The Use of Penalty Functions in Logistics

Upotreba penalty funkcija u logistici

Summary

This article is aimed at the management of interrelated subsystems, through the penalty functions in logistics systems. We compared the results obtained using simulation models on the examples three types of models (production model, supply model and model sales) with use penalty function for their connection with results achieved when optimizing the system as a whole.

Sažetak

Tema ovoga rada je menadžment međusobno povezanih po-sustava kroz penalty funkcije u logističkim sustavima. Autori su uspoređili rezultate dobivene upotrebom simulacijskih modela na primjerima tri tipa modela (proizvodni model, model nabave i model prodaje) i upotrebom penalty funkcije kako bi ih povezali s dobivenim rezultatima u optimizaciji sustava u cjelini.

INTRODUCTION / Uvod

Market and competition create pressure on enterprises to increase utilization of existing resources, reduce costs and optimize overall system management. This pressure increases during economic recession. As shown by the results of a survey of the economic crisis in 2008, which in July 2008 performed by Ernst & Young and the Economist Intelligence Unit, 87% of respondents, the main activity of firms in recession reported revenue optimization and cost reduction. One possibility to improve the situation in terms of costs in the enterprises is an application of exact methods, which should serve as a means of support for managerial decision making.

The paper based on the fundamental philosophy of modern Complexity Theory. Application of Complexity Theory into the economy is the application complexity of scientific problems, as a result of the state of human knowledge, into economic thought. There is no generally valid definition for the complexity of economic systems. The complexity of the economy is built on the foundations that draw inspiration from areas such as behavioural economics, institutional economics and evolutionary economics. Proponents of complexity economics argue that economic systems not having a ‘natural’ tendency to reach equilibrium.

Current research in the field of economic systems, and the possibilities his prediction and management, points to the fact that economic systems are really complicated, with changing patterns of quantitative and qualitative relations. The complexity of managing a system as a whole raises the need for its decomposition into mutually affecting subsystems. This article is aimed at the management of interrelated subsystems, through the penalty functions in logistics systems.

LOGISTICS AND LOGISTICS COSTS / Logistika i troškovi logistike

To the paper we need to specify the logistic area. We come out from the concept of logistics comprehension, where logistics is understood as integrated planning, execution and audit of material flow and appropriate flow of information. These flows are flows from contractor to enterprise, within enterprise and from enterprise to customers. Kortschak’s [3] definition is appropriate here: Logistics is a “knowledge about co-ordination of active and passive elements of enterprise, leading to the lowest costs in time, to the flexibility improvement and adaptability of enterprise to the changing general economic conditions and buyer’s market”. Objects of interest are manufacturing stocks, semi products and finished products. Logistics fulfills several functions. One of them is optimisation of costs.

Logistics costs are associated with the logistics system functioning. These costs are direct by proportional to the rate of consumer satisfaction. Deficit costs are arising as a consequence of insufficient consumer satisfaction (penalty, loss of consumer). The level of cost is indirectly proportionate to consumer’s satisfaction.

The results of researches focused on the share of logistics costs on total costs of enterprise in the last decade are very different. Logistics costs represent from 10 percent to 15 percent of total costs included in the price of the final product [2]. In the automotive industry logistics costs are supplied parts, the starting materials and their subsequent distribution to the production line about 10 percent of the total cost of the car or truck. Results of findings from particular enterprises are shown in Table 1.
The main factors that influence different understanding of logistics costs can be considered:
1. The differing analytical breakdown logistics costs;
2. Responsibilities logistics managers;
3. Changing priorities in assessing logistics costs.
In this paper we start from the breakdown of logistics costs as related to performance.

**METHODOLOGY - MODELS AND USE OF PENALTY FUNCTIONS / Metodologija – Modeli i upotreba penalty funkcija**

The shape of the model depends on organisational structure and management system of enterprise. Our approach came out of classical hierarchy structure. Our example of models includes activity connected with delivery, production and sales, includes price and material costs. Philosophy of the approach comes from the dominant position of the only subsystem, to which the others are subordinated. All models are based on the alternative production options, taking into account changing market prices. Used as operational research methods based on the use of linear programming problems. A basic characteristic of the system is economic result. Economic result is seen as the difference between profit and cost of production, storage and expedition. In developing individual models are used different types of objective functions: the profit function (production model), the cost function (model inventory and model sales) and the penalty function (conflict situations). Variable is the production plan (production volumes by product type during one time period).

**PRODUCTION MODEL / Proizvodni model**
The aim of enterprise, contribution to profit maximisation is modelled by function

$$\max x(\mathbf{x}) = \sum_{i=1}^{m} \left( p_i - \sum_{j=1}^{n} a_{ij} x_j + \sum_{k=1}^{r} b_k w_k \right) \times x_i$$
(1)

where

$$x_i \geq 0 \quad i = 1, 2, ..., m$$
(4)

$$x_i \leq x_i \quad i = 1, 2, ..., m$$
(3)

$$\sum_{j=1}^{n} b_{ij} x_j \leq b_i^H \quad k = 1, 2, ..., r$$
(2)

$x_i$ is the quantity of product $i$ amount per one period,
$a_{ij}$ is the standardised consumption of material $j$ per product $i$,
$p_i$ is the price of product $i$,
$p_j$ is the price of material $j$,
$w_k$ is the wage rate profession $k$,
r is the number of types of professions involved in the production,
b_{ik} is labour intensity of the product $i$ in the profession $k$,
b_{kH} is the upper limit of working time profession $k$.

**SUPPLY MODEL / Model nabave**
The function of total costs $C(T_s)$ is

$$C(T_s) = c_s A + \frac{b_d w_s}{T_s} + T_s \sum_{j=1}^{n} \frac{\lambda_j}{2} c_j + w_s \sum_{j=1}^{n} \lambda_j b_j$$
(5)

$$\frac{b_d}{T_s} + \sum_{j=1}^{n} \lambda_j b_j \leq b_s^H$$
(6)

$$T_s > 0$$
(7)

where

$\lambda_j C(T_s)$ are total costs of supply,
Ts is the delivery cycle,
c_sA are acquisition costs of one supply,
c_j are unit costs connected with storage of material $j$ per one period,
is the need of material $j$ per one period,
bsA is standard performance is a warehouseman in equipment deliveries,
bsj is working standard for handling of one unit material $j$,
ws is the wage rate warehouseman,
bsH is the upper limit of working time profession warehouseman.

<table>
<thead>
<tr>
<th>Types of enterprise (country)</th>
<th>The share of logistics costs on total costs of enterprise (resource)</th>
<th>Author of research (year)</th>
</tr>
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<tbody>
<tr>
<td>Agricultural enterprise (unspecified country)</td>
<td>42.20 percent (<a href="http://ageonsearch.umn.edu/bitstream/24562/1/pp950wa01.pdf">http://ageonsearch.umn.edu/bitstream/24562/1/pp950wa01.pdf</a>)</td>
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<td>Department store (Germany)</td>
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Source: own processing

**Table 1. The share of logistics costs on total costs of enterprise**

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Comparison of Means

Table 2. Test the consistency of the means of economic result

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<tr>
<th>Null hypothesis: mean1 = mean2</th>
<th>Assuming equal variances</th>
<th>Not assuming equal variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Alt. hypothesis: mean1 ≠ mean2</td>
<td>t = -4.00275</td>
<td>P-value = 0.0000626394</td>
</tr>
<tr>
<td></td>
<td>t = -4.00275</td>
<td>P-value = 0.0000626394</td>
</tr>
<tr>
<td>(2) Alt. hypothesis: mean1 &gt; mean2</td>
<td>t = -4.00275</td>
<td>P-value = 0.999969</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>(3) Alt. hypothesis: mean1 &lt; mean2</td>
<td>t = -4.00275</td>
<td>P-value = 0.0000313197</td>
</tr>
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</tr>
</tbody>
</table>

Source: own processing

Table 3. Test the consistency of the standard deviations of economic result

Comparison of Standard Deviations

Table 4. Test the consistency of the medians of economic result

MODEL SALES / Model prodaje

The function of total costs \( C(T_e) \) is

\[
C(T_e) = \frac{c_e}{T_e} + \frac{b_e}{T_e} \sum_{i=1}^{m} x_i + \frac{w_e}{T_e} \sum_{i=1}^{m} x_i \frac{b_e}{T_e} \leq b_e^H \tag{8}
\]

\[
\sum_{i=1}^{m} x_i \leq \frac{b_e^H}{T_e} \tag{9}
\]

\[
T_e > 0 \tag{10}
\]

where

\( C(T_e) \) are total costs of sales, 
\( T_e \) is the expedition cycle, 
\( c_e \) are distribution costs connected with one expedition, 
\( b_e \) are unit costs connected with storage of product \( i \) per one period,  
\( b_e^s \) is standard performance is a salesman in equipment expedition,  
\( b_e^w \) is working standard for handling of one unit product \( i \),  
\( w_e \) is the wage rate salesman, 
\( b_e^H \) is the upper limit of working time profession warehouseman, 
\( x \) is the product i amount per one period.  

The problem is interconnection models. In case capacity problems we used penalty functions. Substitute objective function minimizes the loss of deficit (i.e. penalties). For example in case of capacity problem in supply penalty function is

\[
\min \delta(T_e) = \sum_{i=1}^{m} d_i (x_i - x_i) \tag{11}
\]

\[
x_i \leq x_i \quad i = 1, 2, \ldots, m \tag{12}
\]

\[
\frac{b_a}{T_e} + \frac{\sqrt{a_i}}{T_e} \sum_{j=1}^{m} x_j a_j = b_e^H \tag{13}
\]

\[
x_i \geq 0 \quad i = 1, 2, \ldots, m \tag{14}
\]

where \( d_i \) is the penalty for non-delivery of one unit of product \( i \) to customer,  
\( a_j \) is the standardised consumption of material \( j \) per product \( i \), 
\( b_a \) is standard performance is a warehouseman in equipment deliveries,  
\( b_e^H \) is the upper limit of working time profession warehouseman.
$b_j$ is the working standard for handling of one unit material $j$, $T_d$ is the delivery cycle, $x_i^*$ is the optimum amount of the product $i$ from the optimal production program which not respecting storage capacity limitation, $x_s$ is the amount of the product $i$ capable of supplying process.

RESULTS / Rezultati
Simulation models are processed by EXCEL 7.0 software. Varying input parameters are the selling prices of finished products. We generated 1,000 entries with a normal distribution and a standard deviation of size 30 percent of the mean value. We compared results of models with use penalty function and results of complex models which included the same parameters. We used parametric tests, considering sample size, and nonparametric tests, considering of the outcome of the tests of normality. The results of parametric tests are in Table 2 and Table 3. The results of nonparametric tests are in Table 4 and Table 5.

From Table 2 shows that, on the significance level 0.01, may be accepted third alternative hypothesis: mean of economic result in model with use penalty functions is worse than the mean value of economic result in the complex model. From Table 3 shows that between models are not statistically significant differences among the values of variances. Nonparametric testing should confirm the results of the parametric tests.

Results of Mann-Whitney test for the null hypothesis of identity medians in Table 4 show that, on the significance level 0.01 may be accepted third alternative hypothesis: economic result in model with use penalty functions is worse than economic result in the complex model.

CONCLUSION / Zaključak
Results of tests showed that mean values of economic result are higher in the optimization model as a whole, but for the dispersion and the standard deviation statistically significant differences are not. The problem is that economic systems are really complicated. Often is unrealistic their optimization as a whole. In case their decomposition to subsystems occurs problem their subsequent interface. Use of the penalty functions is one option to solve this problem. We showed example for exploitation of the penalty function on the particular model.

Results of statistical tests confirmed the general conclusions of the theory of cybernetics. Nevertheless, we believe that penalty functions have the potential to solve the questions of modern Complexity Theory.

REFERENCES / Literatura