Macroeconomic Covariates of Default Risk: 
Case of Pakistani Non-Financial Firms

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Bilal Mehmood**
Muhammad Mubashir Hussain Awan**

Abstract: Empirical estimation of default probability through structural approach in the context of macroeconomic dynamics turn out be an emerging idea. However, various aspects of these studies are still needs to be explored to make these models more reliable. This study explored the structural model of default risk (Moody’s KMV) application with macroeconomic dynamics in Pakistani non-financial firm’s context and confirm whether Moody’s KMV model of default prediction could be applicable in Pakistan where the markets are highly speculative and stock markets are highly volatile. The study approximate about the expected default frequency (hereafter EDF) of 307 Pakistani non-financial firms, categorized in 12 industries for a span of 8 years from 2004 to 2011. It further check the macroeconomic variables effects on EDF with the use of generalized method of moments (hereafter GMM). Empirical results compared with the real life scenarios over the said years and on the basis of results we infer that Moody’s KMV model can predict default probability in a much better way than traditional ratio based approach.

Keywords: Expected Default Frequency, KMV Model, GMM, Industry Analysis

JEL Classification: C23, C58, E44

Introduction

Advancements in technology, communication channels and fast track information about every industry or firm leads the world towards globalization, but despite these advancements economies are still getting shocks in shape of bankruptcy of corporate entities.
Enron Corporation (2001), Lehman Brothers (2008) and General Motor Corporations (2009) are some examples which force the world to revisit their strategies to avoid such instances in future and make them serious to think about credit risk management. Furthermore, in the outlook of the 2008 and onward financial uncertainty of subprime mortgage crises, the need of “Credit Risk Management” is extremely important to the corporate entities. Credit risk calculations or the aptitude to forecast business catastrophes is an unqualified compulsion for unwavering financial and economic systems. Higher the accuracy rate that researchers will be able to predicting the default forecast, the less probable it will be for financial prudence to be taken shocks by events such as the 2008 financial turbulence. The precise forecast of default is even more serious given that firms’ default are disparaging events, particularly when a contamination or harmonization influence is concomitant with them. The undesirable magnitude of firm’s catastrophe will be felt by many stakeholders, including but not limited to, creditors, shareholders and employees. So, in this regard, wide range of economy is exaggerated.

For the purpose of predicting firms’ default, it’s vital here to understand the main drivers and factors of default probability for precise default forecasting. There are too many factors that influence the company’s default probability (Wu, 2010). The one of the major factor is economy conditions like: change of interest rate, unemployment, Gross Domestic Product (GDP) growth, last but not the least is inflation which plays important part when forecasting the firm default (Qu, 2008). Logic provide by the researcher to taken into consideration of macroeconomic variable while forecasting the default is that during boom in the economy demand of the products are on increasing trends and resultantly firms profits are on higher side and resultantly these high profits improves firm financial health and ultimately making firm less probable to default. During the recession opposite of the boom have happen. By applying the aforementioned reason, it is reasonable to assume that the economy conditions within which a firm is operating play a substantial role in its default prediction.

**Objective**

This paper mainly focus on the prediction of default risk of non-financial firms with macroeconomic dynamics and further explored that whether the Moody’s KMV model which is basically a structural based approach, captures the default probability of speculative market just like Pakistan.

**Literature Review**

In very recent studies, various scholars have calculated the precision of the Moody’s KMV approach for default risk calculation and try to find means to improve its ef-
ficiency. For this purpose, Bohn (2000) considered the credit spread of corporate bonds and then compared the results by using the rating agencies data and the EDF of the Moody’s KMV model. After Bohn (2000) Kealhofer and Kurbat (2000) simulated Moody’s study results and contended that Moody’s methodology apprehended more information from firm’s data and responded more rapidly as compared to the rating agencies. In addition to these researchers, various researchers have also been interested in KMV methodology. In 2003, Crosbie and Bohn (2003) sum up the KMV default probability methodology after making some revisions to the assumptions of the model. For example, they applied a variant of the Merton model to calculate the market value and volatility of the firm’s assets based on equity values to improve their accuracy in obtaining the distance-to-default (hereafter DD). Duffie et al., (2007) showed that KMV probabilities have significant predictive power in a model of default probabilities over time, which can generate a term structure of default probabilities. Bharath and Shumway (2008) examined the accuracy and the contribution of the KMV default forecasting model by formulating its naive alternative probability. The results showed that the naive predictor performs slightly better models and in out-of-sample forecasts than both the KMV model and a reduced-form model that uses the same inputs. Qu (2008), examined the effect of macroeconomic factors on the probability of default which were calculated through Moody’s KMV model and verified that there is strong impact of macroeconomic variables on EDF of industries. Lu (2009) presented the basic ideals and structures of the KMV model in the framework of both the Merton and Vassicek and Kealhofer models, and also explained some of the necessary conditions before implementing the models. He also extended Merton’s model in hazard to a special case of the KMV. In his empirical study, he used real data to examine the default probability of several firms that have different financial conditions in three industries, and discovered some of the implications for the parameters that we input and derive. Furfine and Rosen (2011) successfully examine the impact of mergers on default risk which was calculated through Moody’s KMV model. Norliza and Maheran (2012) conclude that the Merton model can be adapted in estimating the credit risk of the loan approval towards Malaysian companies by their local banks.

Data and Methodology

Structural model is use in this paper for the calculation of firm’s default probability, which was first introduced by Merton (1974). Merton basically extend the Black-Scholes (1973) option pricing theory. Merton model was based on certain unrealistic assumptions, however, the model was further extended by Vassicek and Kealhofer (VK) on some realistic assumption in 1980’s and named there model as KMV. Furthermore, the said KMV model was successfully commercially launched by the Va-
sicek and Kealhofer. After that one of the biggest rating agency “Moody” acquired the KMV commercial application and named it as “Moody’s KMV”. For ascertaining the volatility of assets through the employment of the complex sequential technique is outside the scope of this paper. However the main four equations of the model is as follows:

\[
d_1 = \frac{\log\left(\frac{D}{F(t)}\right) + \left(r - \frac{1}{2}\sigma_F^2\right)(T-t)}{\sigma_F\sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma_A\sqrt{T}
\]

\[
V_E = V_A N(d_1) - e^{-rT} D N(d_2)
\]

\[
Distance\ to\ Default\ (DD) = \frac{\text{ln}(F(t)/D) + (r - \frac{1}{2}\sigma_F^2)(T-t)}{\sigma_F\sqrt{T-t}}
\]

\[
EDF = N\left(-D\right)
\]

For more detail about Moody’s KMV model, see Crosbie and Bohn (2003).

The access to the Moody’s KMV® EDF-database is not possible, therefore the assumption of normally distributed assets return is used in generating the default probability (Bharath et al., 2004). One year has been set as the time horizon for the default prediction. The Current liabilities and a half of fixed liabilities have been corresponded with the debt amount that is rational for outlining the probability of default as it is a frequently used way (Bharath and Shumway 2008; Chan-Lau and Amadou 2007).

The dataset dimensions consist of 307 listed firms of Pakistan and for 8 years (from 2004 to 2011). Data collection as earlier said is from various sources including World Development Indicator (WDI), Karachi Stock Exchange (KSE) and State Bank of Pakistan (SBP). The selected firms are listed in Karachi Stock Exchange.

Keeping in mind the conditions of Pakistan’s macroeconomic environment, the variables selected for this study include gross domestic product (GDP) growth, interest rate spread (difference between lending and deposit rates), unemployment rate, exchange rate, and stock market index. All these variables reflect the macroeconomic conditions existing in Pakistan and also the lagged value of EDF. For calculations and formulas of variables used in this study, see appendix. The estimable model is as follows:

For the purpose of checking the impact of macro-economic variables on firms EDF, we use GMM in this paper because the presence of endogeneity as well as heteroskedasticity in the panel data. For more recent applications of GMM, see Mehmood and Azim (2013). The estimable model is as follows:
**Expected Default Frequency**

\[ \zeta_{i,t} + \infty (Expected\ Default\ Frequency_{i,t-1}) + \Omega_i (GDP\ Growth_{i,t}) \\
+ \Phi_i (Interest\ Rate\ Spread_{i,t}) + \Theta_i (Log\ of\ Exchange\ Rate_{i,t}) \\
+ \Xi_i (Consumer\ Price\ Index_{i,t}) + \xi_i (Unemployment_{i,t}) + \beta_{1i} (DT_{i,t}) + \beta_{2i} (DF_{i,t}) \\
+ \beta_{3i} (DC_{i,t}) + \beta_{4i} (DOM_{i,t}) + \beta_{5i} (DON_{i,t}) + \beta_{6i} (DMV_{i,t}) + \beta_{7i} (DFE_{i,t}) + \beta_{8i} (DIC_{i,t}) \\
+ \beta_{9i} (DRPi_{t}) + \beta_{10i} (DPi_{t}) + \beta_{11i} (DO_{i,t}) + \beta_{12i} (DEM_{i,t}) + \eta_i + \varepsilon_{i,t} . \]

**Interpretation**

Panel data models with small time series produce biased coefficient estimates using ordinary least squares ‘OLS’, fixed effects ‘FE’ and random effects ‘RE’ (Baltagi, 2008). Moreover, endogeneity can be an issue for which following tests are employed. The statistical significance of the test statistic indicates presence of endogeneity.

**Table 1: Test for Endogeneity**

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistics</th>
<th>Notation</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu-Hausman F test</td>
<td>36.8977</td>
<td>F(1,2449)</td>
<td>0.000</td>
</tr>
<tr>
<td>Durbin-Wu-Hausman (\chi^2) test</td>
<td>36.4540</td>
<td>(\chi^2(1))</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations using Stata 12.0 (SE).

In the presence of endogeneity, 2SLS/IV regression produces more reliable results as compared to above mentioned estimators. But 2SLS/IV also shows biased results in case the heteroskedasticity is present in the variables. So, 2\textsuperscript{nd} test before estimation of the final results is heteroskedasticity test.

**Table 2: All tests for Heteroskedasticity in Presence of Instrumental Variables (IVs)**

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistics</th>
<th>Notation</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pagan-Hall general test statistic</td>
<td>41.952</td>
<td>(\chi^2(5))</td>
<td>0.000</td>
</tr>
<tr>
<td>Pagan-Hall test w/assumed normality</td>
<td>44.989</td>
<td>(\chi^2(5))</td>
<td>0.000</td>
</tr>
<tr>
<td>White/Koenker nR2 test statistic</td>
<td>25.944</td>
<td>(\chi^2(5))</td>
<td>0.000</td>
</tr>
<tr>
<td>Breusch-Pagan/Godfrey/Cook-Weisberg</td>
<td>60.382</td>
<td>(\chi^2(5))</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: Authors calculations using Stata 12.0 (SE).

Test 2 of heteroskedasticity rejects the null hypothesis which shows that the panel data have both the endogeneity and heteroskedasticity problems. In this regard one
step GMM has been used in this article. Table 3 shows the results of variables and industries dummies.

**Table 3: System GMM Estimates**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Robust Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDF_{i,t-1}</td>
<td>0.2024</td>
<td>0.0486</td>
<td>4.16</td>
<td>0.000</td>
</tr>
<tr>
<td>YG_{i,t}</td>
<td>-0.0671</td>
<td>0.0144</td>
<td>-4.67</td>
<td>0.000</td>
</tr>
<tr>
<td>IRS_{i,t}</td>
<td>-0.3702</td>
<td>0.0512</td>
<td>-7.23</td>
<td>0.000</td>
</tr>
<tr>
<td>Ln(ER)_{i,t}</td>
<td>-0.9863</td>
<td>0.1278</td>
<td>-7.72</td>
<td>0.000</td>
</tr>
<tr>
<td>CPI_{i,t}</td>
<td>-0.0212</td>
<td>0.0052</td>
<td>-4.09</td>
<td>0.000</td>
</tr>
<tr>
<td>U_{i,t}</td>
<td>0.1164</td>
<td>0.0140</td>
<td>8.32</td>
<td>0.000</td>
</tr>
<tr>
<td>CONS</td>
<td>6.5727</td>
<td>0.9088</td>
<td>7.23</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Industry Dummies**

<table>
<thead>
<tr>
<th>Industry</th>
<th>Coefficient</th>
<th>Robust Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DT_{i,t}</td>
<td>-0.1551</td>
<td>0.0926</td>
<td>-1.68</td>
<td>0.094</td>
</tr>
<tr>
<td>DF_{i,t}</td>
<td>-0.0707</td>
<td>0.0987</td>
<td>-0.72</td>
<td>0.474</td>
</tr>
<tr>
<td>DC_{i,t}</td>
<td>-0.0295</td>
<td>0.1009</td>
<td>-0.29</td>
<td>0.770</td>
</tr>
<tr>
<td>DOM_{i,t}</td>
<td>-0.0039</td>
<td>0.1020</td>
<td>-0.04</td>
<td>0.969</td>
</tr>
<tr>
<td>DON</td>
<td>-0.1309</td>
<td>0.0937</td>
<td>-1.40</td>
<td>0.163</td>
</tr>
<tr>
<td>DMV</td>
<td>0.0236</td>
<td>0.1062</td>
<td>0.22</td>
<td>0.824</td>
</tr>
<tr>
<td>DFE</td>
<td>-0.1394</td>
<td>0.0986</td>
<td>-1.41</td>
<td>0.158</td>
</tr>
<tr>
<td>DIC</td>
<td>-0.1169</td>
<td>0.0972</td>
<td>-1.20</td>
<td>0.229</td>
</tr>
<tr>
<td>DRP</td>
<td>0.2702</td>
<td>0.1247</td>
<td>2.17</td>
<td>0.030</td>
</tr>
<tr>
<td>DP</td>
<td>-0.0469</td>
<td>0.1052</td>
<td>-0.45</td>
<td>0.656</td>
</tr>
<tr>
<td>DO</td>
<td>-0.0197</td>
<td>0.1156</td>
<td>-0.17</td>
<td>0.865</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations

The estimated coefficient on the lagged dependent variable is 0.202 which is less than 1, which means that the steady-state assumption holds. GDP growth shows negative relationship with the EDF which shows that, higher GDP growth leads to lower EDF. The spread of the interest rate is negative which means that if the yield curve is downward sloping, there is not very much future expectation of growth and resultantly increase in the EDF or vice versa. Exchange rate coefficient is negative which implies that the majority firms are export oriented. Coefficient value of consumer price index is negative which means that majority firms produces necessity items. CPI coefficient is negative which implies that the majority of the industries are based on necessity items, as far as from firms point of view higher the prices leads to higher profit for firms in the horizon of one year which ultimately strengthen their credit worthiness and decreases the EDF. The only positive coefficient of the macroeconomic variables is un-employment which is quite obvious in the context of business activities. If there is un-employment in the country then the firms will not be able to earn maximum profits and this will eventually decrease the credit worthiness of the firms and leads to higher value of EDF.
Table 3 results show overall satisfactory position but the aforementioned results depends on some test. The tests which are going to conducted is Arellano-Bond test for AR (1) in first differences, Arellano-Bond test for AR (2) in first differences, Sar- gan test of override restrictions and Hansen test of override restrictions.

Table 4: Arellano-Bond test for AR (1) and (2)

<table>
<thead>
<tr>
<th>Test</th>
<th>z</th>
<th>Pr &gt; z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arellano-Bond test for AR (1) in first differences:</td>
<td>-7.57</td>
<td>0.000</td>
</tr>
<tr>
<td>Arellano-Bond test for AR (2) in first differences:</td>
<td>-0.53</td>
<td>0.595</td>
</tr>
</tbody>
</table>

Source: Authors calculations using Stata 12.0 (SE)

Table 5: Over Identification Restrictions Test

<table>
<thead>
<tr>
<th>Test</th>
<th>$\chi^2(1)$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sargan test of over identification restrictions:</td>
<td>0.67</td>
<td>0.412</td>
</tr>
<tr>
<td>(Not robust, but not weakened by many instru- ments.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hansen test of over identification restrictions:</td>
<td>0.86</td>
<td>0.355</td>
</tr>
<tr>
<td>(Robust, but weakened by many instruments.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors calculations using Stata 12.0 (SE)

Table 4 and 5 shows the robustness test of overall results and results shows that the instruments are valid and overall results are robust.

This article examine the real life scenarios with the calculated results and found that the results capture the historical events. Following graph shows the trends of all industries with respect to EDF.

Figure 1: Industry trends of the EDF.
Pakistani economy was in the year 2005 was almost in the state of transition from agricultural economy to and industrial state. In 2005 Pakistani economy was growing at a rapid pace of almost 7%. However, national disaster of earthquake in 2005 causing huge shock to the sustained economic growth of the economy. Disastrous earthquake shock hit the growing economy very badly and caused loss of $5.2B which is shocking when we compared this figure with the very next budget of the Pakistan i.e. 2006-07 which was $25B.

Financial crisis in the world economy starts in the year 2008 from the U.S. and Europe. Although these continents are far away from but all other economies are linked to these continents so developing countries are affected. Pakistan being a developing country has also suffered from high current account and fiscal deficits, quick increase in inflation, reserves have been very low and currency becomes weaker and weaker and deteriorating economy that put the country economy in a very tough condition. Death of a political leader was also create instability in the economy.

**Conclusion**

This research paper estimates the EDF of Pakistani firms and determines whether the macro-economic factors affect the default probability of firms or not by using structural approach through Moody’s KMV model. The study extended to further level and analyze the results of industry EDF with real life scenarios. For the purpose EDF calculations this paper analyze 307 Pakistani firms’ data from 2004 to 2011, which are listed on Karachi Stock exchange in various indexes. Moody’s KMV® model is used to approximate the EDF while GMM have been used to determine the relationship between macro-economic variables and EDF.

Empirical estimation results of EDF shows that the default probability trend portrays the events that impact the Pakistani economy i.e. the 2005 earthquake shock which causes a $5B loss to the economy and slow down the whole economy, stock market crashes due to artificial bubble creation, global financial crises in 2008, the death of Benazir Bhutto creates political instability and the war against terrorism etc. Furthermore, results portrays that all these events cause the EDF to increase because all these crises negatively impact the economy and ultimately affect the industry and so on to the firms. The analysis of industries after approximating the EDF shows that the EDF trends of all non-financial sectors which have been included in this thesis depicts similar patterns of ups and downs in EDF trend.


## Appendix

### Data Collection for EDF:

<table>
<thead>
<tr>
<th>Sr. #</th>
<th>Data Type</th>
<th>Source</th>
<th>Use / Extract</th>
<th>Calculation Method / formula if any</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Financial Spreads</td>
<td>SBP Analysis of Non-Financial Firms</td>
<td>Per share price, No. of Shares, Short term Debt, Long term Debt</td>
<td>No calculations used for this</td>
<td>Financial spreads are the annual accounts of the firms.</td>
</tr>
<tr>
<td>2</td>
<td>Equity volatility</td>
<td>KSE historical share prices of each firm from kse.com.pk</td>
<td>Per day closing price of each firm from 2004 to 2011.</td>
<td>=STDEV(Range of closing prices)*SQRT(365)</td>
<td>Equity volatility is the measure of dispersion of equity values in a given time period.</td>
</tr>
<tr>
<td>3</td>
<td>Market value of equity</td>
<td>kse.com.pk</td>
<td>Closing price share as on Dec 31 of each year and multiply with No. of shares outstanding</td>
<td>=No. of shares outstanding at each year*Closing price of the share as on Dec 30 of each year.</td>
<td>Market value of equity is the simply equity price of the firm derived through the market value of share of that particular firm</td>
</tr>
<tr>
<td>4</td>
<td>Risk Free Rate</td>
<td>SBP statistical Hand Book 2010</td>
<td>T Bill Rate</td>
<td>No calculations used for this</td>
<td>Risk free rate is the rate at which investor can invest without taking any risk.</td>
</tr>
<tr>
<td>5</td>
<td>Default Point</td>
<td>Financial Spreads</td>
<td>Total Current liabilities plus half of Long term Liabilities</td>
<td>=Current Liabilities + (Long term Liabilities / 2)</td>
<td>According to the Moody’s KMV model default point of the firm is consist of total current liabilities and one half of long term liabilities.</td>
</tr>
<tr>
<td>6</td>
<td>Market value of Assets</td>
<td>Manually Calculated</td>
<td>Calculated through Black-Scholes option pricing theory (1973)</td>
<td>VB scripts of the same is given in the appendix of this Thesis.</td>
<td>Market value of assets is considered here as the total worth of the firm.</td>
</tr>
<tr>
<td>7</td>
<td>Asset Volatility</td>
<td>Manually Calculated</td>
<td>Calculated through Black-Scholes option pricing theory (1973)</td>
<td>VB scripts of the same is given in the appendix of this Thesis.</td>
<td>The dispersion in the value of assets derived through the dispersion in the value of equity</td>
</tr>
<tr>
<td>8</td>
<td>Distance to Default</td>
<td>Manually Calculated</td>
<td>Value of DD</td>
<td>=(LN(Market value of Assets/Default Point)+(Risk Free Rate - Asset Volatility ^2/2)*1)/ ( Asset Volatility *1)</td>
<td>Distance to default is the instrument which tells the analyst that how much standard deviation away a firm is from default.</td>
</tr>
<tr>
<td>9</td>
<td>EDF</td>
<td>Manually Calculated</td>
<td>Final Value of Expected Default Frequency</td>
<td>=NORMSDIST(-Distance to Default)</td>
<td>EDF gets after normally distributed the Distance to Default</td>
</tr>
</tbody>
</table>
Data Collection for Macroeconomic variables:

<table>
<thead>
<tr>
<th>Sr. #</th>
<th>Variables</th>
<th>Type</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Consumer Price Index</td>
<td>Annual</td>
<td>World Development Indicators (WDI) (2004 – 2011)</td>
<td>Consumer price index is the general measurement of inflation.</td>
</tr>
<tr>
<td>2</td>
<td>Unemployment Rate</td>
<td>Annual</td>
<td>WDI (2012)</td>
<td>Unemployment rate is the percentage of unemployed rate from total civilian labor force of the country.</td>
</tr>
<tr>
<td>3</td>
<td>Interest Rate Spread</td>
<td>Annual</td>
<td>WDI (2012)</td>
<td>Interest spread is the difference between short term interest rate and long term interest rate.</td>
</tr>
<tr>
<td>4</td>
<td>Stock Market Index</td>
<td>Annual</td>
<td>KSE (website)</td>
<td>Stock market index is the aggregate value of which were produced by combining various stocks.</td>
</tr>
<tr>
<td>5</td>
<td>Exchange Rate</td>
<td>Annual</td>
<td>WDI (2012)</td>
<td>The price at which one currency could be exchange with another country currency.</td>
</tr>
<tr>
<td>6</td>
<td>GDP Growth</td>
<td>Annual</td>
<td>WDI (2012)</td>
<td>GDP growth is the increase in the capacity of an economy or country to produce more goods and services.</td>
</tr>
</tbody>
</table>

VB Script for Calculation:

Global Const Pi = 3.14159265358979
Option Base 1
Rem Berechnung der kumulativen bivariaten Normalverteilung nach Drezner (1978)
Public Function kumbinvert(a As Double, b As Double, rho As Double) As Double
  Dim X As Variant, y As Variant
  Dim rho1 As Double, rho2 As Double, delta As Double
  Dim a1 As Double, b1 As Double, Sum As Double
  Dim I As Integer, j As Integer
  X = Array(0.24840615, 0.39233107, 0.21141819, 0.03324666, 0.00082485334)
  y = Array(0.10024215, 0.48281397, 1.0609498, 1.7797294, 2.6697604)
  a1 = a / Sqr(2 * (1 - rho ^ 2))
  b1 = b / Sqr(2 * (1 - rho ^ 2))
  If a <= 0 And b <= 0 And rho <= 0 Then
    Sum = 0
    For I = 1 To 5
      For j = 1 To 5
        Sum = Sum + X(I) * X(j) * Exp(a1 * (2 * y(I) - a1) + b1 * (2 * y(j) - b1) + 2 * rho * (y(I) - a1) * (y(j) - b1))
      Next
    Next
    kumbinvert = Sqr(1 - rho ^ 2) / Pi * Sum
  ElseIf a >= 0 And b <= 0 And rho >= 0 Then
    kumbinvert = kumvert(b) - kumbinvert(-a, b, -rho)
  ElseIf a <= 0 And b >= 0 And rho >= 0 Then
    kumbinvert = kumvert(a) - kumbinvert(a, -b, -rho)
  ElseIf a >= 0 And b >= 0 And rho <= 0 Then
    kumbinvert = Sqr(1 - rho ^ 2) / Pi * Sum
  Else
    kumbinvert = kumvert(b) - kumbinvert(-a, b, -rho)
  End If
kumbinvert = kumvert(a) + kumvert(b) - 1 + kumbinvert(-a, -b, rho)

ElseIf a * b * rho > 0 Then
    rho1 = (rho * a - b) * Sgn(a) / Sqr(a^2 - 2 * rho * a * b + b^2)
    rho2 = (rho * b - a) * Sgn(b) / Sqr(a^2 - 2 * rho * a * b + b^2)
    delta = (1 - Sgn(a) * Sgn(b)) / 4
    kumbinvert = kumbinvert(a, 0, rho1) + kumbinvert(b, 0, rho2) - delta
End If
End Function

Rem Berechnung der kumulativen Standardnormalverteilung
Public Function kumvert(f As Double) As Double
    Dim N As Double, K As Double
    Const a1 = 0.31938153
    Const a2 = -0.356563782
    Const a3 = 1.781477937
    Const a4 = -1.821255978
    Const a5 = 1.330274429
    N = Abs(f)
    K = 1 / (1 + 0.2316419 * N)
    kumvert = 1 - 1 / Sqr(2 * Pi) * Exp(-N^2 / 2) * (a1 * K + a2 * K^2 + a3 * K^3 + a4 * K^4 + a5 * K^5)
    If f < 0 Then
        kumvert = 1 - kumvert
    End If
End Function

Private Sub Worksheet_Change(ByVal Target As Range)
    ' Macro written by Andreas Emmert
    ' July 29th, 2002
    ' Glasgow/Scotland
    If Not Intersect(Target, Range(“B7:B12”)) Is Nothing Then
        found = 0
        While found = 0
            target1 = Cells(17, 7)
            Range(“I17”).GoalSeek Goal:=target1, ChangingCell:=Range(“E20”)
            target2 = Cells(21, 7)
            estimate1 = Cells(17, 9)
            estimate2 = Cells(21, 9)
            If Val(estimate1) = Val(target1) And Val(estimate2) = Val(target2) Then
                found = 1
            End If
        Wend
    End If
End Sub