Algebraic Characterization of Simple Transition-Metal Molecules*

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INTRODUCTION

We seek to characterize simple transition-metal (TM) species by using Diophantine algebra and the stability rules familiar in chemistry. The idea is as follows: if analysis of the millions of known species in chemistry shows that the rule of 18 is widely applicable, then why not assume this rule (and one other) and find what species satisfy it (them)? Later on, quantum chemists and experimentalists can be asked »How stable are the forecasted species?« or »In what environments are they stable or, at the very least, observable?«

The method of this paper is similar to that used for main-group (MG) molecules1–3 except, importantly, that in all but one section there is no limitation on the numbers of atoms in the molecules. We study organo-metallic molecules that possess one central TM atom with ionic,

* Dedicated to Professor Haruo Hosoya in happy celebration of his 70th birthday.

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or covalent and dative, bonds to atomic, functional-group, or π-ring sandwich ligands (the method could be extended to molecules with additional transition-metal atoms). We also study gas phase diatomic molecules that have one or two TM atoms (the method could easily be extended to triatomic or larger species).

**ALGEBRAIC FORMULATION**

**Assumptions**

1. Atoms will possess valence-shell electrons of their own plus (or minus) those due to the bonding that takes place with one or more neighboring ligands;

2. The bonds are atom-to-atom covalent, ionic, van der Waals, and/or dative and involve integer numbers of electrons;

3. The bond order may vary from 0–a quadrupole or higher-order bonds) – for covalent bonding – or more for ionic bonding;

4. Atoms with higher electronegativity will not donate an electron pair to form a dative bond with those of lower electronegativity.

**Maximum Oxidation States**

The maximum oxidation states of transition-metal atoms are 3 to 7 for groups 3 to 7 (in period 4) and 3 to 8 for groups 3 to 8 (in periods 5 and 6), though Shchukarev also allows Fe(VIII). (This phenomenon has led to occasions in which these maximum values are called valences). The maximum values decrease thereafter. A simple algebraic expression is consonant with the maximum oxidation states of these five (six) groups of atoms:

\[ C_1 - \sum_{k} q_k = 0 \]  

\[ (1) \]

\( C_1 \) is the group number of the transition-metal atom (3 to 7), \( q_k \) is the charge transferred to the neighboring atoms or sites in polydentate neighbors, and zero indicates that the TM atom ends up with an argon configuration. For the special molecules to which this formula pertains, it can be said to articulate a «rule of zero», though it really says that the oxidized atoms are described by the rule of 8.

**The Rules of 18 and 12**

The formulae are

\[ C_1 + \sum_{i=0}^{P} v_{im} + 2 \sum_{j=0}^{Q} P_{jn} = R_1 \]  

\[ (0 \leq n \leq 1; \ 0 \leq m + n \leq 3) \]  

\[ (2) \]

\[ C_i + v_{im} = R_2 \]  

\[ (if \ m = 0 \ and \ n = 0, \ then \ C_k \rightarrow Ne; \]  

\[ if \ m = 0 \ and \ n = 1, \ then \ C_k \rightarrow \ any \ entity \ that \ has \ bound-pair \ electrons) \]  

\[ (3) \]

The group number \( C_1 \) can now vary from 3 to 12; \( v_{im} \) indicates that there is a covalent bond of order \( m \) between atom 1 and any of \( P \) ligands \( i \); \( P_{jn} \) means that some ligand \( j \) from among \( Q \) has contributed \( n \) (zero or one) dative bonds of two electrons each to atom 1. The sets of ligands \( i \) and \( j \) are not necessarily disjoint. \( C_i \) is the group number of an electronegative simple ligand \( i \), or the group number that ligand \( i \) could have in view of the number of electrons it desires (e.g., 17 for CN). The rule of 18, which states that after bonding the period-4 TM atom shall have a krypton-like closed valence shell). \( R_2 \) is also 18, but for a totally different reason: it expresses the octet rule, which states that when bonded the period-4 TM atom shall have a neon-like closed valence shell; this configuration, due to the numbering of atomic groups now in vogue, is 18. In Refs. 1 to 3, \( R_2 \) was 8 (or, for group-1 and group-2 atoms, 2).

**TABLE I. A few of the many known period-4 rule-of-18 species**

<table>
<thead>
<tr>
<th>Atom</th>
<th>( C_1 ) electrons needed</th>
<th>π-ring sandwiches(a),(b)</th>
<th>Species with :CO ligands(c)</th>
<th>Other species(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc</td>
<td>3</td>
<td>15</td>
<td>Bis(η5-cyclopentadienyl)titanium dichloride</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>4</td>
<td>14</td>
<td>Vanadium,(η5-benzene)(η7-cycloheptadienyl)</td>
<td>[V(CO)6]3–</td>
</tr>
<tr>
<td>V</td>
<td>5</td>
<td>13</td>
<td>Chromium, bis(η5-benzene)</td>
<td>Cr(CO)6 Cr(PF3)6</td>
</tr>
<tr>
<td>Cr</td>
<td>6</td>
<td>12</td>
<td>Manganese,(η5-benzene)(η5,2,4-cyclopentadien-1-yl)</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>7</td>
<td>11</td>
<td>Ferrocene (i.e., C10H10Fe)</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>8</td>
<td>10</td>
<td>Cobalt,(η5-cyclobutadiene-η5-cyclopentadienyl)</td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>9</td>
<td>9</td>
<td></td>
<td>Fe(CO)5 CoH(CO)4</td>
</tr>
<tr>
<td>Ni</td>
<td>10</td>
<td>8</td>
<td></td>
<td>Ni(CO)4 Ni(PF3)4</td>
</tr>
<tr>
<td>Cu</td>
<td>11</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zn</td>
<td>12</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Ref. 8; (b) Ref. 9; (c) Ref. 10.

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The equations are solved by choosing the bonds and solving for the group numbers. In all the solutions given, atoms from the earliest possible period number are shown.

Example 1: If there are to be nine covalent single bonds and no dative bonds then \( n_{2,1} \) to \( n_{10,1} \) are 1, \( C_1 = 9 \) (i.e. Co), and \( C_2 \) to \( C_{10} \) are all 17. Having nine atoms surrounding the Co atom, providing these single bonds is not possible; however, they can be members of two \( \pi \)-rings, so the molecule can be \( \eta^{4}\)-cyclobutadiene-\( \eta^{5}\)-cyclopentadienyl cobalt, as listed in Table I and shown in Figure 1. The corresponding manganese \( \pi \)-ring sandwich cobalt and vanadium molecules are shown in Figures 2 and 3.

Example 2: If there are to be six covalent double bonds and no dative bonds then \( n_{2,2} \) to \( n_{7,2} \) are 2, \( C_1 = 6 \) (Cr), and \( C_2 \) to \( C_7 \) are all 6. Thus the molecule can be \( \text{Cr(PF}_3\text{)}_6 \), assuming that this radical provides double covalent bonds.

Example 3: If there are to be no covalent bonds but six dative bonds then \( p_{2,1} \) to \( p_{7,1} \) are 1, \( C_1 = 6 \) (Cr again), and \( C_2 \) to \( C_7 \) are all 18. The condition attached to Eq. (3) allows these ligands to be \( :\text{CO} \), so the molecule can be \( \text{Cr(CO)}_6 \) with chromium having a formal charge of zero, as listed in Table I.

Example 4: If there are to be one single bond and separately four dative bonds then \( \alpha_{1,1} \) is 1, \( p_{3,1} \) to \( p_{6,1} \) are 1, \( C_1 = 9 \) (Co again), \( C_2 = 17 \) (e.g. F), and \( C_3 \) to \( C_6 \) are 18 (e.g. :CO again). Thus the molecule can be \( \text{CoH(CO)}_4 \) as listed in Table I. A further example of prediction from the equations is in a later section.

It is not being claimed that all transition-metal atoms will have 18 or 12 valence-shell electrons when present in molecules, but that note should be taken of the many that do.

Isolated zinc, cadmium, and mercury atoms have closed s and d subshells. They behave somewhat like rare gas atoms (the more so in the order listed), as manifested in their dissociation energies, vibration frequencies, and internuclear separations. This behavior suggests that a »rule of 12« might be worth investigating in molecules as well.12 The general formulae for these molecules are the same as those above except that in Eq. (1) \( R_1 = 12 \); the method of solution is similar to that described above, including the combinatorial development that can be built up starting with such formulae as \( \text{CrX}_6 \). Table II shows a very few of the known molecules that can obey this »rule«. One chromium and one iron molecule are shown in Figures 4 and 5.

<table>
<thead>
<tr>
<th>Atom</th>
<th>electrons needed</th>
<th>Group 16 ligands</th>
<th>Group 17 ligands</th>
<th>Other species(^{(a),(b)})</th>
<th>Other species(^{(b)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc</td>
<td>9</td>
<td></td>
<td></td>
<td>( \text{VC}_{12}\text{H}_6\text{Cl}_3\text{N}_5\text{O}_4 )</td>
<td>( \text{VC}<em>{6}\text{H}</em>{12}\text{NO}_4 )</td>
</tr>
<tr>
<td>Ti</td>
<td>8</td>
<td></td>
<td></td>
<td>( \text{CrX}_6)</td>
<td>( \text{CrC}_6\text{H}_5\text{N}_5 )</td>
</tr>
<tr>
<td>V</td>
<td>7</td>
<td>( \text{CrO}_3)(^{(c)})</td>
<td>( \text{CrX}_6)(^{(c)})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>6</td>
<td>( \text{CrO}_3)(^{(c)})</td>
<td>( \text{CrX}_6)(^{(c)})</td>
<td>( \text{VC}_{12}\text{H}_6\text{Cl}_3\text{N}_5\text{O}_4 )</td>
<td>( \text{VC}<em>{6}\text{H}</em>{12}\text{NO}_4 )</td>
</tr>
<tr>
<td>Mn</td>
<td>5</td>
<td></td>
<td></td>
<td>( \text{Cr(CO)}_6 )</td>
<td>( \text{CrC}_6\text{H}_5\text{N}_5 )</td>
</tr>
<tr>
<td>Fe</td>
<td>4</td>
<td></td>
<td></td>
<td>( \text{MnC}_6\text{H}_5\text{N}_5\text{O}_2\text{P}_2 )</td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>3</td>
<td>( \text{NiO})(^{(e)})</td>
<td>( \text{CoX}_3)(^{(c),(d)})</td>
<td>( \text{Fe(CO)}_2(\text{NO})_2 )</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>2</td>
<td>( \text{NiO})(^{(e)})</td>
<td>( \text{NiX}_2)(^{(c),(d)})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>1</td>
<td></td>
<td></td>
<td>( \text{CuX})(^{13} )</td>
<td></td>
</tr>
</tbody>
</table>

\(^{(a)}\) Carbonyls in these molecules attach with a single covalent bond; \(^{(b)}\) Ref. 11; \(^{(c)}\) Ref. 10; \(^{(d)}\) X, any halide, can be replaced by OH, NO\(_3\), or many other functional groups; \(^{(e)}\) Ref. 13.
Covalent bonding is consistent with its conclusion that Cu(I) much prefers and 4 metals usually bond ionically, and the CuX entry consistent with the conclusion of Ref. 14 that group 3 in data bases.13 The Lewis dot rules are less pervasively interest in gas-phase chemistry, and scores of them are listed The molecules to be discussed here are primarily of in-

Atom and Row-2 MG Atom

Gas Phase Diatomic Molecules with a Row-4 TM Atom and Row-2 MG Atom

The molecules to be discussed here are primarily of interest in gas-phase chemistry, and scores of them are listed in data bases.13 The Lewis dot rules are less pervasively relevant for these species; nonetheless, we shall proceed to see where the method leads. The equations for covalent and dative bonding between one TM and one MG atom are special cases of Eq. (2) and (3):

$$C_1 + v + 2p = R_1$$

$$0 \leq p \leq 1; < +p \leq 3$$

(4)

$$C_2 + v = R_2$$

(if \(v = 0\) and \(p = 0\), then \(C_2 \rightarrow Ne\);

if \(v = 0\) and \(p = 1\), then \(C_2 \rightarrow\) any entity that has bound-pair electrons)

(5)

and the equations for ionic and dative bonding are

$$C_1 - q + 2p = R_1$$

$$0 \leq p \leq 1; 1 \leq q \leq 3\text{ if period number = 4}$$

(6)

$$C_2 + q = R_2$$

$$1 \leq q \leq 3\text{ if period number = 4}$$

(7)

The solutions of Eqs. (4) and (5) are not as trivial as it may appear, because row 4 contains not only TM, but also MG, atoms, and so many solutions are void. The italicized values of \(R\) show which atoms are specified as from period 4, whether they be s, p, or d. On the other hand, a simplifying factor is that the TM atom in a covalently-bonded diatomic molecule cannot accumulate the number of electrons needed for a Kr configuration, and so it can only be described by the rule of 12. The solutions follow.

- \((R_1, R_2) = (12,12)\): there can be a van der Waals bond (Zn\(_2\)) or covalent bonds of order 1 (Cu\(_2\)), 2 (Ni\(_2\)), and 3 (Co\(_2\)). No dative bonding will take place. This series is analogous to the series Ne\(_2\), F\(_2\), O\(_2\), N\(_2\) for molecules with two rule of 8 (18) atoms.

- \((R_1, R_2) = (12,2)\): a van der Waals bond gives ZnBe, and no dative bond will be given. A single covalent gives CuLi; a dative bond given by Cu would cause atom 2 to be below row 2. The abnormal notations are in the same order as the solutions to Eqs. (4) and (5).

- \((R_1, R_2) = (2,12)\): the reverse of the preceding case; the same molecules (but in standard notation). The reversal invites for Li in LiCu to form a dative bond, but Li has no bound pair.

- \((R_1, R_2) = (12,0)\) and \((0,12)\): the only solution is van der Waals bonded ZnHe.

- \((R_1, R_2) = (12,18)\): a van der Waals bond yields ZnNe. Covalent bonds of orders 1, 2, and 3 yield FCu, ONi, and NCo; in the first two instances, dative bonds are forbidden due to electronegativity considerations; in the third the dative bond is disallowed because the bond order would exceed 3. ZnNe, CuF, NiO, and CoN are the analogs of Zn\(_2\), Cu\(_2\), Ni\(_2\), and Co\(_2\).

- \((R_1, R_2) = (18,12)\): the same four molecules appear in reverse notation, however, now Cu and Ni can donate a dative bond to make CuN and NiC. Co cannot make CoB because a quadruple bond would be involved.

- \((R_1, R_2) = (18,2), (2,18), (18,0)\) and \((0,18)\): there is no molecule with a TM atom.

The solutions for ionic bonding, Eqs. (6) and (7), are even fewer in number. TM atom in a diatomic molecule cannot give away enough electrons to achieve an argon configuration, so again it can only be described by the rule of 12. The solutions follow.

- \((R_1, R_2) = (0,12)\): Li loses one electron to resemble He and Cu gains one to resemble Zn (LiCu); Be loses two electrons to resemble He and Ni gains two to resemble Zn (BeNi). Neither Li\(^+\) nor Be\(^+\) are able to donate a dative bond. CuLi, from \((R_1, R_2) = (12,2)\), and BeNi are analogs of LiF and BeO for molecules with two rule of 8 (18) atoms.

- \((R_1, R_2) = (0,18)\): Sc loses three electron to resemble Ar, and N gains three to resemble Ne (ArN).

There is a clear equivalence between the two pairs of equations if \(p = 0\) in equation (4) and if \(v = q = p\). Thus, in Eqs. (4) and (5) if \((R_1, R_2) = (12,18)\) and \(v = 1\), then \((C_1, C_2) = (11,17)\) and the molecule is CuF. Cor-
respondingly, in Eqs. (6) and (7) \( q = 1 \) and \( p = 1 \), then again \( C_1, C_2 = (11, 17) \) and again the molecule is CuF (as demonstrated in Figure 6). If \( v = q = p = 2 \) then the molecule is NiO; if \( v = q = p = 3 \) then it is CoN; neither molecule is in Figure 6 due to the imposed limitations. Eqs. (6) and (7) are not redundant; they allow for BeNi and ScN and for the molecules discussed in Section Maximum oxidation states.

Figure 6 shows all of the molecules in the entire solution set except for \( (R_1, R_2) = (12, 12) \) and \( (0,18) \). Several interesting features are seen in this figure – for instance the absence of neutral radicals; the series NeZn, FCu, ONi, CoN that exactly parallels the series NeZn, F2, O2, N2 for MG molecules; the absence of FeC, which would like \( C_2 \) require a quadruple bond; and van der Waals species, NeZn and ZnHe. The comparable plot for MG molecules is given in Ref. 1.

The situation differs somewhat for diatomic molecules where both atoms are transition metals. The molecules Zn2, Cu2, Ni3, and Co2 go down the diagonal from the upper right to lower left.

**PREDICTIONS**

Many more species that satisfy the rule of 18 can be forecasted from the concepts of Section The rules of 18 and 12. To illustrate, consider nickel, to which eight covalent bonds are allowed. In combinatorial fashion one can begin with six single bonds, one double bond and four single bonds, two double bonds and two single bonds, and so on, giving for example the series CrF6, CrOF4, CrO2F2, CrO3, CrNF3, CrNOF, and CrN2. Higher period-number TM atoms should not lightly be substituted for period-4 TM atoms without further investigation.

**COMMENT**

Froeyen and Herdewijn\(^{15}\) have also used algebraic methods, but to opposite effect: the atoms are given and the bond orders are found. Their paper shows that additional non-trivial restrictions, such as bond angles and adjacent charges, could be included in further development of our model. On the other hand, Vukičević and Graovac\(^{16,17}\) use set theory to determine whether a molecular graph can exist when the covalent bonds are given: the goal is similar but the mathematics differs.

**Acknowledgements.** – That which is correct in this work is due in large part to the patiently given advice of Drs. Alexandru Balaban, Danail Bonchev, D. J. Klein and Dragutin Svr\v{t}an. That which is less than fully correct is solely the responsibility of the authors.
REFERENCE

11. Landolt-Börstein Search (http://lb.chemie.uni-hamburg.de/s_text.php?PHPSESSID=7a794f9264ed1822bc474f41f206e490)

SAŽETAK

Algebarska karakterizacija molekula prijelaznih elemenata

Ray Hefferlin i Ken Luk

U radu su ranija istraživanja molekula s atomima glavne skupine proširena na one prijelaznih elemenata, pri čemu su najveća oksidacijska stanja atoma skupina 3 do 7 opisana istom algebarskom jednadžbom koja vrijedi i za ionski vezane atome glavnih skupina. Kovalentno vezane molekule prijelaznih elemenata opisane su jednadžbama koje vrijede i za one glavne skupine osim što se oktetno pravilo zamjenjuje pravilom 18 i ovdje predloženim pravilom 12, što je ilustrirano primjerima organo-metalnih spojeva s jednostavnim ligandima i na atomima prijelaznih elemenata u sendvič-molekulama. Rješenja iste jednadžbe za posebne slučajevne vrijede i za plinski fazu dvoatomskih molekula važnih u atmosferi zvijezda i planeta te u uređajima za spaljivanje.