ANSYS ANALYSIS OF STRESS AND STRAIN AFTER CONES PLASTIC DEFORMATION

Krzysztof Rokosz, Tadeusz Hryniewicz, Winfried Malorny, Jan Valiček, Marta Harničárová

The paper deals with study of stress and strain after cones plastic deformation by ANSYS software. There are shown the new results of FEM analysis, what is the base for the further experimental studies.

Keywords: ANSYS analysis; carbon steel C45; cones shape; plastic deformation

1 Introduction

Numerical analysis of non-linear plastic deformation especially by rolling [1 ÷ 6] with the appointment of team working on stress and strain analysis [7 ÷ 12] was studied. It was also possible to find a solution of contact problem of movable elasto/visco-plastic body [8] and thermal numerical analysis in deformation zone [9, 10] as well as mathematical modelling of deformation of the asperity in the burnishing rolling operation [11]. Based on the available literature on the theory of solid mechanics and finite-element method [14, 15, 16] the authors proposed in this paper the cone deformation analysis using a simulation experiment in ANSYS. The simulation analysis presented below will be performed in the future as experiment using a strain-stress machine. Then, during these measurements, the additional analyses such as non-contact methods of surface analysis [17], measurements uncertainties [18], etc., will be performed. Moreover there is planned a comparison of the stress and strain results after cold rolling of flat plates [18] with other ones concerning e.g. triangle and/or trapezoidal cross sections.

Deformation is characterized by the change in mutual displacement of points on the solid surface due to the action of applied loads. There is an elastic deformation which vanishes just after removal of the load, as well as the plastic/firm deformation which remains after load removal.

The deformation state is characterized by linear and angular deformations (Fig. 1). The linear deformation occurs e.g. during extending the rod of a uniform square cross-section due to a normal stress (Fig. 1a). If the deformation is determined from the dependence:

\[ \varepsilon = \frac{l_c - l_0}{l_0} \]  \hspace{1cm} (1)

where: \( l_0 \) and \( l_c \) – initial and transient/temporary rod length so they are determined as relative/apparent deformations.

In practice, the relative deformations are usually given in per cent:

\[ \varepsilon = \frac{l_c - l_0}{l_0} \times 100\% . \]

The deformation \( \varepsilon \) is an apparent characteristic because while it’s determining no continuous change in the rod length is regarded during the deformation. The real deformation \( \varepsilon_{rz} \), called the logarithmic deformation, in the opposition to the relative deformation \( \varepsilon \), reflects the physical meaning of deformation.

It may be confirmed basing on the following example: if during the extension the initial length of rod \( l_0 \) is increased up to \( 2l_0 \), and during compressing \( l_0/2 \), the relative deformations are equal in both cases as to the value and opponent/adverse as to the sign. In such conditions under extension:

\[ \varepsilon = \frac{2l_0 - l_0}{l_0} = 1, \]
\[
e_{zz} = \ln \frac{2l_0}{l_0} = \ln 2 = 0.69, \\
and under compression:
\[
\varepsilon = \frac{l_0 - l_0}{l_0} = -0.5, \\
\varepsilon_{zz} = \ln \frac{l_0}{2} = \ln 1 = 1 - 2 = \ln 2 = -0.69.
\]

Evaluation/estimation of the deformation state in any point of the body relies on determination of the length change between the point and any other neighbouring point and the change of angle between two directions coming out of the same point, occurring as the result of deformation. Relative deformations in the directions \(x_1\) and \(x_2\) are described by the equations:

\[
\begin{align*}
\frac{u_1 + \frac{\partial u_1}{\partial x_1} dx_1}{dx_1} - u_1 & = \frac{\partial u_1}{\partial x_1}, \\
\frac{u_2 + \frac{\partial u_2}{\partial x_1} dx_1}{dx_1} - u_2 & = \frac{\partial u_2}{\partial x_1}.
\end{align*}
\]

Under introducing the third axis \(x_3\) there will be the third relative deformation \(\frac{\partial u_3}{\partial x_3}\). Additionally, there are two angular deformations \(\frac{\partial u_1}{\partial x_2}\) and \(\frac{\partial u_1}{\partial x_3}\), \(\frac{\partial u_2}{\partial x_2}\) and \(\frac{\partial u_2}{\partial x_3}\) as well as \(\frac{\partial u_3}{\partial x_2}\) and \(\frac{\partial u_3}{\partial x_3}\), against each of the three axes. These deformations form a tensor of relative deformations \([14, 15, 16]\):

\[
\frac{\partial u}{\partial x} = \begin{bmatrix}
\frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\
\frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\
\frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}
\end{bmatrix}
\]

Values and types of stresses depend first of all on the processes of deformations and cracking. In the studies of mechanical properties the characteristics of the stress state in each point of the body are necessary. The values of stresses existing in each cross-section coming through the point should be known. Taking into consideration the whole body volume one may separate just a tiny parallelepiped with the edges forming rectangular coordinate system of axes marked as \(x, y, z\) or \(x_1, x_2, x_3\) (Fig. 2).

In a general case, the mutually balanced stresses which may be easily distributed onto one normal and two tangent components, act on its three non-parallel faces. Each component is marked by two indexes. The first index of the stress denotes the plane on which the stress acts \((x, y, z\) or \(1, 2, 3\)), and corresponds with the index of axis perpendicular to this plane, the second one denotes the direction of stress action \((x, y, z\) or \(1, 2, 3\)).

All the occurring components of stress (3 normal and 6 tangent) acting in the faces of elementary parallelepiped fully determine the stress state in any point.

They may be presented in the form of matrix, called the stress tensor:

\[
T_{ij} = \begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix},
\]

where: \(\sigma_{ij}\) for i ≠ j (e.g. \(\sigma_{xx}, \sigma_{yy}\)) – normal stresses, \(\sigma_{ij}\) for i = j (e.g. \(\sigma_{xx}, \sigma_{yy}\)) – tangent stresses.

Conditions of equilibrium of a body require the summary rotary moment against each of the three coordination axes to be zero. From this, the rule of pairs of tangent stresses results: the symmetric components against the general diagonal of tensor must possess simultaneously identical in pairs absolute values with their turns directed to the common edge or from it \((\tau_{11} = \tau_{22} = \tau_{33}, \tau_{12} = \tau_{23} = \tau_{31})\). Inequality \(\tau_{ij} = \tau_{ji}\) means arising of a rotary moment against the axis \(x_3\) (Fig. 3).

This rule has the essential meaning in formulating conditions of equilibrium of a solid in the studies of mechanical properties (stress state) because the processes with the tangent stresses occur in two groups of the mutually perpendicular planes. The studies of stress state rely mainly on the determination of normal stresses because they are used in calculations of design strength.

It results from the theory of elasticity and plasticity that one may draw three mutually perpendicular planes through each of the solid point being under the state of stresses; they are called the general planes in which the normal stresses attain the extremum values \((\sigma_{max}, \sigma_{min})\), with no tangent stresses occurring. Axes of the
coordination system determined by straight/lines of intersections of these planes are the general directions and the stresses acting in the direction of these axes are called the general stresses \((\sigma_1, \sigma_2, \sigma_3)\). They are denoted usually in a simpler manner: \(\sigma_1 \rightarrow \sigma_1, \sigma_2 \rightarrow \sigma_2, \sigma_3 \rightarrow \sigma_3\). In the mechanical studies the general directions are known, e.g. under axial tension, pressing ball into the material surface during the hardness measurement; they may be assumed as the coordination axes. Taking into account the general coordinates, the tensor of stresses is simplified and takes the form:

\[
\sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix},
\]

with \(\sigma_1 > \sigma_2 > \sigma_3\).

The highest tangent stresses occur in the planes situated at the angle 45° to the general directions and their value is equal to the half of difference of the respective main stresses:

\[
r_1 = \frac{\sigma_2 - \sigma_3}{2}, \quad r_2 = \frac{\sigma_1 - \sigma_3}{2}, \quad r_3 = \frac{\sigma_1 - \sigma_2}{2}.
\]

If the main stresses \(\sigma_1, \sigma_2, \sigma_3\) are equal: \(\sigma_1 = \sigma_2 = \sigma_3\), so the tangent stresses \(r_1 = r_2 = r_3 = 0\).

The stress state in a point may be characterized by six independent parameters: three normal stresses and three tangent stresses, or three normal stresses and three directional cosinuses. The stress state, in which three main stresses act, is called a spatial, volumetric, or three-dimensional state. The stress state in which only two main stresses act (the third one is equal to zero) is called a flat or two-dimensional state. In linear or one-dimensional state of stresses the only one, general stress is different from zero.

2 Materials and method

Carbon Steel C45 (JUSČ.1530, DIN C45, BS 080M46, GOST 45 AISI 1043, NF AF65C45, UNI C45) is the most popular material used in the industry. The ANSYS meshed sample of cone is shown in Fig. 4 and the parameters and properties of ANSYS model of the studied C45 steel are demonstrated in Tab. 1.

![Figure 4 Meshed sample of cone deformation in ANSYS [13]](image)

<table>
<thead>
<tr>
<th>Material parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (\rho)/ kg/m³</td>
<td>7865</td>
</tr>
<tr>
<td>Young’s modulus (E)/ GPa</td>
<td>200</td>
</tr>
<tr>
<td>Poisson’s ratio (\nu)/</td>
<td>0.29</td>
</tr>
<tr>
<td>Strengthening parameter (\beta)/</td>
<td>1</td>
</tr>
<tr>
<td>Yield stress (\sigma_0)/ MPa</td>
<td>342</td>
</tr>
<tr>
<td>Material parameter depending the plastic strain hardening (E_{\text{tan}})/ MPa</td>
<td>241</td>
</tr>
<tr>
<td>Material parameter defining the effect of intensity of plastic strain rate (C)/ 1/s</td>
<td>5</td>
</tr>
<tr>
<td>Material constant defining the material’s sensitivity to plastic strain rate (P)/</td>
<td>10</td>
</tr>
<tr>
<td>Failure strain /</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The ANSYS simulations were performed based on the five-level rotatable plan of experiment that is shown in Tab. 2.
3 Results and discussion

The shapes and deformations of cones as well as stress and strain are shown in Figs. 5, 6, 7, and 8. Maximum intensity of stresses for 10 % strain of cone of the top angle 60° equals $\sigma_{\text{max}} = 0.604$ GPa, whereas the minimum stress $\sigma_{\text{min}} = 0.023$ GPa. The substitute stresses were as follows: $\sigma_{\text{max}} = 0.573$ GPa and $\sigma_{\text{min}} = 0.02$ GPa, respectively. The first degree stresses for 10 % strain equal $\sigma_1 = 4.33$ GPa, the second degree stresses equal $\sigma_2 = 4.23$ GPa, whereas the third degree stresses equal $\sigma_3 = 4.01$ GPa. Deformations are equal to $\varepsilon = 1.754$, and the substitute deformations $\varepsilon_z = 1.677$.

Maximum intensity of stresses for 30 % strain of the top angle 60° equals $\sigma_{\text{max}} = 0.587$ GPa, whereas the minimum intensity of stress $\sigma_{\text{min}} = 0.186$ GPa. Maximum substitute stresses for the studied sample were $\sigma_{\text{max}} = 0.558$ GPa, and minimum substitute stresses $\sigma_{\text{min}} = 0.177$ GPa. The first degree stresses for 30 % strain equal $\sigma_1 = 3.42$ GPa, whereas the second degree stresses are equal to $\sigma_2 = 3.35$ GPa, and the third degree stresses equal $\sigma_3 = 2.87$ GPa. Deformations values are $\varepsilon = 1.906$, and the substitute deformations $\varepsilon_z = 1.721$ (Fig. 6).

Maximum intensity of stresses for 50 % strain of sample of the top angle 60° equals $\sigma_{\text{max}} = 0.586$ GPa, whereas the minimum intensity of stress $\sigma_{\text{min}} = 0.200$ GPa. Maximum substitute stresses for the studied sample were $\sigma_{\text{max}} = 0.561$ GPa, and minimum substitute stresses $\sigma_{\text{min}} = 0.177$ GPa. The first degree stresses for 50 % strain equal $\sigma_1 = 3.63$ GPa, whereas the second degree stresses are equal to $\sigma_2 = 3.59$ GPa, and the third degree stresses equal $\sigma_3 = 2.87$.

The result of maximum intensity of stresses for 10 % strain of cone of the top angle 90° equals $\sigma_{\text{max}} = 0.467$ GPa, whereas the minimum intensity of stresses $\sigma_{\text{min}} = 0.018$ GPa. The substitute maximum stresses for the studied sample were $\sigma_{\text{max}} = 0.451$ GPa and the minimum substitute stresses $\sigma_{\text{min}} = 0.015$ GPa. The first degree stresses for 10 % strain equal $\sigma_1 = 2.94$ GPa, whereas the second degree stresses are equal to $\sigma_2 = 2.92$ GPa, and the third degree stresses equal $\sigma_3 = 2.72$ GPa. The intensity of deformations equals $\varepsilon = 1.113$, and the substitute deformations equal $\varepsilon_z = 0.99$.

The result of maximum intensity of stresses for 30 % strain of cone of the top angle 90° equals $\sigma_{\text{max}} = 0.527$ GPa, whereas the minimum intensity of stresses $\sigma_{\text{min}} = 0.112$ GPa. The substitute maximum stresses for the studied sample were $\sigma_{\text{max}} = 0.482$ GPa and the minimum substitute stresses $\sigma_{\text{min}} = 0.098$ GPa. The first degree stresses for 30 % strain equal $\sigma_1 = 1.29$ GPa, whereas the second degree stresses are equal to $\sigma_2 = 1.23$ GPa, and the third degree stresses equal $\sigma_3 = 0.833$ GPa. For the crush attempt the intensity of deformations equals $\varepsilon = 1.128$, and the substitute deformations equal $\varepsilon_z = 1.002$.

Figure 6 Results of stress and deformation intensity of cones of top angle 60°, with the flat punch and 10 %, 30 %, and 50 % strains [13]

Figure 7 Results of stress and deformation intensity of cones of top angle 90°, with the flat punch and 10 %, 30 %, and 50 % strains [13]

The result of maximum intensity of stresses for 50 % strain of cone of the top angle 90° equals $\sigma_{\text{max}} = 0.517$ GPa, whereas the minimum intensity of stresses $\sigma_{\text{min}} = 0.133$ GPa. The substitute maximum stresses for the studied sample were $\sigma_{\text{max}} = 0.459$ GPa and the minimum substitute stresses $\sigma_{\text{min}} = 0.124$ GPa. The first degree stresses for 50 % strain equal $\sigma_1 = 0.928$ GPa,
K. Rokosz i dr.  

ANSYS analiza naprezanja i deformacije nakon plastične deformacije konusa

whereas the second degree stresses are equal to \( \sigma_2 = 0.896 \) GPa, and the third degree stresses equal \( \sigma_3 = 0.478 \) GPa. The intensity of deformations for the studied sample equals \( \varepsilon = 1.128 \), and the substitute deformations equal \( \varepsilon_z = 1.026 \) (Fig. 7).

The result of maximum intensity of stresses for 10 % strain of cone of the top angle 120° equals \( \sigma_{\text{max}} = 0.457 \) GPa, whereas the minimum intensity of stresses \( \sigma_{\text{min}} = 0.017 \) GPa. The substitute maximum stresses for the studied sample were \( \sigma_{\text{z max}} = 0.432 \) GPa and the minimum substitute stresses \( \sigma_{\text{z min}} = 0.016 \) GPa. The first degree stresses for 10 % strain equal \( \sigma_1 = 0.769 \) GPa, whereas the second degree stresses are equal to \( \sigma_2 = 0.612 \) GPa, and the third degree stresses equal \( \sigma_3 = 0.466 \) GPa. The cone crush experiment shows that the intensity of deformations equals \( \varepsilon = 0.716 \), and the substitute deformations equal \( \varepsilon_z = 0.701 \).

The result of maximum intensity of stresses for 30 % strain of cone of the top angle 120° equals \( \sigma_{\text{max}} = 0.493 \) GPa, whereas the minimum intensity of stresses \( \sigma_{\text{min}} = 0.214 \) GPa. The substitute maximum stresses for the studied sample were \( \sigma_{\text{z max}} = 0.442 \) GPa and the minimum substitute stresses \( \sigma_{\text{z min}} = 0.203 \) GPa. The results of first, second, and third degrees of stresses have been shown in Figs. 2, 3, 4. The first degree stresses for 30 % strain equal \( \sigma_1 = 1.50 \) GPa, whereas the second degree stresses are equal to \( \sigma_2 = 1.44 \) GPa, and the third degree stresses equal \( \sigma_3 = 1.10 \) GPa. The cone crush experiment shows that the intensity of deformations equals \( \varepsilon = 0.738 \), and the substitute deformations equal \( \varepsilon_z = 0.704 \).

Table 3 Five-level rotatable plan of experiment based on [12]

<table>
<thead>
<tr>
<th>Exp. no</th>
<th>Experimental result</th>
<th>Model result</th>
<th>Statistical result [%]</th>
<th>Dependent variable and experiment difference</th>
<th>angle</th>
<th>deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0971</td>
<td>0.0875</td>
<td>0.0980</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.0975</td>
<td>0.0853</td>
<td>0.0984</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>0.1025</td>
<td>0.0972</td>
<td>0.0989</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0.0971</td>
<td>0.0853</td>
<td>0.0984</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>0.1025</td>
<td>0.0920</td>
<td>0.0979</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>0.0801</td>
<td>0.0682</td>
<td>0.0804</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>0.0975</td>
<td>0.0853</td>
<td>0.0984</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>0.0971</td>
<td>0.0853</td>
<td>0.0984</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>0.0971</td>
<td>0.0853</td>
<td>0.0984</td>
<td>0.0810 – 0.020</td>
<td>4.7%</td>
<td>10</td>
</tr>
</tbody>
</table>

The complete results ordered by the five-level rotatable statistical plan are given in Tab. 3. Fig. 9 shows the plane graph with mathematical formula which described about 87 % of the points obtained by ANSYS simulation (Tab. 3). Based on this data processing there was noticed that under the cone deformation the top angle has a significant effect on the stress and strain in the deformed surface layer. It was found that at the top angle of 120° and at 50 % deformation the highest stress was obtained.

\[ S = \exp(-1.075 + 0.0003 \cdot \alpha + 0.0041 \cdot d), \quad R^2=0.87 \]

where: \( S \) – stress, \( \alpha \) – angle, \( d \) – deformation

Figure 9 3D plot of influence of the deformation on stress

4 Conclusion

The following conclusions may be formulated after the studies:

- Under the cone deformation, the top angle has a significant effect on the strain in the deformed surface layer
- Under the cone deformation, the top angle has a significant effect on the stress in the deformed surface layer
- The highest stress was obtained at the top angle 120° and at 50 % deformation.

Acknowledgment

Experiments were carried out partly by DAAD-Forschungsaufenthalt für Hochschullehrer und Wissenschaftler (DAAD research stays for University Academics and Scientists). This paper has been
elaborated in the framework of the Moravian-Silesian Region project no. RRC/01569/2012 and the project RMTVC no. CZ.1.05/2.1.00/01.0040. Thanks also go to the IT4Innovations Centre of Excellence project, reg. no. CZ.1.05/1.1.00/02.0070.

5 References


