ON THE MACRODIVERSITY RECEPTION IN THE CORRELATED GAMMA SHADOWED NAKAGAMI-M FADING

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In this paper an analysis of selection combining (SC) macrodiversity reception performed in correlated Gamma shadowing environment will be presented. At each microlevel maximal ratio combining (MRC) with correlated branches is observed, for mitigating effects of Nakagami-m short-time fading. First, novel closed form expressions are derived for second order statistical measures, level crossing rate (LCR) and average fade duration (AFD). Capitalizing on these expressions, the influence of correlation at macrolevel (shading correlation) will be analysed through their derivatives. Provided analysis could find application in current macrodiversity system design.

Keywords: macrodiversity, correlated, shadowed fading channel, level crossing rate, average fade duration

1 Introduction

Since the usage of diversity techniques applied at single base station (micro-diversity combining) mitigates only influence of multipath (short-time) fading, then in order to deal with overall channel degradation, with concurrently present shadowing (long-term fading), combining between base stations (macro-diversity combining) has to be applied. Macrodiversity combining is used to alleviate the effects of shadowing, since it ensures that different long-term fading is experienced, by signals received at two or more base stations (BS). It has been shown that correlations limit diversity gains in all (time, space, frequency) diversity schemes. When diversity system at single BS is applied on small terminals with multiple antennas, due to insufficient spacing between antennas, correlation arises between the microdiversity branches. However, correlation at macro level is also common phenomenon, which has been measured and shown to be significant in various wireless networks. With single mobile station (MS) and two BSs considered at a given time, shadowing components on the two links often experience correlation, as witnessed by experimental results given in [1, 2]. Level of correlation depends on the separation between the BS, on the surrounding terrain, the angle of arrival of the received signals, and various factors. In [3, 4, 5] has been shown that in cellular radio systems, correlation on links between a MS and multiple BSs significantly affects mobile hand-off probabilities and co-channel interference ratios. Also coverage area and interference characteristics are affected by correlated shadowing, that occurs in digital broadcasting, links between multiple broadcast antennas to a single receiver [6]. Finally, correlated shadowing is significant (correlation coefficients even reach 0,95), and strongly impacts system performance in indoor WLANs [7]. Correlated signals between macrodiversity branches have already been studied at [8, 9]. In [10], LCR (Level crossing rate) and AFD (Average Fade Duration) at the output of SC (Selection Combining) macrodiversity operating over the Gamma shadowed Nakagami-m fading channels were observed. Based on their output signal power values, which are in this case assumed to be uncorrelated, macrodiversity system selects one of two microdiversity structures. Results obtained in [10] are generalized here, with correlation introduced at macrolevel. Novel closed form expressions are derived for same second order statistical measures (LCR and AFD). Capitalizing on these rapidly converging expressions, in order to point out the influence of assumed correlation at macrolevel, LCR and AFD derivatives over macrolevel correlation coefficient are obtained, observed and analysed in the function of various system parameters, such as fading and shadowing severity, microdiversity order and correlation level.

2 System model

Considered two microdiversity systems at the BSs are of MRC type with arbitrary number of branches and subjected to Nakagami-m fading. Treating the correlation between the branches as exponential, the probability density functions (PDF) of the signal-to-noise ratios (SNRs) at the outputs of microdiversity systems are modelled with [12]:

$$P_{\eta}(r) = \frac{1}{\Gamma(M)} \left( \frac{L_m}{q_i \Omega_i} \right)^M r^{M-1} \exp \left( - \frac{L_m}{q_i \Omega_i} r \right)$$  (1)
Order of each microdiversity system is denoted \( L_i \), while Nakagami-m fading severity is determined through parameter \( m_i \), while \( \Gamma(x) \) stands for the Gamma function. Parameter \( \rho_i \), related to the exponential correlation \( \rho_i \) among the branches, and parameter \( M_i \) are defined, respectively as:

\[
g_i = L_i + \frac{2 \rho_i}{1 - \rho_i} \left[ L_i - \frac{1 - \rho_i^{L_i}}{1 - \rho_i} \right],
\]

\[
M_i = \frac{m_i L_i^2}{q_i}.
\]

Since we will observe second order statistical measures, we must take into consideration time derivative characteristics of observed random processes. Processes at the outputs of observed MRC systems follow [12]:

\[
r_i^2 = \sum_{k=1}^{L_i} r_{ik}^2 \quad i = 1, 2
\]

\[
r_i = \sum_{k=1}^{L_i} \frac{r_{ik}}{q_i} \quad i = 1, 2
\]

with \( r_i \) being Gaussian random variable with zero mean and variance defined as bellow [13]:

\[
p_{\tilde{r}}(\tilde{r}_i) = \frac{1}{\sqrt{2\pi\sigma^2_{\tilde{r}_i}}} \exp\left(-\frac{\tilde{r}_i^2}{2\sigma^2_{\tilde{r}_i}}\right),
\]

\[
\sigma^2_{\tilde{r}_i} = \sum_{k=1}^{L_i} \frac{r_{ik}^2}{\sigma^2_{\tilde{r}_i} r_i^2},
\]

where \( f_d \) is a Doppler shift frequency. Joint PDFs of random process and their time derivatives conditioned on \( \Omega_i \), can be expressed as:

\[
P_{\tilde{r}, \tilde{r}'}(\tilde{r}, \tilde{r}') = \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(r, \rho \tilde{r}', \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega' + \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(\rho \tilde{r}, r', \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega' + \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(\tilde{r}, r', \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega'.
\]

Switching between BSs is based on the microcombiners output signal power values, similarly as in [14]:

\[
P_{\tilde{r}, \tilde{r}'}(r, \rho \tilde{r}) = \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(r, \rho \tilde{r}, \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega' + \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(r, \rho \tilde{r}', \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega' + \int_0^{\Omega} \int_0^{\Omega} p_{\tilde{r}, \tilde{r}'}(r', \rho \tilde{r}, \Omega, \Omega') p_{\Omega, \Omega'}(\Omega, \Omega') d\Omega d\Omega'.
\]

Cumulative distributions functions (CDFs) of random processes \( F(r_i / \Omega_i) \) at the microdiversity outputs are equal to:

\[
F(r_i / \Omega_i) = \frac{1}{\Gamma(c)} \int_{0}^{\frac{r_i}{\Omega_i}} t^{c-1} e^{-t} dt.
\]

Here long-term fading is as in [8] described with correlated Gamma distributions, as:

\[
P_{\Omega_1 \Omega_2}(\Omega_1, \Omega_2) = \frac{\rho^2_2}{\Gamma(c)(1 - \rho^2_2)} \Omega_1^{c-1} \Omega_2^{c-1} \exp\left(-\frac{\Omega_1 + \Omega_2}{\Omega_0 (1 - \rho^2_2)} \right) \left( \frac{\Omega_1^{c-1} \Omega_2^{c-1}}{\Omega_0^{c-1} (1 - \rho^2_2)} \right) - 1.
\]
respectively as:

\[ N_r(r) = \frac{\int \rho \left( 1 - e^{-2 \rho r} \right) r \, dr}{\bar{\rho}^2}, \]

\[ T_R(r) = \frac{F_r(r \leq R)}{N_r(r)}, \]

these second order statics measures can be presented respectively as:

\[ N_r(r) = 2 \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

\[ \Gamma(b+c) b^{2b-1} \frac{L_m}{2q_1} \Omega_0(1 - \rho_2) \times \]

\[ \times K_{2+a+b+2c-M_2} \left( \frac{2L_m}{q_2} \Omega_0(1 - \rho_2) \right)^{2^{-1}} \times \]

\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \times K_{2+a+b+2c-M_2} \left( \frac{2L_m}{q_2} \Omega_0(1 - \rho_2) \right)^{2^{-1}} \times \]

\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

\[ \Gamma(b+c) b^{2b-1} \frac{L_m}{2q_1} \Omega_0(1 - \rho_2) \times \]

\[ \times K_{2+a+b+2c-M_2} \left( \frac{2L_m}{q_2} \Omega_0(1 - \rho_2) \right)^{2^{-1}} \times \]

\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \times K_{2+a+b+2c-M_2} \left( \frac{2L_m}{q_2} \Omega_0(1 - \rho_2) \right)^{2^{-1}} \times \]

\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

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\[ \frac{\rho_2}{\Gamma(c)(1 - \rho_2)} \Omega_0^{2^{-1}} \left( \frac{L_m}{q_1} \right)^{M_1} \times \]

\[ \times \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left( \frac{4}{\sqrt{\rho_2}} \right)^{2b+1-1} \frac{1}{\Omega_0^2(1 - \rho_2)} \times \]

\[ \Gamma(b+c) b^{2b-1} \frac{L_m}{2q_1} \Omega_0(1 - \rho_2) \times \]

\[ \times K_{2+a+b+2c-M_2} \left( \frac{2L_m}{q_2} \Omega_0(1 - \rho_2) \right)^{2^{-1}} \times \]
3 Numerical results

In Figs. 1 and 2, we have graphically presented obtained normalized LCR and AFD values in the function of shadowing correlation at macrolevel. LCR and AFD values are normalized by maximal Doppler shift frequency $f_d$. By observing those figures it is clear that lower LCR levels are crossed with the lower level of shadowing correlation between the branches, and that better AFD performance is also achieved at lower values of $\rho_2$. Sensitivity of normalized LCR over $\rho_2$ is presented in Figs. 3 and 4. It is clear from Figs. 3 and 4 that sensitivity grows in the area of high values of $\rho_2$, which means that a small change in $\rho_2$ would result in a significant change in LCR value when higher values of shadowing correlation are observed.

![Figure 1](image1.png)

**Figure 1** Normalized LCR in the function of shadowing correlation $\rho_2$

![Figure 2](image2.png)

**Figure 2** Normalized AFD in the function of shadowing correlation $\rho_2$

![Figure 3](image3.png)

**Figure 3** Sensitivity of normalized LCR over shadowing correlation $\rho_2$

![Figure 4](image4.png)

**Figure 4** Sensitivity of normalized LCR over shadowing correlation $\rho_2$ in the function of fading severity and correlation and microdiversity order

It is also evident from Fig. 4, that sensitivity over this correlation level could be further reduced by increasing order of microdiversity structure, while increasing space at the terminal between diversity branches (reducing correlation level at microstructure), and that obtains lower values when fading is less severe (smaller values of $m$ parameter). By observing Figs. 1 and 2 it can be concluded that sensitivity of AFD over $\rho_2$ due to system parameter change will behave in a similar manner as the sensitivity of normalized LCR over $\rho_2$, and would be reduced by increasing order of microdiversity and fading severity.

4 Conclusion

Influence of shadowing correlation on selection combining (SC) macrodiversity reception performances in correlated Gamma shadowing environment was observed. Rapidly converging infinite-series expressions are derived for LCR and AFD of observed structure. Further, sensitivity of those measures over correlation at macrolevel (shadowing correlation) was observed. Numerically obtained results are graphically presented and discussed in the function of various system parameters.
5 References


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