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The Gestalt Intuition Model: Theory and Practice in Teaching

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Abstract

This article presents the Gestalt Intuition Model, designed as a theoretical and practical model to improve the way of teaching of mathematics in elementary education. Gestalt Intuition Model includes dual treatments and is aimed at the formation of integrating perceptions and gestalt intuition in students. The model has a clear and logical basis. It is based on the instructions of math curricula for elementary education. It is in accordance with all the instructions and knowledge included in the school program. This model is simple and easy to implement in the examples found in math textbooks and could easily be implemented in teaching, following the rules of the teaching process. The model has become trustworthy, reliable, and accepted by experimenting teachers and continues to be applied in their teaching process. The model is easily grasped and comprehended by the students, thus generating new results in teaching. As a model that influences positive critical thinking, Gestalt Intuition Model increases the effectiveness of learning among students, offering teachers a new alternative to achieving effective teaching.

Key words: *dual treatment; elementary education; integrating perception; teaching method.*

Introduction

Experiences and opinions given by scholars of Cognitive Psychology, Philosophy and Multicultural Education (Anderson, Hiebert, Scott, & Wilkinson, 1985; Palincsar & Brown, 1989; Resnick, 1987; Banks, 1988) reach the conclusion that students' learning expands when the teacher uses a variety of strategies involving all the domains of the thinking process. According to Temple, Crawfrord, Saul, Mathews and Makinster (2006) the student learns actively if he/she is curious, asks questions, discovers new things, thinks about a topic or spends time researching the topic, applies prior knowledge for the purpose of problem solving, etc. But, in order to develop critical thinking, these authors add learning how to apply theory in practice from different viewpoints to the above-mentioned list. They also enhance the ability of the students to explore different types and consequences of ideas when reasons are supported by facts. In this context, dual interpretations may be considered as a practice that helps students and teachers develop critical thinking. The theoretical experiences of the above authors and other authors who have worked on critical thinking presented a specific and new view on learning for the Albanian education system at the beginning of the '90s. The inclusion of these experiences in projects about the development of the educational system in Albania started after 1997 reveals new perspectives about the teaching in particular and Albanian education in general.

The aim of the article is to implement strategies that lead to active learning and critical thinking. Based on the real experience of education in Albania, the authors will attempt to present a teaching theory and practice that fits these experiences appropriately. This goal will be achieved through the presentation of the Gestalt Intuition Model, designed and created by authors and the description of the empirical data drawn from testing the model through teaching.

Views and Perspectives

Duality is the form and way of the existence of matter, the law of natural processes. Furthermore, duality is not a new theory in science – it comes from classical times. A concise description of duality can be found in the introduction of the book by Gao (2000), after he gives the meaning of the term duality in daily life as "harmony of two opposite or complementary parts through which they integrate into a whole" (p. xiii); he describes duality in natural sciences as "amazingly beautiful", and mathematics as the science that stands at the foundations of duality. Many authors have based their studies in mathematic duality. They describe the duality in various fields of mathematics. After 2000, the dual properties of mathematics were investigated (Aronov & Znamenskaya, 2006; Yastrebov, 2001), as well as their reflection in this kind of teaching. Therefore, the reflection of dual properties is made present in algebra, solid geometry and trigonometry (Yastrebov, Men'shikova, & Yepifanova, 2006), in mathematics analysis (Kërënxhi, 2009; Kërënxhi & Gjoci, 2010), and dual properties are introduced that link mathematic analysis with mechanics (Gao, 2000). For Artstein-Avidan and Milman (2007) "the notion of duality is one of the central concepts both in geometry and in analysis" (p. 42). These studies refer to the programs of high education and universities. Meanwhile, many dual concepts are studied in the school program, beginning with the first mathematical topics since the first grade of elementary education. Research on the existence of duality in mathematics curricula in elementary education in Albania and on how teachers interpret duality in teaching (Gjoci & Kërënxhi, 2010; 2012; Kërënxhi & Gjoci, 2013) suggests that dual treatments should be included in teaching. The lack of necessary literature on how to use duality effectively leads to a theoretically untreated situation and issue which is closely linked to the question of how teachers should act in order to avoid falling into one-sided interpretations and analyses. The Gestalt Intuition Model answers this question. It helps teachers in teaching and explaining the lesson, and students in acquiring accurate knowledge.

Kuhn (1977) lists five criteria that a theory created from foundations should meet in such ways to be considered 'a good' theory: accuracy, consistency, scope, simplicity and fruitfulness. According to Korthagen (2010, p. 102) "these seem to be five criteria for establishing whether a person has fully developed the theory level". In this paper, Gestalt Intuition Model (hereafter GI-Model) is described as a model that fulfills the following conditions:

– it is accurate and has a clear and logical base,
– it is based on the guidelines of the curricula for the elementary education,
– it is designed in accordance with mathematical textbooks,
– experimenting teachers evaluated it as a model that is easy to implement,
– it is easily understood and applied by the students,
– it generates new results.

GI-Model

The Theoretical Basis of the Model

In order for the GI-Model to have a clear and logical basis, it is initiated and designed taking into consideration the theory of Gray and Tall (1994) based on the process-concept duality; the metaphor of "a new integrating image" described by Schön (1993), and the experience of the Albanian pedagogy. The readers are informed widely on this relevant topic.

In the article "Duality, Ambiguity, and Flexibility in Successful Mathematical Thinking", Gray and Tall (1991) base their empirical study on ambiguity of symbolism for process and concept. In mathematics, a symbol represents both process and the product of that process. Gray and Tall define a "procept" as amalgam of process and concept, in which process and product are represented by the same symbolism. They have developed their theory further in Gray and Tall (1994), and Gray, Pinto, Pitta, and Tall (1999).

In the "Generative metaphor: a perspective on problem-setting in social policy", Schön (1993) presents important ideas connected with vase-faces figure. If some people see the "vase-faces" figure for the first time and are asked what they see in this figure, some of them will answer that they see a vase; others will say that they see profiles of two people. Alongside with this way of looking at the "vase-faces" figure, Schön suggests another way of looking at it; "as two profiles pressing their noses into a vase" (1993, p. 163). The metaphor described by Schön has attracted the attention of scholars, some of whom (e.g. Bereiter, 1997; Korthagen & Lagerwerf, 1996) have based their studies on this metaphor.

How is the GI-Model connected with theories of Gray, Tall and Schön? In the GI-Model, the "procept" and "the proceptual facts" (Gray & Tall, 1994) are treated in dual viewpoint. To clarify the notion of integrating perception, Schön's metaphor is used (1993). Meanwhile, the term 'gestalt' in the 'gestalt intuition' supersedes the classical meaning of gestalt. It is used in a broader context, "as a dynamic and constantly changing entity" (Korthagen, 2010, p. 101).

This research is also based on the experience gained through the involvement in a series of projects and their implementation nationwide. This experience goes back to the project entitled "The Development of Education System in Albania" sponsored by AEDP-1997 (Albanian Education Development Project). This project lasted for three years. A sub-project of AEDP-1997 was the "Development of Critical and Creative Thinking", AEDP-1998. Later on, this was continued with the project "Improving Teaching and Learning in Albania", again from AEDP-1999, which lasted for about a year. During that time smaller projects that were responsible for the application of the previous projects followed. The research for the creation of the GI-Model and the possibility of its implementation in teaching in elementary education, in Albania started as a continuation of the application and implementation of these projects.

The Structure of the Model

GI-Model includes:
– dual treatments,
– integrating perceptions,
– gestalt intuition.

Firstly, the terms such as dual treatment, integrating perception and gestalt intuition must be clarified.

The term 'dual treatment' in elementary education mathematics refers to the dual interpretation, dual analysis, dual solution and dual formulation of mathematical concepts, processes, exercises or problems.

Referring to the definition, the dual treatment in elementary education mathematics includes:

- dual interpretation – an activity through which mathematical concepts and relations are interpreted together with their dual aspects;
- dual analysis – an activity through which mathematical facts and processes are analyzed in two different ways;
- dual solution – an activity through which mathematical exercises are solved in two different ways;
- dual formulation – an activity through which mathematical problems are formulated in two different ways without changing the way of solving; when these possibilities exist.

According to Egan, (1997) dual structuring is so prominent in modern young children's thinking that it requires pointing out. Meanwhile, according to Hallpike (1979), dual oppositions are intrinsic to the process of human thought. If students get used to a structure based on dual treatments, then initial integrating perceptions are formed within them for the first time. The first integrating perceptions come from the dual interpretation of the basic concepts. Later on, these integrating perceptions continue to be completed with other integrating perceptions created by the dual analysis of facts and processes from the dual treatment of exercises and problems. 'Integrating perception' refers to the ability of the student to instantly perceive the existence of duality within the same appearance, when this appearance carries a dual nature itself. Just as people develop the ability to see two figures in "vase–faces", students should be able to perceive the existence of duality in a concept, a process, an exercise or problem. Integrating perceptions for concepts, processes, exercises and problems in students are only formed if a teacher trains this student how to 'see' these concepts, processes, exercises and problems in duality. The human brain is able "to hold two conflicting ideas in constructive tension" (Martin, 2007).

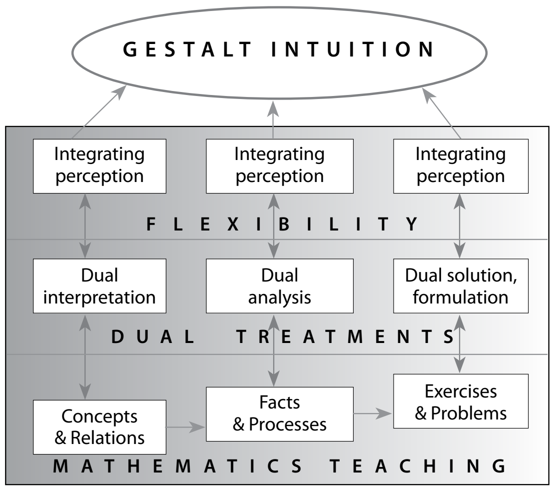

Figure 1. The formation of gestalt intuition through dual treatments and integrating perceptions

Students manifest gestalt intuition at a certain stage, as a result of refining their cognitive abilities by integrated perceptions. 'Gestalt intuition' refers to the ability developed by integrating perceptions that help students to better understand, apply and choose the best solution among several possibilities, as well as making prompt decisions avoiding the intermediate situation. The gestalt intuition appears every time when students face new and unknown situations in which they must choose the best solution out of several alternatives. In order to better explain the GI-Model, this is presented in Figure 1, demonstrating the connection of gestalt intuition to dual treatments and integrating perceptions.

Practical Implementation of the Model

In this section, the GI-Model refers to Gray and Tall's (1994) theory several times in order to demonstrate relationships with other mathematical theories also, where symbol, process, and concept are treated in detail. Despite referring frequently to Gray and Tall's theory, the GI-Model functions independently of that theory.

Concepts are represented by symbols, whereas symbols can evoke either process or concept. Alongside this "ambiguity of notations", which allows the student to think "proceptually" (Gray & Tall, 1994), the GI-Model allows a student to interpret these symbols from the dual viewpoint, creating the possibility to reach integrating perception. The symbol could be interpreted or analyzed in duality. The concept could be interpreted in duality, whereas the process could be analyzed in duality. The interpretation and analysis are dependent on which aspect is more 'apparent' – the concept or the process.

1. Dual Interpretation

Some examples of dual interpretation of concepts:

a. The symbol > is one way of showing the dual existence in mathematical concepts. In the arithmetic inequality $a>b$ the number a is greater than the number b, and at the same time the number b is less than the number a.

b. If the segment AB is longer than the segment CD, at the same time the segment CD is shorter than the segment AB.

c. The triangle is a right angled triangle if that triangle has an angle of 90°, and if a triangle has a 90° angle, then the triangle is a right angled triangle.

d. If Anna is in front of Emma in the queue, at the same time Emma stands behind Anna in the queue.

These examples simplify and explain the words of Schön: "Two different ways of seeing the housing problem are made to come together to form a new integrating image; it is as though, in the familiar gestalt figure, one managed to find a way to see both vase and profiles at once" (1993, pp. 155-156). Referring to the examples above, we can also explain the gradual formation of integrating perception among students. A teacher should teach the students that when they see a sign in an inequality, the students should see it once from one side and another time from the other side of the sign. This flexibility of seeing inequality makes us believe that after several classes, the students may gain the special ability of seeing at the same time both relations: 'greater than', and 'less than'. This means that if students are given an inequality, for example 3>2, both relations; '3 is greater than 2' and '2 is less than 3', should come to mind. Other examples are described in the same way there. As a result of several practices directed and led by the teacher, when the teachers speak of the comparison of segments, the two relations 'longer than', 'shorter than' should come to the students' minds. When a teacher talks about right angled triangles, students should immediately think of the triangle with one of the angles measuring 90°. When teachers speak about the ordering, the student should think of both relations; 'before' and 'after'. Only when the students have gained the above described abilities, it can be said that for the concepts: 'big', 'small; 'long', 'short'; 'right angle', '90°', which are percept through comparative relations in sets of numbers, segments, figures, respectively and for the concepts 'before', 'after' which are percept through the ordering, the formation of integrative perception for related dual interpretation is reached.

2. Dual Analysis

Gray and Tall "consider the duality between process and concept in mathematics, in particular, using the same symbolism to represent both a process (such as the sum of two numbers 3+2) and the result of that process (the sum 3+2)" (1994, p. 116). In addition, they treat 3+2=5 as a "proceptual fact" that could produce new "proceptual facts". Dual analysis done from the moment when the addition process is being introduced, as well as other mathematic actions facilitates grasp of new "proceptual facts".

Examples of dual analysis:

a. In order to clarify the process of addition, arithmetic equality 3+2=5, should be analyzed in duality: 3+2=5 shows that the sum of 3 and 2 is equal to 5, and at the same time 5 could be explained as the sum of two numbers 3, 2.

b. The symbol a/b stands for both the process of division and the concept of fraction. In order to clarify the process of division it should be analyzed in duality: in the process of division when the divisor divides the dividend, the dividend is also being divided by the divisor.

3. Dual Solution and Formulation

Since solving exercises in two different ways is clear to the teachers, we are focusing on the dual formulation of the problem. Experience has shown that critical thinking is promoted more when problem-solving is accompanied by a dual problem. In the dual problem, the solution does not change, only the formulation does (Gjoci & Kërënxhi, 2010). The dual formulation includes the primary problem and the dual problem. If a given problem has a dual problem, it means that it accepts dual formulation.

Below are some examples of problems that accept dual formulation:

a. Anna bought an English Dictionary, whereas Emma bought an Italian Dictionary. Anna paid 21 €, Emma paid 17 €. Who paid more? How much more?
The dual problem is: Anna bought an English Dictionary, whereas Emma bought an Italian Dictionary. Anna paid 21 €, Emma paid 17 €. Who paid less? How much less?
In both cases, regardless of different wording and formulation of the problem, the problem is solved in the same way.

b. Anna is 12 years old. Emma is 2 years younger than Anna. How old is Emma?
The dual problem is: Anna is 12 years old. She is 2 years older than Emma. How old is Emma?

c. The first day Anna read 18 pages. The second day she read 4 more pages than the first day. How many pages did Anna read in both days?
The dual problem is: Anna read 18 pages the first day, which is 4 pages fewer than the number she read on the second day. How many pages did Anna read in both days?

After training students to work with dual formulation of the problem, the teacher may go on to applications combined with formative aspects. Such applications belong to more advanced dual treatments. To fulfill these combined applications, the student should be able to give different ways of solving problems, to build up various schemes for their solution, and to give the dual aspect of the given problem. Such applications help students to be able to apply gestalt intuition.

Concluding the description of the dual treatment in mathematics classes in elementary education for the formation of the student with integrating perceptions, emphasis must be placed on the fact that the GI-Model forms students with gestalt intuition if teachers promote critical thinking utilizing the dual viewpoint. Critical thinking from dual viewpoint for teacher means that the teacher should create and formulate well-thought ideas and illustrate them with examples derived from exploring dual treatment. The teacher should explain, give reasons and illustrate everything with examples applying dual treatment. The teacher must have a clear image of what kind of answer he or she is expecting from the students, before asking the students which method is available for a certain problem. The teacher should promote and facilitate what students learn, and the advantages of applying dual treatment.

Curricula and the GI-Model

In order for the model to be more functional, the GI-Model was designed according to the content of mathematics curricula (Grades 1-4). The questions which were answered in this phase were: to what extend does the mathematics curricula fulfill the conditions for the possibility of dual thinking? Can mathematical textbooks include dual treatments? In what grade of elementary education can dual treatments begin?

The mathematics curricula of elementary education (Grades 1-4) in Albania were studied carefully and it was concluded that there was enough room to apply and interpret research in the textbooks. Referring to the standards of elementary education (ISP, 2003), instructions of curricula (IKS, 2006), program (IKS, 2009) and the textbook Mathematics 1 (Dedej, Spahiu & Konçi, 2009a), there are some examples in which duality is present:

– Dual reciprocal concepts: 'inside' - 'outside', unit 1.4; 'before' - 'after', unit 13.9, etc.
– Dual reciprocal relations: 'more than' - 'less than', unit 1.6; 'larger than' - 'smaller than', unit 2.8; 'shorter than' - 'longer than', unit 3.3, etc.
– Dual equalities: 'a+b=c' - 'c=a+b', unit 3.4, etc.
– Dual inequalities: 'a>b' - 'b<a', unit 2.7; 'a+b>c' - 'c< a+b'', unit 21.4, etc.

These dual concepts, relations, equalities and inequalities should be treated in duality, always together, as they exist in reality. A proper presentation of the dual model in the textbook gives the teacher and students the possibility to apply dual treatment. The teacher is the only person who can modify the content accordingly in order to reach the aim of the lesson.

In Table 1, data about the possibility that mathematics curricula (Grades 1-4) can include dual treatment in the teaching process is shown. Data is compiled from the study of the program (IKS, 2009) and especially from the study of the content of mathematical textbooks in grades 1-4 of elementary education, with authors Dedej et al. (2009a,b,c,d). These categories are present in mathematical textbooks of other authors as well.

Table 1

The distribution of data about the possibility that the mathematics curricula create with reference to dual treatments

		1st category			2nd category			3rd category		
Grades	Type of information	Total number of classes	Accept dual treatment N	Accept dual treatment %	Total number of classes with new concepts	Accept dual treatment N	Accept dual treatment %	Total number of classes with exercises and problems	Accept dual treatment N	Accept dual treatment %
1st grade	Theoretical	175	77	44	34	17	50	19	4	21
2nd grade	Theoretical	175	75	43	46	22	48	60	18	30
3rd grade	Theoretical	175	68	39	60	34	57	53	19	36
4th grade	Theoretical	140	62	44	87	46	53	32	14	44

Analyzing the data in Table 1, it is concluded that the curricula of mathematics for grades 1-4 of elementary education gives teachers possibilities to discuss in duality approximately 41% of the topics with new concepts and 33% of the topics with exercises and problems. The teacher can discuss mathematical dual situation with the students on average in 2 – 2.5 math classes.

Teachers' Evaluation of the GI-Model

The study on the possibility of implementing the GI-Model in teaching among teachers was carried out during the school year 2009-2010 (Gjoci & Kërënxhi, 2010). During the time this study was conducted, 10 teachers of the first 4 grades of the 9-year schools of the city Elbasan, Albania were engaged. This study aimed to answer several questions: Could the dual view be included in teaching and learning mathematics? Which kinds of concepts should the teacher interpret in duality? Which mathematical processes could be analyzed in duality? Are there problems that accept dual formulation? In which grade of elementary education could dual treatment begin? What are some of the difficulties that teachers face in dealing with dual treatments? How should the teacher direct questions to enhance discussions on a dual situation?

In cooperation with the teachers involved in this study, some reciprocal dual concepts (13% of the topics with new concepts) were chosen to be interpreted in duality. Mathematical exercises and problems that accept dual analysis, solution and formulation were chosen and adapted (12% of the total topics with exercises and problems) to be included in teaching processes. The selected models and other models individually chosen by teachers were implemented in their teaching of mathematics. Some of the conclusions drawn from this study about the simplicity of implementation of the GI-Model in teaching are as follows:

– Dual interpretations are easily implemented by teachers in their teaching. This has to do with the fact that dual interpretations are achieved through mathematical concepts which are reciprocally dual, and that these concepts are easily understood in mathematical textbooks, by teachers and students alike.
– The inclusion puts the teachers in a more difficult situation as compared with dual interpretations, since the dual analyses are realized for mathematical processes in teaching dual analyses. The difficulties are mainly observed in defining the moment when a certain dual analysis should be included in teaching, in a way that would make it understandable for the student.
– Dual solutions are implemented by teachers without any difficulties.
– In many cases teachers misinterpret the problem's dual formulation with the opposite of the given problem. After appropriate instruction and training, dual formulation is made understandable for the teachers, and they use it in teaching without any difficulties.

The research of Gjoci and Kërënxhi (2010) showed that experimenting teachers discussed dual reciprocal concepts and relations with their students. This was done in 34% of the topics with new concepts. They solved and formulated the problems with duality in 27% of the topics with exercises and problems. Finally, the experimenting teachers discussed with their students at least one mathematically dual situation in approximately every third math class in the first 4 grades of elementary education. These conclusions showed once again that dual treatments are simple and easy for teachers to implement in their teaching. Meanwhile, the formation of the students' gestalt intuition is a long process, lasting many years, results of which can be seen only when dual treatments are included year after year in mathematics programs. Cooperation and collaboration with experimenting teachers, sharing experiences with them, discussions, interviews, observations, students' tests, all helped us tremendously to improve the GI-Model and enrich it with new experiences, up to the creation of the final structure, which was presented above.

The Impact of the GI-Model in Student Learning

In order to evaluate the effects of GI-Model on the students' results during 2010-2011 the experimental research was conducted. The research questions in this study were:

– How successful are students in the mathematics course after the implementation of the GI-Model?
– Are there any differences between genders in the mathematics course after the implementation of the GI-Model in teaching?
– How could this model be acquired, and for what level of students is the GI-Model appropriate?

– How much time does the teaching process with the GI-Model require for achieving the educational objectives?

Method

Participants

The study sample included 168 students aged 6-7. The students were enrolled in the first grade of elementary 9-year public schools in Elbasan, Albania. The students were divided randomly into classes. The classes were then classified into the experimental group (3 classes) and the control group (3 other classes). The distribution of participants according to gender and groups is presented in Table 2.

Table 2

The distribution of participants according to gender and groups

Gender	Experimental Group		Control Group		Total
	N	%	N	%	
Male	51	30	48	29	99
Female	33	20	36	21	69
Total	84	50	84	50	168

N: number of participants in groups; %: percentage of participants in groups

The teaching of mathematics in the first grade of elementary education is done in 35 weeks, 5 classes (45 minutes) per week. In total, first grade students undergo 175 class hours of mathematics. The teachers who taught in these classes had a college degree with a relatively long teaching experience (over 15 years of teaching) and the same level of qualification. The teachers implemented the curricula approved by the Albanian Ministry of Education and Science for the school year 2010-2011 in these classes. The teaching of mathematics was carried out using the textbook "Matematika 1" by Dedej, Spahiu and Konçi (2009a). The equipment and means used by the teachers were the same. The students had the same conditions for learning. Attempts to minimize the difference between the two groups were made, except for the way of teaching topics, which was different. During the school year 2010-2011, the GI-Model was implemented in the experimental group, but not in the control group.

Research Design

The implementation of the GI-Model in teaching in the experimental group started in the second week of the first term and lasted until the end of the school year. In this study, a quasi-experimental design (NEGD) for non-equivalent groups (Trochim, 2001) was implemented. In order to evaluate the effectiveness of teaching using the GI-Model, pre-tests and post-tests were conducted in both the experimental and control group. Both groups were tested before and after the intervention. The test conducted in the first week of the first term served as a pre-test to measure the initial mathematics achievement of the students. Control variables were prior achievement in mathematics. The independent variable was the intervention (the teaching with the GI-Model and/or traditional teaching). The dependent variable was the post-test for the achievements of the students in the mathematics course. The post-test was conducted in the third week of May.

Instruments and Measurements

The data for this study were collected using the Achievement in Mathematics Course test (AMC). The instrument contained 48 questions. The first 36 questions were grouped into 8 categories which measured: (1) the comparison between the two sets, (2) the comparison of numbers, (3) addition and subtraction of numbers up to 20, (4) the addition of numbers up to 100 without regrouping into the tens column, (5,6) the solution of equations and inequalities with evidence, selecting from a set numbers instead of solving for unknown numbers from a finite set, (7,8) the solution of problem situations, expressed through drawings or words, implementing addition and subtraction within 20. In the second section of the post-test 12 questions we included, grouped into four categories: (1) to compare the sets in duality, (2) to compare the numbers in duality, (3) to identify reciprocal dual pairs, and (4) to write as many equality and inequality expressions as the students could, using the numbers given. Each category was accompanied by questions that enhanced the description of the dual situation. The questions from AMC were designed in accordance with the program and textbooks approved by the Albanian Ministry of Education and Science for the school year 2010-2011. The structure of the test was based on Boriçi (2004).

Results and Discussion

The means and standard deviations for each group on the pre-test were: for the control group M= 33.82, SD= 10.791 and for the experimental group M= 35.19, SD= 10.791. To detect any significant differences between the experimental group and the control group on the pre-test scores, t-test for two independent samples was used. No significant difference was found. The analyses revealed no statistically significant differences in prior achievement of the students in mathematics [t(166)= -.843, p= .498].

The data from the AMC test were analyzed using SPSS 17.0 statistical analysis program. The frequencies (N), percentages (%), means (M), and standard deviations (SD) were calculated for descriptive statistics. For inferential statistics, analysis of covariance (ANCOVA) and t-test for independent samples were used. An alpha level of 0.05 for all statistical tests was used. Table 3 summarizes the descriptive statistics for post-test AMC scores according to each group.

Table 3

Descriptive statistics for dependent variables

Measure	Experimental Group (n=84)		Control Group (n=84)	
	M	SD	M	SD
AMC	43.01	13.679	39.69	13.020

AMC – Achievement Mathematics Course test

Homogeneity of regression slopes was done in order to see if there indeed was an interaction between the covariate, (pre-test) and the independent variable (AMC test). SPSS reported the interaction to be non-significant ($F(1,164)= .412$, $p= .522$) so this assumption has not been violated. After being checked for the existence of a linear relationship between the covariate and the dependent variable, ANCOVA was performed in order to measure how successful the students were during the mathematics course. After using results from the pre-test as covariates, the analyses of results of the AMC test indicated that the GI-Model has had a significant effect on the subjects factor group, $F(1,165)=65.025$, $p< .0005$, partial $\eta^2= .28$ in favor of the experimental group. The results of the test demonstrated that the indicators of the experimental group upon the acquisition of the dual treatment of concepts, relations and exercises, as compared with those of control group were encouraging. In the experimental group, 78% of the students versus 23% of the students of the control group compared the sets in duality. 64% of the students of the experimental group to 15% of the students of the control group included dual interpretation while comparing numbers. 100% of the students of the experimental group compared to 65% of the students of the control group had the ability to distinguish dual models. 48% of the students of the experimental group, compared to 15% of the students of the control group formed simple equality and inequality expressions describing pairs that are reciprocally dual.

A t-test for independent samples was administered to the AMC test scores to determine whether there was any statistical significance between boys and girls. The Levene's Test for Equal variances yields a p-value of .571. This means that the difference between the variances was statistically insignificant. The t-value ($t(82)= .-683$, $p < .496$) indicates that there was not any significant difference between average difficulty for males and females. So the GI-Model had affected boys and girls equally, and the gender factor was not relevant in the mathematics course.

The analysis of the post-test reveals that the very able students of the control group had the ability to understand the existence of duality in a dual situation and were able to describe it clearly and correctly, whereas the average students and those below average of the control group did not have these abilities. This shows that the traditional program interpreted by the teachers in a biased way provides and forms students with partial knowledge. Referring to the students of the experimental group, the study showed that dual treatments are acquired by students. Taking into consideration the results drawn from the observation in mathematics classes, it could be concluded that dual interpretations and dual analyses can be acquired by students of any level. Concerning dual solution and dual formulation of the problems, only the very able students of the first grade understood and acquired them properly. This is linked to the fact that dual solution and dual formulation of the problem were included in the experimental school program only in the last few weeks of the school year, thus it is too early to draw conclusions on them. A complete conclusion on the simplicity of the dual solution and dual formulation of the problem may be drawn if the study continues with the students of the second grade.

The duration of teaching process with the GI-Model to achieve the educational objectives was evaluated through the study of teachers' archives. Studying the notes of the teachers while they were preparing for teaching and while talking to them, it was concluded that the teachers who implemented the GI-Model needed more time to prepare for teaching than the teachers of the control group. From the conversations and observations, time spent on teaching the GI-Model was the same as the time spent on traditional teaching. By the end of the school year 2010-2011, the experimenting teachers of the first grade evaluated the GI-Model as a simple model, easily implemented and easily acquired by the students.

Conclusions Pertaining to the GI-Model

Taking into consideration the five criteria that, according to Kuhn (1977), a 'good' theory should accomplish, we can say that the GI-Model has become totally stable. Until now, the research has showed that the inclusion of dual treatments in the teaching process of mathematics in primary education influences greatly the level of knowledge assimilation of this subject, helping the students towards the creation of gestalt intuition and further promotion of critical thinking. Psychological studies have identified the special ability of the human brain to single out the objects in a periodical way, once in a logical plan, another time in another logical plan. In order to demonstrate this ability of the human brain, Fisher (1995) and Pettijohn (1996) use gestalt figures. Just like Fisher (1995) tries to enable the students through the gestalt figures, our GI-Model enables the students through dual treatments in mathematics. The GI-Model allows the successful students to be flexible in thinking, moving between both viewpoints, thus creating the possibility for the students to understand the existence of opposite realities upon the same scene. The formation of students with integrative perception through dual treatments, as well as the gestalt intuition, is fully achievable due to the dual nature that often lends itself to mathematics. The pedagogical benefits for the use of the GI-Model are mostly in the critical way of solving problems and exercises, generating new ideas during their dual treatments and flexibility of thinking.

There is a possibility for the GI-Model to be included in teaching mathematics beginning from the first grade of elementary school. The first grade teacher can begin from dual treatments that are easier for the student to grasp. Such are interpretations of the basic meanings and relations with their dual side. The research showed that if the first grade teacher teaches the students to use a mental structure based on dual treatments, then such abilities are developed by the students; the students can see both realities of the same view at the same time. These abilities help the student to be equipped with the first integrating perceptions. These integrating perceptions continue to be completed with other integrating perceptions created by dual treatments of problems and exercises. It is recommended that the dual treatments in exercises and the dual formulation of the problems begin in the second term of the 1st grade.

While cooperating with the experimenting teachers, they did not hesitate to express their ideas and opinions. In the beginning phase of the study, some teachers were doubtful about the implementation of the GI-Model in teaching, thinking that they were going to change their way of explaining. That is why it is recommended to explain to the teachers who want to include the GI-Model in teaching, that the way of explaining the GI-Model remains the same as that in the traditional one. Meanwhile, the teacher needs to enrich the way of explaining, where there is any possibility; two-to-three discussions at the most to clarify for the students the existence and the process of duality. Teaching through dual treatments for creating the integrative perceptions and gestalt intuition is completely applicable and achievable by every teacher and it does not cause overload in the teaching process (Kërënxhi & Gjoci, 2013, pp. 50-51). These ideas were validated even more when the results of the tests for the use of the model were analyzed. The results were encouraging, and as the experimenting teachers state, the implementation of the GI-Model is a new practice that is added to their professional experience.

Following the results achieved after the implementation of the GI-Model in teaching mathematics in the first grade, the research will continue, gradually moving, year after year, to the upper grades, in order to give detailed answers to the question of the effectiveness of teaching mathematics through implementing the GI-Model and the pedagogical benefits acquired from implementing this model.

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Gestalt intuicijski model: teorija i praksa u nastavi

Sažetak

U ovom je radu predstavljen Gestalt intuicijski model, osmišljen kao teorijski i praktični model za unapređenje nastave matematike u osnovnoškolskom obrazovanju. Gestalt intuicijski model uključuje dvojnu obradu i usmjeren je na formaciju integrativne percepcije i Gestalt intuicije kod učenika. Model ima jasan i logički temelj. Temelji se na nastavi matematičkog kurikula za osnovnoškolsko obrazovanje. U skladu je s izvođenjem nastave i znanjem u školskom programu. Može se jednostavno i lako primijeniti na primjerima koje nalazimo u matematičkim udžbenicima i u nastavi, slijedeći pravila nastavnog procesa. Navedeni model postao je pouzdan i vjerodostojan te je prihvaćen od nastavnika koji su bili uključeni u eksperiment, a koji su ga nastavili primjenjivati u vlastitom nastavnom procesu. Model je shvatljiv i razumljiv i učenicima, pa se može reći da ostvaruje neke nove rezultate u nastavi. Kao model koji pozitivno utječe na kritičko razmišljanje, Gestalt intuicijski model jača učinkovitost učenja kod učenika, a nastavnicima nudi novu alternativu za postizanje učinkovite nastave.

Ključne riječi: *dvojna obrada; integrativna percepcija; nastavna metoda; osnovnoškolsko obrazovanje.*

Uvod

Iz iskustava i na temelju mišljenja stručnjaka s područja kognitivne psihologije, filozofije i multikulturalnog obrazovanja (Anderson, Hiebert, Scott i Wilkinson, 1985; Palincsar i Brown, 1989; Resnick, 1987; Banks, 1988) dolazi se do zaključka da se kapacitet za učenje kod učenika širi kada nastavnik upotrebljava različite strategije i uključuje sve domene procesa razmišljanja. Prema Temple, Crawfrord, Saul, Mathews i Makinster (2006), učenik aktivno uči ako je znatiželjan, postavlja pitanja, otkriva nešto novo, razmišlja o temi ili posvećuje vrijeme istraživanju neke teme, primjenjuje prethodno stečena znanja u svrhu rješavanja problema i slično. Međutim, s ciljem razvoja kritičkog mišljenja, navedeni autori tom popisu dodaju i učenje kako primijeniti teoriju u praksi s različitih gledišta. Povećavaju i sposobnost učenika za istraživanje različitih tipova i posljedica ideja kada postoji uporište u činjenicama. U ovom kontekstu dvojno tumačenje može se smatrati praksom koja pomaže učenicima i nastavnicima u razvoju kritičkog mišljenja. Teorijska iskustva spomenutih i drugih autora koji su radili na kritičkom mišljenju početkom 1990-ih predstavljala su specifičan i sasvim nov pogled na učenje u albanskom obrazovnom sustavu. Uključivanje navedenih iskustava u projekte na temu razvoja obrazovnog sustava u Albaniji, a što je započelo 1997. godine, otkriva nove perspektive o poučavanju, ali i općenito o obrazovanju u Albaniji.

Cilj je ovoga rada provesti strategije koje vode prema aktivnom učenju i kritičkom promišljanju. Na temelju stvarnog iskustava iz albanskog obrazovnog sustava, autori će pokušati predstaviti nastavnu teoriju i praksu koja odgovara navedenih iskustvima. Taj ćemo cilj postići predstavljanjem Gestalt intuicijskog modela, koji smo sami osmislili i izradili, kao i opisom empirijskih podataka iz testiranja modela u nastavi.

Pogledi i perspektive

Dvojnost je oblik i način postojanja materije, zakon prirodnih procesa. U znanosti dvojnost ne predstavlja novu teoriju – ona postoji još od antičkih vremena. Sažeti opis dvojnosti pronalazimo u uvodu knjige autora Gao (2000), kada određuje značenje pojma dvojnosti u svakodnevnom životu kao „sklada dvaju suprotnih ili komplementarnih dijelova kroz koji se integriraju u cjelinu (str. xiii); dvojnost u prirodnim znanostima opisuje kao „zapanjujuće divnu", a matematiku kao znanost koja se izdvaja u samim temeljima dvojnosti. Brojni autori temelje svoje studije na matematičkoj dvojnosti. Dvojnost je stoga opisana u najraznolikijim područjima matematike. Nakon 2000. godine počinju se istraživati dvojne značajke matematike (Aronov i Znamenskaya, 2006; Yastrebov, 2001), kao i njihov odraz u istom obliku poučavanja. Slijedom toga, odraz dvojnih značajki prisutan je u algebri, geometriji i trigonometriji (Yastrebov, Men'shikova i Yepifanova, 2006), matematičkoj analizi (Kërënxhi, 2009; Kërënxhi i Gjoci, 2010), a uvode se i dvojne značajke koje povezuju matematičku analizu s mehanikom (Gao, 2000). Za Artstein-Avidan i Milman (2007): „pojam dvojnosti jedan je od središnjih koncepata kako u geometriji, tako u analizi"(str. 42). Navedene studije odnose se na programe visokoškolskih ustanova i fakulteta. Istodobno se mnogi dvojni koncepti proučavaju i u školskom programu, počevši s prvim matematičkim temama koje se obrađuju od prvog razreda osnovne škole. Istraživanje na temu postojanja dvojnosti u kurikulu matematike o albanskom osnovnoškolskom sustavu i kako nastavnici tumače dvojnost u nastavnoj praksi (Gjoci i Kërënxhi, 2010; 2012; Kërënxhi i Gjoci, 2013) daje naslutiti da bi dvojnu obradu trebalo uvesti u nastavu. Nedostatak relevantne literature o načinima kako učinkovito primjenjivati dvojnost vodi do teorijski neobrađene situacije te pitanja usko povezanog s time kako bi nastavnici trebali poučavati a da ne padnu u klopku jednostranih tumačenja i analiza. Gestalt intuicijski model nudi odgovor na to pitanje. Naime, on nastavnicima pruža podršku u nastavi i objašnjavanju nastavne jedinice, a učenicima stjecanje točnih znanja.

Kuhn (1997) navodi pet kriterija koje bi teorija postavljena iz temelja trebala zadovoljiti ne bi li je se smatralo „dobrom", a to su: točnost, dosljednost, opseg, jednostavnost i plodnost. Prema Korthagenu (2010, str. 102), „tih pet kriterija služi za provjeru je li pojedinac do kraja razvio teorijsku razinu“. U ovome radu Gestalt intuicijski model (nadalje: GI-Model) je opisan kao model koji ispunja sljedeće uvjete:

– Točan je te ima jasan i logički temelj
– Temelji se na smjernicama kurikula za osnovnoškolsko obrazovanje
– Osmišljen je u skladu s matematičkim udžbenicima
– Nastavnici uključeni u eksperiment ocijenili su ga kao lako primjenjiv model
– Učenicima je shvatljiv i primjenjiv
– Ostvaruje nove rezultate.

GI-model

Teorijski temelj modela

Kako bi GI-Model imao jasna i logički temelj, osmišljen je i započet uzimajući u obzir teoriju autora Gray i Tall (1994) koja se temelji na procesno-koncepcijskoj dvojnosti, metafori „nove integrativne slike“ koju opisuje Schön (1993) i iskustvima albanske pedagogije. Upućujemo čitatelje na tu opsežnu i relevantnu temu.

U članku pod naslovom „Dvojnost, dvoznačnost i fleksibilnost u uspješnom poučavanju matematike“, Gray i Tall (1991) temelje svoju empirijsku studiju na dvoznačnosti simbolizma za proces i koncept. Naime, u matematici neki simbol može predstavljati i proces i produkt toga procesa. Stoga Gray i Tall definiraju „procept“ kao amalgam procesa i koncepta u kojemu su proces i produkt predstavljeni istim simbolizmom. Svoju su teoriju dodatno razvili u Gray i Tall (1994) i Gray, Pinta, Pitta i Tall (1999).

U članku „Generativna metafora: perspektiva na postavljanje problema u društvenoj politici“, Schön (1993) predstavlja važne ideje povezane s likovima vaze i lica. Kada se ljudima po prvi put predstavi lik vaze-lica te ih se upita što vide na slici, jedni će odgovoriti da vide vazu, a drugi da vide profile dviju osoba. Osim ta dva načina promatranja istog lika, Schön predlaže još jedan: „kao dva profila koja nosom dodiruju vazu“ (1993, str. 163). Ta metafora koju opisuje Schön zaokupila je pažnju znanstvenika i stručnjaka pa su neki od njih (primjerice Bereiter, 1997; Korthagen i Lagerwerf, 1996) upravo na toj metafori temeljili vlastita istraživanja.

No kako je GI-Model povezan s teorijama Graya, Talla i Schöna? U sklopu GI-Modela, „procept“ i „proceptualne činjenice“ (Gray i Tall, 1994) tretiraju se s dvojnog stajališta. S ciljem pojašnjavanja pojma integrativne percepcije, koristi se Schönova metafora (1993). Istodobno, pojam gestalt s područja gestalt intuicije nadređen je klasičnom značenju gestalta. Koristi se u širem kontekstu „kao dinamična jedinka u neprestanoj mijeni“ (Korthagen, 2010, str. 101).

Ovdje predstavljeno istraživanje također se temelji na iskustvu stečenom kroz uključivanje u niz projekata te njihovu primjenu diljem države. Iskustvo seže do projekta pod naslovom „Razvoj obrazovnog sustava u Albaniji“ koji je financirao AEDP-1997 (Projekt za razvoj albanskog obrazovanja). Navedeni projekt trajao je tri godine. Pod-projekt navedenog AEDP-1997 proveden je pod nazivom „Razvoj kritičkog i kreativnog mišljenja", AEDP-1998. Poslije je isto nastavljeno kroz projekt „Unapređenje poučavanja i učenja u Albaniji", opet pod okriljem AEDP-1999, a trajao je otprilike jednu godinu. Tijekom tog razdoblja uslijedili su manji projekti zaduženi za primjenu prethodnih. Istraživanje kreacije GI-Modela i mogućnosti njegove implementacije u nastavu u osnovnoškolskom sustavu u Albaniji počelo je kao nastavak uvođenja i primjene navedenih triju projekata.

Struktura modela

GI-Model uključuje:
– Dvojnu obradu
– Integrativne percepcije
– Gestalt intuiciju.

Najprije valja pojasniti pojmove dvojne obrade, integrativne percepcije i gestalt intuicije.

Pojam „dvojna obrada" u osnovnoškolskoj nastavi matematike odnosi se na dvojno tumačenje, dvojnu analizu, dvojna rješenja i dvojnu formulaciju matematičkih pojmova, procesa, vježbi i problema.

Prema definiciji, dvojna obrada u osnovnoškolskoj nastavi matematike uključuje:
- Dvojno tumačenje – aktivnost kroz koju se matematički pojmovi i odnosi tumače zajedno s njihovim dvojnim aspektima;
- Dvojnu analizu – aktivnost kojom se matematičke činjenice i procesi analiziraju na dva različita načina;
- Dvojno rješenje – aktivnost kojom se matematičke vježbe rješavaju na dva različita načina;
- Dvojnu formulaciju – aktivnost kojom se matematički problemi formuliraju na dva različita načina bez promjene u načinu rješavanja ukoliko ta mogućnost postoji.

Prema Eganu (1997), dvojno strukturiranje dominira načinom razmišljanja suvremene djece te ga je potrebno istaknuti. Istodobno, prema Hallpike (1979), dvojne opozicije urođene su u procesu ljudskog razmišljanja. Naviknu li se učenici na strukturu utemeljenu na dvojnoj obradi, tada se u njima prvi put oblikuje početna struktura integrativne percepcije. Prve integrativne percepcije potječu iz dvojnog tumačenja osnovnih pojmova. Poslije se dopunjuju drugim integrativnim percepcijama koje nastaju uslijed dvojne analize činjenica i procesa iz dvojne obrade vježbi i problema. „Integrativna percepcija" podrazumijeva sposobnost učenika da trenutačno uoči postojanje dvojnosti unutar jedne pojave kada je ta pojava dvojne naravi. Naime, kao što ljudi razvijaju sposobnost da na slici vaze i likova vide dva lika, učenici bi trebali biti sposobni percipirati postojanje dvojstva nekog pojma, procesa, vježbe ili problema. Integrativne percepcije za pojmove, procese, vježbe i probleme kod učenika se formiraju jedino ako učitelj pouči toga učenika kako te pojmove, procese, vježbe i probleme dvojno sagledati. Ljudski je mozak, naime, sposoban „pojmiti dvije suprotstavljene ideje u konstruktivnoj tenziji" (Martin, 2007).

Učenici iskazuju gestalt intuiciju u određenoj dobi kao posljedicu unaprijeđene uporabe kognitivnih sposobnosti zbog integrativne percepcije. „Gestalt intuicija" podrazumijeva sposobnost razvijenu putem integrativnih percepcija, a koja omogućuje učenicima bolje shvaćanje, primjenu i odabir najboljeg rješenja između nekoliko mogućnosti, kao i donošenje brzih odluka bez upotrebe posredničkih rješenja. Gestalt intuicija iskazuje se svaki put kada je učenik suočen s novom i nepoznatom situacijom u kojoj mora odabrati najbolje od nekoliko rješenja. Kako bi se bolje shvatio GI-Model, navedeno je predstavljeno u Slici 1, na kojoj se vidi povezanost gestalt intuicije, dvojne obrade i integrativne percepcije.

Praktična primjena modela

U ovome dijelu GI-Model nekoliko se puta referira na teoriju Grayja i Tall (1994) kako bi jasno demonstrirali odnos s drugim matematičkim teorijama u kojima se simbol, proces i pojam detaljno obrađuju. Usprkos činjenici da se spomenuta teorija često navodi, GI-Model funkcionira neovisno o njoj.

Pojmovi su predstavljeni simbolima, pri čemu simbol priziva ili proces ili pojam. Osim te „dvoznačnosti bilježenja" koja omogućuje učeniku „proceptualno" mišljenje (Grey i Tall, 1994), GI-Model učeniku nudi i mogućnost tumačenja tih simbola s dvojnog stanovišta, čime se otvara mogućnost postizanja integrativne percepcije. Simbol se, dakle može tumačiti i analizirati u dvojnosti. Pojam se može samo tumačiti, a proces analizirati u dvojnosti. Tumačenje i analiza ovisni su o tome koji je aspekt „očitiji" – pojam ili proces.

1. Dvojno tumačenje

Slijedi nekoliko primjera dvojnog tumačenja pojmova.

a. Simbol > jedan je način demonstracije dvojnog postojanja matematičkih pojmova. U aritmetičkoj nejednakosti $a>b$ broj a je veći od broja b, a istodobno je broj b manji od broja a.
b. Ako je segment AB duži od segmenta CD, istodobno je segment CD kraći od segmenta AB.
c. Trokut je pravokutni ako sadrži kut od 90 stupnjeva, a ako trokut sadrži kut od 90 stupnjeva, trokut je pravokutni.
d. Ako Anna stoji u redu ispred Emme, istodobno Emma u redu stoji iza Anne.

Navedeni primjeri pojednostavljuju i objašnjavaju Schönove riječi: „Dva različita načina viđenja problema stanovanja objedinjuju se i tvore novu integriranu sliku; kao da je, da se izrazimo poznatom gestalt slikom, netko pronašao način kako istodobno vidjeti i vaze i profile" (1993, str. 155-156). Navedenim primjerima možemo se poslužiti i pri objašnjenju postupne formacije integrativnih percepcija kod učenika. Nastavnik bi trebao poučiti učenike da kada u nejednakosti uoče određeni znak, da ga trebaju jednom promotriti s jedne strane, a drugi put s druge strane. Ta fleksibilnost u promatranju nejednakosti navodi na zaključak da bi nakon nekoliko takvih nastavnih sati učenici trebali razviti posebnu sposobnost sagledavanja obaju odnosa istodobno: „veći od“ i „manji od“. To znači da bi, kada se učenicima predstavi nejednakost, primjerice 3>2, njima istodobno na pamet trebala pasti oba odnosa: „3 je veći od 2“ i „2 je manji od 3“. Ostali su primjeri opisani na isti način. Dakle, kao rezultat nekoliko vježbi koje učitelj usmjerava i vodi, kada učitelj spomene usporedbu segmenata, učenici bi se trebali dosjetiti obaju odnosa „duži od“ i „kraći od“. Kada učitelj spomene pravokutni trokut, učenici bi smjesta trebali pomisliti na trokut kojemu jedan od kutova mjeri 90 stupnjeva. Kada učitelj govori o poretku, učenici bi trebali pomisliti na dva odnosa: „prije“ i „poslije“. Tek nakon što učenici steknu opisane sposobnosti, može se reći da je za pojmove: „veliko“, „malo“, „dugo“, „kratko“, „pravokutno“, „devedeset stupnjeva“, koji su viđeni usporedbama odnosa u nizu brojki, segmenata i znamenki, kao i za pojmove „prije“ i „poslije“ koji su viđeni u poretku postignuta formacija integrativnih percepcija za vezano dvojno tumačenje.

2. Dvojna analiza

Gray i Tall „razmatraju dvojnost između procesa i pojma u matematici, osobito upotrebom istog simbolizma za predstavljanje i procesa (kao što je zbroj dvaju brojeva 3+2) i rezultat toga procesa (zbroj 3+2)“ (1994, str. 116). Nadalje, 3+2=5 tretiraju kao „proceptualnu činjenicu“ koja može proizvesti nove „proceptualne činjenice“. Dvojna analiza koja započinje od trenutka predstavljanja procesa zbrajanja, kao i ostalih matematičkih radnji, omogućuje razumijevanje novih „proceptualnih činjenica“.

Primjeri dvojne analize:

a. S ciljem pojašnjenja postupka zbrajanja, aritmetička jednadžba 3+2=5 trebala bi se dvojno analizirati: 3+2=5 pokazuje da je zbroj 3 i 2 jednak 5 te istodobno da se 5 može prikazati kao zbroj dvaju brojeva, 3 i 2.
b. Simbol a/b označava i postupak dijeljenja i pojam razlomka. S ciljem pojašnjenja postupka dijeljenja, trebalo bi ga dvojno analizirati: u postupku dijeljenja kada djelitelj postane djeljenik, djeljenik se isto tako dijeli djeliteljem.

3. Dvojno rješenje i formulacija

Kako je rješavanje zadataka na dva različita načina učiteljima jasno, usmjeravamo se na dvojnu formulaciju problema. Iskustvo nas uči da se kritičko mišljenje razvija kada je rješavanje problema popraćeno dvojnim problemom. Kod dvojnog problema ne mijenja se rješenje, već samo formulacija (Gjoci i Kërënxhi, 2010). Dvojna formulacija sadrži primarni i dvojni problem. Ima li zadani problem dvojni problem, to znači da isti prihvaća dvojnu formulaciju.

U nastavku predstavljamo primjere nekoliko problema koji prihvaćaju dvojnu formulaciju:

a. Anna je kupila engleski rječnik, a Emma talijanski rječnik. Anna je platila 21 €, a Emma 17 €. Tko je platio više? Koliko više?

Dvojni problem je sljedeći: Anna je kupila engleski rječnik, a Emma talijanski rječnik. Anna je platila 21 €, a Emma 17 €. Tko je platio manje? Koliko manje?
U oba slučaja, neovisno o različito postavljenom problemu i različitoj formulaciji problema, problem se rješava na isti način.

b. Anna ima 12 godina. Emma je 2 godine mlađa od Anne. Koliko godina ima Emma?
Dvojni problem je sljedeći: Anna ima 12 godina. Dvije godine je starija od Emme. Koliko godina ima Emma?

c. Tijekom prvog dana Anna je pročitala 18 stranica. Drugoga dana pročitala je 4 stranice više nego prvoga dana. Koliko je stranica Anna pročitala u oba dana?
Dvojni problem je sljedeći: Tijekom prvog dana Anna je pročitala 18 stranica, 4 stranice manje nego drugog dana. Koliko je stranica Anna pročitala u oba dana?

Nakon što učenike osposobi za rad s dvojnim formulacijama istoga problema, učitelj može nastaviti s primjenom u kombinaciji s formacijskim aspektom. Takva primjena pripada naprednijoj dvojnoj obradi. Kako bi riješio kombiniranu primjenu, učenik bi trebao moći ponuditi različite načine rješavanja problema, izraditi raznolike sheme za njihovo rješavanje te postaviti dvojni aspekt zadanog problema. Takva primjena pomaže učenicima u primjeni gestalt intuicije.

U zaključku opisa dvojne obrade u osnovnoškolskoj nastavi matematike za formaciju učenika s integrativnim percepcijama, naglasak se mora staviti na činjenicu da GI-Model tvori učenika kojega odlikuje gestalt intuicija ako nastavnik promiče kritičko mišljenje uporabom dvojnog viđenja. Kritičko mišljenje s dvojnog gledišta za nastavnika znači da bi on ili ona trebao ili trebala osmisliti i formulirati promišljene ideje i potkrijepiti ih primjerima izvedenima iz istraživanja dvojne obrade. Nastavnik bi dakle trebao objašnjavati gradovi, ponuditi razloge i sve potkrijepiti primjerima primjenom dvojne obrade. Usto mora imati jasnu viziju o tome kakav odgovor očekuje od učenika prije nego što ih upita koja je metoda prihvatljiva za rješavanje nekog problema. Trebao bi promicati i omogućavati učenje i prednosti primjene dvojne obrade.

Kurikul i GI-model

Kako bi bio što funkcionalniji, GI-Model je osmišljen prema sadržaju kurikula matematike (od prvog do četvrtog razreda). Pitanja na koja su u ovoj fazi ponuđeni odgovori su sljedeća: Do koje mjere kurikul matematike ispunja uvjete za mogućnost dvojnog razmišljanja? Može li se dvojna obrada uključiti u matematičke udžbenike? U kojem razredu osnovne škole bi trebalo započeti s dvojnom obradom?

Kurikul matematike za osnovnu školu (prvog do četvrtog razreda) u Albaniji detaljno je proučen te je izveden zaključak da ima prostora za primjenu i interpretaciju istraživanja u udžbenicima. Na temelju standarda za osnovnoškolsko obrazovanje (ISP, 2003), uputama u kurikulu (IKS, 2006), programu (IKS, 2009) i udžbenika Matematika 1 (Dedej, Spahiu i Konçi, 2009a) navodimo primjere u kojima je prisutna dvojnost:

– Pojam dvojne recipročnosti: „unutra“- „vani“, jedinica 1.4, „prije“ – „poslije“, jedinica 13.9, itd.
– Pojam recipročnog odnosa: „više od“ – „manje od“, jedinica 1.6; „veće od“ – „manje od“, jedinica 2.8; „kraće od“ – „duže od“, jedinica 3.3, itd.
– Dvojne jednadžbe: „a+b=c“ – „c=a+b“, jedinica 3.4, itd.
– Dvojne nejednadžbe: „a>b“ – „b<a“, jedinica 2.7; „a+b>c“ – „c<a+b“, jedinica 21.4, itd.

Navedene dvojne pojmove, odnose, jednadžbe i nejednadžbe trebalo bi obrađivati prema načelu dvojnost, uvijek zajedno, baš kao što postoje u stvarnosti. Ispravno predstavljanje dvojnog modela u udžbeniku nastavniku i učenicima daje mogućnost da primijene dvojnu obradu. Nastavnik je jedina osoba koja može modificirati sadržaj s ciljem ostvarenja cilja nastavnog sata.

U tablici 1 prikazani su podaci o mogućnosti uključivanja dvojne obrade u nastavni proces u kurikulu matematike (od prvog do četvrtog razreda). Podaci su prikupljeni studijom programa (IKS, 2009) te studijom sadržaja matematičkih udžbenika od prvog do četvrtog razreda osnovne škole, autora Dedej i sur. (2009a, b, c, d). Iste kategorije prisutne su i u matematičkim udžbenicima drugih autora.

Tablica 1.

Iz analize podataka u Tablici 1 izvodi se zaključak da kurikul matematike od prvog do četvrtog razreda osnovne škole nastavnicima omogućuje dvojno predstavljanje otprilike 41% tema s novim pojmovima i 33% tema s vježbama i problemima. Dakle, nastavnik na u prosjeku 2-2,5 nastavna sata matematike može predstaviti matematičku dvojnu situaciju.

Nastavnička procjena GI-modela

Studija o mogućnosti provedbe GI-Modela u nastavi među učiteljima je provedena tijekom školske godine 2009.-2010. (Gjoci i Kërënxhi, 2010). U provedbi studije sudjelovalo je 10 učitelja prva četiri razreda devetogodišnjih škola iz grada Elbasan u Albaniji. Studijom su dobiveni odgovori na nekoliko pitanja: Može li se dvojno gledište uključiti u poduku i učenje matematike? Koje vrste pojmova bi nastavnici trebali tumačiti uporabom dvojnosti? Koje matematičke procese bi se moglo dvojno analizirati? Postoje li problemi koji su pogodni za dvojnu formulaciju? U kojem razredu osnovnoškolskog obrazovanja se može započeti s dvojnom obradom? Kakva pitanja bi nastavnik trebao upućivati s ciljem poboljšanja rasprave koja se tiče dvojnih situacija?

U suradnji s nastavnicima uključenima u studiju, za dvojno tumačenje odabrani su neki recipročni dvojni pojmovi (13% tema s novim pojmovima). Matematičke vježbe i problemi koji prihvaćaju dvojnu analizu, rješenje i formulaciju su odabrani i prilagođeni (12% ukupnog broja tema s vježbama i problemima) te uključeni u nastavni proces. Odabrani i ostali modeli koje su nastavnici samostalno izabrali uvedeni su u njihovu nastavu matematike. Neki od zaključaka izvedenih iz studije u pogledu jednostavnosti upotrebe GI-Modela u nastavi su sljedeći:

– Nastavnici u nastavi jednostavno primjenjuju dvojna tumačenja, ponajprije zbog činjenice da se dvojna tumačenja postižu kroz matematičke pojmove koji su recipročno dvojni i da su ti pojmovi jasni i razumljivi u matematičkim udžbenicima, kako nastavnicima, tako i učenicima.
– Uključivanje stavlja nastavnike u teži položaj u usporedbi s dvojnim tumačenjima jer se dvojne analize ostvaruju za matematičke procese u poučavanju dvojnih analiza. Teškoće se ponajprije odnose na određivanje trenutka u kojemu bi se određena dvojna analiza trebala uvesti u nastavu na način shvatljiv učenicima.
– Nastavnici primjenjuju dvojna rješenja bez ikakvih teškoća.
– U mnogim slučajevima nastavnici pogrešno tumače dvojnu formulaciju problema sa suprotnošću zadanog problema. Nakon odgovarajućih uputa i obuke nastavnicima dvojna formulacija postaje razumljiva te je koriste u nastavi bez ikakvih teškoća.

Istraživanje Gjoci i Kërënxhi (2010) pokazuje da nastavnici uključeni u eksperiment s učenicima raspravljaju o dvojnim recipročnim pojmovima i odnosima. Navedeno se iskazuje u 34% tema s novim pojmovima. Rješavali su i formulirali probleme u dvojnosti u sklopu 27% tema s vježbama i problemima. Posljednje, nastavnici uključeni u eksperiment s učenicima su raspravljali o najmanje jednoj matematički dvojnoj situaciji na otprilike svakom trećem nastavnom satu matematike u prva četiri razreda osnovnoškolskog obrazovanja. Njihovi zaključci još jednom dokazuju da je nastavnicima vrlo jednostavno i lako primjenjivati dvojnu obradu u nastavi. Istodobno, formacija gestalt intuicije kod učenika dugotrajan je proces koji traje niz godina, a čiji su rezultati vidljivi samo kada se dvojna obrada dosljedno uključuje u program nastave matematike iz godine u godinu. Suradnja s nastavnicima uključenima u eksperiment, razmjena iskustava, rasprave, intervjui, promatranje, učenički testovi, sve to iznimno nam je pomoglo u poboljšanju GI-Modela i njegovu obogaćivanju novim iskustvima sve do kreiranja konačne, prije predstavljene strukture.

Učinak GI-modela na učenje učenika

S ciljem procjene učinka GI-Modela na učeničke rezultate tijekom 2010. – 2011. provedeno je eksperimentalno istraživanje. Istraživačka pitanja ove studije bila su sljedeća:

– Kakav uspjeh iskazuju učenici u pogledu nastavnog predmeta matematike nakon uvođenja GI-Modela?
– Postoje li razlike po spolu u pogledu nastavnog predmeta matematike nakon uvođenja GI-Modela?
– Kako se taj model može usvojiti i za koji je stupanj učenika prikladan?
– Koliko vremena zahtijeva nastavni proces s GI-Modelom za postizanje nastavnih ciljeva i ishoda?

Metoda

Ispitanici

U studiji je sudjelovalo 168 učenika u dobi između 6 i 7 godina. Svi učenici bili su upisani u prvi razred devetogodišnje državne osnovne škole u gradu Elbasanu u Albaniji. Učenici su nasumce podijeljeni u razrede. Učenici spomenutih razreda podijeljeni su u eksperimentalnu skupinu (3 razreda) i kontrolnu skupinu (druga 3 razreda). Raspodjela ispitanika prema spolu i skupinama prikazana je u Tablici 2.

Tablica 2.

Nastava matematike u prvom razredu osnovnoškolskog obrazovanja izvodi se u 35 tjedana, 5 nastavnih satova (u trajanju od 45 minuta) tjedno. Dakle, učenici prvog razreda ukupno imaju 175 nastavnih sati matematike. Nastavnici koji izvode nastavu u tim razredima odreda su imali završen sveučilišni studij s razmjerno dugim nastavničkim iskustvom (15 godina rada u nastavi) s istim stupnjem kvalifikacija. Nastavnici su provodili kurikul odobren od albanskog Ministarstva obrazovanja i znanosti za školsku godinu 2010. – 2011. Nastava matematike izvođena je uz pomoć udžbenika „Matematika 1" autora Dedej, Spahio i Konçi (2009a). Svi nastavnici koristili su se istom nastavnom opremom i sredstvima. Svi učenici imali su iste uvjete za učenje. Obavljene su sve radnje kako bi među skupinama došlo do najmanje moguće razlike, osim u pogledu samog načina izvođenja nastave prema određenim temama, koji se razlikovao. Tijekom školske godine 2010. – 2011. GI-Model se primjenjivao u eksperimentalnoj, ali ne i u kontrolnoj skupini.

Istraživanje

Uvođenje GI-Modela u nastavu u eksperimentalnoj skupini počelo je u drugom tjednu prvog polugodišta te trajalo do kraja školske godine. U ovoj studiji korišten je eksperimentalna izvedba za nejednake skupine (NEGD) (Trochim, 2001). Kako bismo procijenili učinkovitost nastave uz primjenu GI-Modela, i u eksperimentalnoj i u kontrolnoj skupini provedena su pred- i post-testiranja. Dakle, obje grupe ispitane su prije i poslije intervencije. Test proveden u prvom tjednu prvog polugodišta poslužio je kao pred-testiranje za mjerenje početnih matematičkih postignuća učenika. Kontrolne varijable bile su prethodna postignuća u matematici. Nezavisna varijabla bila je intervencija (nastava uz primjenu GI-Modela i/ili tradicionalna nastava). Zavisna varijabla bilo je post-testiranje postignuća učenika iz nastavnog predmeta matematike. Post-test proveden je u trećem tjednu svibnja.

Instrumenti i mjerenja

Podaci za ovu studiju prikupljeni su uporabom testa Postignuća iz matematike (Achievement in Mathematic Course, AMC). Taj instrument sadržavao je 48 pitanja. Prvih 36 pitanja grupirano je u 8 kategorija kojima se mjerila: (1) usporedba dva skupa, (2) usporedba brojeva, (3) zbrajanje i oduzimanje brojeva do 20, (4) zbrajanje brojeva do 100 bez ponovnog grupiranja u desetice, (5,6) rješavanje jednadžbi i nejednadžbi bez dokaza, odabir iz niza brojeva umjesto rješavanja nepoznatih brojeva iz ograničenog skupa (7,8) rješavanje problemskih situacija izraženih crtežima ili riječima, primjena zbrajanja i oduzimanja do 20. U drugom dijelu post-testiranja nalazilo se 12 pitanja grupiranih u četiri kategorije: (1) usporedba skupova u dvojnosti, (2) usporedba brojeva u dvojnosti, (3) određivanje recipročnih dvojnih parova i (4) ispis što više izraza jednakosti i nejednakosti iz zadanih brojeva. Svaka kategorija bila je popraćena pitanjima koja su pojačavala opis dvojne situacije. Pitanja u AMC-u osmišljena su u skladu s programom i udžbenicima odobrenima od albanskog Ministarstva obrazovanja i znanosti za školsku godinu 2010. –2011. Struktura testa utemeljena je na Boriçi (2004).

Rezultati i rasprava

Srednje vrijednosti i standardne devijacije za svaku skupinu na predtestiranju bile su: za kontrolnu skupinu M= 33,82, SD= 10,791, a za eksperimentalnu skupinu M= 35,19, SD= 10,791. U cilju identifikacije značajnih razlika između eksperimentalne i kontrolne skupine glede rezultata pred-testiranja korišten je jedan t-test za dva neovisna uzorka. Nisu ustanovljene značajne razlike. Analizama također nisu otkrivene statistički značajne razlike u pogledu prethodnih postignuća učenika matematike [t(166)= -,843, p= ,498].

Podaci AMC testa analizirani su uporabom programa za statističku analizu SPSS 17.0. U svrhu opisne statistike izračunate su učestalosti (N), postotci (%), srednje vrijednosti (M) i standardne devijacije (SD). U svrhu inferencijalne statistike provedena je analiza kovarijacija (ANCOVA) i t-test za neovisne uzorke. Alfa razina od 0.05 korištena je za sve statističke testove. U tablici 3 sažeto je prikazana opisna statistika za rezultate posttestiranja AMC za pojedine skupine.

Tablica 3.

Provedena je homogenizacija regresijskog otklona s ciljem utvrđivanja postoji li doista interakcija između kovarijable (pred-testa) i neovisne varijable (AMC testa). Prema SPSS, ta interakcija nije značajna (F(1,164)= ,412, p= ,522), dakle ta se pretpostavka nije pokazala netočnom. Nakon provjere postoji li linearna veza između kovarijable i ovisne varijable, provedena je ANCOVA kako bi se izmjerilo koliko su učenici bili uspješni tijekom nastave matematike. Nakon što su rezultati pred-testa upotrijebljeni kao kovarijable, analiza rezultata AMC testa ukazuje na to da je GI-Model ostvario značajan učinak na faktorsku skupinu ispitanika F(1,165)=65,025, p< ,0005, djelomični η^2= ,28 u korist eksperimentalne skupine. Rezultati testa upućuju na to da su pokazatelji za eksperimentalnu skupinu nakon usvajanja dvojne obrade pojmova, odnosa i vježbi u usporedbi s onima u kontrolnoj skupini uistinu dojmljivi. Naime, u eksperimentalnoj skupini 78%, a u kontrolnoj 23% učenika usporedilo je skupove dvojno. 64% učenika eksperimentalne i 15% učenika kontrolne skupine uključilo je dvojno tumačenje u usporedbu brojeva. 100% učenika eksperimentalne skupine u odnosu na 65 % učenika kontrolne skupine iskazalo je sposobnost razlikovanja dvojnih modela. 48% učenika eksperimentalne skupine i samo 15% učenika kontrolne skupine oblikovalo je jednostavne iskaze jednakosti i nejednakosti opisujući recipročno dvojne parove.

T-test za neovisne uzorke primijenjen je na rezultate testa AMC radi utvrđivanja postoji li statistički značajna razlika između djevojčica i dječaka. Levene Test za jednake varijacije pokazao je p-vrijednost od ,571. Navedeno je oznaka da je razlika između varijacija bila statistički beznačajna. Naime, t-vrijednost ($t(82)= ,-683$, $p < ,496$) ukazuje na to da nema statistički značajne razlike između prosječne težine zadataka za dječake i djevojčice. Dakle, GI-Model je utjecao na dječake i djevojčice podjednako te se tako čimbenik spola nije pokazao relevantnim u nastavi matematike.

Analiza posttestiranja otkriva da su samo najvrsniji učenici u kontrolnoj skupini bili sposobni pojmiti postojanje dvojnosti u dvojnoj situaciji te isto opisati jasno i točno, dok prosječni i ispodprosječni učenici u kontrolnoj skupini to nisu bili spodobni. To dokazuje da tradicionalni program koji nastavnici tumače na pristran način formira učenike s nepotpunim znanjem. Kada je riječ o učenicima iz eksperimentalne skupine, studija je pokazala da su učenici usvojili dvojnu obradu. Uzimajući u obzir rezultate dobivene promatranjem nastavnih satova matematike, dolazimo do zaključka da učenici bilo kojeg stupnja znanja mogu pojmiti dvojno tumačenje i dvojne analize. Međutim, samo najvrsniji učenici prvog razreda potpuno su shvatili i usvojili dvojna rješenja i dvojne formulacije problema. Navedeno je povezano s činjenicom da su dvojna rješenja i dvojne formulacije uključeni u eksperimentalni školski program tek u posljednjih nekoliko tjedana nastavne godine ps je stoga prerano izvoditi zaključke. Cjelovit zaključak o jednostavnosti dvojnog rješenja i dvojne formulacije problema moći će se izvesti ako se studija nastavi na učenicima drugog razreda.

Trajanje nastavnog procesa s GI-Modelom radi postizanja obrazovnih ciljeva procijenjeno je proučavanjem arhive nastavnika. Proučivši bilješke nastavnika, odnosno njihove pripreme za nastavu, a i kroz razgovor s učiteljima, zaključeno je da je onim nastavnicima koji su uveli GI-Model trebalo više vremena za pripremu nastavnog sata u odnosu na nastavnike kontrolne skupine. Iz razgovora i hospitacija razvidno je da je vrijeme utrošeno na poučavanje putem GI-Modela jednako vremenu utrošenom na isto u tradicionalnoj nastavi. Pred kraj nastavne godine 2010.-2011. nastavnici prvih razreda uključeni u eksperiment ocijenili su GI-Model kao jednostavan, lako primjenjiv i jednostavan za usvajanje od učenika.

Zaključci vezani uz GI-model

Uzimajući u obzir pet kriterija koje prema Kuhnu (1977) svaka „dobra" teorija mora zadovoljiti, možemo reći da je GI-Model postao potpuno stabilan. Dosadašnja istraživanja pokazala su da je uvođenje dvojne obrade u nastavni proces matematike u osnovnoškolskom obrazovanju ostvarilo velik utjecaj na stupanj usvajanja znanja ovoga nastavnog predmeta te pomoglo učenicima u formiranju gestalt intuicije te daljnjeg promicanja kritičkog mišljenja. Psihološke studije ustanovile su posebnu sposobnost ljudskog mozga da izdvoji predmete na periodičan način; jednom u jednom logičnom planu, drugi put u drugome. Kako bi pokazali tu sposobnost ljudskoga mozga, Fischer (1995) i Pettijohn (1996) koristili su gestalt likove. Baš kao što Fischer (1995) pokušava osposobiti učenike korištenjem gestalt likova, tako i GI-Model osposobljuje učenike dvojnom obradom u matematici. GI-Model, naime, omogućuje učenicima da u razmišljanju budu fleksibilni, da se kreću od jednog do drugog gledišta te time otvara učenicima mogućnost da pojme postojanje suprotstavljenih situacija u istom prizoru. Formacija učenika s integrativnom percepcijom kroz dvojnu obradu, kao i gestalt intuicije u potpunosti je ostvariv cilj zbog dvojne naravi koja je u matematici često prisutna. Pedagoške koristi uporabe GI-Modela uglavnom se tiču kritičkog načina rješavanja problema i vježbi, stvaranja novih ideja tijekom dvojne obrade i fleksibilnog načina razmišljanja.

GI-Model moguće je uključiti u nastavu matematike počevši s prvim razredom osnovnoškolskog obrazovanja. nastavnici prvog razreda mogu započeti s dvojnom obradom koju je učenicima lakše shvatiti. Primjer toga su tumačenja osnovnih značenja i odnosa s dvojnom stranom. Naše istraživanje dokazuje da ako učitelj prvoga razreda uči učenike da koriste mentalne strukture utemeljene u dvojnoj obradi, tada ti učenici razvijaju spomenute sposobnosti; učenici istodobno vide dvije realnosti. Navedene sposobnosti doprinose opremanju učenika prvim integrativnim percepcijama. Te integrativne percepcije nastavljaju se dopunjavati drugim integrativnim percepcijama nastalima uslijed dvojne obrade problema i vježbi. Preporučuje se da se dvojnom obradom vježbi i dvojnom formulacijom problema započne u drugom polugodištu prvog razreda.

U suradnji s nastavnicima uključenima u eksperiment, ti nastavnici nisu oklijevali izraziti vlastite ideje i mišljenja. U početnoj fazi studije neki su nastavnici bili sumnjičavi u pogledu uvođenja GI-Modela u nastavu te su smatrali da će to utjecati na njihov stil poučavanja. Upravo je zato preporučljivo nastavnicima koji žele uključiti GI-Model u nastavu objasniti da stil poučavanja ostaje isti kao i njihov ustaljeni. No istodobno se od nastavnika zahtijeva da obogati svoj stil poučavanja postoji li za to mogućnost; najviše dva do tri razgovora s učenicima u kojima im se pojašnjava postojanje i proces dvojnosti. Nastava s uporabom dvojne obrade radi stvaranja integrativne percepcije i gestalt intuicije u potpunosti je primjenjiva i lako ostvariva svakom nastavniku te ne donosi novo opterećenje u nastavni proces (Kërënxhi & Gjoci, 2013, str. 50-51). Te su ideje dodatno potvrđene kada su analizirani rezultati testova za uporabu ovog modela. Rezultati su bili poticajni, a prema izjavama nastavnika uključenih u eksperiment, uvođenje GI-Modela nova je praksa koja je doprinijela njihovu profesionalnom iskustvu.

Nakon rezultata postignutih uvođenjem GI-Modela u nastavu matematike u prvom razredu, istraživanje će se nastaviti te postupno kretati godinu za godinom do viših razreda, a sve s ciljem davanja detaljnih odgovora na pitanja o učinkovitosti nastave matematike uz uvođenje GI-Modela i pedagoških koristi dobivenih primjenom toga modela.