THEORETICAL DEVELOPMENT OF THE ESTIMATE OF DIAMOND WIRE SAVING PLANT

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The article deals with possible estimates which would make the choice of the diamond wire sawing plant easier. Based on the principle law of friction of thin elastic bodies, the estimate procedure intended for the choice of a known plant, and the estimate procedure for verification of the known plant were established. Orientation and detailed way of estimating the pulling force of the diamond wire sawing plant by the method of passing along the contour is theoretically developed. The advantages and disadvantages of these estimates are discussed, as well as the problems in establishing their basic parameters.

Introduction
Stone sawing with the diamond saw wire cutter is an affirmed and unavoidable technology in dimension stone exploitation. Efficient development and application of diamond wire sawing plant were followed by numerous publications presenting the acquired experiences and results of application. However, this development was not adequately accompanied by theoretical discussions and research in the application of these machines, or by mathematical estimates which would enable easier introduction and choice of the plant.

Technological compatibility of the sawing plant with the technology of concrete operations in the dimension stone exploitation, technological possibilities and characteristics, general quality and economical distinctions of the concrete type of sawing plant, are the factors influencing the right choice and application of available equipment, adequate planning and introduction of new mechanization. The economy of the diamond wire sawing plant application, as well as of other mechanization, implies first of all a professional choice and then its professional use. Even the most rational plant application will not eliminate basic failures in unprofessional choice of the plant. The knowledge on accurate plant estimate procedures helps, among others, in making the right choice.

As far as I know, in specialist and scientific literature there have not been elaborated any estimates which would make easier the right choice of a diamond wire sawing plant for dimension stone exploitation. This report should therefore fulfill the existing emptiness in this field and initiate further similar theoretical and especially experimental research in these problems.

Theoretical assumptions of the basic estimate
In estimating the diamond wire sawing plant it is important to establish the values influencing the capacities of the sawing plant considering the character and size of a cut in certain rock type. The power of diamond saw wire cutter equals the multiplication product of force and velocity, which means that the power needed for the diamond wire drive will be calculated according to the formula:

\[ P_0 = F_{\text{max}} \cdot v \]  (1)

where

\[ P_0 \] ... power needed for diamond wire drive in W,
\[ F_{\text{max}} \] ... force needed for diamond wire drive in N,
\[ v \] ... wire velocity in m/s.

The power needed for diamond wire drive is smaller than the one of the motor for the value of losses created in power transfer from the motor to the periphery of driving pulley. The motor power has to be increased in relation to the power needed for diamond wire drive, by introducing into the estimate also the coefficient of safety or of the power reserve. By this reserve additional power will be provided necessary to overcome unpredicted stresses, i.e. powers which will enable eventual damage, e.g. getting stuck of wire in the cut a.o.

Accordingly, the power needed for the driving motor of diamond wire sawing plant will be:

\[ P_m = k_1 \cdot k_2 \cdot P_0 \]  (2)

where
\[ P_m \] ... power of driving motor in W,
\[ k_1 \] ... coefficient of efficient action by transmission system,
\[ k_2 \] ... coefficient of power reserve.
The coefficient of efficient action of the transmission system includes, beside the coefficient of efficient action by transmission from the motor to the axis of driving pulley, also the losses created due to the friction resistance in the bearings on the driving pulley axis as well as on orientation wheel-bearings by which enclosing angles are increased and the wire is directed. On the basis of experienced values it can be accepted, that power losses amount approximately 15% total power, so that the coefficient \( k_1 = 1.2 \). It can also be assumed, that such an amount of reserve power will also be needed and the coefficient of power increase will amount \( k_2 = 1.2 \).

This means, that by the known driving motor power, the needed power for diamond wire drive is defined according to the formula:

\[
P_n = \frac{P_m}{k_1 \cdot k_2} \quad \text{(W)}
\]

From the power needed for diamond wire drive (formula 1), the maximum circumferential pulling force needed for diamond wire drive is calculated:

\[
F_{\text{max}} = \frac{P_n}{V} \quad \text{(N)}
\]

Accepting the assumption that the movement transmission from the driving pulley to the diamond wire undergoes the basic law of friction of thin elastic bodies (Havliček, 1950, Dubbel, 1956, Dundaua, 1991) we obtain that the biggest pulling force on the periphery of driving pulley equals the difference between the biggest strain of the wire arriving wire branch and the strain of the leaving wire branch:

\[
F_{\text{max}} = F_a - F_0
\]

The relation between the strain force of the coming wire branch arriving \( F_a \) on the driving pulley and the leaving branch \( F_0 \) whereby the driving pulley by friction transmits movement to the wire without sliding taking place, amounts:

\[
F_a = F_0 \cdot e^{\mu \alpha}
\]

where

- \( e \) ...base of natural logarithms,
- \( \mu \) ...coefficient of friction between wire and pulley,
- \( \alpha \) ...angle of wire encirclement on the pulley (in arc).

By solution of the equations (5) and (6), the strain in leaving part of diamond wire is obtained:

\[
F_a = F_{\text{max}} \cdot \frac{1}{e^{\mu \alpha} - 1}
\]

In the same way the strain in arriving wire branch is calculated:

\[
F_a = F_{\text{max}} \cdot e^{\mu \alpha}
\]

The static force generated by previous wire tightening has to be added to these strains, generated due to static stress of the drive. By tightening device the needed tightening force is realized, which keeps, in certain limits, the tightening strain of diamond wire in its contour points, compensing eventual stretching during traction and limiting wire deflection on the line from the wire outlet from the rock to its arrival on the driving pulley.

If this total force, generated by a tracting device, is devided into the leaving and the coming wire branch, the real strain will be obtained on the first and on the second side, i.e. maximum tractive force in diamond wire will be calculated.

Accordingly, real strain of the leaving part of diamond wire is:

\[
F'_a = F_a + \frac{F_s}{2}
\]

where

- \( F_s \) ...total tightening force provoked by tightening mechanism.

Real strain of the coming wire part, or maximum tractive force in diamond wire is:

\[
F' = F_a + \frac{F_s}{2}
\]

This maximum static tractive force must take the strength of line wires with certain safety, i.e. the following relation has to exist:

\[
F' = \frac{\sigma_b}{\nu} \cdot f
\]

where

- \( \sigma_b \) ...nominal breaking strength of the wires in line, in Pa,
- \( f \) ...metal cross-section of the line (summary surface of all line wires) in \( \text{m}^2 \),
- \( \nu \) ...arithmetical coefficient safety.

To find the mass of the line \( p \), i.e. to find the interdependence between the values \( p \) and \( f \), the concept of fictive density of the line will be used in the estimate. Namely, the relation among these values can be expressed in this way:

\[
p = f \cdot \gamma \cdot c = f \cdot \gamma_0
\]

where

- \( p \) ...mass of the line per length meter, \( \text{kg/m} \),
- \( \gamma \) ...specific mass of wire metal, \( \text{kg/m}^3 \),
- \( c \) ...coefficient considering longer wires in entangled line than the line length,
- \( \gamma_0 \) ...fictive density of line, \( \text{kg/m}^3 \).

Value \( f \) is calculated from the equation (13):

\[
f = \frac{p}{\gamma_0}
\]

If this value is used in equation (11) we obtain:

\[
F'_a = \frac{\sigma_b}{\nu} \cdot \frac{p}{\gamma_0}
\]

Solving this equation with \( p \) we obtain:

\[
p = \frac{F'_a \cdot \gamma_0}{\sigma_b \cdot \nu}
\]
According to the calculated value \( p \), the appropriate line is chosen and according to the arithmetical summary breaking force by all wires \( F_b \), the safety coefficient is controlled again with the formula:

\[
v' = \frac{F_b}{v'}
\]  

(16)

**Alternative estimation procedure**

From the previous estimation one can see, that this way of estimating is suitable for verification of the existing plants, and it is nowadays also the most frequent case, because diamond wire sawing plants are bought without any previous estimates on the basis of various criteria.

A diamond wire sawing plant should be thoroughly studied before being bought, and the essential part of such studies is a certain estimate.

Since the machine power is not known, and the defining of this power is the end objective, the estimate should have the reverse order than before presented.

As the diamond wire is formed mainly at the quarry, the estimation should start with the known bearing capacity of the diamond wire line, chosen on the basis of estimate of possible resistances to required cut sizes.

Accordingly, this way of estimating could be performed as follows.

**Maximum static tractive force in the line:**

\[
F'_s = \frac{F_b}{v'}
\]  

(17)

where

- \( F_b \) ... arithmetical breaking force of the line, N,
- \( v' \) ... coefficient of safety for the new line.

\[
v' = k_w \cdot \nu
\]  

(18)

where

- \( k_w \) ... the factor which takes into account the increase of the line coefficient due to breaking of particular wires during the lifetime,
- \( \nu \) ... prescribed coefficient of safety.

In the estimation of diamond wire sawing plant, the coefficient of safety for the new line should be assumed somewhat higher than the regular "proscribed" one, which so far has not been regulated by any standards, but which will get its value in some future norms and regulations. This increase of such a prescribed coefficient of safety is needed in this phase of estimation, because this coefficient of safety must keep its prescribed value during the whole time of line exploitation. During operation, the total breaking capacity of the line will be decreased due to breaking of individual wires which can then not be considered as bearing.

Strain in the the coming wire branch

\[
F_a = F'_a - \frac{F_a}{2}
\]  

(19)

where

- \( F'_a \) ... maximal tractive force in the line,
- \( F_a \) ... total tightening force.

By solution \( F_a \) in the formula (6) through \( F_0 \) and using this solution in the formula (5), the expression for the greatest peripheral pulling force on the driving pulley is obtained, e.g. the pulling force for driving of diamond wire:

\[
F = \frac{F_a \cdot e^{-\mu} - 1}{e^{-\mu}} (N)
\]  

(20)

where

- \( \mu \) ... coefficient of friction between diamond wire and sheathing of driving pulley,
- \( \alpha \) ... enclosing angle of diamond wire on driving pulley.

**Needed power for diamond wire drive**

\[
P_0 = F \cdot \nu (W)
\]  

(1)

where

- \( F \) ... peripheral pulling force, N,
- \( \nu \) ... accepted wire velocity, m/s.

**Power of driving motor**

\[
P_m = k_1 \cdot k_2 \cdot P_0 (W)
\]  

(2)

where

- \( k_1 \) ... coefficient of efficient section of transmission system,
- \( k_2 \) ... coefficient of reserve power,
- \( P_0 \) ... power on periphery of driving pulley, N.

**Theoretical considerations of the estimation by the method of passing along the contour**

The presented estimates of pulling force by diamond wire sawing plant could be called "traditional procedures" because of the analogy with "similar" plants, because the estimates started with the known data: the motor power or the breaking strength of diamond wire. A more complex and more precise estimation procedure should cover the influence of a larger number of factors, e.g. it should in more details establish particular forces of resistance to rock sawing in relation to complex dependences in different working conditions.

In stone sawing the resistances occur, which have to be overcome by some force in order to enable sawing in foreseen direction with desired effect. Total resistances consist of several individual resistances with different intensities and kinds on particular places of cutting trajectory. This means, that the general force of resistance to be overcome by tractive
force of diamond wire sawing plant consists of a number of components. By summing up of separate components, the general coefficient of resistance is obtained.

Diamond wire covers round the cutting surface and is closed by the driving pulley presenting a continuous closed flow. In defining total resistances for any plant with continuous action by elastic working organ, it is convenient to apply so called methods of estimation "along contour" or "along points".

This method can apply two procedures of estimating the pulling force: approximate and detailed ones.

**Approximate estimate**

For estimating the pulling force in estimate, the general coefficient of sawing resistance $w'$ is applied. As result, the approximate power values of machine and wire tension in desired contour points are obtained.

General coefficient of resistance by diamond saw wire cutter would be obtained in such a way, that individual resistance of one bead would be multiplied with the number of beads existing in the cut. To this resistances those resistances will be added, which occur on the driving pulley and the deflection wheels. This added gathering resistances beginning as a results bending diamond wire along driving pulley and deflection wheels.

Accordingly, general coefficient of resistance $w'$ will be:

$$w' = w_{c} + w_{d}$$  (21)

where

- $w_{c}$ ... total cutting resistances,
- $w_{d}$ ... concentrated resistances of driving part.

Total cutting resistances $w_{c}$ would be calculated according to the formula:

$$w_{c} = w_{p} \cdot n \cdot l_{c}$$  (22)

where

- $w_{p}$ ... partial cutting resistance of one bead,
- $n$ ... number of beads per m in diamond wire,
- $l_{c}$ ... length of diamond which covers the cut at the beginning of sawing.

The length of diamond wire in cut equals:

$$l = 2a + h$$  (23)

where

- $a$ ... length (width) of cut, m,
- $h$ ... height of cut, m.

Length of wire out of cut

$$l'_{a} = 2c' + l_{a}$$  (24)

where

- $c'$ - distance between sawing plant and working face (or block) from 3 to 4 m,

- $l'_{a}$ - diamond wire contact arc on the driving pulley which is equal:

$$l'_{a} = \frac{r \cdot \pi \cdot \alpha}{180}$$  (25)

where

- $r$ ... radius of driving pulley,
- $\alpha$ ... enclosing angle of diamond wire on the driving pulley in degrees.

Total length $L$ of diamond wire amounts:

$$L = l'_{c} + l'_{a} + c' (l'_{c} + l'_{a})$$  (26)

where

- $c' = 0.02$ to $0.03$ ... coefficient of contour loosing $(2$ to $3\%$).

According to the analogy with similar estimates for the plants of continuous acting with the elastic working organ, concentrate resistances of the driving part will be replaced by the coefficient of resistance $k_{k}$. This means, the increase of resistance force on the part of diamond wire at the outlet from the cut to the inlet in the cut by means of the coefficient of resistance: $k_{k} = 1.03$ (at the enclosing angle 180°) to 1.05 (at the enclosing angle 270°) will be taken into account.

Periphery pulling force which is transmitted on the diamond wire must be at least equal or bigger than general force of resistance without any resistance in driving mechanism. Accordingly, periphery pulling force $F$ on this estimate is:

$$F = k_{c} \cdot w'$$  (27)

where

- $k_{c} = 1.2$ ...coefficient of pulling force increase, which considers secondary resistances in rock sawing by diamond wire.

After establishing the pulling force according to the formula (27) the force of diamond wire sawing plant is calculated according to the formulas (1) and (2).

The way of defining the total cutting resistance could be considered as a basic disadvantage of such approximate estimate, because by simple multiplication of the resistance of one bead by the total number of working beads, it is assumed that resistances are constant along the whole trajectory of sawing i.e. the resistance increase regulary along the wire contour in direction of its movement, starting from the point of inlet into the cut to the pint of its outlet from the cut. Sawing resistances along the whole cut trajectory are not constant values; they change along the contour depending on many factors, e.g. trajectory shape, wire velocity, wire construction etc.

Since by such simple estimate some average values of total resistances are still obtained. I think that just such an estimate would be suitable and sufficient for the choice of diamond wire sawing plants produced in
series, when the project of dimension stone exploitation will be performed.

The basic problem with this estimate is the establishment of partial resistance of one bead. Accepting insufficiently accurate value of this simple resistance, the failure in defining the total resistance increases proportionally with the sawing surface increase. Experimental testing and defining of partial resistance of certain bead in concrete rock type, in the basis of this estimate. This means, that the accuracy in estimating the tractive stresses in wire depends, in each concrete case, on regulatory and accurately defined basic value of partial resistance to moving and cutting of one bead. The disadvantage is, as in all empiric examinations, that it gives sufficiently accurate results only when the rock and sawing conditions differ a little bit from the conditions under which tests and measurements are performed, on the basis of which this single resistance is established. Even if it is accurately established for some type of rock in certain working conditions, it cannot be generally applied for all cases. The advantage of such defining of the partial resistance of a bead is the fact, that the result is obtained directly from practice.

Detailed estimate

A more complex estimation of the pulling force and the motor power in function of contour strain forces uses the method of gradual passing the diamond wire contour or passing the characteristic points of contour. The contour closed by diamond wire decomposes into linear and curvilinear parts following one another. Connecting points of these parts are accepted as characteristic points of the contour and are marked by ordinal numbers from 1 to n. Characteristic point are thus the points in which sawing conditions change, i.e. the direction of path-line changes, the stressed part of wire is replaced by the unstressed part of wire, a.o. Following these points in direction of diamond wire moving, the strains in leaving and coming wire branches on the diving pulley are determined, and according to the value of these strains the total pulling force is established. The passing of the contour starts from the point, where diamond wire leaves the driving pulley, which is in the same time the point of its least stress.

The essential conception of detailed estimate for the tractive force consists of adding up the resistances along sawing trajectory, starting from the point where the wire leaves the driving pulley and in which the force \( F_0 = F_1 \) acts, to the point of its coming to the driving pulley, where the force \( F_2 \) acts. In establishing of the strain in all other contour points, the following general rule of estimation in used:

The diamond wire strain in each following point, going along the contour in direction of wire movement,

equals the strain in previous point, increased for the stress on the part between these two points.

This rule is expressed by the formula:

\[
F_i = F_{i-1} + w_{(i-1) to i}
\]

where

\[
\begin{align*}
F_i & \quad \text{... strain in point } i, \\
F_{i-1} & \quad \text{... strain in previous point, } i - 1, \\
w_{(i-1) to i} & \quad \text{... stress on the part between the mentioned points.}
\end{align*}
\]

It results from the formula, that in passing along the contour in direction opposite to the diamond wire moving, the strain in each following point will equal the difference between strain value in the previous point and the resistance force value on the part between these points. If the contour is devided into n parts, and if the strain on the leaving part of wire is marked by \( F_1 = F_0 \), and the strain of the coming part of diamond wire on the driving wheel is marked by \( F_3 \) then the pulling force is calculated according to the formula (5) or with estimating of the resistance on the driving pulley axis:

\[
F_{\text{max}} = F_3 - F_0 + W_{n-1}
\]

The needed power on the driving pulley rim is calculated according to the formula (1), and needed power the machin by the formula (2). The coefficient of efficient action of the transmitting system \( k_l \) (formula 2) includes in determination of \( F \) according to the formula (5) also the losses on the driving pulley axis. In determination of \( F \) according to the equation (29) these losses are not included in \( k_l \), because they are included in \( w \).

In order to show in detail the determination of required pulling force or resistances, which have to be overcome by this force, the scheme of sawing with the contour in noose at the vertical height cut will be discussed. The simplified curve (Fig. 1) can be devided into basic characteristic points of transition from individual curved parts. For start of passing along the contour appropriate points 1 where the diamong wire leaving the driving pulley. Point 2 is the wire entrance into the rock. Point 3 is placed at the end of "the horizontal" part on the transition point to the curved part. Point 4 is located at the end of the curved part with the beginning on the "vertical" part. The first curved part is marked by points 5 and 6, and the rectilinear part next to the other upper curved part is marked by points 6 and 7. From the point 7 to the point of the wire leaving the cut 8, there is the curved part formed by rounding off the face top. Point 9 presents the point of coming on the driving pulley.

Strains in some contour points will be established starting from the point of leaving the driving pulley:

\[
F_0 = F_1
\]
The last equation (38) in the series of equations expressed by $F_1$ and of the sum of all resistances is:

$$F_9 = F_8 = F_1 + \Sigma w$$  \hspace{1cm} (40)

By equalization of the equation (40) with the equation (39) the value of original strain is obtained:

$$F_1 = \frac{\Sigma w}{e^{\mu \alpha} - 1}$$  \hspace{1cm} (41)

After the forces in the initial and in the final contour point have been estimated, the rim pulling force is calculated according to the formula (5), and the power of diamond wire sawing plant according to the formulas (1) and (2).

To estimate the diamond wire sawing plant by the detailed method of passing along the contour, it is necessary to determine and to sum up the resistances on individual parts of the contour. The contour shaped by diamond wire consists of linear and curved parts with the final part where the wire wraps the driving pulley. The first problem in this estimation is to establish the contour shape and the resistance value along this contour. Establishing of an accurate shape of the contour is a complex task, because this shape is not constant, but it differs for each cut, and it also changes permanently as the sawing advances. This is the reason why in estimating by the method of passing along the contour, the shape of contour at the beginning of sawing has to be accepted as the most disadvantageous case. The diamond wire is then with its biggest length in the rock, why the total resistance obtained as the sum of individual ones will be the biggest. Based on the total resistance the pulling force and the driving motor power of diamond wire plant are defined, and if this is determined for the most unsuitable case, it will satisfy in less unsuitable cases, too.

A more difficult task is to establish strengths resisting the cutting. There are numerous causes of resistance generation which are complex and interrelated (Dundar, 1992). Some of them can be established and determined, but in some cases this is very difficult or almost impossible. Further, individual resistances of beads are not constant along the whole trajectory of sawing, they change along the contour in dependence on the trajectory shape. Because of this, for the estimation of diamond wire sawing plant by the method of passing along the contour, the approximate estimate is more real than a detailed estimate. This way of estimating requires only the determination of single cutting resistance, because the average value of the general coefficient of resistance is obtained in such a way, that single resistance of one bead is multiplied by the number of beads in the cut. The disadvantage of such approximate determination of the resistance, i.e. approximate estimate is, that by simple multiplication of the resistance of a single bead by the total number of working beads it is assumed, that
the resistances are constant along the whole sawing trajectory.

**Conclusion**

The developed basic procedures of estimating the diamond wire sawing plant facilitate a perfect choice of plant depending on sawing proportions or a perfect choice of sawing proportions for the existing plants.

In a more complex estimation by "the method of passing along the contour" the approximate estimate based on the determination of partial sawing resistance of a single bead has been evaluated as more suitable (due to the problems in establishing the resistance on individual parts of the diamond wire contour). Accurate proportions determination of resistances resisting the sawing is the most important accuracy parameter of all developed estimates. Therefore the priority of further research in this field is experimental determination of the resistance in different exploitation conditions. Gathering of a greater number of empirical data will enable the development of patterns with a more universal character of satisfying accuracy, which can be used in estimations of the sawing plant in different conditions of dimension stone blocks mining.

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