GENETIC ALGORITHM APPLIED TO OPTIMIZATION OF THE SHIP HULL FORM WITH RESPECT TO SEAKEEPING PERFORMANCE

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Summary

Hull form optimization from a hydrodynamic performance point of view is an important aspect in preliminary ship design. This study presents a computational method to estimate the ship seakeeping in regular head waves. In the optimization process, the genetic algorithm (GA) is linked to the computational method to obtain an optimum hull form by taking into account the displacement as a design constraint. New hull forms are obtained from the well-known S60 hull and the classical Wigley hull taken as initial hulls in the optimization process at two Froude numbers (Fn=0.2 and Fn=0.3). The optimization variables are a combination of ship hull offsets and main dimensions. The objective function of the optimization procedure includes the peak values for vertical absolute motion at the centre of gravity (CG) and the bow point (0.15Lwl) behind the forward perpendicular (FP).

Key words: Optimization, Hull Form, Genetic Algorithm, Seakeeping, Strip Theory

1. Introduction

Prediction of ship performances in calm and rough waters is one of imperative concerns of naval architects, already at the earliest design stage. From this point of view, seakeeping performance is one of the most important performances in the ship hull form optimization. It is possible to accomplish considerable enhancement in terms of habitability, operability and survivability by means of changes in the hull form even when displacement and main dimensions have been fixed.

It is worth noting that for a comprehensive and detailed ship hydrodynamic optimization all objective functions such as resistance, stability, seakeeping, etc., must be considered because it is clear that the consideration of one objective function without the others gives unrealistic and impractical results.

Some researchers have considered two or three objective functions for optimizing the hull form, while some others, only one objective function. For example Gammon [8] uses three objective functions in his study, Biliotti et al. [2] and Grigoropoulos & Chalkias [9] employed two objective functions in their work and many researchers considered only one objective function [11, 20, 27].
Optimization techniques are most widely used in hull shape modification. Zhang [29], Kim et al. [16-17] and Saha et al. [23] employed different types of nonlinear linear programming as optimization techniques. Evolutionary algorithm (EA) and artificial neural networks (ANN or NN) offer effective methods for conducting optimization and data analysis. EA techniques may be divided into genetic algorithms (GAs), evolutionary strategies (ESs), and evolutionary programming (EP). However, GAs and ESs are most widely used in hull form modification. A review of the structure of multi-objective optimization by multi-objective evolutionary algorithm (MOEA) was presented by Zamarin et al. [28]. In this study, the GA was used to determine the optimization problem. Day & Doctors [4] studied the hull form optimization using a GA technique in which the objective was to minimize resistance. A single-objective optimization algorithm based on the genetic algorithm to improve the hull form of a catamaran [14] and a passenger ship [26] was presented there.

Optimization of the seakeeping performance of the destroyer-type hull form in head seas and at various speeds was carried out by Bales [1]. Griogoropoulos & Loukakis [10] developed a numerical method, based on a nonlinear direct search algorithm to minimize response amplitude operator (RAO) peak values in head regular waves. Similar studies were carried out by Hearn et al. [12] who developed an inverse design procedure based on the optimum hull nonlinear direct search process. Kukner & Sariöz [18] optimized the seakeeping qualities of a high speed vessel, using the Lackenby method [19] to generate several hulls, each having different form parameters as regards to those of the parent hull. Peacock et al. [22] defined a mathematical model based on a multi-objective research algorithm for displacement mono-hulls. Sariöz & Sariöz [25] proposed a new optimization procedure based on a nonlinear problem solved by direct search techniques. Campana et al. [3] proposed a new optimization technique for the heave motion of the S175 container ship, adopted by the ITTC seakeeping committee as a benchmark test, considering two different optimization procedures, namely the filled function based algorithm and the particle swarm optimization method. Diez&Peri [5] presented a new approach to the robust optimization of the conceptual design of a bulk carrier subjected to uncertain operating and environmental conditions. In the approach, the standard deterministic formulation for design optimization is extended to take into account the uncertainty related to design variables, operating conditions, and computational results of the simulations. Özüm et al. [21] investigated the seakeeping qualities of fast ships, systematically varying both main dimensions and hull form parameters. Kukkanen, & Matusiak presented experimental and numerical results for the Wigley and RoPax ships using the nonlinear time domain method [15]. Anyway, in almost all the cases, the optimization procedures were based on the assumption that the optimum hull is found when the vertical plane motions and absolute vertical acceleration in regular head waves due to combined pitch and heave motions are minimized.

In this research, the strip theory and a particular form of the optimization algorithm (genetic algorithm) are employed for obtaining the optimized hull form. A program MATLAB code was prepared and two models are examined. Main dimensions as well as offset data may be changed to generate various hull models and to determine the seakeeping performance results. Finally, based on the objective functions, the optimum hull can be found. It should be noted that the current design procedure is restricted to the minimization of vertical plane motions.

The outline of the paper is as follows. In section 2, optimization problem and GA are introduced. Section 3 is devoted to the seakeeping analysis. Procedure of the optimization of hull forms is described in section 4. Numerical results for two cases are discussed in section 5 and finally, conclusions are presented in section 6.
2. Optimization Problem and GA

The general mathematical form of a constrained optimization problem is presented here. Design variables and constraint conditions are used to characterize the problem. The role of design variables in hydrodynamic optimization problems is to control the geometry of the hull during the optimization procedure. Constraints are the values by which the design variables are restricted. They may be divided into two types, i.e. equality and inequality constraints. A function being maximized or minimized by users is known as the objective function and the value of this function is a criterion for determining the efficiency of design optimization methodology. If only one objective function is used in an optimization problem, the optimization is known as single objective and if two or more objective functions are used, the optimization is known as multi objective. The standard formulation of an optimization problem expressed mathematically is as follows:

\[ \text{Optimize } F(\overline{x}) = [f_1(x), f_2(x), ..., f_m(x)] \quad x \in \mathbb{R}^n \]  

Subject to

\[
\begin{align*}
& h_i(\overline{x}) = 0 \quad i = 1, ..., q \\
& g_i(\overline{x}) \leq 0 \quad i = 1, ..., p
\end{align*}
\]

where \( f_i(\overline{x}) \) is the objective function, \( m \) is the number of objective function, \( q \) is the number of equality constraints, \( p \) is the number of inequality constraints and \( \overline{x} = (x_1, ..., x_n) \in F \subseteq S \) is a solution or individual. The set \( S \subseteq \mathbb{R}^n \) defines the search space and the set \( F \subseteq S \) defines a feasible search space. The search space \( S \) is defined as an \( n \)-dimensional rectangle in \( \mathbb{R}^n \) (domains of variables defined by their lower and upper bounds):

\[ l(i) \leq x_i \leq u(i), 1 \leq i \leq n \]

The constraints define the feasible area. This means that if the design variables vector \( \overline{x} \) is in agreement with all \( h_i(\overline{x}) \) constraints (equality constraints) and \( g_i(\overline{x}) \) constraints (inequality constraints), it belongs to the feasible area.

In this study, the design variables vector includes the main parameters (length, breadth, and draft) and the hull offsets, which are limited by the lower and upper bounds. The ship hull displacement is also an inequality constraint.

Among the class of evolutionary algorithms, the genetic algorithm (GA) is the most popular algorithm for solving continuous optimization problems, i.e. for optimizing the real-valued function \( f \) defined on a subset of \( \mathbb{R}^n \) for some dimension \( n \). Genetic algorithm can be applied to combinatorial problems as well. Genetic algorithm is inspired by the evolution theory (Darwin’s theory of biological evolution) by means of a process that is known as the natural selection and the "survival of the fittest" principle. The common idea behind this technique is similar to other evolutionary algorithms: consider a population of individuals; the environmental pressure causes natural selection which leads to an increase in the fitness of the population. It is easy to see such a process as optimization. Consider an evaluation function to be minimized (the lower, the better). A set of candidate solutions can be randomly generated and the objective function can be used as a measure of how individuals have performed in the problem domain (an abstract fitness measure). According to this fitness, some of the better solutions are selected to seed the next generation by applying recombination and/or mutation operators to them. The recombination (also called crossover) operator is used to generate new candidate solutions (offspring) from existing ones by taking two or more selected candidates.
(parents) from the population pool and by exchanging some of their parts to form one or more offspring. The mutation operator is used to generate one offspring from one parent by changing some parts of the candidate solution. The application of the recombination and mutation operators causes a set of new candidates (the offspring) to compete based on their fitness with the old candidates (the parents) for a place in the next generation.

This procedure can be iterated until a solution with sufficient quality (fitness) is found or a previously set computational time limit is reached. In other words, the end conditions must be satisfied. The composed application of selection and variation operators (recombination and mutation) improves fitness values in the consecutive population. A general flowchart of genetic algorithm is shown in Figure 1.

![General flowchart of genetic algorithm](image)

**Fig. 1** General flowchart of genetic algorithm

Genetic algorithm variables are divided into two categories: object and genetic variables. Variables in the genetic algorithm are commonly taken as real-valued vectors because this algorithm is usually used for continuous parameters. A form of an individual in GA is as follows:

\[ < x_1, ..., x_n > \]

where \( x_i \) is the object variable. In the mutation of object variables, each gene (biological name of a vector) is changed with a mutation rate (genetic variable) in the range of its lower and upper bounds. The mutation methodology for \( i \in \{1, ..., n\} \) is as follows:

\[ < x_1, ..., x_n > \rightarrow < x'_1, ..., x'_n > \quad x_i, x'_i \in [l(i), u(i)] \tag{3} \]

Scatter recombination is one of the main types of recombination (crossover) used in GA. This type of crossover creates a random binary vector. The genes are selected from the first parent where the vector is a 1, and from the second one where the vector is a 0. The \( (\mu, \lambda) \) survivor selection scheme has advantages over its competitor, the \( (\mu + \lambda) \) selection scheme, but the \( (\mu + \lambda) \) selection scheme is an elitist mechanism that can maintain the best solution in each generation (Eiben and Smith, [6]).
3. Seakeeping Calculation

The determination of hydrodynamic forces acting on a ship can be formulated as a linear boundary value problem in potential theory. On the assumption that motion responses are linear, or at least can be linearized and are harmonic, equations of motion for the advancing ship in waves may be written in the following general form:

$$L_{kj}(H, \omega, U) \ddot{z}_j = F_k, \quad k, j = 1, 2, \ldots, 6$$  \hspace{1cm} (4)

$$\sum_{k=1}^{6} \left[ (M_{jk} + A_{jk}) \ddot{z}_k + B_{jk} \dot{z}_k + C_{jk} z_k \right] = f_{ex,j} e^{i\omega t}, \quad j = 1, 2, \ldots, 6$$  \hspace{1cm} (5)

where $H$ represents the hull geometry, $\omega$ is the wave frequency, and $U$ is the forward speed. Typically, the operator $L_{kj}$ is of the form.

$$L_{kj} = -(M_{kj} + A_{kj}) \omega^2 - iB_{kj} \omega + C_{kj}$$  \hspace{1cm} (6)

where $M$ is the generalized mass matrix, $A$ and $B$ represent the added mass and the fluid damping matrices associated with forces/moments induced in the $k$th mode as a consequence of motion in the $j$th mode, and $C$ is the hydrostatic restoration matrix. The degrees of freedom, $j$, correspond to surge, sway, heave, roll, pitch, and yaw as $j$ assumes the value 1-6, respectively. The dependence of the hydrodynamic coefficients and the hydrostatic restoration upon the hull form shape may be expressed as:

$$\begin{align*}
B_{kj} &= B_{kj}(H, \omega, U) \\
A_{kj} &= A_{kj}(H, \omega, U) \\
C_{kj} &= C_{kj}(H)
\end{align*}$$  \hspace{1cm} (7)

The wave excitation $F_k$ is also a function of wave heading parameters as follows:

$$F_k = F_k(H, \omega, U, \beta)$$  \hspace{1cm} (8)

The added mass, damping, restoring force and wave exciting force terms can be calculated by using well established numerical procedures. In order to reduce the computing time, a linear strip theory approach is adopted as described by Salvesen et al. [24]. The sectional added mass and damping coefficients are calculated by using the well-known Frank Close-Fit method [7]. The seakeeping responses in head seas are generally the most important responses for mono-hulls. Thus, all calculations were done for vertical motions and the related kinematics. The computed ship responses include vertical motion and acceleration at the bow region (at a point 0.15 LBP behind the forward perpendicular). All the results are given for regular head waves.

The dimensions of the Wigley hull model are 3 m in length with $L/B = 10$ and $L/T = 16$. That is the same model used by Journée [13] at Delft University of Technology (DUT). The wave condition is a head sea with a wave height of 2 cm. For this model and conditions, the heave and pitch response amplitude operator (RAO) results together with experimental data are shown in Figure 2. Using the numerical method described above for computing the ship vertical motion results in good agreement with experimental results and errors in predictions compared to the experimental results (with respect to the case when the linear theory was employed) are within 10% for the design Froude number (Fn=0.3). It should be noted that according to Figure 2, the heave and pitch RAOs at $\lambda / L = 1.2$ that at which the peak value occurs, exhibit a 110 degree phase difference, approximately. The vertical bow motion (objective function) is a function of the main dimensions (length, breadth, and draft) and the hull offsets of the ship. The vertical bow motion as an objective function must be minimized in the optimization process.
4. Procedure of hull form optimization

The procedure of optimizing a ship hull form in order to find a hull shape with minimum bow vertical motion is as follows. The optimization of hull form can be performed by evaluating the hull forms that are generated by variation operators and then by selecting the best forms of lower vertical motion at the bow region in each generation. The seakeeping computational flowchart is shown in Figure 3.

The Wigley and S60 hull forms are considered as initial hull forms. Each chromosome (biological name of a solution) in the optimization algorithm consists of ship offsets, length, breadth, and draft. Because of a large number of variables, the genetic algorithm is a winning skill for the hull form optimization problems from a seakeeping point of view. The design constraint that was used for this study is that the optimizer allowed no change in the total displacement of the ship. In addition, sinkage and trim effects are not considered as a hydrodynamic design constraint, which means that the ship is always in even-keel condition. Some limits have been imposed on the principal dimensions and the hull offsets. In order to restrict the search space and to keep the optimal hull near the original one for comparison, the length, breadth, and draft are limited to ±10% variations in the principle dimensions and the offset points are limited to ±3% of the initial hull offsets. Table 1 presents the variation percent of variables used in test cases.

![Fig. 2 Comparison of the heave and pitch RAOs for the models of the Wigley hull at Fn=0.3](image)

![Fig. 3 Seakeeping computational flowchart](image)
Table 1 Variation percent of variables used in test cases

<table>
<thead>
<tr>
<th>Variables</th>
<th>Length (L)</th>
<th>Breadth (B)</th>
<th>Draft (T)</th>
<th>Displacement (Δ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation Percent</td>
<td>±3%</td>
<td>±10%</td>
<td>±10%</td>
<td>±3%</td>
</tr>
</tbody>
</table>

The Wigley model is a popular and well-known model in ship hydrodynamics experiments. Many experimental and numerical results can be found in the literature for this model. This model is selected to compare the numerical results. The standard Wigley hull is a mathematical displacement hull form, the geometric surface of which can be defined as:

\[
y = \frac{B}{2} \left[1 - \left(\frac{2x}{L}\right)^2 \right] \left[1 - \left(\frac{Z}{T}\right)^2 \right]
\]

where \(B\) is the ship breadth, \(L\) is the ship length, \(T\) is the ship draft, and \(-T \leq z \leq 0\). Vertical motions of hull sections are predicted by the coupled strip theory and the Frank method. The hull form optimization is carried out at a single Froude number \((F_n = U / \sqrt{gL})\) that is constant for each model and that is 0.3 for the Wigley model and 0.2 for the S60 model, where \(U\) and \(L\) are the speed and the waterline length of the model, respectively. Table 2 shows the main characteristics of the Wigley and S60 hulls.

Table 2 Main characteristics of the Wigley and S60 hulls

<table>
<thead>
<tr>
<th>Item</th>
<th>Model type</th>
<th>Wigley hull form (model)</th>
<th>S60 hull form (full scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length /m</td>
<td>3</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>Breadth /m</td>
<td>0.3</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Draft /m</td>
<td>0.1875</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Volume of displacement (V)</td>
<td>0.078</td>
<td>10201</td>
<td></td>
</tr>
<tr>
<td>Centre of gravity /m</td>
<td>(0,0,0.17)</td>
<td>(0.6,0,6.7)</td>
<td></td>
</tr>
<tr>
<td>Radius of gyration /m</td>
<td>0.25L</td>
<td>0.25L</td>
<td></td>
</tr>
<tr>
<td>Froude number</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

The process of optimization is performed by the genetic algorithm. The offset points and principal dimensions can be represented by real-valued vectors in the limits already mentioned. The results of optimization carried out in this study by means of a genetic algorithm with the stochastic uniform sampling as the selection operator, the scatter crossover as the crossover operator, and the uniform mutation as the mutation operator. The recombination rate was 0.80, while the mutation rate was 0.02. According to the results of tests carried out by the authors, the \((\mu, \lambda)\) scheme has proven to be an appropriate survivor selection mechanism for the test cases using the Wigley hull and S60 as mother models.

5. Result and Discussion

5.1 Case of Wigley hull

The first example is the Wigley hull form. This model, 3 m in length, with the length to breadth ratio \((L/B=10)\) and the length to draft ratio \((L/T=16)\) is optimized at \(F_n=0.3\). The hydrodynamic optimization is to minimize the peak value for absolute vertical motion at a point 0.15LBP behind the FP. The offset values and main dimensions of the hull are changed in the domains of ±3% and ±10% of initial ones, respectively. Also, displacement is varied around ±3% of the initial value. In each generation, 130 hull forms are created and the best 10 hull forms are selected to seed the next generation based on the fitness, i.e. the less vertical bow motion the better hull form. Figure 4 depicts a body plan of the optimal hull form (dashed lines) generated by the use of the genetic algorithm optimization technique and a
The body plan of the initial Wigley hull (solid lines). The optimization procedure improved the initial hull and produced a reasonable hull form. The main dimensions of the parent and optimized hull forms are shown in Table 3.

### Table 3 Main dimensions of the parent and the optimized Wigley hull

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parent Hull</th>
<th>Optimized Hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ /m</td>
<td>3</td>
<td>3.29</td>
</tr>
<tr>
<td>$B$ /m</td>
<td>0.3</td>
<td>0.29</td>
</tr>
<tr>
<td>$T$ /m</td>
<td>0.1875</td>
<td>0.17</td>
</tr>
<tr>
<td>$V$ /m$^3$</td>
<td>0.078</td>
<td>0.0765</td>
</tr>
</tbody>
</table>

As can be seen in Figure 4, the breadth of the optimized hull is greater than that of the initial Wigley hull in fore and aft parts and smaller in the amidships. The draft of the optimized hull has decreased dramatically. The three-dimensional view of the initial and the optimized Wigley hull form is presented in Figure 5.

During the execution of the optimization algorithm, in addition to the hull offsets, the length, breadth, and draft of the hull are changed. The variations of the main dimensions of the hull versus the iteration number are shown in Figure 6. The two figures (6-b and 6-c) confirm that the hull has a tendency toward a shallower draft and approximately fixed breadth during the optimization algorithm. The length of the hull is sharply increased in the initial iterations and then it is fixed. The changes in the variable parameters of the hull are made to reach the minimum vertical motion peak value and to satisfy the constraint for the displacement.
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Fig. 6 Variation of dimension parameters and the history of the fitness value of the Wigley hull by iteration number

Fig. 7 Heave and Pitch RAOs for the initial and optimum Wigley hull form

Fig. 8 Absolute vertical motion and acceleration for the initial and the optimum Wigley hull form
As can be seen in Figure 7, the heave and pitch RAOs for the optimum Wigley hull are reduced in a range of wave lengths (and also the peak value of RAOs). The changes in the fitness value versus iteration number of the vertical motion are shown in Figure 6-d. The reduction in heave and pitch RAOs leads to the reduction in vertical motion and acceleration in a range of wave lengths, as shown in Figure 8.

5.2 Case of S60 hull

In this example, a length of 122 m of the S60 hull form $C_B=0.7$ with a length to breadth ratio $L/B = 7$ and a breadth to draft ratio $B/T = 3$ is chosen in order to derive a hull with minimum bow vertical motion at $F_n=0.2$. The variation range in the offset values is between ±3% of the initial S60 hull offsets and the main dimensions are changed in the range of ±10% of the main dimensions of the initial hull and the displacement is also changed within ±3% of the initial one. The body plan of the initial hull form (solid lines) and the optimized hull form (dashed lines) are shown in Figure 9. In each generation, 200 hulls are created and then the best 15 hull forms are selected to go to the next generation based on the lower vertical motion peak value at the bow of the ship. The breadth and draft of the optimized hull are approximately fixed, with a deeper draft than that of the initial S60 hull. The main dimensions of the parent and the optimized hull forms are given in Table 4.

Table 4 Main dimensions of the parent and the optimized S60 hull

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parent Hull</th>
<th>Optimized Hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ /m</td>
<td>122</td>
<td>115.3</td>
</tr>
<tr>
<td>$B$ /m</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>$T$ /m</td>
<td>7</td>
<td>7.67</td>
</tr>
<tr>
<td>$\nabla$ /m$^3$</td>
<td>10201</td>
<td>10701</td>
</tr>
</tbody>
</table>

Fig. 9 Body plan of the original S60 ($C_B=0.7$) hull and optimal hull
(Solid line is the initial and dashed line is the optimal hull)

The three-dimensional view of the initial and the optimized S60 hull form is shown in Figure 10. During the implementation of the optimization algorithm, in addition to hull offsets, the length, breadth, and draft of the hull are varied and the changes of them by evaluation number are as in the previous case, but the difference is the hull has a tendency towards approximately smaller length (see Figures 11-a,11-b and 11-c). As can be seen in Figure 11, the significant changes in four characteristics of the hull are noted at the early evaluation stage of the objective function and after that, they remain fixed.
Figure 11-d demonstrates the changes in the fitness value of hull by iteration number. The heave and pitch RAOs are shown in Figure 12, which illustrates the reduction in both RAO values in a range of wave lengths (in pitch RAO in particular). As can be seen in Figure 13, during the optimization process, vertical motion and acceleration decreased together with the vertical motion peak value in the entire range of wave lengths. This is due to the variation of the length and draft and the hull form in the optimization process. Evaluation of the hydrodynamic performance of the initial hull and the optimized hull in terms of the vertical motion and acceleration is shown in Figure 12.

Fig. 10 Three-dimensional hull forms (Red is the initial hull and blue is the optimized hull)

Fig. 11 Variation of dimension parameters and the history of the fitness value of the S60 hull by iteration number
6. Conclusions

A numerical method was employed for hydrodynamic hull form optimization in regular head waves with respect to vertical motion at the CG and the bow of the ship as the only objective function. The genetic algorithm is combined with a numerical method for minimizing the peak value of vertical motion characteristic (the ship motion in waves based on strip theory and the sectional added mass and damping coefficients calculation by the well-known Frank Close-Fit method). The design variables include the hull offsets and the main dimensions (length, breadth and draft) and the displacement is used as the design constraint during the optimization with the Wigley and S60 hull forms as standard models at a constant speed Froude number (i.e. \( \text{Fn}=0.3 \) for the Wigley model and \( \text{Fn}=0.2 \) for the S60 model) to develop optimized ship hull forms. Based on the numerical finding, the following conclusions can be drawn:

1. In the case of the S60 hull form optimization, due to a great length of the hull, the pitch motion has a strong influence on the bow vertical motion and a change in vertical motion far from the CG depends on the pitch variation.
2. Compared with the initial hull, the peak value of vertical motion of the improved hull is reduced by 33% in the Wigley hull and by 27% in the S60 hull.
3. Reduction in vertical motion and its acceleration is significant for both ship hull forms.
4. The gains in terms of fitness value reductions were considerable in both cases, especially for the S60 ship hull form. Therefore, we can make a conclusion that the genetic algorithm used in this study is an effective and robust technique for hull form optimization.
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