ORTHONORMAL FUNCTIONS BASED MODEL PREDICTIVE CONTROL OF PH NEUTRALIZATION PROCESS

Amila Dubravić, Zenan Šehić, Mustafa Burgić

This paper presents use of Legendre and Laguerre orthonormal functions for representation of the control trajectory in discrete model predictive control, on a pH neutralization process, which is a process with static non-linearity. Performance is tested with plant-to-model mismatch present. Orthonormal functions are used for efficient parameterisation of the difference of control signal as in the case of linear process. This approach has better computational efficiency compared to the classical predictive control algorithms.

Keywords: model predictive control, non-linear process, orthonormal functions, pH neutralization

1 Introduction

Recently proposed approaches of model predictive control suggest use of different model descriptions: hybrid fuzzy model [13, 15], Wiener-model based on PWL [14], hybrid MPC based on genetic algorithms [16], probabilistic neural-network [17]. Approach used in this article proposes use of orthonormal functions as in [2] tested on a non-linear pH neutralization process.

As described in [1] the technique used in the design of discrete model predictive controller is based on modeling the control trajectory, control signal \( u(k) \) or the difference of control signal \( \Delta u(k) \) by forward shift operators, which can lead to possible large number of forward shift operators used for the description of control trajectory if complicated dynamics of the process, fast sampling or high demands on closed-loop performance are present. Fast changes of control signal are also possible as there is no structural constraint on the future control signal. Performance of the model predictive controller, based on orthonormal Laguerre and Legendre functions, is tested on a pH neutralization process, having static non-linearity. It can be concluded that plant-to-model mismatch is present, so robustness of this approach is tested.

As described in [1, 2] advantage of such approach is reduction in number of parameters used for description of the control trajectory compared to the classical approach. The change of control trajectory is managed through adjustment of scaling factor present in orthonormal function.

Paper consists of Section 1, being Introduction, Sections 2 and 3, present algorithm for model predictive controller design using Laguerre and Legendre functions, respectively. Section 4 presents pH neutralization process. In Section 5 results of simulation examples are presented. After simulation section, follow conclusions.
Orthonormal functions based model predictive control of pH neutralization process

A. Dubravić, Z. Šehić, M. Burgić

\[ A_t = \begin{bmatrix}
    a & 0 & 0 & 0 & 0 \\
    \beta & a & 0 & 0 & 0 \\
    -a\beta & \beta & a & 0 & 0 \\
    a^2\beta & -a\beta & \beta & a & 0 \\
    -a^3\beta & a^2\beta & -a\beta & \beta & a
\end{bmatrix}. \]

Design of discrete MPC using the pulse operator corresponds to the case where the parameter \( a = 0 \) in the Laguerre polynomial.

At time \( k_i \), the control trajectory \( \Delta u(k_i), \Delta u(k_i+1), \Delta u(k_i+2), \ldots \), is regarded as the impulse response of a stable dynamic system. A set of Laguerre functions, \( l_1(k), l_2(k), \ldots, l_N(k) \) is used to capture the dynamic response with a set of Laguerre coefficients that are to be determined from the design process. Based on this,

\[ \sum_{j=1}^{N} c_j (k_i) l_j(k), \quad (5) \]

where \( k_i \) is the initial time of the moving horizon window and \( k \) is the future sampling instant, \( N \) is the number of terms used in the expansion, \( c_j, j = 1, 2, \ldots, N \), are the coefficients being functions of the initial time of the moving horizon window, \( k_i \).

Eq. (5) can also be expressed in a vector form:

\[ \Delta u(k_i + k) = L(k)^T \eta, \quad (5a) \]

where \( \eta \) has \( N \) Laguerre coefficients:

\[ \eta = [c_1, c_2, \ldots, c_N]^T. \quad (5b) \]

So, the coefficient vector \( \eta \) is optimized and computed in the design.

\[ \begin{array}{c}
\begin{array}{c}
\sqrt{1-a^2} \\
1 \\
z^{-1}-a \\
1-a z^{-1} \\
n-1-a (1-a z^{-1})^{-1}
\end{array}
\end{array} \]

Figure 1 Laguerre network

3 Legendre orthonormal functions

The \( z \)-transforms of the discrete-time Legendre networks are written as

\[ B_n(z) = z^d \left( \frac{1-z^{-2}}{z-\xi_n} \right) \prod_{k=0}^{d-1} \frac{1-z^{-2}}{z-\xi_k}, \quad d = 0 \text{ or } 1, \]

\[ \xi_k = \frac{2-a(2k+1)}{2+a(2k+1)}. \quad (6) \]

\[ p_i(k) \] denotes the inverse \( z \)-transform of \( B_i(z, \xi_1, \ldots, \xi_i) \). The set of discrete-time Legendre functions is expressed in a vector form as

\[ P(k) = [p_1(k) \ p_2(k) \ldots \ p_N(k)]^T. \quad (7) \]

Set of discrete Legendre functions satisfies the following difference equation

\[ P(k+1) = \Omega P(k), \quad (7a) \]

similar to the Laguerre case.

By analysing Legendre network given in Fig. 2, or by analyzing construction given by Eq. (6) one obtains

\[ \Omega = \begin{bmatrix}
    \xi_1 & 0 & 0 & \ldots & 0 \\
    0 & \xi_2 & 0 & \ldots & 0 \\
    0 & 0 & \xi_3 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & \ldots & \xi_N
\end{bmatrix}, \quad (7b) \]

\[ P(0) = \begin{bmatrix}
    \sqrt{\beta_1} \\
    -\xi_1 \sqrt{\beta_2} \\
    \xi_1 \xi_2 \sqrt{\beta_3} \\
    -\xi_1 \xi_2 \xi_3 \sqrt{\beta_4} \\
    \xi_1 \xi_2 \xi_3 \xi_4 \sqrt{\beta_5} \ldots
\end{bmatrix}, \quad (7c) \]

\( \beta_i \) being \( \beta_i = 1-z_i^2 \).

At time \( k_i \), the control trajectory \( \Delta u(k_i), \Delta u(k_i+1), \Delta u(k_i+2), \ldots, \Delta u(k_i+k) \), is regarded as the impulse response of a stable dynamic system. A set of
Legendre functions, $p_1(k), p_2(k), \ldots, p_N(k)$ is used to capture the dynamic response with a set of Legendre coefficients that are to be determined from the design process. Based on this,

$$\Delta u(k_i + k) = \sum_{j=1}^{N} c_j(k_i) p_j(k),$$  \hspace{1cm} (8)

where $k_i$ is the initial time of the moving horizon window and $k$ is the future sampling instant, $N$ is the number of terms used in the expansion, $c_j, j = 1, 2, \ldots, N,$ are the coefficients being functions of the initial time of the moving horizon window, $k_i$.

Eq. (8) can also be expressed in a vector form:

$$\Delta u(k_i + k) = P(k)^T \eta,$$  \hspace{1cm} (9)

where $\eta$ has $N$ Legendre coefficients:

$$\eta = [c_1, c_2, \ldots, c_N]^T.$$  \hspace{1cm} (10)

So, the coefficient vector $\eta$ is optimized and computed in the design.

![Figure 2 Legendre network](image)

As thoroughly explained in [1], an alternative formulation of the cost function is used and formulated, for both, Laguerre functions presented in Section 2, and Legendre functions presented in this section. The task is finding the coefficient vector $\eta$ to minimize the cost function:

$$J = \sum_{m=1}^{N_P} \xi_m(k_i + m|k_i|) Q_s(k_i + m|k_i|) + \eta^T R_L \eta,$$  \hspace{1cm} (11)

$Q \geq 0$ and $R_L > 0$ being weighting matrices.

Having optimal parameter vector $\eta$, the receding horizon control law is realized as

$$\Delta u(k_i) = P(0)^T \eta.$$  \hspace{1cm} (11a)

Design parameter $N$ is the number of terms used in capturing the control signal, in both, Laguerre and Legendre case.

Parameter $\xi_m$ defined in Eq. (6) is like parameter $a$ in the Laguerre case, except it has progression.

Stability of the closed loop system can be guaranteed under certain circumstances. An approach that uses terminal constraints on the state variables, which forces the terminal state variables to be zero is thoroughly explained in [1] for Laguerre case, and can be used in the Legendre case.

4 pH neutralization process

As presented in [3] the pH neutralization process is explained as follows. An acid stream (flow $Q_1$), a buffer stream (flow $Q_2$) and a base stream (flow $Q_3$) are mixed in a tank $T_1$. The acid and base streams are equipped with flow control valves. Before mixing, the acid stream passes through a tank $T_2$, which introduces additional flow dynamics. The liquid levels $h_1$ and $h_2$ and effluent pH are measured variables. The single-input single-output control scheme considers the effluent pH to be the controlled variable $y$ and the base stream flow $Q_3$ to be the manipulated variable $u$.

A more detailed description of this process can be found in [4].

Fig. 3 shows block diagram of the process described above.

Model of the process described with cascade of a discrete linear block and static nonlinearity $f$ is estimated and presented in [3]. Both Laguerre and Legendre based predictive controllers were obtained based on the discrete linear block and gain equaling one. So, in different working points, there is plant-to-model mismatch present as controllers were obtained for gain equaling one, and true gain is anywhere between approximately 0.2 and 2 in different working points, as can be seen in [3].
4 Simulation

Fig. 4 presents block scheme used for simulation of pH neutralization process.

Fig. 5 shows reference trajectory. Figs. 6 and 7 show output of the pH neutralization process obtained using MPC based on Laguerre and Legendre orthonormal functions.
As can be seen from results presented, proposed use of Laguerre and Legendre functions in model predictive control in the case of pH neutralization process that has static non-linearity, is justified. As working point moves away from the one used for obtaining the MPC, output of the process becomes slower, in both cases, Laguerre and Legendre MPCs. In Fig. 6 one can see that there is a slight difference in outputs, Legendre based MPC produces output with slightly smaller overshoot than the Laguerre one. Both MPCs were obtained for equal number of functions \( N = 5 \), and equal prediction horizon \( N_p = 15 \). In Laguerre case, \( a = 0.998 \); in Legendre case middle \( \xi \) is \( \xi_{sr} = 0.989 \). Also, disturbance is present with amplitude of 0.1, whose pulse width is 30 % of its period, so after each reference step, follows step of disturbance as can be seen in Figs. 6 and 7. Disturbance rejection can be noticed.

6 References


Authors’ addresses

Dubravić Amila, Teaching Assistant
Faculty of Electrical Engineering, University of Tuzla Franjevačka 2, 75000 Tuzla, Bosnia and Herzegovina
Tel.: +387 35 259 624
E-mail: amila.dubravic@untz.ba

Šehić Zenan, Associate Professor
Faculty of Electrical Engineering, University of Tuzla Franjevačka 2, 75000 Tuzla, Bosnia and Herzegovina
Tel.: +387 35 259 627
E-mail: zenan.sehic@untz.ba

Burgić Mustafa, Full Professor
Faculty of Technology, University of Tuzla Univerzitetska 8, 75000 Tuzla, Bosnia and Herzegovina
Tel.: +387 35 320 754,
E-mail: mustafa.burgic@untz.ba
Location
Situated on the eastern coast of Spain, Valencia was founded in 1388 as a Roman colony. The city, which is the third largest in Spain, has the biggest port on the Mediterranean Sea. A large historic city centre means Valencia is a popular tourist destination with many ancient monuments, museums and sites of interest. Valencia is famous for "Paella" - four days and nights of citywide celebrations held each year during March in commemoration of Saint Joseph. Visitors are also drawn to the region for its food with Paella having originated from the city.

Conference Venue
The TRYP Valencia Oceanic Hotel is situated near the centre of Valencia. Only a short distance from the City of Arts and Sciences, the harbour and downtown, the hotel is close to all that the city has to offer. With 300 rooms all with high speed internet access, a 24 hour fitness centre, pool, sauna, bar and restaurant, the hotel has all the amenities guests may require.

Conference Secretariat
Irene Moreno Milan
irenom@wessex.ac.uk
Wessex Institute
Ascot Lodge, Arden
Southampton SO19 7DA, UK
Tel: +44 (0) 23 8029 3799
Fax: +44 (0) 23 8029 2893
For more information visit wessex.ac.uk/materials2015

Submission Information
Papers are invited on the topics outlined and others falling within the scope of the meeting. Abstracts of no more than 300 words should be submitted as soon as possible.

Abstracts should clearly state the purpose, results and conclusions of the work to be described in the final paper. Final acceptance will be based on the full-length paper, which accepted for publication must be presented at the conference.

The language of the conference will be English.

Online submission:
wessex.ac.uk/materials2015

Email submission:
irenom@wessex.ac.uk

Submit your abstract with "Materials Characterisation 2015" in the subject line. Please include your name, full address and conference topic.

The aim of the conference is to facilitate interactions within the research community and discuss the latest developments in this rapidly advancing field. The meeting responds to the demand for high quality products for both industry and consumers, which has led to rapid developments in materials science and engineering. This requires the characterisation of the physical and chemical properties of the materials. Consideration of different experimental techniques as well as computer simulation methods is essential to achieve a proper analysis. A very wide range of materials, starting with metals through polymers and semiconductors to composites, necessitate a wide spectrum of material characterisation experimental techniques and numerical methods.

All papers to be published in the conference volume as well as those from previous meetings, are permanently archived in the Wessex Institute Library (http://library.wessex.ac.uk) where they are available to the international community.

Conference Chairman
C A Breslin
Wessex Institute of Technology, UK
International Scientific Advisory Committee
I Camara Instituto Nacional de Técnica, Brasil
P De Wilde Free University of Brussels, Belgium
S Hernandez University of A Coruña, Spain
M Horgole Letistry Research Center, France
H Huh Korea Advanced Institute, South Korea
A Klima
Glasgow Caledonian University, UK
A Mahur
University of Northumbria, UK
D Northwood University of Windsor, Canada
E Soltan Kardan Institute of Technology, Germany
S Syngellas Wessex Institute, UK
A Treda University of Coimbra, Portugal

Materials Characterisation 2015
22-24 April 2015
Valencia, Spain
Wessex Institute
Organised by
Sponsored by
WIT Transactions on Engineering Sciences
International Journal of Computational Methods and Experimental Measurements

Benefits of Attending
Conference Proceedings Papers presented at Materials Characterisation 2015 will be published by WIT Press in Volume 90 of WIT Transactions on Engineering Sciences (ISSN: 1743-4471 Digital ISSN: 1743-4503). WIT Press ensures maximum worldwide dissemination of your research through its own offices in Europe and the USA, and via its extensive international distribution network.

Delegates will have the choice of receiving the conference book as either hard cover or digital format on a USB flash drive. The USB flash drive will, in addition, contain papers from previous conferences in this series.

Indexing and Archiving Papers presented at Wessex Institute conferences are referenced by CrossRef and regularly appear in notable reviewers, publications and databases, including referencing and abstracting services such as SCOPUS, Compendex, Thomson Reuters Web of Knowledge and ProQuest. All conference books are archived in the British Library and American Library of Congress.

Digital Archive All conference papers are archived online in the WIT Library (http://library.wessex.ac.uk) where they are easily and permanently available to the international scientific community.

Journal Papers After the conference, presenters at Materials Characterisation 2015 will be invited to submit an enhanced version of their research for possible publication in the International Journal of Computational Methods and Experimental Measurements published by the Wessex Institute.

Review: Abstracts and papers are reviewed by members of the International Scientific Advisory Committee and other experts.

Open Access Open Access allows for the full paper to be downloaded from the WIT Library archives, offering maximum dissemination. Authors who choose this option will also receive complimentary access for one year to the entire WIT Archives.

Network: Participants can present their research and interact with experts from around the world, becoming part of a unique community.

Reduced Fee For PhD Students The Wessex Institute believes in the importance of encouraging PhD students to present and publish innovative research at their conferences. As a result, the institute offers PhD students a much reduced conference fee.

Citizenship When referring papers presented at this conference please ensure that your citations refer to Volume 90 of WIT Transactions on Engineering Sciences as this is the title under which papers appear in the indexing services.