OPTIMIZATION OF PARKING PUBLIC TRANSPORT VEHICLES IN OSTRAVA

ABSTRACT

A typical trait of public transport is a spatially scattered demand. A route net that is operated by a carrier (or several carriers) has to be adapted to the demand. Public transport vehicles that are not used during a period of a day are usually parked in defined parking lots that have a given capacity. When the vehicle goes from the place where its schedule ends (usually a terminus of the last connection served by the vehicle) to the place where the vehicle should be parked, a non-productive journey occurs. The same occurs at the beginning of the vehicle schedule as well. The main goal of the paper is to present a mathematical model that enables minimization of the total length of all the non-productive journeys. Functionality of the proposed mathematical model was tested in the conditions of a real bus public transport network.

KEY WORDS

optimization; mathematical model; linear programming; public transport;

1. INTRODUCTION – WHY IS IT IMPORTANT TO SOLVE THE PROBLEM?

In 2005 an essential operating change occurred in the conditions of bus public transport in the city of Ostrava. Garages and parking places for vehicles located in the central urban district Moravska Ostrava were closed down. This fact resulted in the need for a decision how the vehicles that had been parked in this locality should be assigned to other parking lots.

However, the problem of vehicle parking optimization in public transport is not only the problem arising from such changes. Public transport operation is a dynamical process that is determined by national deadlines enabling changes in transport organization and operation. In these deadlines some minor or major changes can be done in public transport organization. The most frequent changes are in the route net, in times of individual connections or in vehicle scheduling. These are the operational factors that can influence the total length of all the unproductive journeys. It is obvious that the high sum of the unproductive journey lengths (deadhead kilometres) influences negatively the public transport effectiveness and increases the total amount of necessary subsidies. If public transport is financed so that a provider of subsidy contracts the number of kilometres being subsidized per year with a carrier, the high amount of the non-productive journeys has an impact on the productive journeys because the carrier has fewer kilometres for the productive journeys.
2. STATE OF THE ART

Mathematical programming offers very effective methods that were employed for solving a lot of practical problems in transport in the past. In the area of public transport, mathematical programming was most often used for planning a route net \([1–6]\) or vehicle scheduling planning \([14–17]\). Mathematically programming was also used for example to solve a problem of assigning the vehicles to individual routes depending on round-trip times \([18]\) or with acceptance of a request on approximate homogeneity from the point of view of the vehicle capacity \([19]\). Mathematical programming was also successfully used for time coordination of public transport connections \([12]\) or designing zone tariffs in integrated transport systems \([11]\). The methods of mathematical programming are often combined with heuristic methods for solving the problems of public transport; for example sources \([1], [2] or [10]\) can be mentioned. Thanks to the fact that a lot of universal computationally high-performance tools (such as Xpress-IVE) are available, calculation times are not a limiting factor when solving extensive practical problems and therefore mathematical programming has a big potential for solving the practical problems of public transport.

The problem of assigning the vehicles to the defined parking lots can be considered to be a partial part of a more general task about vehicle scheduling \([14–17]\). Such type of the task was successfully programmed in the past (the program KASTOR \([14]\)) and repeatedly applied in suburban public transport \([4]\). It resulted in substantial savings in the running costs of bus carriers that were caused by minimizing the number of vehicles and the value of non-productive kilometres. But in practice there can be some situations when it is not possible to essentially decrease the number of vehicles by planning their journeys among the routes (the vehicles do not go from a connection of a route to a connection of a different route). In such cases the main advantage of the optimization process that the system KASTOR utilized is lost. The main reasons are obvious – the number of the vehicles cannot be decreased and the total sum of the non-productive kilometres increases. An exact border which would say when the program KASTOR can bring substantial savings is not known to the authors of the paper. The value of potential savings probably depends on several operating parameters of the routes that are run (headways, distances between the termini and so on). The authors estimate that the significance of the effects got by the program KASTOR strongly decreases in the situations when the headway in public transport does not exceed 30 minutes.

In such situation it makes sense to search for an alternative approach to optimization of savings when planning vehicle scheduling. The savings can be found when arrivals of vehicles in the beginning of their daily schedule and their returns to a depot are planned. It is not difficult to find out that such alternative approach does exist. It is known that in our case places from which the vehicles have to go to serve the first connection (the parking lots) and places where the vehicle starts its first productive journey (usually the bus terminus where the vehicle starts its daily schedule) are given. On the other hand the places where the vehicles finish their daily schedule (usually the bus terminus of the last connection the vehicle has to serve) and the places where the vehicles are parked are also known. Let us assume that the lengths of both types of non-productive journeys are given. It is more than obvious that this task resembles the transportation problem that was formulated in \([8]\) and for its solving we know all the necessary data. The parking lots from which the vehicles depart to serve the first connection of their daily schedule and the bus terminus where the vehicles finish their last productive journey according to their schedule represent sources in the transportation problem. On the other hand, the bus termini where the vehicles start their first productive journey (serving the first connection) and the places where the vehicles are parked after serving the last connection represent destinations of transportation problem. The lengths of the non-productive journeys correspond to the unit costs that are used in the transportation problem. A decision should be made about the number of the non-productive journeys among the individual places so that the total length of all the non-productive journeys is minimal.

Due to the fact that the vehicles run non-productively in two phases (before the first connection and after the last connection), the original mathematical model of the transportation problem needs not be sufficient for solving the task and therefore it is necessary to modify the original model for the conditions of the concrete carrier.
The paper is limited only to optimization of nonproductive journeys before the first connection and after the last connection of the planned vehicle schedule during working days. This means that the vehicle schedule between the first connection and the last one is not changed. Moreover, we also optimize parking of only one kind of vehicles (trams, buses, trolleybuses...). This is because it is not usually possible (or it is very complicated) to use parking lots intended for one kind of the vehicles for parking other kinds of the vehicles. For example, the tram parking lots cannot be used for parking the buses.

3. MATHEMATICAL MODEL

It is possible to take into consideration two modifications of the transportation problem for solving our task – the balanced transportation problem and the unbalanced transportation problem with the excess of the capacities over the demand. However, our task is more complicated. The complications follow from the operational conditions of the concrete carrier and they are the following (please note that the following constraints are the constraints for the Ostrava Transport, joint-stock co.):

1. The carrier uses different vehicles but they are the same kind (for example only buses).
2. The concrete vehicle is assigned to the single parking lot (it is due to record keeping, for example all the technical documentation of the concrete vehicle is available in this parking lot) – the vehicle departs from the parking lot to serve the first connection and after serving the last connection the vehicle returns back.
3. Sometimes it can happen that the vehicle starts to serve the first connection and finishes the last connection at different termini.
4. For some kinds of the vehicles it is necessary to build up specialized facilities, therefore it is requested to park all of the vehicles in the same parking lot. Let us consider the situation that the carrier owns more kinds of vehicles that must be parked in the same parking lot. For solving such problem two variants exist. The first variant enables parking of all vehicles in the same parking lot. The second variant admits that parking of the vehicles can be in several parking lots.

Let us pay attention to the transportation problem and its modification that is necessary for solving the problem. At first, let us characterize what destinations (or customers) of the original transportation problem and their demands represent our task. In our case, the destinations correspond to the places where non-productive journeys begin. From the previous text it is known that there are two types of the sources. The first type is represented by the parking lots to be the sources of our task. The second type corresponds to the parking lots in which the vehicles are parked after finishing their daily schedules. Note that the places need not be the same. Thus it is necessary to use a different way of how to formulate the destinations and their demands (or requests). At first, let us define the term – cumulated place. The cumulated place has its importance in the cases when the bus terminus where the vehicle starts the schedule differs from the bus terminus of the last connection. In such cases the fictional cumulated place represents both of the termini described in the previous sentence. The request of each cumulated place is given by the number of vehicles for which their schedule starts or terminates at the bus terminus represented by this cumulated place. Establishing the fictional cumulated places is very important. Using them we are able to model various non-productive journeys of the vehicles and the original information about the real value of non-productive covered distances is kept.

Let us explain the cumulated places on an example. Let us have two bus termini A and B. If the vehicle starts its daily schedule at the bus terminus A and its schedule ends at the same bus terminus A, then the cumulated place for this vehicle is the original bus terminus A. The non-productive covered distance corresponds to a sum of the distance from the parking lot to the bus terminus A and back in this case. If the vehicle starts at the bus terminus A and finishes at the bus terminus B, then the cumulated place for this vehicle is a new fictional place A-B. The non-productive covered distance is the distance from the parking lot to the bus terminus A plus the distance from the bus terminus B to the parking lot.

Now let us discuss what the sources and their capacities of the original transportation problem represent in our case. The sources correspond to the places at which the non-productive journeys begin. From the previous text it is known that there are two types of the sources. The first type is represented by the parking lots from which the vehicles depart to serve their first connections in the frame of the planned schedule. The second type comprises the bus termini of the last connections according to the schedules. Thanks to the definition of the cumulated place, that also includes information about the second type of the sources; we consider only the parking lots to be the sources of our modified transportation problem. Finally, we can define the meaning of the last group of entry data – the rates that influence the value of the objective function. In the original transportation problem the rates represent how much a unit carriage from the source to the destination costs. In our case the rates will correspond to the distances among the sources (the parking lots) and the destinations (the cumulated places). Let us presume that the vehicle begins to serve the first connection in the frame of its
schedule at the same bus terminus where the vehicle finishes the service of the last scheduled connections. In this case the rate for the individual parking lot is calculated as a sum of the distance from the parking lot to the bus terminus and back. Now let us consider that the vehicle begins to serve the first connection at the bus terminus that differs from the bus terminus at which the vehicle finishes the last scheduled connection. In this case the rate for the individual parking place is calculated as a sum of the distance from the parking lot to the first bus terminus of the vehicle schedule and the distance from the last bus terminus of vehicle schedule to the parking lot.

Formulation of the mathematical model:

Let a set of the vehicle types \( I \) be given for the solved kind of the vehicles. For each type of solved kind of the vehicles \( i \in I \) its parking norm \( p_i \) is defined and it is given if all the vehicles of the individual type have to be parked together in the same parking lot or not. The parking norm represents how many parking places the individual vehicle needs for parking. Using the parking norm in the model enables us to model parking of vehicles with different needs of parking places if it is necessary. Therefore, the vehicle set \( I \) is divided into two subsets denoted \( I_1 \) and \( I_2 \). Let us assume that subset \( I_1 \) contains all the vehicle types for which it is requested that all the vehicles from the individual vehicle type have to be parked together in a single parking lot. Subset \( I_2 \) includes such vehicle types for which the carrier does not request that all of them have to be parked in the same parking lot. For the sets \( I_1 \), \( I_2 \) and \( I_2 \) it holds that \( I_1 \cup I_2 = I \) and \( I_1 \cap I_2 = \emptyset \).

For each vehicle type a set of the possible parking lots \( J \) and a set of cumulated places \( K \) are given. For each parking lot \( j \in J \) its capacity \( a_j \) is known. For each cumulated place \( k \in K \) its request \( b_k \) for the vehicles of the type \( i \in I \) is defined. For each journey between the parking lot \( j \in J \) and the cumulated place \( k \in K \) the distance \( c_{jk} \) in kilometres is known (the distance does not depend on the vehicle type). Our task is to decide about how many vehicles run between the individual parking lots and the individual cumulated places so that the total non-productive covered distance is minimal.

In order to write down the mathematical model it is necessary to establish two groups of variables:

- \( x_{ijk} \) - the number of vehicles of type \( i \in I \) running between the parking lot \( j \in J \) and the cumulated place \( k \in K \),
- \( y_j \) - an auxiliary bivalent variable that models the decision about parking the vehicles of the type \( i \in I \) in the parking lot \( j \in J \). If \( y_j = 1 \) then the vehicles of type \( i \in I \) will be parked in the parking lot \( j \in J \). If \( y_j = 0 \) then the vehicles of type \( i \in I \) will not be parked in the parking lot \( j \in J \).

The mathematical model has the following form:

\[
\min f(x,y) = \sum_{j \in J} \sum_{k \in K} c_{jk} \sum_{i \in I} x_{ijk}
\]

subject to:

\[
\sum_{i \in I} p_i \sum_{k \in K} x_{ijk} \leq a_j \quad \text{for} \quad j \in J,
\]

\[
\sum_{j \in J} x_{ijk} = b_k \quad \text{for} \quad i \in I_1 \cup I_2 \text{ and } k \in K,
\]

\[
\sum_{j \in J} x_{ijk} \leq y_j T \quad \text{for} \quad i \in I_1 \text{ and } j \in J,
\]

\[
\sum_{j \in J} y_j = 1 \quad \text{for} \quad i \in I_1,
\]

\[
x_{ijk} \geq 0 \quad \text{for} \quad i \in I_1 \cup I_2, \quad j \in J \text{ and } k \in K,
\]

\[
y_j \in \{0,1\} \quad \text{for} \quad i \in I_1 \text{ and } j \in J.
\]

It is supposed in the presented model that the parking lots have their capacities so that all the vehicles can be parked in the parking lots.

Function (1) expresses the total distance that is covered non-productively (deadhead kilometres). Let us briefly discuss why the non-productive covered distance is used as the objective criterion. It is clear that it is possible to consider also other criteria such as fuel consumption or emission of pollutants when deadheading (please note that using a different optimization criterion can change the results of the model). However, the city of Ostrava, which is the founder and the subsidizer, evaluates traffic performance of Ostrava Transport, joint-stock co. according to the total covered distance. Currently in the Czech Republic, it is usual that the amount of subsidy is lower than it was in the past. Ostrava Transport, joint-stock co. is therefore forced to reduce the total covered distance due to the decreasing subsidy. Reducing the non-productive covered distance is the most natural way how to reduce the total covered distance with no influence on the productive covered distance.

The group of constraints (2) ensures that the given capacity of each parking lot will not be exceeded. The group of constraints (3) assures that all the planned vehicle schedules will be served. The groups of constraints (4), (5) and (7) are active only when subset \( I_1 \) is not empty – \( I_1 \neq \emptyset \). The group of constraints (4) ensures logical links among the groups of the variables \( x_{ijk} \) and \( y_j \). The group of constraints (5) assures that the request that all the vehicles of type \( i \in I \) must be parked in the same parking lot will be satisfied. The groups of constraints (6) and (7) define domains of definition for all the variables of the model.

Symbol \( T \) represents a suitably chosen prohibitive constant. To assess its value before the beginning of the optimization calculation for example formula (8) can be applied:

\[
T = \max_{j \in J, k \in K} \{b_k\}
\]

\[\text{symbol} \quad T \quad \text{represents a suitably chosen prohibitive constant. To assess its value before the beginning of the optimization calculation for example formula (8) can be applied:}\]

\[T = \max_{j \in J, k \in K} \{b_k\}\]
4. GENERAL DESCRIPTION OF OPTIMIZATION ALGORITHM

The optimization algorithm consists of several steps:

Step 1 – create a list of the parking places (set \( J \)). The list of the parking places forms a list of the sources in the presented transportation problem modification.

Step 2 – define capacity \( a_j \) for each parking place \( j \in J \) (how many vehicles can be assigned to the parking place).

Step 3 – for each vehicle find out the bus terminus where the vehicle starts its daily schedule (the initial bus terminus) and the bus terminus where the vehicle ends its daily schedule (the final bus terminus).

Step 4 – on the basis of existing combinations of the initial and final bus termini create a list of the cumulated places (set \( K \)). The list of the cumulated places forms the list of the customers in the presented transportation problem modification.

Step 5 – for each cumulated place \( k \in K \) define its request \( b_k \) – the request represents how many schedules are served by the vehicle type \( i \in L \) starts at the initial bus terminus or ends at the final bus terminus that form the cumulated place \( k \).

Step 6 – calculate the non-productive covered distances between the individual parking lots and the individual cumulated places – the values represent the coefficients of the objective function.

Step 7 – create the mathematical model and implement it in suitable software on the basis of input data you have got in the previous steps. After successful implementation solve the mathematical model.

Step 8 – calculate the savings in the non-productive covered distance (NCDS) according to formula (9):

\[
\text{NCDS} = \frac{\text{NCD}_a \cdot \text{NCD}_b}{\text{NCD}_b - \text{NCD}_a} \cdot 100,
\]

where:

- \( \text{NCDS} \) – the savings in the non-productive covered distance,
- \( \text{NCD}_b \) – the non-productive covered distance before the optimization experiment,
- \( \text{NCD}_a \) – the non-productive covered distance after the optimization experiment.

5. NUMERICAL CALCULATION

The presented mathematical model was tested using an example from the practice. The model was applied to plan the non-productive journeys of the buses in the conditions of Ostrava Transport, joint-stock co.

The total number of the buses that were included in the calculation experiment is equal to 104; their types are as follows: ten buses are Fiat or Mercedes, seven buses are Renault City Bus and the other vehicles are of other types. For the buses Fiat and Mercedes there is a request that they have to be parked together in the same parking lot; the same must be ensured for all the buses Renault City Bus as well. According to the notation we established in the previous section it holds that \( |I| = 3, |A| = 2 \) and \( |L| = 1 \). For the experiments, the applied parking norm was equal to \( p_i = 1 \) for \( i = 1, \ldots, 3 \), because all the vehicle types have the same need for parking places. In this case the original model can be simplified and the parking norm \( p_i \) can be omitted in formula (2).

The vehicles can be parked in three parking lots that are situated in different urban districts, which means \( |J| = 3 \). For the purposes of the experiment the parking lots were named as follows – Slezska Ostrava, Martinov and Poruba. The total capacity of the parking lots is 160 parking places; please note that in this case parking for 104 vehicles is planned. The capacity of the parking lots is listed in Table 1. Due to the fact that the number of vehicles is less than the total capacity of all the parking lots, it is necessary to use the modified mathematical model that has the group of the constraints in form (2).

For each vehicle schedule it was necessary to find two important bus termini of the vehicle schedule – the initial bus terminus of the first connection and the final bus terminus of the last connection. In total there were 53 different bus termini. From these bus termini 73 cumulated places were created in order to apply the presented mathematical model; that means \( |K| = 73 \). Instructions how to get the cumulated places are explained by way of a simple example in Section 3. The requests of the individual cumulated places for the individual vehicle types were in the range from 1 to 6.

For each combination of the parking lot and the cumulated place the corresponding distance was computed. Consequently, we created a matrix of objective function coefficients that had \( 3 \times 73 = 219 \) elements.

Detailed information about the creation of the cumulated places and the values of distances covered non-productively can be found in publication [13].

The mathematical model has the following parameters: Objective function (1) has 219 terms. The set of the constraints consists of 84 constraints in total. The group of constraints (2) includes 3 constraints; the group of constraints (3) contains 73 constraints. The group of constraints (4) is formed by 6 constraints and the group of constraints (5) includes 2 constraints. The total number of the non-negative variables \( y_{ij} \) for \( i \in I_1 \) and \( j \in J \) is 6. The model has 79 variables in total. Numerical experiments were performed on PC equipped with the processor Intel® Core™2 Duo E8400 and 3.25 GB of RAM (hardware configuration is important regarding calculation times). The results obtained by the experiment are summarized in Tables 1 and 2.
Applying formula (9) the savings in the non-productive covered distance were calculated after the optimization experiment. The rounded decrease of the total non-productive covered distance expressed as a percentage was 4% after the optimization experiment. The non-productive covered distance before the optimization experiment was 1,166.09 km, the non-productive covered distance after the optimization experiment is 1,120.67 km. The difference seems to be small but if we realize that a year has approximately 250 working days we get the savings of about 11,355.00 km.

The information about searching the optimal solution is depicted in Figure 1. The important pieces of information in Figure 1 are: information about the value of the optimization criterion (the total non-productive covered distance) and the time of the optimization calculation. As you can see in Figure 1, the calculation time is insignificant. Moreover, we can see in Figure 1 that the solution we have got is optimal.

### 6. CONCLUSION

The presented paper is focused on the problem of assigning the vehicles to the defined parking lots. The paper contains the mathematical model of the task; the functionality of the model was tested on an example from practice – the model was applied to the selected group of the Ostrava Transport’s vehicles. The calculation experiment showed that the model is functional. Applying the model to our problem brought a 4% decrease of the total non-productively covered distance. It can be assumed that including other vehicles in the optimization process could bring an additional decrease of the non-productively covered kilometres.

The mathematical model formed by (1) – (7) does not consider additional costs that are associated with moving the specialized facilities for the types of the vehicles that have to be parked together in the same parking lot (in our case such moving did not occur). However, the additional costs can be easily imple-
mentioned in the presented model; it can be incorporated in the additional costs in the objective function. It can be modified analogously to a model of a location problem [9].

Ing. DUŠAN TEICHMANN, Ph.D.
E-mail: dusan.teichmann@vsb.cz
VŠB – Technická univerzita Ostrava
Ing. MICHAL DORDA, Ph.D.
E-mail: michal.dorda@vsb.cz
VŠB – Technická univerzita Ostrava
Fakulta strojní, Institut dopravy
17. listopadu 15/2172, 708 33 Ostrava, Česká republika
E-mail: michal.dorda@vsb.cz
Ing. HELENA BÍNOVÁ, Ph.D.
VŠB – Technická univerzita Ostrava
Fakulta strojní, Institut dopravy
17. listopadu 15/2172, 708 33 Ostrava, Česká republika
E-mail: ludvik@hsfsystem.cz
Ing. MARTIN LUDVÍK
VŠB – Technická univerzita Ostrava
Fakulta strojní, Institut dopravy
17. listopadu 15/2172, 708 33 Ostrava, Česká republika
E-mail: michal.dorda@vsb.cz

ABSTRAKT

OPTIMALIZACE PARKOVÁNÍ VOZIDEL MĚSTSKÉ HROMADNÉ DOPRAVY V OSTRAVĚ

Typickým problémem veřejné hromadné dopravy je prostorově roztroušená poptávka. Linková síť, která je provozována dopravcem (nebo skupinou dopravců), musí být pro tuto poptávku adaptována. Zvidí městské hromadné dopravy, která nejsou využívána v průběhu některé části dne, jsou obvykle parkována na definovaných parkovacích plochách s danou kapacitou. Pokud vozidlo přejíždí z místa, ve kterém vozidlo končí svou službu (zpravidla se jedná o konečnou zastávku posledního spoje, který vozidlo obsluhuje), do místa, kde je vozidlo zaparkováno, vzniká neproduktní jízda. Tento problém nastává i při zahájení obsluhy prvního spoje. Hlavním cílem článku je představit matematický model, který umožňuje minimalizovat celkovou sumu všech neproduktně ujatých kilometrů. Funkčnost navrženého matematického modelu byla testována v podmínkách reálně sítě veřejné autobusové dopravy.

KLIČOVÁ SLOVA

optimalizace; matematický model; lineární programování; veřejná doprava;

REFERENCES
