

MINIMUM CORRECT THERMO-ELECTRIC MODEL FOR TRANSIENT BEHAVIOUR OF INCANDESCENT LAMP

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Accurate description of thermo-electric systems and their behaviour during transient represents a demanding task. Because of high nonlinearity of elements in such system and if linearization cannot be done during transient, corresponding differential equation(s) are also nonlinear. Development of personal computers (PCs) and the reduction of their price, provide application of the numerical techniques and procedures for solving such nonlinear systems instead of the analytical solving techniques. However, knowing of analytical expression(s) which describes behaviour of nonlinear system during the transient is beneficial. In this article development of minimum correct transient thermo-electric model of incandescent lamp is described in detail. Procedure for obtaining analytical solution with exact solution which describes behaviour of incandescent lamp during the transient is shown. Obtained analytical solution and presented model are verified with numerical solution of non approximated nonlinear differential equation and measurement results. The application of presented thermo-electric model for description transient behaviour of incandescent lamp is shown on obtaining inrush current of well known H7 car light bulb. Presented thermo-electric model and its analytical solution are also applicable for determining inrush current of all incandescent lamps and infrared heaters.

Keywords: *incandescent lamp, inrush current, nonlinear differential equation, thermo-electric model, transient*

Minimalno korektni termo-električni model tranzijentnog ponašanja žarulje

Izvorni znanstveni članak

Točan opis termo-električnih sustava i njihovog ponašanja tijekom tranzijenata predstavlja zahtjevan zadatak. Zbog nelinearnosti elemenata u takvom sustavu i ukoliko nije moguće provesti linearizaciju tijekom tranzijenta, odgovarajuća diferencijalna jednačba(e) su također nelinearne. Razvoj osobnih računala (PC) te pad njihove cijene, omogućavaju primjenu numeričkih tehnika i postupaka za rješavanje takvih nelinearnih sustava, umjesto analitičkih tehnika rješavanja. Međutim, poznavanje analitičkog izraza koji opisuje ponašanje nelinearnih sustava tijekom tranzijenta je blagodatno. Razvoj minimalno korektnog tranzijentnog termo-električnog modela žarulje pobliže je opisan u ovom članku. Prikazan je postupak određivanja analitičkog rješenja s egzaktnim rješenjem koje opisuje ponašanje žarulje tijekom tranzijenta. Dobiveno analitičko rješenje i prikazani model verificirani su numeričkim rješenjem neapksimirane nelinearne diferencijalne jednačbe i mjernim rezultatima. Primjena prikazanog termo-električnog modela za opisivanje tranzijentnog ponašanja žarulje pokazana je na primjeru određivanja uklopne struje dobro poznate automobilske H7 žarulje. Prikazani termo-električni model i njegovo analitičko rješenje su primjenjivi za određivanje uklopnih struja svih žarulja i infracrvenih grijača.

Cljučne riječi: *nelinearna diferencijalna jednačba, termo-električni model, tranzijent, uklopna struja, žarulja*

1 Introduction

In engineering and science, a special class of tasks are those with inherent nonlinearity [1], which is due to various circumstances that must be taken into account. Within this class of tasks, there is a special subset of the tasks for which by some reason cannot or should not be made a linearisation for the purpose of obtaining solutions. Such tasks can only be simulated and analyzed on the computer [1]. In this case, the experience of the behavior of such nonlinear systems can be acquired only by carrying out multiple simulations on the computer with varying system parameters and initial and boundary conditions.

Although this approach usually meets the needs of the majority of those who conducted it knowledge of analytical solutions, if one could get to it, would be equally informative with the difference that experience of the system behavior can be acquired without conducting multiple simulations on the computer. In addition, it avoids a permanent presence in doubt of the validity of the model and numerical calculation (simulation) on the computer, i.e. in such a way obtained results. Furthermore, it is possible to use analytical solutions for validation of complex simulation (larger number of variables and the parameters) that is performed on the personal computer. For these reasons, whenever possible, it is beneficial to have at least an approximate analytical expression which describes the behavior of certain nonlinear systems.

The process of determining an analytical expression for the behavior of some variables in the nonlinear system traditionally is a very demanding task, with an uncertain outcome. The success of its determining depends largely on previous experience with similar nonlinear systems [2]. This paper presents all theories, with all steps in the deriving the expressions, that give a clear and unambiguous insight into one way of thinking and analytical procedures to determine the expression for transient behavior of one very widespread nonlinear system. In this paper without loss of generality in the case of ordinary (H7) automotive light bulbs [3] is provided unambiguous and clear guidance on how to approach modelling, and form a thermo-electric model that minimum correctly describes the transient behavior of incandescent lamp and infrared heaters. Also, in the paper are given the clear limits of validity of the analytical expressions obtained by the presented thermo-electric model. All theoretical results obtained are further evaluated by measurement results.

2 Problem description

The transient event that occurs during energization of incandescent bulbs and infrared heaters is accompanied by a unique change of current, which is in literature mostly sketched or shown by oscillograms. Inrush current of incandescent bulbs and infrared heaters during the transient, and the temperature of their filaments are described by approximate expressions. The circumstances

and the conditions under which these expressions were obtained are unclear. Also, there is no conducted analysis and determination of the limits of validity of such expressions. To obtain analytical expressions which can relatively accurately describe the transient event and thus the waveforms of the inrush currents for incandescent bulbs and infrared heaters, it is necessary to solve a differential equation and skillfully get an expression for the temporal temperature change of the tungsten filament. This expression is then used to obtain acceptable correct expressions for the temporal change of tungsten filament electrical resistance, and ultimately, the inrush current. The paper discusses the construction of a thermo-electric model valid for the transient state and transient analysis on the example of automotive bulbs H7, so that without loss of generality presentation was simpler, measurement cheaper, and measurement results easily reproducible and verifiable. All conditions under which is derived analytical expression for the temporal change of temperature for incandescent bulbs during the transient are described and commented. All important steps in obtaining an analytical expression which describes the temporal change of tungsten filament temperature are provided. On the basis of this expression, the temporal change of filament resistance is determined and ultimately the temporal current change (inrush current). The values obtained by this analytical expression were compared with values obtained by numerically solving the exact differential equations, and compared with measurement results. Logic of thinking, derived expressions, and differential equation which describes the transient behavior of the lamp after its energization is in such a form that it is understandable to a wider range of engineers, appropriate for use in very widespread and affordable software packages for mathematics, and general-purpose engineering simulator programs like: Mathematica, Mathcad or the very popular MATLAB [4 ÷ 6].

3 Inrush transient current

Inrush current of infrared heater and incandescent lamp has a unique waveform that results from changes in the electrical resistance of filament. Therefore, at the model level, represented by circuit elements, both infrared heater and incandescent lamp can be considered as nonlinear resistors. Depending on whether it is for an incandescent light bulb or infrared heaters, i.e. for which portion of the electromagnetic spectrum they work (visible or infrared radiation), their filaments in steady state conditions after transients have different temperatures. In the infrared heater, the operating temperature of the filament in steady state is smaller than the incandescent light bulb. This difference in temperature filaments in the steady state conditions is reflected in the difference between the minimum and maximum electrical resistance filament. For this reason, in advance can be expected lower rates of peak value of inrush current and the rated current (i_p / I_r) for infrared heaters than for incandescent light bulb. Rarely is the energization of a single lamp or an infrared heater problematic. However, in industry or large farms, where energization of a group includes a large number of individually powerful sources,

inrush currents are significant, and based on this knowledge, the protection of electrical installations must be properly configured. The prevailing electrical protection is by fuse, which, due to its nature (and therefore $I-t$ characteristics [7, 8]) can easily cope with this kind of switching transients. A more modern solution of protection is by switches. In this case it is necessary to pay greater attention to their proper selection of the higher sensitivity of their $I-t$ characteristics [7, 8].

The initial data for the proper selection and adjustment of the protection is the peak value of inrush current and duration of the transient. All data can be obtained by using the theory and derived expressions presented in this paper.

4 Thermo-electrical model assumptions and simplifications

Construction of minimally correct thermo-electric model, that describes the transient behavior after energization of the lamp, begins by simplifying the actual geometry of the lamp to the elements of interest and describing their behavior and interactions. The actual geometry of the H7 bulb is shown in Fig. 1.

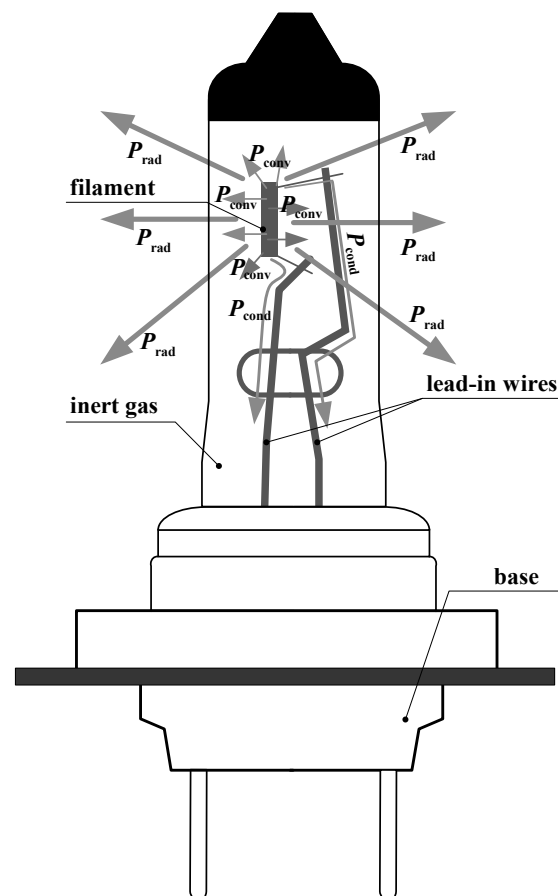


Figure 1 Cross section of H7 light bulb and main radiation, conduction and convection heat paths

The filament coil is simplified to the shape of a small cylinder (Fig. 2). Into its volume the Joule heat will be generated. Lead in wires is simplified to the shape of a cylinder of constant cross section (Fig. 2). Glass envelope is simplified to the shape of transparent cylinder. This assumption allows ignoring heat reflection from the top

and bottom of glass envelope. The gas inside the bulb is at a pressure independent of temperature, and does not circulate due to the temperature gradient inside bulb. This assumption means that the convection coefficient is constant.

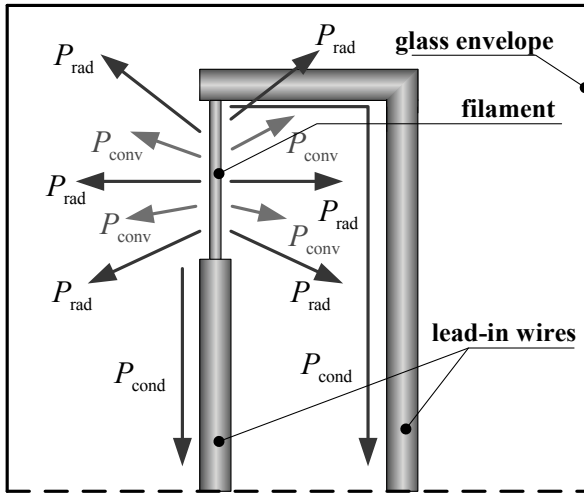


Figure 2 Simplified geometry of the H7 bulbs for thermal analysis and the main path of conduction, convection and radiation of heat transfer

To distinguish thermal quantities from electrical quantities, electrical quantities have the subscript "el" and thermal "th". The fundamental modes of heat transfer involved in incandescent light bulb and infrared heater are: conduction, convection and radiation.

Heat Conduction

This is the transfer of energy between objects that are in physical contact. This occurs in the case of the infrared heater and incandescent lamp between tungsten wire and lead-in wires (Figures: 1 and 2), and lead in wires to the base. Thermal resistance due to conduction of each lead in wire [9-13]:

$$R_{cond}^{th} = \frac{1}{\lambda_{LW}} \frac{l_{LW}}{A_{cond}} \left(\frac{K}{W} \right), \tag{1}$$

where: λ_{LW} - lead in wire conduction coefficient ($W \cdot m^{-1} \cdot K^{-1}$), l_{LW} - lead in wire length (m), A_{cond} - area involved in heat transfer by conduction (m^2) ie average lead in wire cross-sectional area.

Heat Convection

This is the transfer of energy between an object and its environment. This occurs in the case of the infrared heater and incandescent lamp between tungsten wire and fill gas. The fill gas is usually argon to prevent filament evaporation, with some nitrogen to eliminate arcing, (Figs. 1 and 2). Thermal resistance due to convection of each lead in wire and tungsten wire [9 ÷ 13]:

$$R_{conv}^{th} = \frac{1}{h_c A_{conv}} \left(\frac{K}{W} \right), \tag{2}$$

where: h_c - convection coefficient ($W \cdot m^{-2} \cdot K^{-1}$), A_{conv} - surface involved in heat transfer by convection (m^2).

Heat transfer model of lead in wires

Heat transfer by lead in wires includes conduction and convection. Part of the heat generated by the tungsten filament enters into one top end of lead in wires and it is conducted to the metal end of the light bulb (Fig. 1). Part of that heat is lost by convection to the gas inserted in the bulb to slow down evaporation of the filament. Depending on the needed accuracy, heat transfer by lead in wires can be described by lumped element model or distributed element model. For transient analysis, the lumped element model of heat transfer by lead in wires is sufficient. Elements of lumped model can be calculated by equations (1) and (2). The distributed element model (Fig. 3) is more accurate but a little more complex than the lumped element model. Quantities r_{LW}^{th} and g_{LW}^{th} in Fig. 3 denote lead in wire thermal conduction and convection resistance per length.

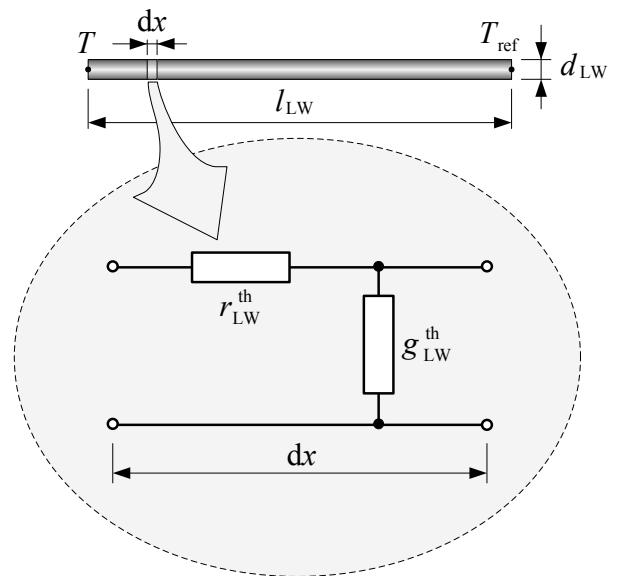


Figure 3 Lead in wire simplification and its heat transfer model with distributed parameters

Fortunately, because of simplicity of geometry, it can be recognized as a rod attached to a wall of constant temperature, surrounded by an ambient constant temperature for which equivalent heat resistance equations are well known [12, 13]:

$$R_{LW}^{th} = \frac{2}{\pi \sqrt{h_c \lambda_{LW} d_{LW}^3}} \text{th} \sqrt{\frac{4h_c l_{LW}^2}{\lambda_{LW} d_{LW}}} \left(\frac{K}{W} \right), \tag{3}$$

where: λ_{LW} - lead in wire conduction coefficient ($W \cdot m^{-1} \cdot K^{-1}$), l_{LW} - equivalent lead wire length (m), d_{LW} - equivalent lead wire diameter (m).

Electrical resistance of lead in wires and filament

Considering that the complete thermo-electric model is based on the knowledge of electrical filament resistance, it is necessary to devote a little more attention to its determination. The total electrical resistance $R^{el}(T)$ measured from the contacts outside the bulb is equal to

the sum of electrical resistance of lead in wires $R_{LW}^{el}(T)$ and electrical resistance of filament $R_F^{el}(T)$:

$$R^{el}(T) = R_{LW}^{el}(T) + R_F^{el}(T) . \tag{4}$$

Physical opening of the lamp allows easy electrical resistance measurements of the tungsten filament and each lead in wire at temperature of 300 K.

The exact value of the amount of electrical resistance of heated filament in the steady state is determined indirectly by measuring the lamp voltage and current in steady state. To do that, the exact value of the electrical resistance of lead in wires is replaced by an estimated value based on the following logical thinking.

Measurements by a thermovision camera of the filament [14] show that maximum temperature near the end of tungsten filament in steady state conditions is approximately 1500 °C (1225 K) (Fig. 3 in [14]). This value can be used to determine the average electrical resistance of lead in wires. In Fig. 3 in [14], lead in wire is not visible, so the temperature of lead in wire in steady state at point of contact with tungsten filament can only be assumed to be less than 1225 K. In this paper we assumed a value of 1000 K. Average electrical resistance of lead in wires can easily be calculated as:

$$\langle R_{LW}^{el} \rangle_{t \rightarrow \infty} = \frac{1}{l_{LW}} \int_{x=0}^{x=l_{LW}} r_{LW}^{el}(T, x) dx , \tag{5}$$

where: $r_{LW}^{el}(T, x)$ lead in wire electrical resistance per length ($\Omega \cdot m^{-1}$) at place x and temperature T .

Assuming a linear distribution of temperature along the lead in wire, from $T = 300$ K to $T_{max} = 1000$ K, then the average steady state lead in wire resistance is:

$$\langle R_{LW}^{el} \rangle_{t \rightarrow \infty} = \frac{1}{2} (R_{LW}^{el}(1000) + R_{LW}^{el}(300)) . \tag{6}$$

Assuming a similar temperature dependence of lead in wire resistance as tungsten (Tab. 2), according to which $R_{LW}^{el}(1000) \approx 5R_{LW}^{el}(300)$, then the average steady state lead in wire resistance is:

$$\langle R_{LW}^{el} \rangle_{t \rightarrow \infty} \approx \frac{1}{2} (5R_{LW}^{el}(300) + R_{LW}^{el}(300)) = 3R_{LW}^{el}(300) . \tag{7}$$

Accordingly, it can be assumed that the electrical resistance of lead in wires at steady state conditions ($t \rightarrow \infty$) is approximately 3 times greater when compared to the initial electrical resistance at 300 K.

At the steady state, the expected approximate temperature of tungsten filament is about $T = 2700$ K, so the expected steady state electrical resistance of tungsten filament is according to Tab. 2, approximately 14,5 times greater than the initial electrical resistance at 300 K.

At $t = 0^+$ ($T = 300$ K), the ratio of electrical resistance between tungsten filament and lead in wires is approximately $R_F^{el} / R_{LW}^{el} \approx 0,194 / 0,074 = 2,62$, while at

the steady state condition ($t \rightarrow \infty$), the ratio R_F^{el} / R_{LW}^{el} increases approximately five times, i.e. $R_F^{el} / R_{LW}^{el} \approx 14,5 \cdot 0,194 / 3 \cdot 0,074 = 4,8 \cdot 2,62 = 12,576$. This means that in steady state conditions the electrical resistance of tungsten filament is approximately 13 times greater than electrical resistance of lead in wires. So, at the end of transient (ie at the steady state), without significant error, it can be assumed that the total electrical resistance is equal to the electrical resistance of tungsten filament.

Heat Radiation

This refers to the transfer of energy to or from a body by means of the emission or absorption of electromagnetic radiation. This occurs between the tungsten wire and open space (out of bulb) in the case of infrared heater and incandescent lamp, (Figs. 1 and 2). The emissive power leaving an opaque black surface, commonly called blackbody emissive power, can be determined from quantum statistics as:

$$E_{b\lambda}(T, \lambda) = \frac{2\pi hc_0^2}{n^2 \lambda^5 (e^{hc_0/n\lambda kT} - 1)} \quad (n = \text{const.}) , \tag{8}$$

where it is assumed that the black surface is adjacent to a nonabsorbing medium of constant refractive index n . The constant $k = 1,3806 \times 10^{-23}$ J/K is the Boltzmann's constant, and $h = 6,626 \times 10^{-34}$ J·s is the Planck constant. The spectral dependence of the blackbody emissive power into vacuum ($n=1$) is shown for a number of emitter temperatures in Fig. 4 [12].

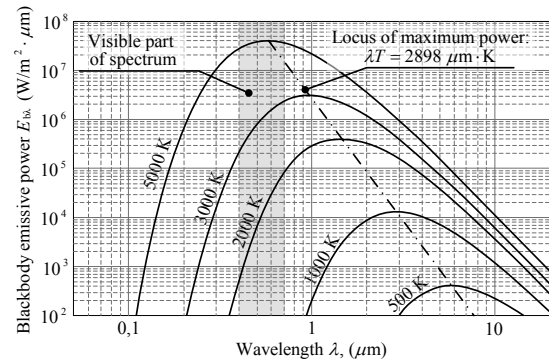


Figure 4 Blackbody emissive power spectrum

The integration of Planck's law over all frequencies provides the total radiated energy of a black body:

$$E_b(T) = n^2 \sigma T^4 . \tag{9}$$

The radiated power from the tungsten wire can be obtained by multiplying the blackbody emittance with the total tungsten emissivity and the coil surface area as:

$$P_{rad} = \varepsilon \sigma A_{rad} \Delta T^4 , \tag{10}$$

where: ε - total emissivity, σ - Stefan–Boltzmann constant ($5,67 \times 10^{-8}$ W·m⁻²·K⁻⁴), A_{rad} - surface included into radiation (m²), ΔT - temperature difference between surrounding ambient and source of radiation (K).

Since the emissivity of Tungsten varies as a function of temperature and surface finish, the values in Tab. 1, [15] should be used only as guidelines.

Table 1 Total emissivity of Tungsten [15]

<i>T</i> / K	300	375	775	1275	1775	2275	3025
ε	0,02	0,03	0,07	0,15	0,23	0,28	0,35

The temperature dependence of the emissivity described by the data in Tab. 1 can be described by a first-order polynomial of the form:

$$\varepsilon(T) = \varepsilon_{300K} (\gamma_0 + \gamma_1 \Delta T). \tag{11}$$

Applying least-squares routines available within the software package Mathcad 14 [4] on the data summarized in Tab. 1, the polynomial coefficients in the sense of least squares are obtained: $\gamma_0 = 1,331$, $\gamma_1 = 6,196 \times 10^{-3} \text{ K}^{-1}$.

The resistance of Tungsten filaments also vary as a function of temperature as shown in Tab. 2 [16].

Table 2 Temperature dependence of the tungsten wire resistance [16]

<i>T</i> (K)	300	600	900	1200	1500	1800	2100	2400	2700	3000	3300	3600
$\frac{R}{R_{300K}}$	1,00	2,34	3,88	5,48	7,14	8,86	10,63	12,46	14,34	16,29	18,28	20,35

Tungsten filaments radiate mostly infrared radiation at temperatures where they remain solid i.e. below 3683 K (3410 °C). Only for better description of temperature dependence data for 3600 K are given. Temperature dependence of the tungsten filament electrical resistance described by the data in Tab. 2 can be described by first and second order polynomial of the form:

$$R_F^{el}(\Delta T) = R_{300K} (\beta_0 + \beta_1 \cdot \Delta T), \tag{12}$$

$$R_F^{el}(\Delta T) = R_{300K} (\alpha_0 + \alpha_1 \Delta T + \alpha_2 \Delta T^2). \tag{13}$$

Applying least-squares routines available within the software package MATHCAD 14 on the data summarized in Tab. 2, the polynomial coefficients obtained for the first and second order polynomial: $\beta_0 = 1$, $\beta_1 = 5,61 \times 10^{-3} \text{ } \Omega \cdot \text{K}^{-1}$ and $\alpha_0 = 1$, $\alpha_1 = 4,636 \times 10^{-3} \text{ } \Omega \cdot \text{K}^{-1}$, $\alpha_2 = 3,778 \times 10^{-7} \text{ } \Omega \cdot \text{K}^{-2}$.

5 Mathematical description and analytical solutions

The previously stated assumptions and simplifications allow mathematical description of transient behavior of filament heating and restitution of steady state conditions [17]. With the aid of the obtained analytical equations, the description of other physical quantities of interest, such as inrush current is possible. Equivalent thermal network represented by circuit elements in the Kirchhoff circuit are shown in Fig. 5.

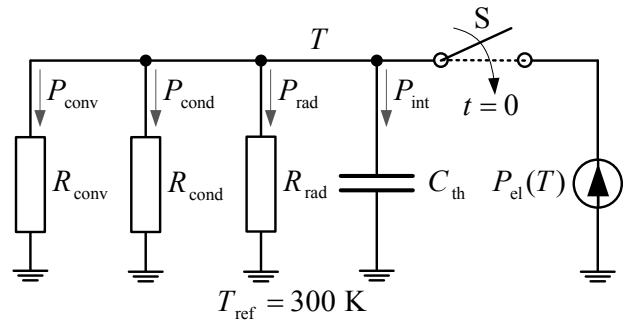


Figure 5 Equivalent thermal network represented by circuit elements in the Kirchhoff circuit

After switching on, for each $t \geq 0^+$, holds:

$$P_{el} = P_{rad} + P_{cond} + P_{conv} + P_{int}. \tag{14}$$

where: P_{el} - electric power (W), P_{rad} - heat loss by radiation (W), P_{cond} - heat loss by conduction to the leads (W), P_{conv} - heat loss to the gas by convection (W), P_{int} - heat loss by increasing internal energy (W).

From (14) follows the differential equation:

$$P_{el}(T) \cdot dt = \varepsilon(T) \sigma A_r \Delta T^4 dt + R' \Delta T dt + m c d(\Delta T), \tag{15}$$

where $R' = R_{cond} + R_{conv}$, $P_{el}(T)$ - the electrical power (W), m - mass of tungsten wire (kg), c - tungsten filament specific heat capacity ($\text{W} \cdot \text{s} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$), T - temperature (K), A_r - surface included into radiation (m^2), σ - Stefan-Boltzmann constant, $\varepsilon(T)$ - tungsten emissivity, ΔT - temperature difference with respect to the ambient temperature (K).

Electrical power is dependent on the electrical resistance of the filament:

$$P_{el}(\Delta T) = I^2(\Delta T) R^{el}(\Delta T) = \frac{U^2}{R^{el}(\Delta T)}. \tag{16}$$

Substituting (16) in (15), gives:

$$\frac{U^2}{R^{el}(\Delta T)} dt = \varepsilon(T) \sigma A_r \Delta T^4 dt + R' \Delta T dt + m c d(\Delta T). \tag{17}$$

Taking substitution ($G' = 1/R'$) and substituting the expressions (11) and (12) into (17), and after separation of variables the differential equation is obtained:

$$dt = \frac{m c R_{300K} (1 + \beta_1 \Delta T) d(\Delta T)}{U^2 - G' \Delta T - \varepsilon_{300K} \sigma A_r R_{300K} (\gamma_0 + \gamma_1 \Delta T) (1 + \beta_1 \Delta T) \Delta T^4} \tag{18}$$

The solution obtained by the differential Eq. (18) describes the transient heating of the tungsten filaments. The numerator on the right side of the differential Eq. (18) is of the first order, and the denominator of the sixth order. Unfortunately, an analytical solution of the differential equation does not exist. For this reason, it is

necessary to reduce it to a form for which there is a solution. Care must be taken in advance so that the temperature as function of time can be explicitly determined from the obtained analytical solution.

Physically, it is reasonable to assume that during the transient, heat conduction and heat convection can be ignored. Taking this into account, Eq. (18) becomes:

$$dt = \frac{mcR_{300K}(1+\beta_1\Delta T) d(\Delta T)}{U^2 - \varepsilon_{300K}\sigma A_r R_{300K}(\gamma_0 + \gamma_1\Delta T)(1+\beta_1\Delta T)\Delta T^4}. \quad (19)$$

Furthermore, let the emissivity be constant during the transients and equal to average emissivity:

$$\bar{\varepsilon} = \frac{1}{T_m - T_{ref}} \int_{T_{ref}}^{T_m} \varepsilon(T) dT. \quad (20)$$

Then Eq. (19) becomes:

$$dt = \frac{mcR_{300K}(1+\beta_1\Delta T) d(\Delta T)}{U^2 - \bar{\varepsilon}\sigma A_r R_{300K}(1+\beta_1\Delta T)\Delta T^4}. \quad (21)$$

Putting $T_{ref} = 0$ K instead of $T_{ref} = 300$ K, then $\Delta T = T$. This assumption simplifies Eq. (21), and it becomes:

$$dt = \frac{mcR_{300K}(1+\beta_1T) dT}{U^2 - \bar{\varepsilon}\sigma A_r R_{300K}(1+\beta_1T)T^4}. \quad (22)$$

Eq. (22) can easily be solved when electrical resistance of the filament is expressed in the form:

$$R_F^e(T) = \frac{R_{300K}}{300} T. \quad (23)$$

Then the differential Eq. (22) takes the form:

$$dt = \frac{mc \frac{R_{300K}}{300} T dT}{U^2 - \bar{\varepsilon}\sigma A_r \frac{R_{300K}}{300} T^5} = \frac{mc}{\bar{\varepsilon}\sigma A_r} \frac{TdT}{\frac{300 U^2}{\bar{\varepsilon}\sigma A_r R_{300K}} - T^5}. \quad (24)$$

Eq. (24) can be written in the form:

$$dt = k \frac{TdT}{a^5 - T^5}, \quad (25)$$

where:

$$k = \frac{mc}{\bar{\varepsilon}\sigma A_r}, \quad (26)$$

$$a = \sqrt[5]{\frac{300U^2}{R_{300K}\bar{\varepsilon}\sigma A_r}}. \quad (27)$$

Since the parameter a refers to maximum filament temperature i.e. $a = T_m$, its more accurate value can be

obtained if $\bar{\varepsilon}$ in Eq. (27) is replaced with $\varepsilon(T_m)$. Because $\varepsilon(T_m)$ is unknown, it is necessary to first calculate T_m according to Eq. (27), then from Tab. 1 put emissivity for that temperature into Eq. (27).

Most of the time during the transient, the approximation $\Delta T^5 \approx \Delta T^4$ is valid, so Eq. (25) becomes:

$$dt = k \frac{TdT}{a^5 - T^4}. \quad (28)$$

The right side of the differential Eq. (28) has a solution of the form [18, 19, 20]:

$$\int \frac{x dx}{b^4 + x^4} = \frac{1}{4b^3} \ln \frac{b^2 + x^2}{b^2 - x^2}. \quad (29)$$

Integrating both sides of Eq. (28), gives:

$$t = \frac{k}{4b^3} \ln \frac{b^2 + T^2}{b^2 - T^2} + C. \quad (30)$$

Where C is the integration constant which is determined from the initial condition. Parameter b must be identified with parameter a determined by Eq. (27). The Eq. (30) adapted for taking the inverse logarithm function is:

$$\frac{4a^3}{k} t = \ln C' \frac{a^2 + T^2}{a^2 - T^2}, \quad (31)$$

where $C = \ln C'$. Taking the inverse logarithm function of Eq. (31), and replacing a with T_m gives:

$$e^{\frac{4a^3}{k}t} = C' \frac{T_m^2 + T^2}{T_m^2 - T^2}. \quad (32)$$

From expression (32), can easily be explicitly determined the temperature as function of time:

$$T = T_m \sqrt{\frac{e^{\frac{4a^3}{k}t} - C'}{C' + e^{\frac{4a^3}{k}t}}}. \quad (33)$$

The unknown constant of integration remains to be determined. For $t = 0^+$ filament temperature is 300 K, so Eq. (33) gives:

$$300 = T_m \sqrt{\frac{e^0 - C'}{C' + e^0}}. \quad (34)$$

After minor editing, the unknown constant of integration is:

$$C' = \frac{T_m^2 - 300^2}{T_m^2 + 300^2}. \quad (35)$$

Finally, an analytical expression which describes the temperature change of filament is:

$$T(t) = T_m \sqrt{\frac{1 - \frac{T_m^2 - 300^2}{T_m^2 + 300^2} e^{-4\frac{a^3}{k}t}}{1 + \frac{T_m^2 - 300^2}{T_m^2 + 300^2} e^{-4\frac{a^3}{k}t}}} \quad (36)$$

The term $k / 4a^3$ can be identified as a time constant:

$$\tau = \frac{k}{4a^3} = \frac{1}{4} \frac{mc}{\bar{\epsilon}\sigma A_r} \left(\frac{R_{300K} \bar{\epsilon}\sigma A_r}{300U^2} \right)^{\frac{3}{5}} = \frac{K}{U^{\frac{6}{5}}} \quad (37)$$

Once the temporal change of filament temperature is known, it is easy to determine the temporal change of filament electrical resistance, and thus the current that flows through it. A temporal variation of filament electrical resistance after energization of the lamp is obtained by inserting Eq. (36) in Eq. (12):

$$R^{el}(t) \approx R_{300K} (\beta_0 + \beta_1 \cdot T(t)) + R_{LW}^{el}(300) \quad (38)$$

Current during a transient:

$$i(t) = \frac{U}{R^{el}(t)} = \frac{U}{R_{300K} (\beta_0 + \beta_1 \cdot T(t)) + R_{LW}^{el}(300)} \quad (39)$$

In Eqs. (38) and (39) ($T(t)$) is determined by the Eq. (36).

Data from steady state conditions

In the steady state ($t \rightarrow \infty$), after the end of transients nearly all electric power is equal to the radiated power:

$$P_{el}(\infty) = \frac{U^2}{R^{el}(T_m)} = \epsilon(T_m) \sigma A_r \Delta T_m^4 \quad (40)$$

where: $P_{el}(\infty)$ - electric power at steady state conditions, $R^{el}(T_m)$ - electric resistance at steady state i.e. at a maximum temperature of tungsten filament.

By measuring the lamp voltage ($U(\infty)$) and current ($I(\infty)$) in steady state it is possible to indirectly determine the filament electrical resistance at maximum temperatures:

$$R^{el}(T_m) = \frac{U(\infty)}{I(\infty)} \quad (41)$$

By using Eq. (13) it is possible to indirectly determine filament overtemperature

$$\Delta T = \frac{1}{2\alpha_2} \left(-\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2 \left(\alpha_0 - \frac{R^{el}(T_m)}{R_{300K}} \right)} \right) \quad (42)$$

Knowing the electrical resistance of the tungsten wire and its temperature in the steady state, it is possible indirectly from Eq. (40) to determine the radiation area:

$$A_r = \frac{U^2(\infty)}{R^{el}(T_m) \epsilon(T_m) \sigma T_m^4} \quad (43)$$

6 Laboratory measurements

Laboratory measurements include measurements of electrical and non electrical quantities. Details of each measurement are stated separately.

The weight of Tungsten filament was measured using a laboratory scale and was found that it is $m = 0,03318$ g.

Measuring the diameter of lead in wires was performed using a micrometer and the measured value is $d_{LW} = 0,58$ mm.

Measuring the length of each lead in wire (Figs. 1 and 2) was performed using Vernier Caliper and the measured values: $l_{LW1} = 1,5$ cm (shorter lead in wire) and $l_{LW2} = 2,5$ cm (longer lead in wire).

Since the total electric resistance of H7 light bulb ($R^{el}(300)$) at the temperature of 300 K is approximately $0,27 \Omega$, by neglecting internal source resistance, it is possible to estimate the worst case peak value of inrush current i.e. $i_p \approx 12 / 0,27 = 44,44$ A. Because of that, a significant voltage drop is expected during transient on: contacts, internal resistance of laboratory source and power supply wires. To avoid this, as much as possible, instead of a laboratory source, lead acid automotive battery ($U_n = 12$ V, $C_n = 45$ A·h) is used, and the light bulb power supply wires with a cross section of $4,0$ mm².

According to the sheme in Fig. 7, by using a universal digital measurement instrument METEX M-4650, the current and voltage of H7 light bulb at the steady state condition one minute after energization is measured with measurement resistor in circuit: $I_{60s} = 4,2$ A and $U_{60s} = 12,2$ V.

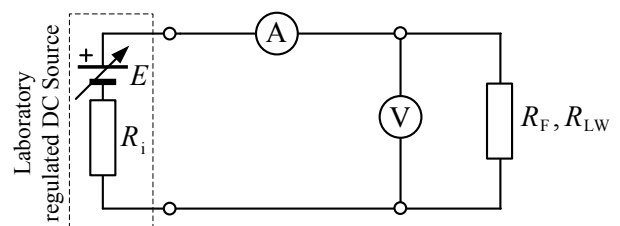


Figure 6 Schematic diagram of small resistance measurement setup

Since the light bulb has a small electric resistance, instead of direct measurement by universal instrument, its value is measured by $U-I$ method adopted for measurement of small resistance according to the sheme presented in Fig. 6 [21, 22]. Measurement gives values for total electric resistance of H7 light bulb $R^{el}(300) = 0,268 \Omega$, el. resistance of its lead in wires $R_{LW}^{el}(300) = 0,074 \Omega$, and el. resistance of tungsten filament $R_F^{el}(300) = 0,194 \Omega$ (measured with removed glass of another H7 light bulb).

With the aid of a digital scope FLUKE 123, the inrush current wave form is recorded using the measurement setup presented in Fig. 7 and its photograph is shown in Fig. 8.

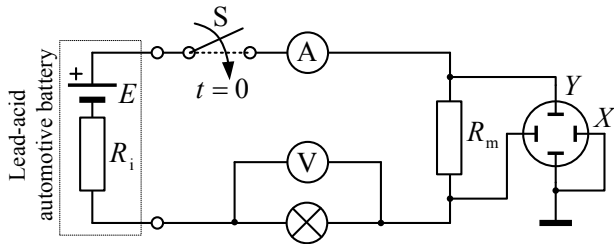


Figure 7 Schematic diagram of an inrush current measurement setup

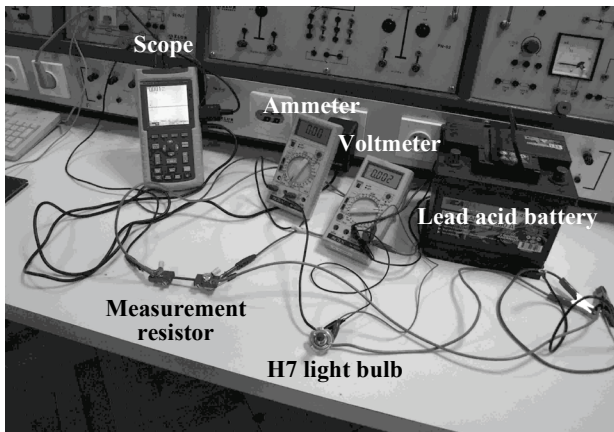


Figure 8 Photograph of an inrush current measurement setup

Transient inrush current is measured indirectly according to the schematic diagram presented in Fig. 7, by measuring the voltage drop on the measurement resistor with rated data ($I_n = 40 \text{ A}$, $U_n = 60 \text{ mV}$, $R_m = 1,5 \text{ m}\Omega$, $K = 0,5$) and it is determined by equation:

$$i = \frac{u_y}{R_m} = \frac{u_y}{0,0015} = 666,7 \cdot u_y \quad (44)$$

7 Numerical solution

Unlike the analytical approach in describing the transient's changes of filament temperature, it is not necessary to carry out any omissions in the numerical approach. Hence, the results of the numerical approach allow testing the limits of validity of the analytical approach, which was done in this article. The differential Eq. (18) takes into account: heat transfer by convection, heat transfer by conduction, heat radiation, variation of filament resistance, variation of filament emissivity, filament heat capacitance. During the development of the analytical expressions it is considered that during the transients there is no heat transfer interaction between gas and the lead in wires and vice versa. Also, heat conduction is completely ignored. It is an objective assumption which is valid only during transients. In the numerical solution, a key assumption is that the transient heat transfer from lead in wires and gas is unidirectional, i.e. from lead in wires to gas. In that case, lead in wires thermal resistance must be calculated by Eq. (3). In that

way, heat conduction and heat convection by lead in wires are taken into a count.

Solving the differential Eq. (18) was done using ready-made software routines "Odesolve" within the software package MATHCAD 14 that returns the solution to an ordinary differential equation (ODE), subject to initial value or boundary value constraints. Similar ready-made routines exist in the software packages MATLAB and "Mathematica" [4 ÷ 6].

Input data used by the numerical procedure for solving the differential Eq. (18) are: $\lambda_{LW} = 170 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$, $h_c = 50 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$, $l_{LW1} \approx 2,5 \text{ cm}$ (longer lead in wire), $l_{LW2} \approx 1,5 \text{ cm}$ (shorter lead in wire), $d_{LW} = 0,58 \text{ mm}$, $m = 0,03318 \text{ g}$, $c = 130 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$, $R_{300K} = 0,194 \Omega$, $U_{60s} = 12,153 \text{ V}$, $I_{60s} = 4,22 \text{ A}$.

A detailed list of commands and their meanings used in the software package MATHCAD 14 for solving differential Eq. (18) is described in [23].

8 Results and discussion

Temporal variation of filament temperature and bulb current obtained by Eqs. (36) and (39) and by numerical procedure, both by using software package MATHCAD 14, is shown in Fig. 9. Graphs in Fig. 9 indicate no significant difference between analytic approaches according to Eqs. (36) and (39) with corrected value of maximum filament temperature (see comment after Eq. (27)) and numerical approach. According to Fig. 9, the analytic approach, in which the corrected maximum filament temperature T_m is obtained, is very close to that obtained by numerical approach.

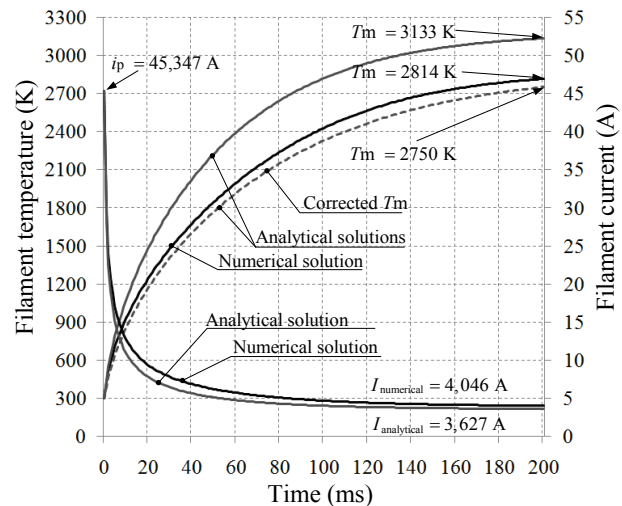


Figure 9 Current and temperature transients obtained by analytical expressions and numerical procedure

Because currents in analytical approach with and without corrected T_m are similar, for clarity, the current obtained by analytical approach with corrected T_m is omitted. Small differences exist between maximum filament temperatures. Eq. (42) gives $T_m = 2762 \text{ K}$, Eq. (42) but with lead in wire resistance at T_m taken into account gives $T_m = 2584 \text{ K}$, while numerical approach

gives $T_m = 2864$ K at $t = 0,3$ s, analytical approach by Eq. (27) gives $T_m = 3178$ K and corrected value for T_m gives $T_m = 2872$ K (see comment after Eq. (27)). All these values are close to those in Fig. 9, which means that at $t = 200$ ms transients in all approaches are almost done. As can be seen from Fig. 9 maximum filament temperature in analytical approach should be calculated by the procedure described in comment after Eq. (27). All of the above suggests that if measurement gives similar results for values, waveforms and trend then all the assumptions in the theoretical models are correct. Oscillogram of the voltage waveform measured at the measuring resistor is shown in Fig. 10, and voltage waveform measured at the H7 light bulb in Fig. 11.

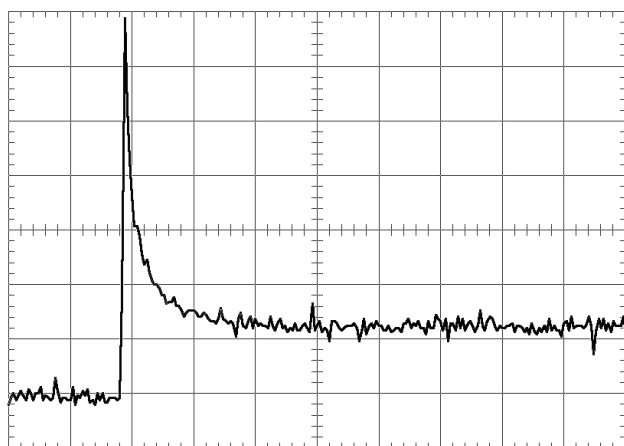


Figure 10 Oscillogram of the voltage waveform measured at the measuring resistor R_m (Y : 5 mV/DIV , X : 100 ms/DIV)

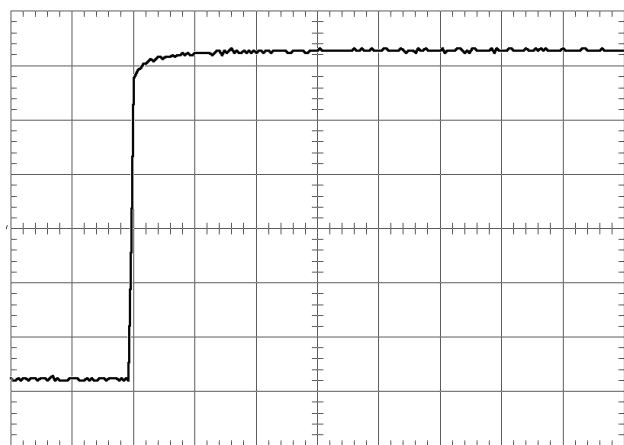


Figure 11 Oscillogram of the voltage waveform measured at the H7 light bulb (Y : 2,0 V/DIV , X : 50 ms/DIV)

By reading measurement data (Fig. 10) from digital oscilloscope transferred into a personal computer, it is found that the measured voltage peak value is $u_p = 34,8$ mV. The corresponding recalculated peak current according to Eq. (44) is $i_p = 23,20$ A. Taking the average of the last ten voltage samples gives the value $\langle u_m \rangle = 6,34$ mV. The corresponding recalculated steady state current according to Eq. (44) recorded by oscilloscope is $I_{0,ss} = 4,23$ A. Steady state current and voltage measured by ammeter and voltmeter one minute after energization are $I_{60s} = 4,22$ A, and $U_{60s} = 12,153$

V. Because the measured voltage on the measurement resistor actually represents current (Eq. (44)), it is possible by comparison of the waveform in Fig. 9 with that in Fig. 10. to conclude that in both cases current shapes and trends are similar. The differences are in peak values and transients duration. According to Fig. 10, the initial voltage ($t = 0^+$) on the measurement resistor is $u_y = u_m = 34,8$ mV, that recalculated to current by Eq. (44) gives $i_p = 23,20$ A. This is in disagreement with the theoretical prediction by both analytical and numerical procedures which predicted (Fig. 9) $i_p = 45,347$ A. This difference in current peak values is caused by voltage drops on: measurement resistor, wires that supply light bulb, ammeter and battery internal resistance. One must keep in mind that the theoretical model presented in this paper considers a constant voltage on the light bulb. This assumption is not valid during transient if significant voltage drops exist. The difference in transient duration is caused by several reasons. During filament heating, part of the energy realized by filament is accumulated as internal energy in tip of lead in wires that is in contact with the filament. The presented theoretical models (numerical and analytical) completely ignored the heat capacitance of lead in wires. Voltage drops also have influence on transient duration. According to Eq. (37), for example, if bulb voltage is $U' = 0,8U$, then time constant is $\tau'(0,8U) = 1,31\tau(U)$. The voltage measured at the H7 light bulb shown in Fig. 11 indicates this influence exists, but only at the beginning of transient. Since the voltage transient is very short, the bulb voltage recovered very fast. There also exist small differences between results for steady state current between results predicted by theoretical model and measured values. Steady state current predicted by theoretical model with numerical approach is 4,046 A and by analytical approach 3,627 A, while measurement gives $I_{0,ss} = 4,23$ A. Although this difference is acceptable (less than 5 %, comparing numerical result and measurement) it deserves to be commented. During measurement, before the recorded waveforms presented in Fig. 10 and Fig. 11 several unsuccessful measurements were made. After each, we notice slightly increasing of battery voltage. We believe that this change of battery voltage influenced the measured current by increasing it.

9 Conclusion

The obtained measurement results of the H7 bulb show an acceptable correspondence with the theoretical model (analytical solution) in which conduction and convection heat are neglected during transients. Of course, they have an impact on the steady state, but as the measurements confirm they are negligible during the transient. Measurements also suggest that the heat capacitance of the tip of lead in wires that is in contact with filament and voltage drops has an impact which is in theoretical models neglected. According to that, further improvements of theoretical model should be done in that direction. Nevertheless, one should keep in mind that the initial goal was to develop a minimal correct model of transient behavior of incandescent lamp, which according

to the results was successfully achieved. The confirmed authenticity of the model allows it to be confidently used to determine the inrush current of lamps and infrared heaters. The presented logic of reasoning and theoretical model are easily expandable to more complex thermo-electrical systems with feedback control. In that case, analysis is only possible using numerical simulations.

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