_{ij} \times TV_{ij}) \quad (1)$$

Where:

roads(i,j) means the set of all undirected segments connecting node i and node j,

arcs(i,j) means the set of all directed segments connecting node i to node j.

Coefficients and constants:

- V Total volume of timber cut in the forest,
- C Capacity of a skidding spur,
- D_n Volume to be left at demand exit node n (equals zero if n is not an exit node),
- Vol_n Amount of timber generated in node n,
- SC_{ij} Unit cost of skidding on arc (i,j),
- TC_{ij} Unit cost of trucking on road (i,j),
- BC_{ij} Capital cost to construct road (i,j),
- MC_{ij} Unit cost of maintenance for road (i,j).

Binary Decision Variables:

- B_{ij} Equals 1 if the segment connecting nodes i and j is built as a road, 0 otherwise,
- S_{ij} Equals 1 if timber is skidded on an access spur between node i and node j, 0 otherwise.

Continuous decision variables:

- SV_{ij} Volume of cut carried on arc ij through access spur k (skidded volume),
- TV_{ij} Volume of cut carried by trucking on arc (i,j).

Network Flow Variables:

- SVI_n Volume skidded into node n from adjacent nodes,
- SVO_n Volume skidded out of node n to adjacent nodes,
- TVI_n Volume trucked into node n from adjacent nodes,
- TVO_n Volume trucked out of node n to adjacent nodes,
- $SVLeft_n$ Timber volume skidded into node n that is left to be trucked on roads exiting node n.

Constraints:

$$R_{ij} + S_{ij} \le 1, \forall \text{ undirected arcs } (i,j) \quad (2)$$

Constraint 2 means that a link can be unused, a road, or an access spur. It cannot be used for both skidding and trucking.

$$TV_{ij} + TV_{ij} \le V \times B_{ij}, \forall \text{ undirected arcs } (i,j) \quad (3)$$

Constraint 3 links the binary variable B_{ij} to TV_{ij} and TV_{ji}, so that if any trucking is carried out on road (i,j), then B_{ij} must be equal one.

$$SV_{ij} \le C \times S_{ij}, \forall i,j \quad (4)$$

Constraint 4 links the binary variable S_{ij} to SV_{ij} so that if SV_{ij} is > 0, then S_{ij} must be equal to 1.

$$SVI_n = \sum_{(j,n) \text{ in arcs}} \times SV_{jn} \quad (5)$$

$$SVO_n = \sum_{(n,j) \text{ in arcs}} \times SV_{nj} \quad (6)$$

$$TVI_n = \sum_{(j,n) \text{ in arcs}} \times TV_{jn} \quad (7)$$

$$TVO_n = \sum_{(n,j) \text{ in arcs}} \times TV_{nj} \quad (8)$$

Constraints 5, 6, 7, and 8 simply calculate volumes skidded and trucked in and out of each node.

$$SVI_n + TVI_n + Vol_n = SVO_n + TVO_n + D_n \quad (9)$$

Constraint 9 is the standard flow conservation equation. Volume entering a node plus timber volume generated at a node must equal the volume leaving plus any demand volume left.

$$D_n = 0 \ \forall \text{ interior nodes } n \quad (10)$$

D_n must be equal to zero if n is an interior node in the network, but may be positive if n is an exit node.

$$SVI_n + Vol_n = SVO_n + SVLeft_n \ \forall \text{ interior nodes } n \quad (11)$$

$$TVI_n + SVLeft_n = TVO_n \ \forall \text{ interior nodes } n \quad (12)$$

Constraints 11 and 12 maintain the integrity of timber product transportation by the different systems. Timber can be skidded from one arc to the next, but timber cannot be carried on a road link, off-loaded, and then skidded along the next link.

$$\sum_{\{e\}} D_e = V, \text{ where } \{e\} \text{ is the set of all exit nodes} \quad (13)$$

Equation 13 ensures that all timber from harvesting sites is transported to exit nodes.

Equations 1–13 define the network flow model with hierarchical constraints. This formulation was improved by adding trigger constraints. These constraints are redundant in the sense that they do not change the optimal integer solution to the model. They provide improved bounds and performance in the branching used by the mixed integer solver (Guignard et al. 1998).

First, constraints are added to trigger construction of an existing road segment if there is volume skidded to the node and left for trucking.

$$SVLeft_{n} \leq \sum_{(n,i)} V \times B_{ni}, (n,i) \text{ means all undirected arcs that are incident on node n.} \tag{14}$$

Second, if there is volume to be harvested and transported at a node, then at least one skid spur or road that is incident to it must be built.

$$\sum_{(n,i)} (B_{ni} + S_{ni}) \geq 1, \forall\, n \text{ with } V_{n} > 0 \tag{15}$$

Third, no isolated road segments should be built.

$$B_{ij} \leq \sum_{(j,k)} B_{jk} + \sum_{(i,n)} B_{in}, \text{ when i and j are interior nodes} \tag{16}$$

2.2 Study area and scenarios – *Područje istraživanja i scenariji*

To examine the model performance, it was used in designing a forest road network in a mountainous area in Iran. The area is located between 53° 16 ′ 57″ E to 53° 21′ 35″ and 36° 12′ 11″ N to 36° 15′ 40″ N (Fig. 3). The 1 428 hectare mountainous hardwood forest is managed under sustainable forest management. The harvesting method is single tree selection with a continuous return cycle of 10 years. Forest roads and their related skid trails are permanent. The potential network design was constructed by requiring one road link for every 2.27 hectares and one access spur for every 1.14 hectares.

Fig. 4 and 5 show an exploded view of a portion of the region with its feasible road links and feasible access spurs. In all, the model contains 252 harvesting sites, 883 potential spurs, and 440 road segments. One exit node is located at the top centre of the forest. The resulting MIP model has 69 420 constraints, 33 085 variables, and 300 083 non-zero coefficients. To check the validity and performance of the model, we used the same spa-

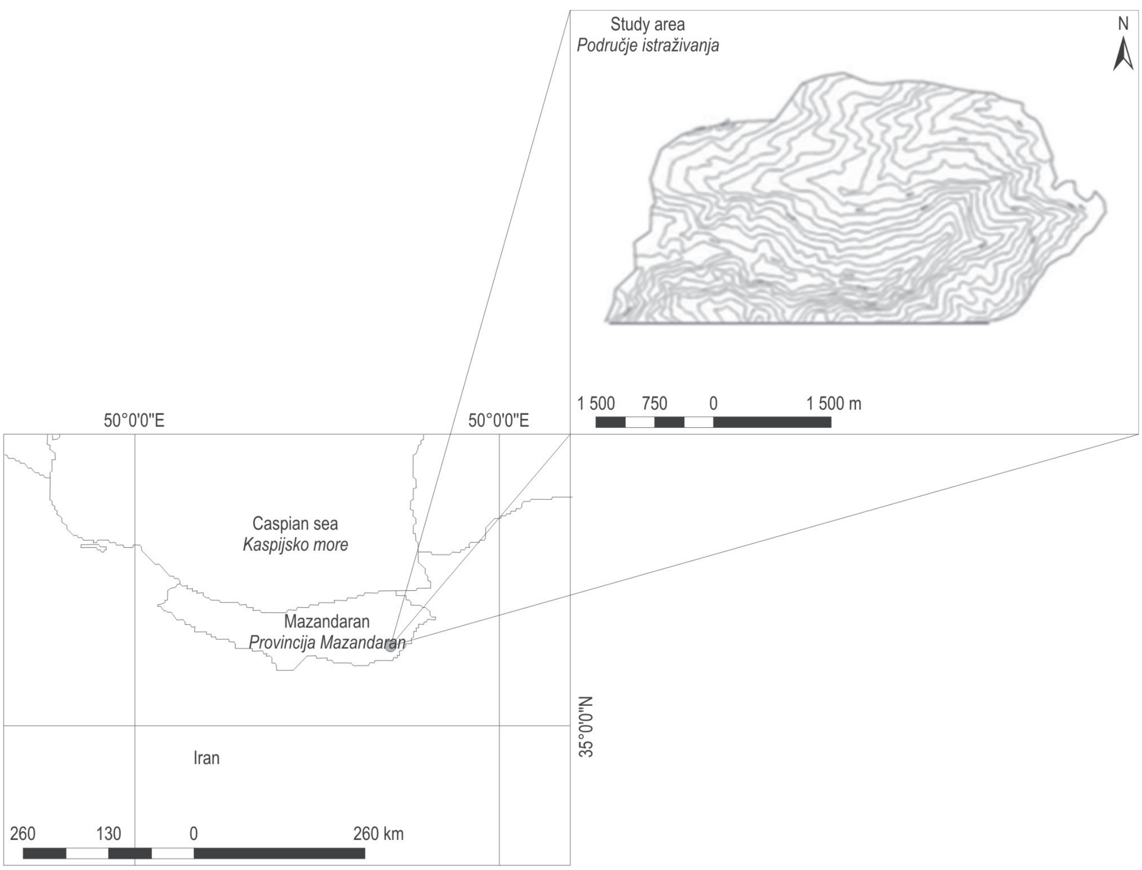

Fig. 3 Study area
Slika 3. *Područje istraživanja*

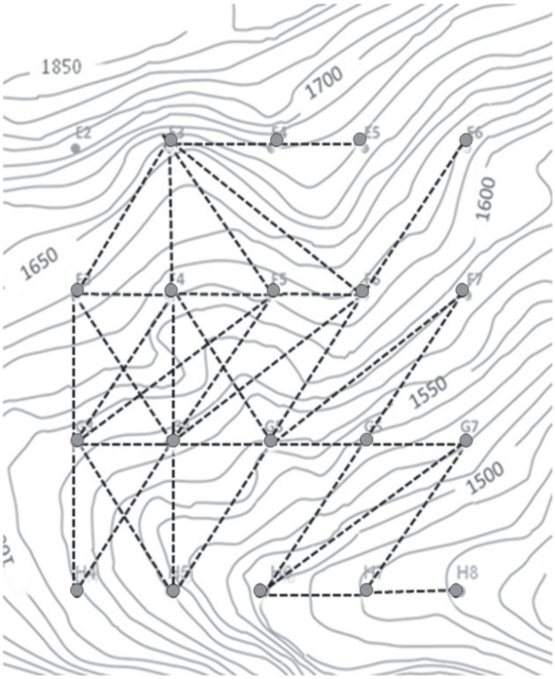

Fig. 4 Typical road feasibility; due to slope, swampy areas, or no volume to be extracted, there are typically less than 20 links for each node

Slika 4. *Izvedivost šumske ceste. Zbog nagiba terena, močvarnih područja ili gospodarske neisplativosti svaka mjerna točka obično ima manje od 20 veza s ostalim mjernim točkama*

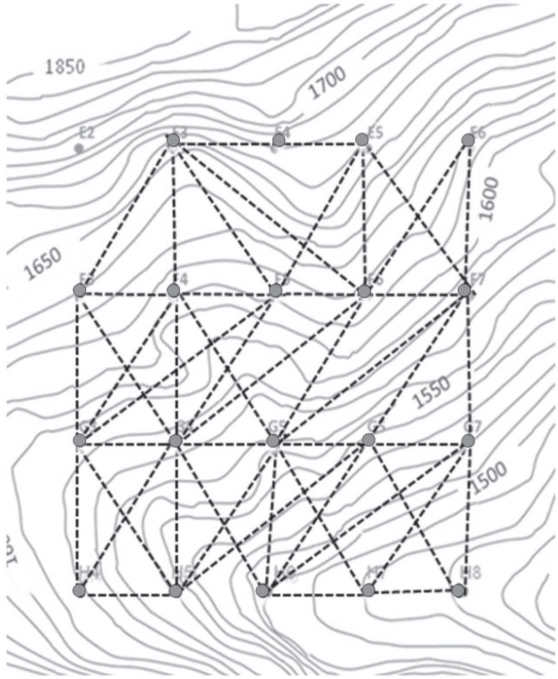

Fig. 5 Access spur feasibility; there may not be 20 links for each node because of steep terrain, swamp areas, or no volume at the systematically placed harvest node

Slika 5*. Izvedivost traktorskoga puta. Moguće da će zbog nagiba terena, močvarnih područja ili gospodarske neisplativosti biti manje od 20 veza među mjernim točakama*

Table 1 Solution Cost Composition, Solution Times and Quality. *Rfact* is a factor applied to the base road construction cost

Tablica 1. *Rješenje troškovne sastavnice, utroška vremena i kakvoće.* Rfact *je faktor koji se primjenjuje pri prikazivanju osnovnih troškova izgradnje*

Rfact	Total Cost *Ukupni troškovi*	Trucking Activity *Prijevoz*	Skidding Cost *Troškovi privlačenja*		Maintenance Cost *Troškovi održavanja*		Trucking Cost *Troškovi prijevoza*		Road Construction Cost *Troškovi izgradnje šumskih prometnica*		CPU Time *Računalno vrijeme*	Final MIP Gap *Završna MIP praznina*
	$	1000 m^3/km	$	%	$	%	$	%	$	%	s	%
1.3	539 392	160 298	165 510	31%	62 516	12%	208 388	39%	102 978	19%	< 100	< 1
1.15	496 013	158 586	162 542	33%	54 712	11%	182 374	37%	96 386	19%	200	< 1
1	452 264	161 226	156 096	35%	48 368	11%	161 226	36%	86 574	19%	3600	1.67
0.85	407 011	165 526	148 170	36%	42 209	10%	140 697	35%	75 934	19%	3600	3.87
0.7	362 138	166 491	143 114	40%	34 963	10%	116 544	32%	67 517	19%	3600	6.52
0.55	313 038	173 609	131 071	42%	28 645	9%	95 485	31%	57 837	18%	3600	10.06
0.4	259 411	193 310	102 092	39%	23 197	9%	77 324	30%	56 798	22%	3600	15.99
0.3	210 244	219 220	67 080	32%	19 730	9%	65 766	31%	57 668	27%	3600	12.63
0.2	162 371	235 438	57 118	35%	14 126	9%	47 088	29%	44 039	27%	3600	15.83
0.1	100 914	251 818	34 279	34%	7 555	7%	25 182	25%	33 899	34%	3600	12.68

tial configuration and skidding costs, but varied road construction costs by factors 1.3, 1.15, …, to 0.40.

The values, assigned to skidding and road construction costs in the original scenarios, were 500$ and 18 400$, respectively. Then, to further test computational performance, we generated a number of random instances of the problem by using Monte Carlo sampling to assign skidding and road construction costs. Probability distributions were fit to the original skidding and cost data, which were then sampled to create the random instances. The skidding cost distribution fit best to a three-parameter Beta, with minimum 4.00, size parameter 0.839, and shape parameter 2.81. Road construction costs were sampled from the uniform distribution on [2700, 14400]. In these random instances, all potential road segments and skid trails were assumed to be feasible. The resultant models have 9 520 skidding, 9 520 trucking, and 5 340 road construction feasible decision variables. These constitute a much denser network of potential arcs for both transportation systems.

Finally, seven of the original scenarios were modified to add a second exit point at the bottom of the landscape.

The MIP models were created using OPL Studio 4.2 and solved using CPLEX version 10.0, both products of Ilog, Inc. They were solved using a Pentium 4 3.4 GHz CPU computer with 4 GB RAM, running Windows XP Professional. All models were run until the MIP gap was less than or equal to one percent, or one hour of CPU time was consumed, whichever occurred first.

3. Computational results – *Računalni rezultati*

Table 1 shows solution values, their cost composition, solution times and final MIP gap for each of ten base scenarios. The scenarios were achieved by varying the costs of road construction by a multiple called *Rfact*. Optimality gaps ranged from less than one percent to 16 percent, and solution times ranged from less than 100 seconds to the cutoff point of 3 600 seconds. Solution quality, as measured by the final MIP gap, worsened as the road construction and trucking costs were reduced to 2 times the first case, then when *Rfact* was further reduced to 0.1. As expected, increasing road costs causes a corresponding decrease in construction and use of hauling roads, with more timber gathered through access spurs. Total trucking activity, as measured in volume-distance units, ranged from 160 298 to 251 818 m^3/km. Fig. 6, 7, and 8 show road network solutions achieved for three of the 10

Table 2 MIP Optimality Gap (%) for 10 randomly generated instances, with varied road construction costs. Model execution was terminated in one hour

Tablica 2. *Optimalna MIP praznina (%) za deset nasumično odabranih slučajeva s različitim troškovima gradnje šumskih cesta. Vrijeme računanja modela iznosilo je sat vremena*

	Road Cost Change Factor *Različiti troškovi gradnje šumskih cesta*						
Case – *Slučaj*	0.4	0.55	0.7	0.85	1	1.15	1.3
1	16.3	10.1	3.76	< 1	< 1	< 1	< 1
2	13.45	8.89	2.53	< 1	< 1	< 1	< 1
3	15.05	10.25	5.12	< 1	< 1	< 1	< 1
4	16.4	10.93	4.55	< 1	< 1	< 1	< 1
5	15.33	9.21	4.11	< 1	< 1	< 1	< 1
6	15.9	8.6	2.73	< 1	< 1	< 1	< 1
7	17.69	15.09	7.63	< 1	< 1	< 1	< 1
8	16.06	10.7	10.63	5.2	< 1	< 1	< 1
9	14.88	9.27	2.17	< 1	< 1	< 1	< 1
10	17.43	10.8	3.57	< 1	< 1	< 1	< 1
Mean *Aritmetička sredina*	15.849	10.384	4.68	1.42	< 1	< 1	< 1
Minimum *Minimum*	13.45	8.6	2.17	< 1	< 1	< 1	< 1
Maximum *Maksimum*	17.69	15.09	10.63	5.2	< 1	< 1	< 1

Table 3 CPU time (seconds), 10 random cases, optimality tolerance 1%

Tablica 3. *Računalno vrijeme (sekunde), 10 nasumično odabranih slučajeva, optimalna tolerancija 1 %*

	Road Cost Change Factor *Različiti troškovi gradnje šumskih cesta*						
Case – *Slučaj*	0.4	0.55	0.7	0.85	1	1.15	1.3
1	3 600	3 600	3 600	400	< 1	< 1	< 1
2	3 600	3 600	3 600	274	< 1	< 1	< 1
3	3 600	3 600	3 600	< 1	< 1	< 1	< 1
4	3 600	3 600	3 600	< 1	< 1	< 1	< 1
5	3 600	3 600	3600	400	< 1	< 1	< 1
6	3 600	3 600	3 600	< 1	< 1	< 1	< 1
7	3 600	3 600	3 600	300	< 1	< 1	< 1
8	3 600	3 600	3 600	3 600	< 1	< 1	< 1
9	3 600	3 600	3 600	< 1	< 1	< 1	< 1
10	3 600	3 600	3 600	< 1	< 1	< 1	< 1

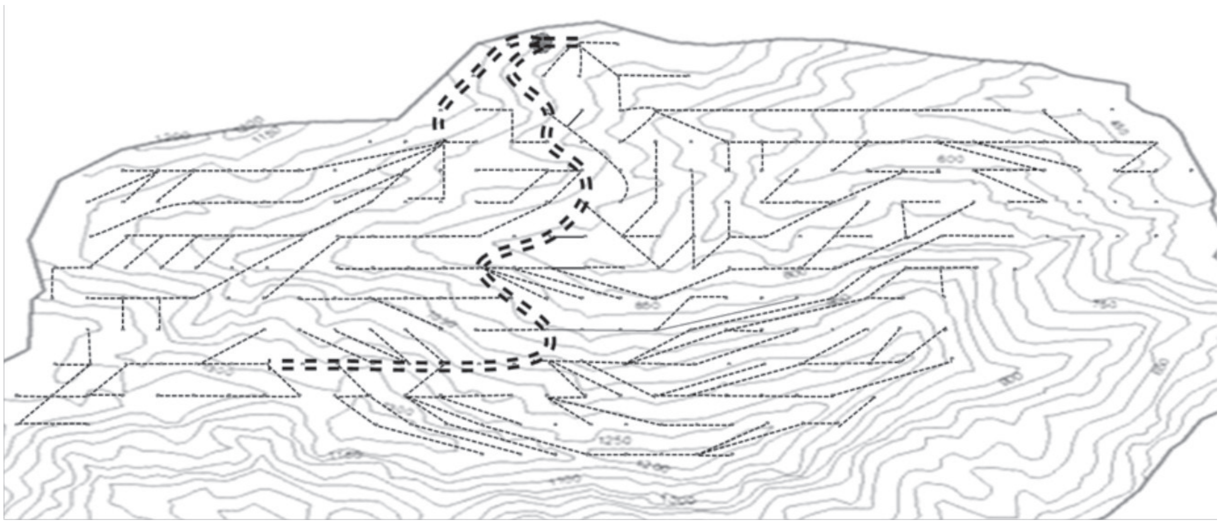

Fig. 6 Solution showing the road and related access spurs (*Rfact* = 1.3)
Slika 6. *Planirana šumska cesta s pripadajućim traktorskim putovima (*Rfact = *1,3)*

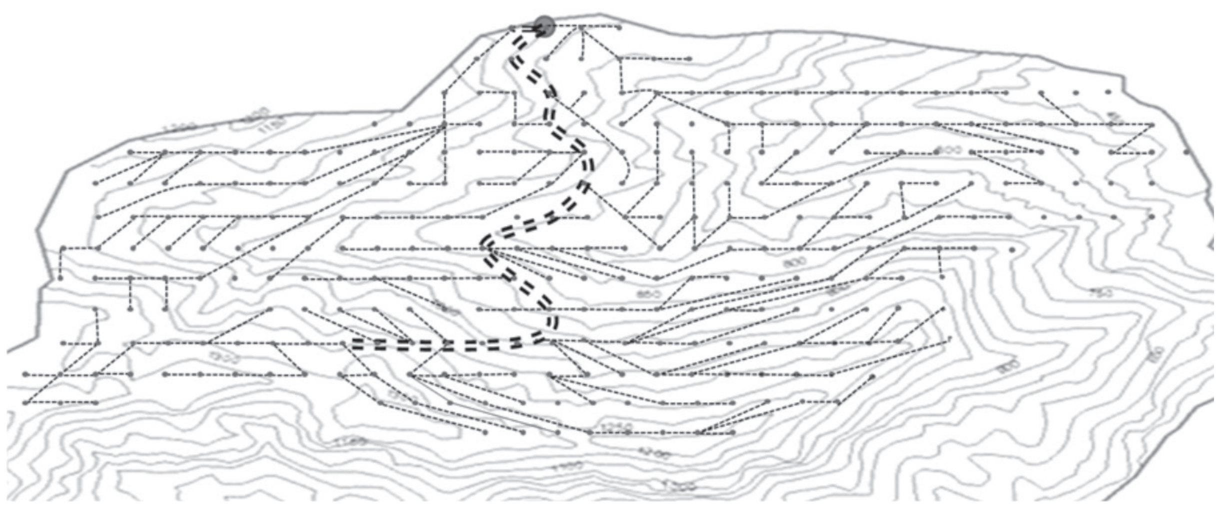

Fig. 7 Solution showing the road and related access spurs (*Rfact* = 1.0)
Slika 7. *Planirana šumska cesta s pripadajućim traktorskim putovima (*Rfact = *1,0)*

scenarios. Dark lines indicate road segments, and lighter lines indicate the skidding paths. One can see that the solutions are feasible with respect to continuity of transport system and connection to the exit point.

Table 2 shows MIP gaps for the ten randomly generated cases. Again, as the relative cost of roads decreased, solution quality degraded. Gaps ranged from less than one percent to 17.69 percent. Table 3 shows CPU times for these 10 random cases with *Rfact* values ranging from 1.3 to 0.4. CPU times ranged from less than one minute to the cutoff point of one hour.

Table 4 shows solution cost structure and computational performance for seven scenarios with two exit points. Solution times ranged from 44 seconds to one hour, and MIP gaps of less than one percent to 16.54 percent. Total costs were reduced significantly from the original scenarios where only one exit point was allowed. Fig. 9 shows the solution for *Rfact* equal to 1.0, and Fig. 11 the solution for *Rfact* equal to 0.4. These solutions are markedly different from the cases of one exit point, with roads clustered at each of the two exit points and considerable reduction in skidding and truck transport costs.

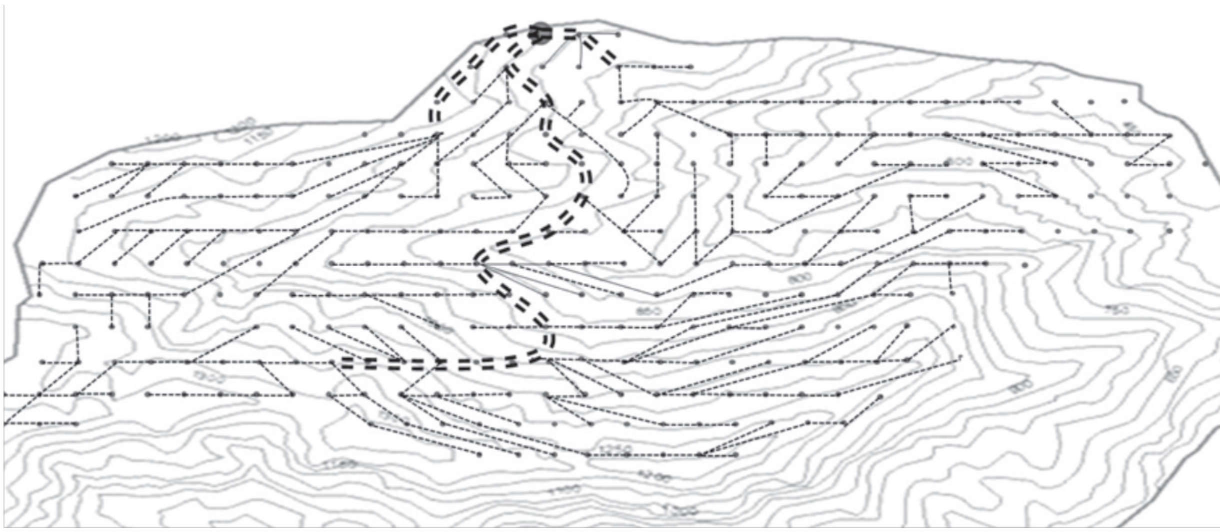

Fig. 8 Solution showing the road and related access spurs (*Rfact* = 0.70)
Slika 8. *Planirana šumska cesta s pripadajućim traktorskim putovima (*Rfact = *0,70)*

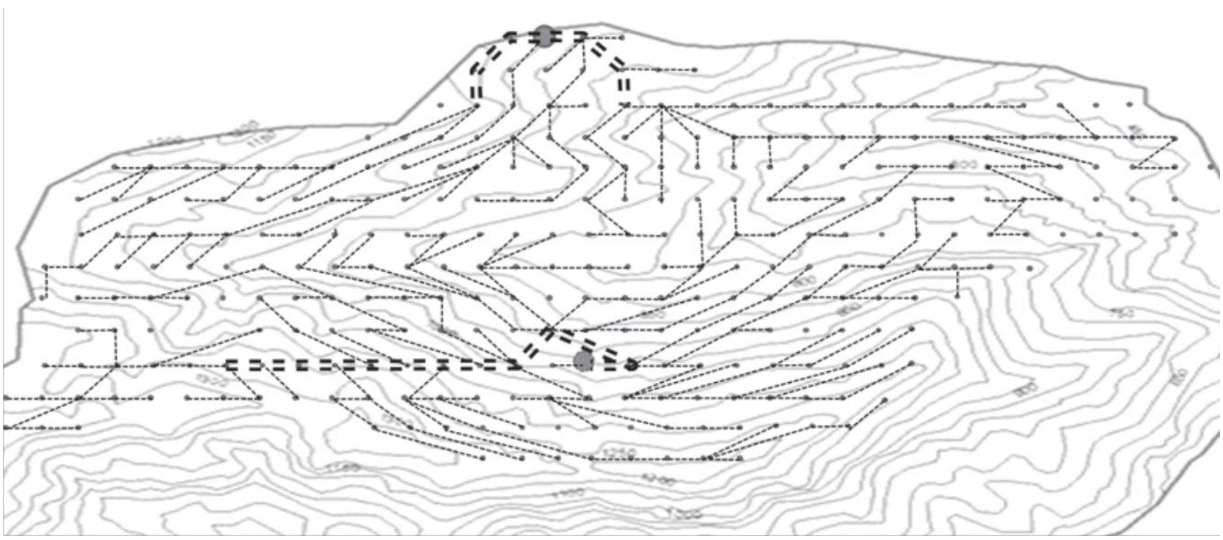

Fig. 9 Solution showing the road and related access spurs (*Rfact* = 1.00, 2 Exit Points)
Slika 9. *Planirana šumska cesta s pripadajućim traktorskim putovima (*Rfact = *1,00 s dvije izlazne točke)*

4. Discussion – *Rasprava*

The model produces valid results for the hierarchical transportation model, with integrity of transport system and road system maintained. Moreover, the results are of very reasonable quality when compared to known upper bounds on optimality.

We have used the MIP gap to assess the quality of the solutions achieved. This is the percentage gap between the best integer solution achieved and the best bound on the optimal solution found by the solver. This does not mean that a solution with MIP gap of 15% is fifteen percent worse than the optimal solution. Rather, the measure guarantees that it is at least as good as 85% of the optimal.

In all cases, the MIP model was able to produce good solutions to the problem in very reasonable computational time. The final MIP gap was less than 20 percent for all cases in up to one hour of computation time, with many cases solved to less than 1 percent optimality in less than one minute.

Table 4 Road network solutions with 2 exit points; solution cost composition, times and quality; *Rfact* is a factor applied to the base road construction cost

Tablica 4. *Planiranje mreže šumskih prometnica s dvije izlazne točke. Rješenje troškovne sastavnice, utroška vremena i kakvoće.* Rfact *je faktor koji se primjenjuje pri prikazivanju osnovnih troškova izgradnje*

Rfact	Total Cost *Ukupni troškovi*	Trucking Activity *Prijevoz*	Skidding Cost *Troškovi privlačenja*		Maintenance Cost *Troškovi održavanja*		Trucking Cost *Troškovi prijevoza*		Road Construction Cost *Troškovi izgradnje šumskih prometnica*		CPU Time *Računalno vrijeme*	Final MIP Gap *Završna MIP praznina*
	$	1000 m^3/km	$	%	$	%	$	%	$	%	Sec.	%
1.30	321 285	48 590	169 917	52.89	18 950	5.90	63 167	19.66	69 251	21.55	44	1.00
1.15	303 761	50 061	167 659	55.19	17 271	5.69	57 570	18.95	61 260	20.17	121	1.00
1.00	285 625	52 944	158 927	55.64	15 883	5.56	52 944	18.54	57 870	20.26	375	0.91
0.85	266 467	54 037	157 567	59.13	13 780	5.17	45 932	17.24	49 190	18.46	3 367	1.00
0.70	246 787	61 780	141 687	57.41	12 974	5.26	43 246	17.52	48 881	19.81	3 600	7.50
0.55	221 520	75 287	121 593	54.89	12 422	5.61	41 408	18.69	46 098	20.81	3 600	11.07
0.40	188 503	98 322	86 407	45.84	11 799	6.26	39 329	20.86	50 969	27.04	3 600	16.54

The quality of solutions attainable in one hour CPU time varied with the cost structure of the problem instance. That is, when road costs were reduced relative to skidding costs, more CPU time was consumed to achieve solutions of the same quality as for instances with a higher road to skidding cost ratio. This behavior was observed in the original scenarios and in the randomly generated cases.

One of our research objectives was to determine whether it was possible to solve the MIP model in reasonable time for problems that had many feasible routing choices. We allowed 20 incoming and outgoing arcs at each node of the network. Our first examples were constructed assuming a mountainous terrain, with many node-to-node connections infeasible due to slopes. However, for the ten randomly generated cases, all of these arcs were feasible. These models were solved to approximately the same quality of solution in much the same computational time as the original ten scenarios. This indicates that density of the network is not a limiting factor in attaining a solution. Also, although we have not reported results using other formulations,

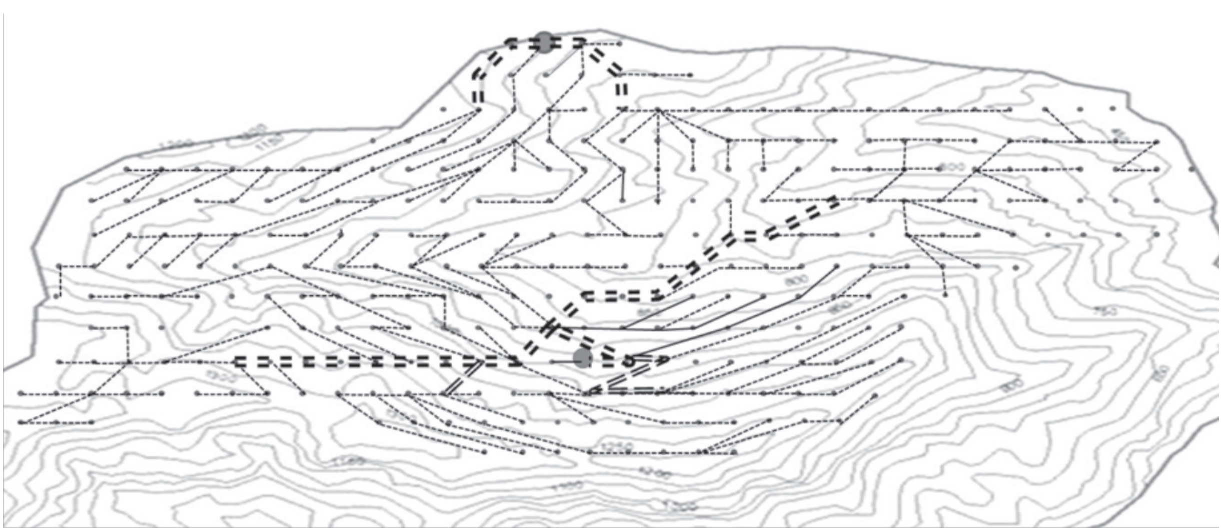

Fig. 10 Solution showing the road and related access spurs (*Rfact* = 0.40, 2 Exit Points)

Slika 10. *Planirana šumska cesta s pripadajućim traktorskim putovima (*Rfact *= 0,40 s dvije izlazne točke)* we found that adding trigger constraints was important in improving MIP performance.

We can see from the network solutions (Fig. 8–10) that the solutions make sense spatially. However, one might question the potential for this model to produce long skidding routes that are feasible in practice. There are no direct constraints on skidding distances in the solution. Instead, arc capacity constraints were used to reduce the likelihood that timber would be skidded too far, by constraining the accumulated flow through each skidding trail. It should be noted that the capacity method works only approximately since timber volumes vary between stands and skidding volumes may accumulate to the imposed capacity over distances which are not acceptable. This implies that it might be necessary to subject the network to some post–optimization fixing if excessive skidding distances occur.

5. Conclusion – *Zaključci*

This model is designed to create access networks for areas to be continuously harvested by selection harvesting, and hence require a permanent road network. In such forests there are generally two choices of road standard, the skidding and the hauling road standards. The cost considerations are that the higher standard roads have higher construction costs but lower transportation costs. Moreover, once timber is skidded to a road location, it must be then transported over hauling roads until it reaches an exit point of the network. To minimize total cost, costs in both networks should be considered simultaneously. This model solves this problem in reasonable computational time and produces realistic solutions of reasonable quality. Solver performance was robust with respect to density of the input network, but sensitive to the cost structure. Solving models with two exit points was not appreciably more difficult than solving those with one exit, but the solutions attained were radically different in structure and in cost.

The examples and the discussion have focused on skidding and hauling transportation systems. This model could also be used to design access systems where any two different transportation systems exist, to create a primary network from which the goods are transported to the secondary network. We did not test systems with more than two transportation systems standards, but the model is easily extended to accommodate more transportation systems and road standards.

The CPLEX, commercial MIP solver was able to solve these problems easily, attaining optimal solutions in less than one minute in most cases, and attaining solutions with optimality bound less than 20% in all cases. Forecasting performance for branch and bound methods using a few instances is risky, since results can vary widely. We found that the cost structure had a significant impact on performance, while the network density did not.

In conclusion, we have presented a model and study to show a mixed integer programming method to design an access network for hierarchically constrained transport systems. The system can minimize costs of construction, transportation, and maintenance for a dense network of potential arcs, and can solve the problem in reasonable time. These results are quite positive, and it is important for decision makers to realize that models of this size, which would have been considered intractable a short time ago, can be solved in a reasonable time, and can provide useful decision support.

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Sažetak

Planiranje mreže primarnih šumskih prometnica primjenom modela mješovitoga cjelobrojnoga programiranja

Troškovi su izgradnje i održavanja šumskih prometnica veliki te je zbog toga važno razviti metode projektiranja šumskih prometnica koje u obzir uzimaju minimalan utrošak novčanih sredstava.

Ovaj je članak optimizirani model mješovitoga cjelobrojnoga programiranja (MIP) koji služi za planiranje mreže šumskih prometnica uzimajući u obzir minimalan utrošak novčanih sredstava pri izgradnji i prijevozu drvnih sortimenata.

Problem pri optimizaciji, uz primjenu modela MIP, bio je odabrati najučinkovitiju mrežu šumskih prometnica koja omogućuje sve aktivnosti u pridobivanju drva. Cilj je spomenutoga modela smanjiti troškove gradnje i održavanja šumskih cesta te troškove primarnoga i sekundarnoga transporta drva.

Pri definiranju sustava za planiranje mreže šumskih prometnica odabrano je 298 smještenih točaka u prostoru. Za svaki točku poznat je drvni volumen za sječu i privlačenje. Kako bi se točke spojile (u buduću šumsku prometnicu), za svaku je sniman broj lukova (poveznica) prema susjednim točkama, a troškovi izgradnje šumske ceste izračunati su pomoću podataka iz digitalnoga modela terena.

Dakle, planiranje je mreže šumskih prometnica temeljeno na smjeru pružanja lukova koji povezuju mjerne točke. U skladu s terenskim ograničenjima svaki luk (buduća prometnica) može služiti kao primarna ili sekundarna šumska prometnica ili se potpuno izostavlja iz modela.

O ovom radu svaka mjerna točka ima 20 mogućih susjednih točaka, a samim time i 20 odlaznih i 20 dolaznih lukova stvarajući na taj način vrlo gustu mrežu šumskih prometnica. Cilj je minimizirati gustoću mreže šumskih prometnica te troškove primarnoga i sekundarnoga transporta drva.

Primjenjivost modela ispitivana je pri planiranju mreže šumskih prometnica u planinskom području u Iranu. Šumske su ceste i njihovi pripadajući traktorski putovi trajne građevine. U model su ugrađena ukupno 252 radilišta (mjesta sječe), s 883 potencijalna traktorska puta i 440 segmenata šumskih cesta. Izlazna je točka iz sastojine smještena na gornjem dijelu istraživanoga područja. U konačnici model MIP sadrži 69 420 terenskih ograničenja, 33 085 varijabli i 300 083 koeficijenta koji svi zajedno određuju mrežu šumskih prometnica. Pri provjeravanju valjanosti i učinkovitosti modela korišteni su isti prostorni podaci i troškovi transporta uz različite faktore za izračun troškova gradnje (u rasponu 1,3; 1,15,..., do 0,40). Dobiveno je sedam mogućih modela (scenarija) mreže šumskih promentica te je dodana i druga izlazna točka pri dnu istraživanoga područja. Kao što se i očekivalo, povećanjem novčanih sredstava utrošenih u izgradnju šumskih cesta smanjuju se novčana sredstva za izgradnju traktorskih putova te se zbog toga na njima povećava drvni volumen u primarnom transportu. Ukupni transport drvnoga volumena istraživanoga područja kretao se u rasponu od 160 298 do 251 818 m^3/km. Ukupni su troškovi značajno smanjeni u usporedbi s prvotnim scenarijem u kojem je bila određena samo jedna izlazna mjerna točka (na gornjem dijelu sastojine). Sustav daje valjane rezultate za hijerarhijski transportni model s cjelovitim transportnim sustavom uz održavanja šumskih cesta. Rezultati su vrlo visoke kakvoće ako ih uspoređujemo s poznatim gornjim granicama optimalnosti. To je post-

otna razlika između najbolje dostignutoga cjelobrojnoga rješenja i najbolje veze s optimalnim rješenjem pronađene putem rješavača (tzv. solver). U svim slučajevima model MIP u mogućnosti je proizvesti dobro rješenje problema u vrlo razumnom računalnom vremenu. Jedan od ciljeva ovoga istraživanja bio je odrediti je li moguće pronaći rješenje problema vezanih uz planiranje trasa modelom MIP u razumnom računalnom vremenu. Stoga su prvi primjeri bili rađeni za planinski teren, s mnogo neostvarivih veza među mjernim točkama zbog ograničavajućega nagiba. Sustav može minimizirati troškove izgradnje, transporta i održavanja kod guste mreže prometnica s mnogo mogućih lukova povezivanja i riješiti problem u razumnom vremenu, što ga čini prikladnim sustavom za potporu odlučivanja u šumarstvu. Ovaj model može biti primijenjen pri planiranju pristupnih pravaca, gdje postoje dva različita transportna sustava, za planiranje sekundarne mreže s kojih se dobra (drvo) transportira na primarnu mrežu šumskih prometnica. Model nije testiran na više od dva različita transportna sustava, ali se vrlo lako može proširiti na druge transportne sustave i prometne standarde. Struktura je troškova važan čimbenik na učinkovitost modela, dok gustoća mreže šumske prometne infrastrukture nije.

Ključne riječi: optimizacija, troškovi pridobivanja drva, gustoća mreže šumskih prometnica, trakorski putovi, planiranje pristupa šumi

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