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On the Causal Relationship between Public Debt and GDP Growth Rates in Panel Data Models
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Abstract

The influential and controversial paper by Reinhart and Rogoff (2010) triggered a debate on the effects of public debt on economic growth. Subsequent papers provide more convincing results. However, one of the key assumptions implied in these studies is that lower economic growth is spurred by high debt. If the reverse causality holds, the usual estimation of the model can yield biased estimators because of a feedback effect. We formally examine the causal relationship between public debt and economic growth in the panel VAR model using Granger causality test. Results show that the inter-temporal causal relationship is bi-directional. These findings provide a warning regarding the estimation results in many previous studies that might have ignored the role of the feedback effect.

Key words
Public debt; Feedback Effect, Reverse Causality, Panel VAR models

JEL classification
O13; C22
1. Introduction

The influential paper by Reinhart and Rogoff (2010) has triggered a debate about the effects of public debt on economic growth. The main argument of their paper is that there is an adverse effect on economic growth when the public debt is greater than 90 percent of GDP. Subsequent studies attempt to provide robustness checks for their claim. For example, Cecchetti et al. (2011) obtained the result that there is a threshold effect of public debt over 96 percent of GDP. Baum et al. (2013) obtained a similar result around a threshold level of 95 percent. Checherita-Westphal and Rother (2012) also note public debt can affect economic growth rates in a nonlinear fashion that becomes relevant only after a certain threshold has been reached. Most of these studies are aimed at investigating the effects of public debt on GDP growth rates.

One of the key assumptions implied in these studies is that lower economic growth is spurred by high debt. Increase in a budget spending leads to a crowding out effect, or even debt overhang. But in theory, causality can go both ways. A classical textbook example is that in recessions, debt raises because of automatic stabilizers. Countercyclical fiscal policy decreases taxes and increases spending in order to increase GDP growth. Besides, debt is usually measured as a debt to GDP ratio. In that case, when GDP falls, there is a mechanical increase in debt ratio. One should be aware that both debt and growth could be influenced by a third factor. For example, wars or economic crises both lower GDP growth, and increase debt. This is an important endogeneity issue.

The usual regression model is based on the specification where the economic growth rate is regressed on public debt and other independent variables. Consider a usual model specification,

\[ GROWTH_{i,t-(t-\eta)} = \alpha_i + \beta DEBT_{i,t-k} + \gamma X_{i,t-k} + \tau_t + \epsilon_{it} \]  

(1)

where \( GROWTH_{i,t-(t-\eta)} \) is per-capita GDP growth rate of country \( i \) over the time period from \( t-\eta \) to \( t \), \( \alpha_i \) is the unobserved heterogeneity of country \( i \), \( DEBT_{i,t-k} \) is the ratio of public debt to GDP, \( \tau_t \) are time fixed effects and \( X_{i,t-k} \) includes a set of control variables. The focus might be to estimate the inter-temporal effect from public debt to economic growth by choosing a proper value of \( k \) and \( \eta \). But the estimation procedures do not take into account a possible reverse inter-temporal relationship where high debt is spurred by lower economic growth.

Dube (2013) notes this point and finds evidence of reverse causality where the debt ratio is more clearly associated with the 5-year past average growth rate, rather than the 5-year forward average growth rates (also see Pescatori et al. (2014)). If this specification is valid, a problem can arise when estimating the model in equation (1) using any of the usual panel estimation methods based on fixed effects (FE), random effects (RE) and first difference (FD) approaches, since \( E[DEBT_{i,t-k}\epsilon_{i,t-s}] \neq 0 \) with \( s > k \) and \( k > 0 \). In this case, past error terms are correlated with regressors, which is a violation of the required assumptions in a regression model. If so, the usual estimation of the model in (1) can be biased. This is known as a feedback effect.

To mitigate the problems of feedback effects, Cecchetti et al. (2012) estimate equation (1) using 5-year overlapping growth rates, and Kumar and Woo (2010) use 5-year non-
overlapping growth rates. Other papers try to cure for inter-temporal endogeneity by using instrumental variables techniques or Generalized Method of Moments (GMM) procedures; see Panizza and Presbitero (2012).

However, the dynamic inter-temporal relationship between public debt and economic growth has not been fully examined. In particular, no previous paper offers a formal examination for an inter-temporal causal relationship between public debt and economic growth. As such, this paper tries to fill a gap in the literature by using an innovative Granger causality test in a panel VAR (P-VAR) framework. Our key findings suggest that the causality between GDP growth and debt runs both ways, both when annual and 5-years frequency is used. It implies that feedback effects are significant in a growth and debt regressions. These results generalize the findings of Dube (2013).

2. Testing for Causality in Panel Models

A conventional view of public debt is that it can stimulate aggregate demand and output in the short-run but crowds out capital and reduces output in the long-run, implying the debt overhang hypothesis; see Kumar and Woo (2010). From this perspective, consider a panel model to examine causality from public debt \( x_{it} \) to economic growth \( y_{it} \) using the following equation

\[
y_{it} = \alpha_i + \tau_t + \alpha_1 y_{it-1} + \ldots + \alpha_k y_{it-k} + \delta_1 x_{it-1} + \ldots + \delta_k x_{it-k} + \epsilon_{it}
\]  

(2)

where \( y_{it} \) denotes economic growth, \( x_{it} \) denotes public debt, \( \alpha_i \) is the unobserved heterogeneity of country \( i \), and \( \tau_t \) reflects time fixed effects. Granger non-causality hypothesis from \( x_{it} \) to \( y_{it} \) implies: \( H_0 : b_1 = \ldots = b_k = 0 \). On the other hand, the effects economic growth rates on debt can be examined in the panel model

\[
x_{it} = \delta_i + \tau_t + \delta_1 y_{it-1} + \ldots + \delta_k y_{it-k} + \epsilon_{it}
\]  

(3)

to examine causality from economic growth to public debt. Here, the Granger non-causality hypothesis from \( y_{it} \) to \( x_{it} \) implies: \( H_0 : d_1 = \ldots = d_k = 0 \). We consider a reduced form model.

Testing for causality has been rarely done in the literature. Primarily because, unlike in the time series analysis, simple OLS estimators cannot be used. Indeed, the estimation of equations (2) and (3) in the panel model becomes complicated. Above all, there is an issue of controlling for the unobserved heterogeneity \( \alpha_i \) and \( \delta_i \). One may employ dummy variables following the fixed effects estimation strategy, but the FE estimator becomes biased in the presence of the lagged dependent variable on the right hand side, unless \( T \) is big, because \( E[x_{it-m} \epsilon_{it-s}] \neq 0 \) with \( s > m \) for \( m = 1, \ldots, k \). Nevertheless, estimating the above model using the FE estimator if the time period of the data (\( T \)) is large enough is informative. However, a serious issue of estimating equations (2) and (3) is that it involves a feedback effect given the construction of this system of equations.

This paper posits that Arellano and Bond’s (1991) GMM estimation can be used for the reduced form P-VAR models in equations (2) and (3), when feedback effects are present in each equation. Because the P-VAR model is a special case of dynamic panel models, the above equations can be estimated with GMM using first differences, which exploits the orthogonality conditions between the errors and lagged values of the dependent variables.
Upon the optimal selection of lags, we can use the lagged dependent variable in level, $y_{i,t−m}$, $m > k$, as instruments. It is recommended to use a parsimonious number of lags for instruments due to a concern for weak instruments.

We invoke the so-called sequential moment restriction assumption for equation (2)

$$E(e_{i,t} | y_{i,t−k−1}, \ldots, y_{i,1}, x_{i,t−k−1}, \ldots, x_{i,1}, \alpha_t) = 0, \ t = 1, 2, \ldots, T,$$

(4)

which implies that the instruments are sequentially exogenous conditional on the unobserved effects and lagged values of the dependent variable. This implies dynamic completeness conditional on $\alpha_t$ and a proper dynamic specification. A similar assumption holds for equation (3). Notably, the above condition allows for possible feedback effects, which we find in our analysis of equations (2) and (3). To determine the optimal lag, we employ the standard topdown method starting with a max lag of $m=5$ for lagged $x_{i,m}$ and $y_{t,m}$ using the 5% significance level for both variables using a F-test.

3. Data and Panel Causality Test Results

For the GDP data, we use the long-run sample (1880-2009) data from the Maddison (2010) database, as it covers more countries than others, and GDP data (1960-2009) from the World Bank (2014), which uses local currency in constant prices. For the data for public debt, we use the public debt to GDP ratio obtained from Abbas et al. (2010). We consider a few different data sets for the GDP growth rates. First, we use annual growth rates for the long-span data from 1880 to 2000 (Model A). Second, we use annual growth rates for the short-run data which is focused on the time period from 1970 to 2009 (Model B). Third, we employ the 5-year frequency data which takes 5-year non-overlapping averages over the period from 1960 to 2000 (Model C).

To estimate the panel VAR models, we use the GMM estimators of Arrelano and Bond (1991). As a robustness check, we have employed the usual FE estimators, which would be valid only when $T$ is big. We use a maximum 5 lags for the GMM estimation in all cases rather than using all possible orthogonality conditions. In all cases, we include time fixed effects, which capture cross-correlations and the effects of business cycles.

The panel causality test results are presented in Table 1. Lag length and it’s $p$ – value and the $p$–value of the null of no-causality are provided. The causality results for economic growth to public debt are given on the left side of the table, while the results for causality from public debt to economic growth are on the right side. “*”s denote the optimal lag length. We begin our discussion with Model A, in the top third of Table 1. It is clear that causality runs in both directions, regardless of the lag length. The optimal lag is chosen as 5 for causality from public debt to economic growth, and 3 for the opposite direction of causality. The $p$ – value for the hypothesis of no-causality is close to 0.00 in all cases for both directions of causality. For Model B, using the short-run annual data (1960-2009), results are similar to the Model A sample. Clearly, causality runs in both directions with an optimal lag of 5 in both cases. Model C employs the 5-year frequency data which takes 5-year non-overlapping averages over the period from 1960 to 2000. Given that inter-temporal relationships should be absorbed in five year non-overlapping averages, the selected optimal lag is small. Again, the null of no-causality is rejected for both cases.

Our results are consistent throughout and demonstrate that causality runs in both directions.
They also upgrade those of Dube (2013) who notes the reverse causality issue, while he does not offer a formal causality test result. First, our causality test from economic growth to public debt confirms his result that the debt ratio is associated with the 5-year past average growth rate. But Dube concludes that using the 5-year frequency of non-overlapping data mitigates the inter-temporal relationship. However, we find that causality runs in both directions even when using 5-year frequency data. Therefore, merely using the 5-year frequency non-overlapping data is not sufficient to cure for the feedback effects.

**Robustness Check**

Perhaps, one way to mitigate the problem of the feedback effect is to allow for the initial level of the dependent variable; see Kumar and Woo (2010). The motivation of adding the initial level of economic growth rate or public debt is related to the conditional convergence literature. Adding the initial values does not necessarily resolve fully the inter-temporal endogeneity problem but is worth including in equations (2) and (3). As such, we examine whether the panel causality test results are affected by including the initial values of the endogenous variables. These results are provided in Table 2. They show that the main results are not changed. Again, it is clear that causality runs in both directions in all cases.

We also employed fixed effect estimators, which may be unbiased when the sample period is sufficiently long. We omit these results to save space, but find that the main findings are unchanged. Additionally, one may want to control the effect of other variables by considering the channel through which the public debt can affect economic growth, such as savings and long-term sovereign interest rates. As an additional robustness check we examined these cases by adding exogenous control variables, but again the results for bi-directional causality are unaffected.

**4. Concluding Remarks**

In this paper, we test for a causal relationship between public debt and economic growth in panel data models. We note that the panel VAR model is a special case of dynamic panel data models, and one can employ the GMM estimation of Arrelano and Bond (1991). Our results show that the inter-temporal causal relationship is bi-directional. They imply that the feedback effects are significant in the regression for economic growth as well as in the regression for public debt. These results provide a warning on many of previous studies that might have ignored the role of feedback effects. We also find that using the 5-year frequency of non-overlapping data does not affect the bidirectional causal relationship, and therefore is not sufficient to cure for the feedback effects.
REFERENCES


Table 1. Panel Causality Test Results using GMM Estimation

<table>
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<tr>
<th>Lag</th>
<th>p-value</th>
<th>Causality</th>
<th>Lag</th>
<th>p-value</th>
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Model A. Long-run Annual Data (1880-2009)

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Model B. Short-run Annual Data (1960-2009)

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Model C. 5-Year Frequency Non-overlapping Data (1960-2009)

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Notes: All estimates include time fixed effects; * denotes the optimal lag using the topdown approach. Five lags are of the lagged dependent variables used to employ the moment conditions in the above results using the GMM estimation.
Table 2. Panel Causality Test Results using GMM Estimation: Includes Initial Values of Endogenous Variables

<table>
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<tr>
<th>Lag</th>
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Model A. Long-run Annual Data (1880-2009)

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<th>Causality p-value</th>
<th>Growth → Debt p-value</th>
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Model B. Short-run Annual Data (1960-2009)

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Model C. 5-Year Frequency Non-overlapping Data (1960-2009)

Notes: Same as above