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Alan KLANAC[1]
Jasmin JELOVICA[2]

Vectorization in the Structural Optimization of a Fast Ferry

Authors' addresses:
1 *Helsinki University of Technology, Ship Laboratory*, PL 5300, FIN-02015 TKK, Finland, tel:+358 9 451 4172, fax:+358 9 451 4173, email: alan.klanac@tkk.fi
2 *Helsinki University of Technology, Ship Laboratory,* visiting from the *Faculty of Engineering, University of Rijeka*, Croatia, email: jasmin.jelovica@riteh.hr

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Vectorization assumes the conversion of constraints into objective functions. It turns a single-objective, or scalar, optimization into a search for a Pareto optimal set, which will enhance the search for the optimum. Vectorization is studied here within a structural optimization of fast ferry's midship section for the minimum of steel weight. Optimization applies a simple genetic algorithm (GA), whose performance is observed over both scalar and vectorized problem formulations. The obtained results show that the applied GA can improve the referenced design, and that the improvement can be significantly better if vectorization is applied.

Keywords: *ferries, genetic algorithms, structural optimization, vectorization.*

Vektorizacija u strukturalnoj optimizaciji brzoga trajekta

Izvorni znanstveni rad

Vektorizacija podrazumijeva promjenu ograničenja u funkcije cilja. Time se jednokriterijalna, ili skalarna, optimizacija generalizira u potragu za *Pareto* optimalnim skupom što poboljšava potragu za optimumom. Ovaj rad proučava vektorizaciju na primjeru strukturalne optimizacije glavnog rebra brzoga trajekta. Optimizacija je izvršena pomoću jednostavnoga genetskog algoritma, čije su značajke promotrene za skalarnu i vektoriziranu formulaciju problema. Rezultati optimizacije pokazuju poboljšanja u odnosu na prototip, osobito pri uporabi vektorizacije.

Ključne riječi: *genetički algoritam, strukturalna optimizacija, trajekti, vektorizacija*

1 Introduction

Design optimization is an important part of modern shipbuilding. Its mathematical formalization of the design process helps a designer to determine the best combination of design parameter values in order to fulfil the design objectives using an appropriate search algorithm. In this paper we elaborate on the possibility to improve this process within the aspect of structural design of an 88 m long fast passenger/car ferry.

Problems involving the optimization of ship structures are often spanned in highly nonlinear design spaces. Strength and stiffness formulations are polynomials of higher order, while the objective function is rarely linear. This can then result in a formation of a non-convex feasible space that cannot be readily handled using the classical gradient-based methods as the concavities can end the search prematurely. Yet, these methods are efficient in their search for the local minima, so if considering the optimization in the phase of preliminary design these problematic features disappear. The amount of applied constraints drastically limit the feasible space, so that it can be either sequentially linearized and solved with linear programming, see [1] and [2], or separated into a series of convex problems and solved accordingly with the non-linear gradient based method [3]. Zanic *et al*. [4] and Rigo [5] show on different practical cases, including thousands of constraints and hundreds of design variables, that these approaches result in significant improvements within an acceptable computational time period.

However, as indicated in the ISSC report on design principles and criteria [6, p.588], optimization often becomes a computational "nightmare" when considering constraints such as vibrations, which cause infeasibility holes within the feasible space, or when variables are non-continuous as in the considered case study. The aid can be found then in the application of genetic algorithms (GA). Gas can deal with such problems as they work solely with the objectives functions and not with their derivatives or with constraints. Gas based on the binary coding of variables can also deal with integer or discrete values, but they can experience problems of local minima [7]. If within a population of designs a sufficient diversity is not maintained, the Gas will have difficulty to map the entire design space, and can therefore generate designs in the close neighbourhood of local minima. These problems often occur when the problem needs to be solved in as few generations as possible and with small population size, thus forcing the algorithm to strictly follow the fittest design within a population. In the design of ship structures this is often demanded as the computation of structural response is costly. Within this study we will explore the possibilities to enhance the performance of Gas for structural optimization. We will concentrate on the approach addressed as *vectorization*, by which all the constraints are converted into additional objectives and a single-objective optimization problem is solved as a multi-objective problem.

These theoretical concepts and their implementation to a GA are further elaborated within the next two chapters. In chapter four, the postulated approach is applied to the structural design of a fast ferry, the results of which are further discussed in the fifth chapter.

2 Vectorization

Let a constrained single-objective, or scalar problem (SO), over a set of design variables $\mathbf{x}$

$$\mathbf{\Omega} = \left\{ \mathbf{x} \in \mathbf{X} \subseteq \Re^n \mid g_j(\mathbf{x}) \geq 0, \forall j \in 1..l \right\}, \min_{\mathbf{x}\in\mathbf{\Omega}} f_0(\mathbf{x}), \quad (1)$$

be understood as a multi-objective, or vector optimization problem (VO)

$$\min_{\mathbf{x}\in\mathbf{X}} \left\{ f_0(\mathbf{x}), f_1(\mathbf{x}), ..., f_l(\mathbf{x}) \right\}, \quad (2)$$

in which the constraints g_j are relaxed and represented as additional objectives f_j, $j \in 1...l$. Let us address this as *vectorization*. Constraint representation in (2) needs to enable minimization towards a desirable value, which is taken at zero, following the feasibility condition of (1). Therefore, Deb [9] and Osyczka *et al*. [10] choose to represent the constraints using the heavy side function, as

$$f_j(\mathbf{x}) = \begin{cases} -g_j(\mathbf{x}), & \text{if } g_j(\mathbf{x}) < 0 \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Even though the constraints are relaxed, this representation differentiates between feasible and infeasible designs. Yet it does not preserve the information on the constraint magnitude for the feasible designs. Hence, the relative distances between the points in the feasible design space are lost, and the designs become ranked solely on f_0. To avoid this drawback, an alternative "absolute" representation can be proposed:

$$f_j(\mathbf{x}) = \left| g_j(\mathbf{x}) \right|. \quad (4)$$

All the information on the position of feasible designs is now preserved, but the information on the feasibility is dropped. By adopting these two constraint representations, two types of vector problem of (2) are formed, the VO-1, which applies representation in (4), and the VO-2 applying the constraint representation of (3).

According to Steuer [11], by solving the VO for a Pareto front it is possible to solve the SO for a scalar optimum $\mathbf{x}^*$, so that each optimum in SO is at least weakly Pareto optimal solution in VO, while one scalar optimum is efficient, or strongly Pareto optimal. Generally, any alternative $\mathbf{x}$ belonging to Pareto front $\hat{\mathbf{X}}$ is weakly Pareto optimal, where $\hat{\mathbf{X}}$ is defined as

$$\hat{\mathbf{X}} = \left\{ \mathbf{x} \in \mathbf{X} \mid \nexists \mathbf{x}^k, \mathbf{f}(\mathbf{x}^k) < \mathbf{f}(\mathbf{x}), \forall \mathbf{x}^k \in \mathbf{X} \setminus \mathbf{x} \right\}, \quad (5)$$

and some alternatives are strongly Pareto optimal if

$$\nexists \mathbf{x}^k, \mathbf{f}(\mathbf{x}^k) \leq \mathbf{f}(\mathbf{x}), \forall \mathbf{x}^k \in \mathbf{X} \setminus \mathbf{x}. \quad (6)$$

Here $\mathbf{f}$ stands for a vector of objectives defined in (2), and the applied inequalities "<" and "≤" are vector inequalities.

The validity of this statement is easy to prove for VO-1 and -2 as $\mathbf{x}^*$ by definition possess the least value of f_0. During the optimization there might exist infeasible designs which possess lower values of f_0 than $\mathbf{x}^*$, hence these might dominate over it. In the case of VO-2, this is avoided by assigning $f_j(\mathbf{x}) = 0, \forall j \in 1...l$ for every feasible design, but in the case of VO-1, $\mathbf{x}^*$ will be guaranteed Pareto optimality only if it lies on any constraint boundary, where $f_j(\mathbf{x}^*)=0$, $j \in 1...l$. Thus, the solution of VO-1 will from the set of feasible designs prefer those which are on the boundaries, as well as those which are in their neighbourhood as the constraint representation will maintain the information on the relative position of designs. This is particularly significant for the studied problem of weight minimization as $\mathbf{x}^*$ in such problems is regularly found on the boundary of the feasible space.

VO-2 will, on the other hand, give the advantage to any feasible design over infeasible, but without specific classification among them. Hence, the VO-2 could lead towards more inconsistent search in which no specific advantages will be given to a design on the feasible boundary, where $\mathbf{x}^*$ might be expected. It is also easy to see that for VO-2 any feasible design $\mathbf{x}^k$ will always be contained in the axis of f_0, and if $\mathbf{x}^k = \mathbf{x}^*$, $\mathbf{x}^*$, will be efficient, due to the constraint representation of (3).

3 Implementation to a genetic algorithm

Within this paper we apply a single GA to perform optimization in both vectorized and scalar form. The concept of solution is applied through fitness φ calculation. If dealing with the SO, it is possible to apply the following simple formula:

$$\varphi(\mathbf{x}^i) = \begin{cases} Big - f_0(\mathbf{x}^i), & \text{if } \mathbf{x}^i \in \Omega \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

which is a simplified piecewise form of the Penalty function approach, see [12] for more, in which now all the infeasible solutions are equally, but also maximally penalized to a value of zero. *Big* is a large enough constant used to assign higher fitness to designs having smaller objective function, since SO involves minimization. This simplified formulation of a penalty function is beneficial as it preserves the nature of the objective function and avoids possible non-linearities due to the exclusion of constraints, but it also neglects the information on the rate of infeasibility, as all infeasible designs receive the same fitness of zero. All the information on designs which are close to optimum, but are infeasible, cannot be then used to further improve the feasible designs.

If solving the vectorized problem, the formulation of the fitness function should account for the performance of the designs over multiple objectives, which in this case are the original objective function and the constraints. The convenient measure of merit is then the Pareto optimality. Yet, in this case, where we desire to obtain the scalar optimum, this criterion is insufficient. Therefore, the following fitness function is proposed:

$$\varphi(\mathbf{x}^i) = \begin{cases} \max d(\mathbf{x}) + \dfrac{1}{d(\mathbf{x}^i)}, & \text{if } \mathbf{x}^i \in \hat{\mathbf{X}} \\ \max d(\mathbf{x}) - d(\mathbf{x}^i), & \text{otherwise} \end{cases} \quad (8)$$

which ranks designs on the basis of the obtained Pareto optimality within a population and the distance d to the reference point $\mathbf{I}$ in an objective space $\mathbf{Y} = \left\{ \mathbf{f}(\mathbf{x}) \mid \forall \mathbf{x} \in \mathbf{X} \right\}$. In this case, $\mathbf{I}$ is the point containing the minimum values of every objective within a population. As the problem deals with objectives of different physical meaning, such as weight, thickness, stress, etc., the objective space needs to be normalized to avoid pitfalls caused by large differences in the magnitudes of functions in (2). The normalization is performed by bounding this space within a unit interval, where 0 now presents the minimum of the objective for the current population, and 1 its maximum. Hence, **I**={0}, and if, similarly to [10], we apply the weighted Euclidean function as measure, the distance to **I** is found as

$$\begin{gathered} \mathrm{d}(\mathbf{x}) = \left\{ \sum_j w_j \left[\mathrm{f}'_j(\mathbf{x}) \right]^2 \right\}^{1/2}, \\ \text{s.t. } 0 < w_j < 1, \\ \sum_j w_j = 1 \end{gathered} \tag{9}$$

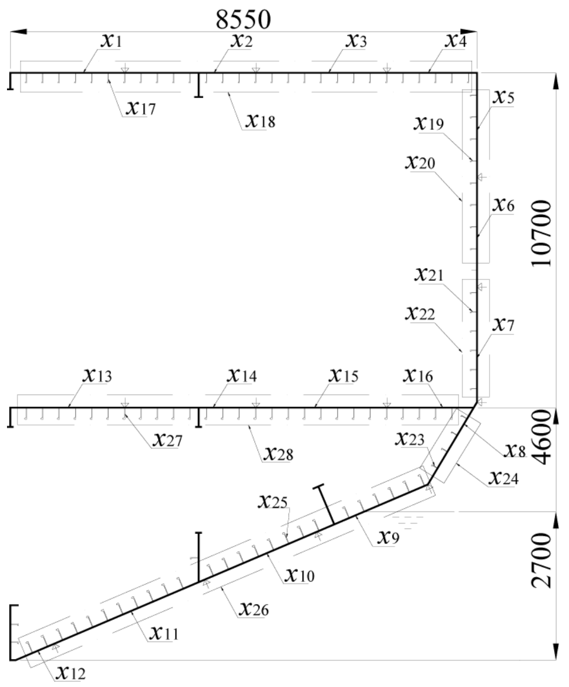
where f´ stands for a normalized value of an objective.

The choice of the weighting coefficient w enables now a biased search for the part of the Pareto front assumed to contain the scalar optimum. It is dependent, however, on the choice of the problem and constraint representation. For VO-2, the weighting coefficient w_0 of the objective function should be taken as $w_0 \le w_j$, $\forall j \in 1...l$, depending also on the number of constraints, in order to concentrate the search within a neighbourhood of points with $g_j \cong 0$. For VO-1, the choice of the w_j is more difficult and several options might have to be tried before obtaining the acceptable result. A possible strategy is to first approach the points bearing minimal values of f_0, using $w_0 >> w_j$, $\forall j \in 1...l$ and then, if the amount of obtained feasible designs is low, gradually reduce the values of w_0 to allow for finding more feasible designs. Additionally we modify the fitness function in (8) by penalizing the infeasible designs, see (10), and preventing them to enter into next generation to force generations consisting of mostly Pareto optimal points, out of which one would be $\mathbf{x}^*$.

$$\varphi(\mathbf{x}^i) = \begin{cases} \max \mathrm{d}(\mathbf{x}) + \dfrac{1}{\mathrm{d}(\mathbf{x}^i)}, & \text{if } \mathbf{x}^i \in (\Omega \cap \hat{\mathbf{X}}) \\ \max \mathrm{d}(\mathbf{x}) - \mathrm{d}(\mathbf{x}^i), & \text{otherwise} \end{cases} \tag{10}$$

Figure 1 **A half of the ferry's midship section with marked design variables x**

Slika 1 **Prikaz oznaka projektnih varijabli x na polovini glavnog rebra trajekta**



4 Optimization of a fast ferry

The presented vectorization concepts are shown on a practical example of the minimum weight design of a fast ferry's midship section; see Figure 1. The results are compared with the classical scalar approach of (1) with the fitness function of (7).

4.1 The structural design model

The ship is designed under the rule requirements of DNV for a high speed, light craft and a naval surface craft [13]. It is considered in fully loaded condition, for both crest and hollow landing, with the amplitudes of M_{CREST} = 143778 kNm and M_{HOLLOW} = 157572 kNm.

The axle load of 1.0 t/axle for the car deck, at 4600 mm from the keel, is applied on the tyre print areas of 115 x 88 mm. The load on the passenger deck is taken as for the weather deck following the assumption that the superstructure does not contribute to the global strength of the ship. Other local loads, such as water pressure, are applied according to the Rules. The applied aluminium alloys 5083 and 6082 are used respectively for plating and stiffeners, with the yield strength of 106 MPa and 84 MPa and the material factor f_1 of 0.44 and 0.35. The Young modulus of 70 GPa and the Poisson coefficient of 0.28 are the same for both alloys.

Design variables include the scantlings of all the longitudinal elements except girders, as well as the spacing of the longitudinal stiffeners. Table 3 lists all the design variables with minimal and maximal bounds. Generally, the minimum plate thickness of 5 mm was chosen due to a possible severe increase in deformations during the welding of thinner plates. Same wise, the minimum longitudinal spacing is selected at 200 mm.

The objective function f_0 is defined trough the total area of all longitudinal elements in one half of the midship section, including girders. Applied minimal requirements for the thickness and the size of longitudinals are given through Pt.3 Ch.3 Sec.5 of [13], namely paragraphs B100 and C100 and Tables B1 and C1. Additionally, all the structural elements are checked for the buckling due to the longitudinal global hull girder loads, see Pt.3 Ch.3 Sec.10 [13, p.27]. The distribution of global hull shear loads is not specifically studied, but it is accounted for through the requirement for minimal thickness of plates.

4.2 The GA model

Variables are binary coded with 4 bit long strings, based on their integer representation, with the step of 0.5 mm for the plate thickness, 0.7 cm² for the size of longitudinals and 10 mm for their spacing. A population of 50 design alternatives, or individuals, is created within each generation following the randomly generated initial population. Based on the computed fitness of an individual, the GA uses a weighted roulette wheel to select designs for the mating pool. Individuals' chromosomes, or a binary string of variable values, are mated with a probability of 0.8 using the randomly selected single point cross-over between two consecutive individuals in the mating pool. Subsequently, the individuals' chromosomes are mutated bit-wise with a probability of 0.03.

Following the conclusions of chapter 3, VO-1 uses high weighting coefficient w_0 of 0.5 for f_0, while w_j's equally share the difference to 1, so that the biased search of Pareto optimal designs in **Y** will concentrate on the lowest attainable values of f_0. VO-2 uses the weighting coefficient w_0 of 0.05 to bring the search closer to the axes of f_0 with w_j's equally sharing the difference to 1.

4.3 The results

We analyze 10 optimization runs of each of the applied approaches. The following 13 measures are applied to capture the particular performance:

- The minimum of the objective function $f_0(\mathbf{x}^{**})$ and its generation $G_{x^{**}}$ for the best run,
- The difference between the $f_0(\mathbf{x}^{**})$ and $f_0(\mathbf{x}^{ref})$ in per cent,
- The mean μ_{x^*} and the dispersion σ_{x^*} of the fittest designs $\mathbf{x}^*$ for the best run,
- Generation of the first crossing of the $\mathbf{x}^*$ below the m_{x^*}, $G_{f_0(\mathbf{x}^*)<mx}$,
- The objective function value $f_0(\mathbf{x}^*_{1\%})$ of the top 1% designs $\mathbf{x}^*_{1\%}$, and their generations of obtainment $G_{\mathbf{x}^*_{1\%}}$,
- The objective function value $f_0(\mathbf{x}^{**}_{1\%})$ of the fittest designs $\mathbf{x}^{**}_{1\%}$ within 1% of the $f_0(\mathbf{x}^{**})$, and their generations of obtainment $G_{\mathbf{x}^{**}_{1\%}}$,
- The mean ${}^{10}_{1}\mu_{\mu_{x^*}}$ and the dispersion ${}^{10}_{1}\sigma_{\mu_{x^*}}$ of the μ_{x^*} for all 10 runs,
- The mean ${}^{10}_{1}m_{f_0(\mathbf{x}^{**})}$ and the dispersion ${}^{10}_{1}s_{f_0(\mathbf{x}^{**})}$ of the $f_0(\mathbf{x}^{**})$ for all 10 runs,
- The mean ${}^{10}_{1}m_{G_{\mathbf{x}^{**}_{1\%}}}$ and the dispersion ${}^{10}_{1}S_{G_{\mathbf{x}^{**}_{1\%}}}$ of the $G_{\mathbf{x}^{**}_{1\%}}$ for all 10 runs.

Table 1 and Table 2 present these measures for the best run, and for all 10 runs respectively. Furthermore, Figures 2 to 4 illustrate the optimization history for the best runs.

Table 1 **Optimization results for the best runs**
Tablica 1 **Rezultati optimizacije za najbolje prolaze**

	VO-1	VO-2	SO
$f_0(\mathbf{x}^{**})$ [m^2]	0.3905	0.3922	0.4129
$G_{x^{**}}$	99	278	34
$f_0(\mathbf{x}^{ref})$-$f_0(\mathbf{x}^{**})$ [%]	7.5	7.1	2.2
$\mu_{\mathbf{x}^*}$ [m^2]	0.4123	0.4154	0.4457
$\sigma_{\mathbf{x}^*}$ in per cent of $\mu_{\mathbf{x}^*}$	3.3	2.7	2.2
$G_{f_0(\mathbf{x}^*)<mx}$	16	32	8
$f_0(\mathbf{x}^*_{1\%})$ [m^2]	0.3905 0.3905 0.3915 0.3930 0.3954	0.3922 0.3942 0.3949 0.3964 0.3965	0.4129 0.4130 0.4191 0.4195 0.4207
$G_{\mathbf{x}^*_{1\%}}$	99 100 101 103 308	278 279 405 296 382	34 35 320 60 160
$f_0(\mathbf{x}^{**}_{1\%})$ [m^2]	0.3905 0.3905 0.3915 0.3930	0.3922 0.3942 0.3949	0.4129 0.4130
$G_{\mathbf{x}^{**}_{1\%}}$	99 100 101 103	278 279 405	34 35

Table 2 **Optimization results for all the computed runs**
Tablica 2 **Rezultati optimizacije za sve izvedene prolaze**

	VO-1	VO-2	SO
${}^{10}_{1}\mu_{\mu_{x^*}}$ [m^2]	0.4153	0.4184	0.4465
${}^{10}_{1}\sigma_{\mu_{x^*}}$ in per cent of ${}^{10}_{1}\mu_{\mu x^*}$	0.57	0.64	0.43
${}^{10}_{1}m_{f_0(\mathbf{x}^{**})}$ [m^2]	0.3940	0.3950	0.4175
${}^{10}_{1}s_{f_0(\mathbf{x}^{**})}$ in per cent of ${}^{10}_{1}m_{f_0(\mathbf{x}^{**})}$	0.7	0.6	0.9
${}^{10}_{1}m_{G\mathbf{x}^{**}_{1\%}}$	282.40	296.70	250.60
${}^{10}_{1}S_{G\mathbf{x}^{**}_{1\%}}$ in per cent of ${}^{10}_{1}m_{G\mathbf{x}^{**}_{1\%}}$	56.3	35.6	62.3

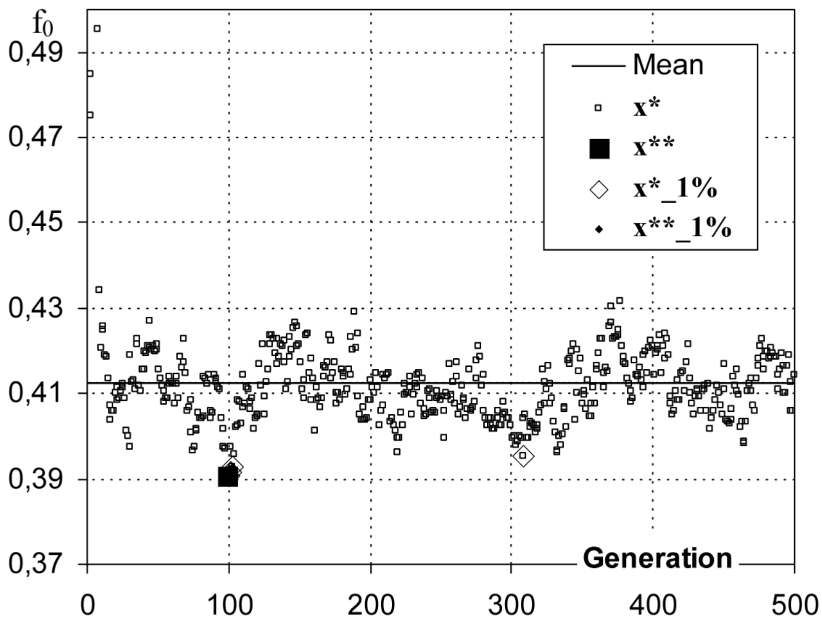


Figure 2 **Optimization history for the VO-1 best run with marked minimum weight designs per generation x* and its mean, global minimum x**, top 1 per cent best performing designs x*_1%, and designs within 1 per cent of the obtained minimum weight x**_1%**

Slika 2 **Povijest optimizacija za VO-1 najbolji prolaz s označenim projektima najmanje težine po generaciji x* i njihovi srednji, globalni minimum x**, gornjih 1 % najboljih projekata x*_1%, i projekata unutar 1 % t x**_1% dobivenih najmanjih težina.**

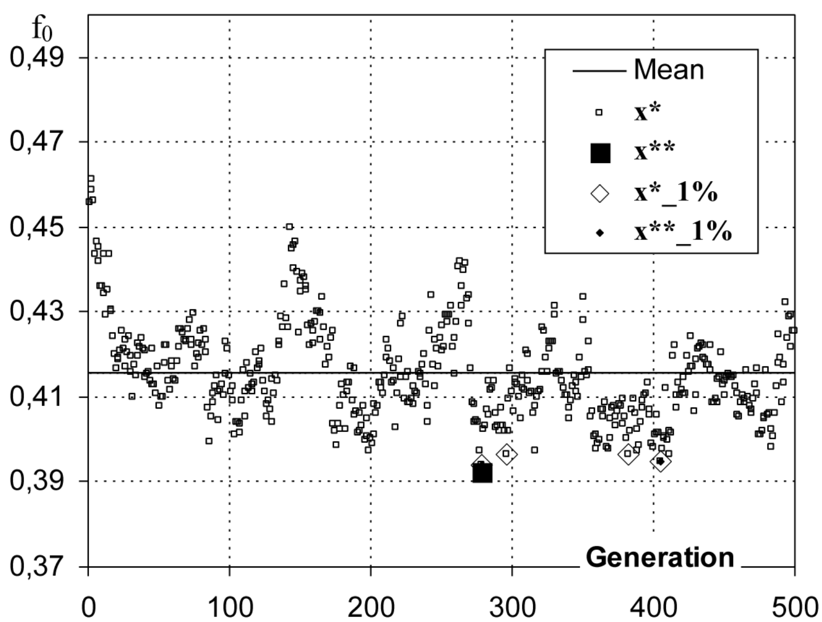


Figure 3 **Optimization history for the VO-2 best run with marked minimum weight designs per generation x* and its mean, global minimum x**, top 1 per cent best performing designs x*_1%, and designs within 1 per cent of the obtained minimum weight x**_1%**

Slika 3 **Povijest optimizacija za VO-2 najbolji prolaz s označenim projektima najmanje težine po generaciji x* i njihovi srednji, globalni minimum x**, gornjih 1 % najboljih projekata x*_1%, i projekata unutar 1 % t x**_1% dobivenih najmanjih težina.**

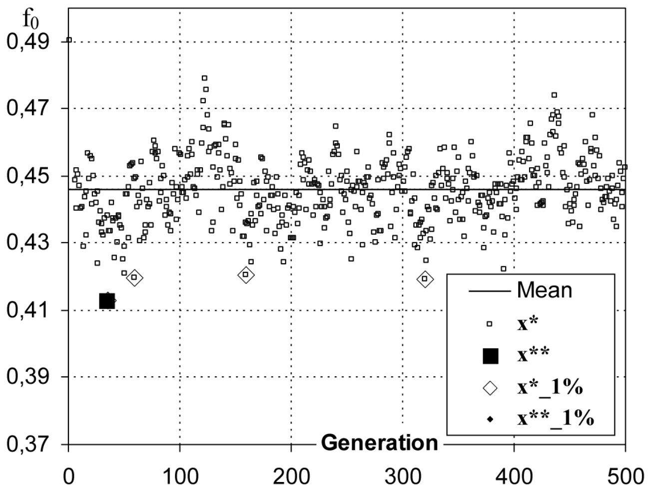


Figure 4 **Optimization history for the SO best run with marked minimum weight designs per generation x^* and its mean, global minimum x^{**}, top 1 per cent best performing designs x^*_1%, and designs within 1 per cent of the obtained minimum weight x^{**}_1%**

Slika 4 **Povijest optimizacija za SO najbolji prolaz s označenim projektima najmanje težine po generaciji x* i njihovi srednji, globalni minimum x**, gornjih 1 % najboljih projekata x*_1%, i projekata unutar 1 % t x**_1% dobivenih najmanjih težina.**

5 Discussion

Clearly, the results show that GA managed to improve on the referenced design using all the presented approaches. The VO-1 has performed the best by reducing the cross sectional area of the midship section by 7.5 per cent. It is closely followed by the VO-2 with a 7.1 per cent improvement, while the SO failed to perform better than 2.2 per cent. The obtained improvements are therefore in line with the typical improvements in literature, as e.g. in [4].

Solving the vectorized problems obtains better results than solving the problem in the original representation of SO. The VOs find the smaller minimums of the objective function, but also generally fitter designs throughout generations, see $f_0(\mathbf{x}^{**})$ in Table 1 and ${}_1^{10}m_{f_0(\mathbf{x}^{**})}$ and ${}_1^{10}\mu_{\mu_{x^*}}$ in Table 1. See also Figures 2 to 4 for the illustration of the differences. The dispersion of these values is small for all the approaches, hence the significance of means is high, see ${}_1^{10}s_{f_0(\mathbf{x}^{**})}$ and ${}_1^{10}s_{m_\mathbf{x}}$. In Table 1 and Figure 2 we can see that for the best run of VO-1, a series of four designs within 1 per cent of the minimum are obtained already around the 100th generation, see $f_0(\mathbf{x}^{**}_{1\%})$ and $G_{\mathbf{x}^{**}_{1\%}}$.

Table 3 provides a comparison between the obtained minimum weight design, now taken to be the minimum of VO-1 $\mathbf{x}^{**}_{\text{VO-1}}$, its standardized version $\mathbf{x}^{**}_{\text{stand}}$ and the referenced design $\mathbf{x}^{\text{ref}}$. In comparison with the reference, the optimization reduced the spacing of longitudinals, which then generally caused the reduction of plate thicknesses and size of longitudinals. Due to the application

Table 3 **Design variables with min – max bounds and values for the reference $\mathbf{x}^{\text{ref}}$, computed optimum $\mathbf{x}^{**}_{\text{VO-1}}$ and its standardized version $\mathbf{x}^{**}_{\text{stand}}$**

Tablica 3 **Projektne varijable sa min-maks graničnim vrijednostima za odnosne $\mathbf{x}^{\text{ref}}$, proračunati optimumi $\mathbf{x}^{**}_{\text{VO-1}}$ i njihove standardizirane vrijednosti $\mathbf{x}^{**}_{\text{stand}}$**

Design variable	Min	Max	$\mathbf{x}^{\text{ref}}$	$\mathbf{x}^{**}_{\text{VO-1}}$	$\mathbf{x}^{**}_{\text{stand}}$
Thickness of passenger deck – strake 1, x_1 [mm]	5	12.5	8	5.5	**7**
Thickness of passenger deck – strake 2, x_2 [mm]	5	12.5	8	9.5	**8**
Thickness of passenger deck – strake 3, x_3 [mm]	5	12.5	8	9	9
Thickness of passenger deck – strake 4, x_4 [mm]	5	12.5	8	9	9
Thickness of shear strake 1, x_5 [mm]	5	12.5	9	5	**7.5**
Thickness of side shell – strake 1, x_6 [mm]	5	12.5	8	5.5	5.5
Thickness of side shell – strake 2, x_7 [mm]	5	12.5	8	7.5	**5.5**
Bilge strake, x_8 [mm]	6	13.5	9	6.5	6.5
Thickness of bottom shell – strake 1, x_9 [mm]	7	14.5	10	8	8
Thickness of bottom shell – strake 2, x_{10} [mm]	7	14.5	11	9	9
Thickness of bottom shell – strake 3, x_{11} [mm]	7	14.5	12	13	13
Keel plate, x_{12} [mm]	8	15.5	12	14	14
Thickness of car deck – strake 1, x_{13} [mm]	5	12.5	8	6	6
Thickness of car deck – strake 2, x_{14} [mm]	5	12.5	8	6.5	**6**
Thickness of car deck – strake 3, x_{15} [mm]	5	12.5	8	6	6
Thickness of car deck – strake 4, x_{16} [mm]	5	12.5	8	6	6
Size of passenger deck longitudinals, x_{17} [cm^2]	5.4	15.9	9.31	6.80	**6.75**
Spacing of passenger deck long's, x_{18} [mm]	200	350	300	210	210
Size of upper side shell longitudinals, x_{19} [cm^2]	5.4	15.9	6.20	5.40	5.40
Spacing of upper side shell long's, x_{20} [mm]	200	350	400	230	230
Size of lower side shell longitudinals, x_{21} [cm^2]	5.4	15.9	6.20	8.20	**5.40**
Spacing of lower side shell long's, x_{22} [mm]	200	350	350	230	230
Size of bilge longitudinals, x_{23} [cm^2]	5.4	15.9	6.20	7.50	**7.74**
Spacing of bilge longitudinals, x_{24} [mm]	200	350	350	230	230
Size of bottom shell longitudinals, x_{25} [cm^2]	5.4	15.9	12.40	13.10	**13.83**
Spacing of bottom shell longitudinals, x_{26} [mm]	200	350	300	280	280
Size of car deck longitudinals, x_{27} [cm^2]	5.4	15.9	12.4	5.4	5.4
Spacing of car deck longitudinals, x_{28} [mm]	200	350	300	200	200
Total area of a half of the midship section [m^2]			**0.4221**	**0.3905**	**0.3885**

of different classification rules within this study, the reference breaks the constraints of longitudinal strength as well as the minimal requirements for the side shell and bilge plate thickness and their stiffener size. Following that, the obtained minimum weight design is actively constrained by the longitudinal strength, thus some of the passenger deck strakes, as well as the lowest parts of the bottom shell, including the keel, are thickened in comparison to the reference, while the spacing of their stiffeners is reduced with additional increase in the size of the bottom stiffeners.

After the optimization was completed we reduced some of the obtained variable values from $\mathbf{x}^{**}_{VO\text{-}1}$, namely the size of the lower side longitudinals and the thickness of the 2nd car deck strake. Within this process we additionally standardized the size of longitudinals, interchanged the thicknesses of the shear strake and the 2nd side shell strake to obtain a rational distribution of the plate thickness over the side, and reduced the difference between the neighbouring strakes in the passenger deck. We attained a lower value of f_0 by the additional 0.5 per cent. Obviously, $\mathbf{x}^{**}_{VO\text{-}1}$ is not then the global minimum, but it has approached active constraints the most, which is typical for GAs as convergence to the global minimum cannot be guaranteed for some finite number of generations.

6 Conclusion

Within this paper we presented an alternative approach to single-objective optimization problems, addressed as *vectorization*. Using the large-scale problem as an example, we have shown that the consideration of constraints as additional objectives alongside the original objective function can provide benefits regarding the achieved minimum weight design.

Two approaches to vectorization have been compared, one considering the formulating of constraints with a heavy side function, in which feasible designs receive the minimum value of a constraint function, and the other with the constraint function values represented as their absolutes. These were then implemented into a simple genetic algorithm and confronted in the case study with the single-objective conventional approach. The obtained results seem to encourage vectorization as both of the vectorization approaches outmatched the conventional approach. The compared minimum weight design is 8 per cent better than the referenced design.

Vectorization, as presented in this study, might however face difficulties if e.g. the ship structural design problem expands over the overall ship structure, in which case the number of constraints explodes into thousands. Thus, vectorization should be further studied and improved in order to tackle such problems. But, following on the results presented here, vectorization can already find its use and offer efficient design optimization.

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