

The Estimation Method of the Characteristics of an Aircraft with Electromechanic Analogue

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Summary

The paper concentrates on the options of MATLAB programming environment in processing experimentally obtained results of measurements by physical analogue. Analogue dynamics is described by a differential equation, the shape of which is identical with the shape of a model aircraft rolling differential equation.

KEY WORDS

identification
similarity
analogue
model
maximum probability positional angle
force
electric voltage

INTRODUCTION

In general, the analysis of physical systems is connected with the process of identification of their characteristics which are presented by an abstract mathematical model. The shape of this mathematical model, presented by the way of its input, is usually well-known as well as the number of parameters the values of which need to be identified in the process of physical system identification. Each method of identification of an object as a physical system is based on observation (connected with measurement) of those parameters, which are dominant for the determination of object characteristics. The abstract mathematical object model can take a shape of an analytical function, data table, value sequence, etc. One of frequently used methods of object identification uses the state space theory which is described in the work in detail. [2]

No mathematical model gives exact concept of an object feature, which means that experimental data obtained by measurement do not match its mathematically determined outputs.

The reason is that in measuring the parameters of interest, real sensors, which possess a certain measurement preciseness error, are used. It is necessary to estimate the stated measurement error for the mathematical model. Repeated measurement makes error estimation more precise, the error is smaller and the mathematical model becomes more exact. The paper describes the estimation

method of flight characteristics of the aviation ergatic system (AES), which represents „live „connection of ties and influences: aircraft – operator pilot.

The principle of gradual refining flight characteristics estimation is based on minimizing the differences between mathematically expected reactions of AES and actually measured flight parameter constants. The evaluation of the differences between the expected and measured reaction of AES is performed by the method of the least squares. A satisfying result can be reached if the requirement of a minimal value of an observed reaction difference, which represents the minimal error, is fulfilled. The presented method is used in the theory of management and is also defined in literature [2].

DETERMINATION OF FLIGHT CHARACTERISTICS RESULTING FROM THE FLIGHT TESTS

Aerodynamic constants presented as state space variable constants [1] are obtained from the calculation of measured values in aerodynamic tunnels. The values taken during flight tests are compared with the presented ones and when the differences are satisfying, they are used in practical calculations. The results of the performed flight experiment (for this purpose) are presented in the MATLAB program, for different values of the figure M as well as in [1] and [2].

The task of different coefficient estimation is simple in a general form.

The universality is determined by the existence of differential equations with the measured coefficients necessary to determine. For this purpose, the measured values (e.g. acceleration, angle speeds, a position of stabilized angles) are substituted into differential equations and a type signal is activated by a sample type signal, which functions on the right side of the same system equation. Activated system reactions are then compared with the real ones, which represent a flight experiment. Gradually, the measurement is done more precisely and missing constants are completed. The described method enables to acquire a concept about the physical object properties. The presented process is complicated by the problems which need to be highlighted and which make the simplicity of the used method more difficult. [7]

The initial problem is the impossibility to obtain perfect (positive) precise reactions of an arbitrary real object. Common errors of sensors are reflected in the model as auxiliary (additional) errors – audible noises resulting in cardinal changes in the theoretical concept of the whole system work. The result is the impossibility to identify the system precisely and therefore it is necessary to use statistic methods of identification, which does not simplify the problem.

The second indicator of the problem complexity is the existence of immeasurable noise, which is visualized by incidental „excitation“ of the system

Atmospheric turbulence is a standard example of state noise in solving the tasks of AES stability and operability. The presented reasons show the impossibility to obtain an exact mathematical model of aircraft features. In most cases of mathematical model creation, the existing errors are simply ignored or approximated to acceptable values due to the fact that the suggested model can be expressed mostly with errors identical to ones occurring under the influence of random constants. In spite of the presented problems, there are methods based on a probability theory which allows quite precise construction of mathematical models. The first step is to write the system of differential equations of AES movement into a continuous – discrete form:

$$\begin{aligned} x(t_0) &= x_0 & (1) \\ \dot{x}(t) &= f[x(t), u(t), z] + F(z)n(t) & (2) \\ z(t) &= g[x(t), u(t), z] + G(z)\eta_i & (3) \end{aligned}$$

where:

„ $\dot{}$ “ - is derivation according to time.

$f, g, g_{(i)}$ - general functions,

z - vector of known parameters

η - vector of measurement noise,

t - time

i - discrete (time) sequence

u - vector of control

x - state variable

g - matrix of measured noise,

f - matrix of unknown parameters, which need to be determined

In solving the presented equations, the state noise is regarded as the Gauss process with the zero mean value. The noise vector of measurement equations (1÷3) is shown in the solution as the sequence of the Gauss independent random variables with the unit covariation and zero mean value. As a result of presented limitations, each estimation value of unknown parameters can be written as a probability of the fact that aircraft measurements (observations) will be performed at the time of the aircraft reaction to the disturbing signal. It means that if the state noise is ignored and equations (1 ÷ 3) are linearised [1], their presentation can be performed in the following form:

$$x(t_0) = x_0 \quad (4)$$

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (5)$$

$$Z(t_i) = Cx(t_i) + Du(t_i) \quad (6)$$

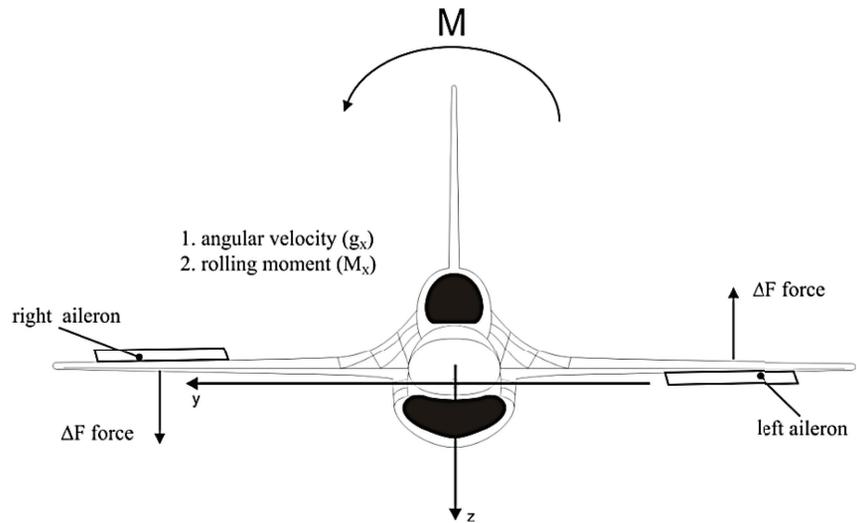


Fig. 1. Forces and moments influencing the aircraft in rolling (a clockwise coordinate system)

axis x – aircraft is oriented along the flight direction
axis y – aircraft is positioned in the median

MATHEMATICAL MODEL CONSTRUCTION

In the first step of the method, we analyse the usability of the supposed mathematical model of a rolling aircraft [1], by the following linearised equation:

$$\ddot{\gamma} + c_1 \dot{\gamma} = c_2 \delta_{kr} + c_3 \beta + M_x I_x^{-1} \quad (7)$$

which can be used as a model when the stated presumptions are accepted [4]. Straightforward, horizontal aircraft flight, the position of which is illustrated in Fig. 1, is considered:

For balanced straightforward flight the last equation element for the value has $M_x = 0$ (a rolling moment is zero – an aircraft is balanced). Similarly, when the side angle β is zero, the following is valid: $c_3, \beta = 0, (7)$.

The differential equation of the second order is a new form of the equation (7):

$$\ddot{\gamma} + c_1 \dot{\gamma} = c_2 \delta_{kr} \quad (8)$$

Let the force P be performed by an operator – pilot (OP) to move the control stick- by the measure of angle acceleration and angle speed then [4]:

$$\ddot{\gamma} + c_1 \dot{\gamma} = c_2 P \quad (9)$$

By the transfer (9) into the state space, where $x_1 = \gamma$, the following shows:

$$\dot{x} + \dot{\gamma} = x_2 \quad (10)$$

will be

$$\dot{x}_1 = \dot{\gamma} = x_2 \quad (11)$$

$$\dot{x}_2 = -c_1 x_2 = c_2 P \quad (12)$$

In the output solution, similarly to the flight experiment, the states $x_1 = \gamma, x_2 = \dot{\gamma}$ are the measured values [5].

Then :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -c_1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} \times P \quad (13)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad (14)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -c_1 \end{bmatrix}; \quad B = [0, c_2]; \quad C = [1, 0; 0, 1], D = [0, 0]$$

The symbols c_1, c_2 are the expression of the ratios (see Fig. 1).

$$c_1 = -\frac{M_x^{\omega_x}}{I_x}, \quad c_2 = -\frac{M_x^P}{I_x}, \quad \omega_x = \omega_x \cdot bV^{-1},$$

which are expressions of the reaction to the evoked changes of the aircraft position initiated by an operator - pilot.

$M_x^{\omega_x}$ - is rotational movement of an aviation ergatic system around its longitudinal axis.

The value of the moment M is determined by the angular velocity ω_x , the non-dimensional shape of which is. The effect of the torque can be expressed by the moment of inertia $I_{x1'2'}$, when $\beta=0$, then the following form is valid:

$$\gamma \cong \omega_x = \gamma, \quad \dot{\omega}_x = \varepsilon_x, \quad (15)$$

where:

ε_x - is angle acceleration

The symbol of aerodynamic constant c_2 highlights the need of an increased force needed for the movement of a control stick at increasing values x of inertia moment I_x . It follows from the coefficient dimensional analysis of c_1, c_2 [2]:

$$c_1 = \frac{1}{T_1}, \quad c_2 = \frac{1}{T_2}, \quad (16)$$

where:

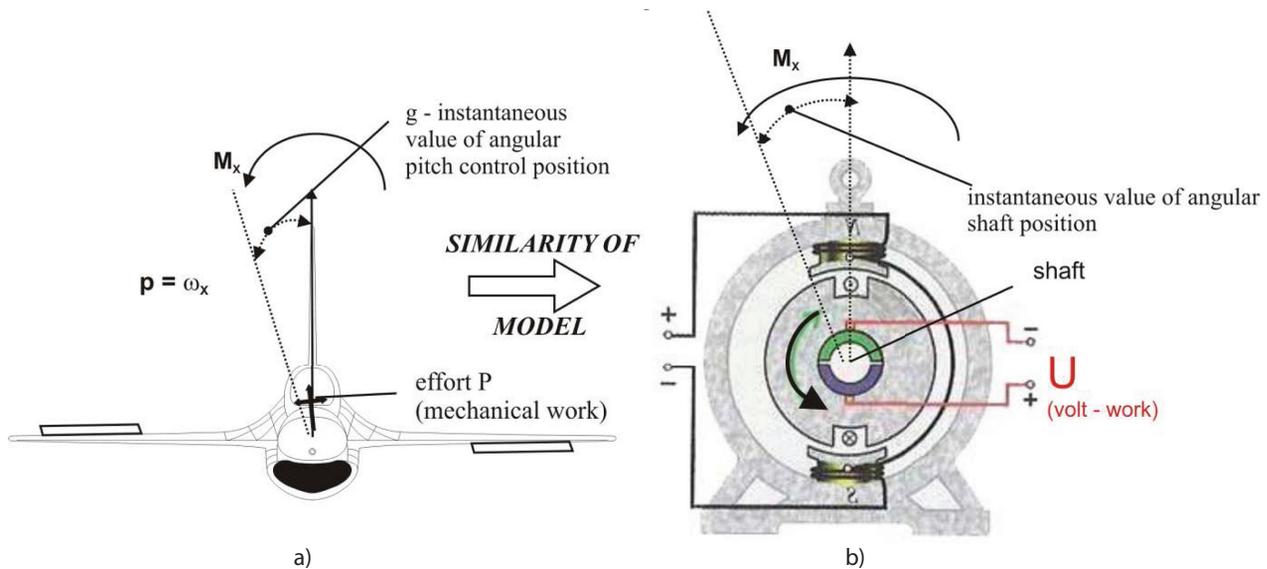


Fig. 2. Analogue of measured physical items: a position angle γ , of an aircraft (a), a position of the DC shaft (b).

T_1 – is time constant of angle speed change ω_x ,

T_2 – is time constant of a control stick position change (deflector).

Let us present the task of equation mechanic analogue creation (13), (14), the representation of which is a single-shift rotary electric motor, for which the symbol DC motor is used in MATLAB program environment. Let maximum agreement probability be the criterion of the mathematical models (13), (14). The scheme and illustration of the correlation identification characteristics of a single-shift motor, the differential equation of which has an equal form as in the AES operation regime under the condition $\beta=0$, is specified by MATLAB in the part „Linear Modelling of Time – Continuous SIMO system“. The analogy is illustrated in Fig.2.

The DC motor (Fig. 2b) generates vector values of measured parameters according to the formula (3)

Let us apply the similarity of input (control) constants $P = U$ (effort on a control stick), [i.e. force used by the pilot to control = to control voltage U (volts)], which also represents a physical activity. To be general: let the momentarily input values bear the labeling u and y of output, where y_1 is angle deflection (γ) and y_2 angular velocity ($\dot{\gamma}$). An operator-pilot compensates random failures at AES control by changes in his effort. Let wavy output voltage recorded by the converter be the model of those failures [2]. The task solution requires labeling of the observed constants, which are compliant with MATLAB language. They are inserted into program by the following orders.

Program in MATLAB environment:

```
load(fullfile(matlabroot, 'toolbox', 'ident',
'idemos', 'data', 'dcmotordata'));
Stick control movement in observed time
period is represented by the following data:
z=iddata(y, u, 0.1, 'Name', 'AES');
set(z, 'InputName', 'Force', 'InputUnit',
'V');converter A/D (analog – digital);
set(z, 'OutputName', {'angle position
(gama)'; 'angle speed(dergama)'});
set(z, 'OutputUnit', {'order', 'order/s'});
set(z, 'Tstart', 0, 'TimeUnit', 's'); start orend
of measurement.
Measurement :position angle y1, angle
speed y2;Distinctive illustration is:
figure('Name', [z.Name ' : Forceinput(Volts)
-> angle position output']);
Illustration of a control stick angle position
measurement y1:
plot(z(:, 1, 1));
figure('Name', [z.Name ' : Voltage input
->angle speed-output']);
Illustration of a control stick angle speed
measurement y2:
plot(z(:, 2, 1));
Measured values show AES reactions, who
observe casual control stick positions.
Let us design mathematical model, the
features of which will remind the measured
ones.
Linear model AES at pitching-design.
In equations (8),(9) there are presented
state variables x1 (gama), x2(derivation
gama).
If the meaning of time constants T1 T2 is
accepted, then (exact recording):

$$\frac{d}{dt}x(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1/T_1 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ K/T_2 \end{bmatrix}u(t)$$

The used A/D converter on the input
enables the transfer of discrete time space,
for which A,B,C,D,K,X0 are defined, where
```

K is matrix of initial conditions, $X0$ is matrix of initial states. In MATLAB the following order is used for transfer:

$m=idss(A,B,C,D,K,X0,...'Ts',T..)$.

Equations (8),(9) do not contain initial conditions. The representation of a state equation, which accepts them is:

$$dx/dt = Ax + Bu + Ke,$$

$$y = Cx + Du + e,$$

where K is X is the matrix of initial conditions, $X0$ is the matrix of initial states, e -is values on output

The representation of a state equation with unknown parameters:-1/T1;k/T2 is:

$$As = [0 \ 1; \ 0 \ NaN];$$

$$Bs = [0; NaN];$$

$$Cs = [1 \ 0; \ 0 \ 1];$$

$$Ds = [0; 0];$$

$$Ks = [0 \ 0; \ 0 \ 0];$$

$$X0s = [0; 0];$$

$$\frac{d}{dt}y(t) = \begin{bmatrix} 0 & 1 \\ 0 & N_a N \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ N_a N \end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x(t)$$

Knowledge of handling AES enables to estimate the size 1/T1=1,k/T2=0.3;

Then:

$$A = [0 \ 1; \ 0 \ -1];$$

$$B = [0; 0.3];$$

$$C = eye(2);$$

$$D = zeros(2,1);$$

The first nominal, estimated model is determined:

$$ms = idss(A,B,C,D),$$

$$tf(ms),$$

$$step(fdy2)$$

Model with estimated parameters has the following structure:

$$setstruc(ms,As,Bs,Cs,Ds,Ks,X0s),$$

$$set(ms, 'Ts',0),interconnected state space,$$

$$msFirst interconnected model, Estimation$$

of parameters by maximum probability method. The method is algorithmised so that it enables to predict estimation errors.

$dcmodel = pem(z, ms, 'trace', 'on'),$

$dcmodel$

Result is compared with measured values: $compare(z, dc\ model),$

Let us use obtained outputs to evaluate AES quality. Estimation $\beta = 0$ (zero

(side angle) secures straightforward flight.

It means that the zeroes and poles of transfer functions must be placed on 'x' coordinate of AES:

$pzmap(dcmodel, 10)$

The credibility of results of used method enables the possibility of solutions modification creation. Let us suppose that analogue-digital (A/D) sensors are not calibrated, i.e. do not have the same shift coefficients. Let x_2 be the derivation of x_1 . Then:

$dcmodel2 = dcmodel,$

$dcmodel2.As(1,2) = NaN,$

$dcmodel2 = pem(z, dcmodel2, 'trace', 'on'),$ illustration of state variables.

Let us compare models quality (see: Loss (final) function of both models as well as columns 'Cost':

$dcmodel2 = pem(z, dcmodel2, 'trace', 'on'),$ illustration of state variables.

Proximity of measured and model values are preconditions for the representation of sought transfer functions:

$A=[0\ 0.99745;0\ -4.0113]; B=[0\ 1.0043]; C=[1\ 0;0\ 1]; D=[0\ 0];$

$mimo_sys=ss(A,B,C,D);$

$G=tf(mimo_sys),$

CONCLUSION

The method presented in the paper is aimed at the doctoral study subject of Automatic Control in the part: Modelling of ergatic systems. The method is interactive, open and as the reaction of students has shown, considerably motivating.

The applications are also suitable in

the area of symbiotic training machine circuits, where it is directly applicable in modelling of arbitrary aircraft movement [6]. Description of the concept of "maximum probability" is not explained in the paper in details - students are taught the terms before the training.

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