ANALYTICAL, NUMERICAL AND EXPERIMENTAL STRESS ASSESSMENT OF THE SPHERICAL TANK WITH LARGE VOLUME

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This paper presents designing of spherical tank using combination of analytical procedure with FEM analysis and experimental testing in order to minimize design time and verify design strength. Analytical procedure for calculation of the tank strength in the initial stages of design process is briefly presented. Based on analytical results, tank is dimensioned and FEM model is created. FEM analysis is used to identify areas with high concentration of stresses. FEM results showed that equivalent value of stress at the points of spherical tank support exceeds the values of yield stress, but this exceedance is not significant and in very small area, so overall design was deemed worthy. Experimental measurements verified FEM results that it is not necessary to reinforce the spherical tank at the points of support. After 8 months experiments were repeated giving the same results as the original measurements, thus justifying decision not to reinforce tank supports.

Keywords: analytical stress assessment; experimental testing; Finite Element Method (FEM); spherical tank

1 Introduction

The spherical tank (Fig. 1) belongs to the group of stable elevated tanks designed for storing butane, propane or mixture of propane-butane with medium pressure [1÷3].

These highly flammable gases need to be stored in tanks designed with safety as their utmost priority [4]. The spherical tank is loaded with the fluid pressure, hydrostatic pressure [5] and forces that arise due to its own weight. In addition to these constant loads, other loads may occur due to the action of wind force [6], snow [7] as well as seismic loads [8, 9]. To prevent leakage or fire of those hazardous gases, detection of damage in tank structure is crucial [5]. However, good tank design, and adhering to safety protocols can prevent critical damage from ever occurring in tank structure. To ensure there are no flaws in their design, engineers cannot rely solely on analytical results, they need to verify their design by numerical simulations and experimental testing as well, which is methodology presented in this paper.

Well known analytical procedure used in designing spherical tanks is briefly explained [10]. Detailed derivation of expressions for membrane forces and stresses in the direction of the tangent to the circle of the parallel and the meridian using the membrane state of stress and equilibrium equations for the shell in the form of surface of revolution is given in [11]. Analytical solution is used in the initial phase of design because the basic dimensions of the spherical tank can be obtained in a relatively short period of time [10], but this solution does not account for specificity of areas with high concentration of stress, such as points of connection between tank and its supports, and therefore more detailed numerical analysis is required in order to be certain that proposed design will meet safety criteria. This analysis is done using Finite Element Method (FEM) and its results show that equivalent value of stress at some points exceeds the values of yield stress. Areas of plastic deformation are not significant in comparison to the entire spherical tank construction, so based on FEM results it is concluded that cumulating of plastic deformation will not
occur. Since FEM results should always be used with caution, having in mind that their accuracy depends on number of factors such as mesh quality, proper constraints, loads and boundary conditions, experimental verification of FEM results in the most critical areas of the tank structure was required in order to verify conclusions drawn from FEM results. Achieved high level of correspondence between results obtained analytically, with FEM analysis, and with experimental tests of spherical tank show that the proposed design meets safety standards. After eight months of exploitation, experimental testing was repeated, and no changes in stress values were observed. Combination of analytical procedure with FEM analysis and experimental testing of spherical tank gives more detailed insight in behavior of tank construction in most critical areas in comparison to design process which rely solely on analytical results and thus ensuring optimum design and safe usage during envisioned operating life time.

### 2 Methods

#### 2.1 Analytical procedure

Stress state in elements of the shell in the form of surface of revolution can be determined by Equilibrium equations for the shell elements.

Forces acting on the element which is a part of shell in the form of surface of revolution are presented in Figure 2. The element is defined by two parallel circles, with radii $r_0$ and $r_0 + dr_0$ and two adjacent meridians determined by the angles $\theta$ and $\theta + d\theta$. Position of the element belonging to the shell in the form of surface of revolution for spherical tank is shown in Fig. 2a as well as components of the external surface load $X, Y, Z$. Fig. 2b shows internal forces acting on that element [10, 11]. The tank operates in moderate climate conditions, so temperature influence can be neglected [12].

**Equilibrium equation for shell section is given by**

\[
2\pi RN_{\phi}\sin^2 \varphi + Q = 0, \quad (4)
\]

where $Q$ represents resultant of external loading.

**2.1.1 Own weight loading**

The load on the spherical tank due to its own weight is shown in Fig. 4.

Internal forces due to tank own weights above supports are:

\[
N_\phi = -\frac{gR}{1 + \cos \varphi}, \quad (5)
\]

\[
N_\theta = -gR \left( \cos \varphi - \frac{1}{1 + \cos \varphi} \right). \quad (6)
\]

Internal forces due to tank own weight below supports are:
The operating pressure in the tank is: $p_g = 1,67$ MPa. For safety reasons, experimental testing was performed with water ($\gamma = 9810$ N/m$^3$) instead of propane-butane mixture. Analytical calculation and FEM simulations were also done with water in order to obtain comparable results. Using (7-13) and expressions for stresses:

$$\sigma_\phi = \frac{N_\phi}{\delta}, \quad \sigma_\theta = \frac{N_\theta}{\delta},$$

(14)

corresponding values of forces and stresses for different angles are obtained. The total value of stresses can be calculated by adding the own weight and internal pressure components. The resulting values are presented in Tab. 1 and Fig. 5.

<table>
<thead>
<tr>
<th>Angle $\phi$ /º</th>
<th>$\sigma_\phi$ /MPa</th>
<th>$\sigma_\theta$ /MPa</th>
<th>$\sigma_e$ /MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>185.3</td>
<td>185.3</td>
<td>185.3</td>
</tr>
<tr>
<td>10</td>
<td>185.4</td>
<td>185.5</td>
<td>185.4</td>
</tr>
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<td>185.7</td>
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<tr>
<td>180</td>
<td>199.1</td>
<td>199.1</td>
<td>199.1</td>
</tr>
</tbody>
</table>

To verify the accuracy of numerical results, a 3D model of the spherical tank was formed by synthesis of 3D models of all structural parts [17-20]. The model represents a continuum discretized by 10-node tetrahedral elements for the purpose of creating the FEM model (45 124 nodes, 25 016 elements). The equivalent
stress field is presented in Fig. 6. The distribution of equivalent stresses of the FEM analysis corresponds to the distribution of stresses obtained by analytical calculation.

The highest values of stresses were obtained at the points of support of the spherical tank (Fig. 7) [20].

These values are higher than yield stress and could cause failure of tank supports. Since the area of this high stress concentration is very small it is concluded that the overall strength of supports will not deteriorate with time due to cumulating of plastic deformation. Deformations of the spherical tank are presented in (Fig. 8). Since FEM analysis identified dangers in the proposed design, experimental verification was needed in order to be absolutely certain that plastic deformations in supports will not cause failure of the structure.

### 2.3 Experimental verification

Verification of analytical and the results obtained using the FEM model was performed by experimental testing of the spherical tank. As stated in previous section, for safety reasons, experiments were performed with water instead of propane-butane mixture.

The experiment was carried out by measuring stresses at 7 measuring points with 21 strain gauges, by using the measuring equipment HBM UPM 100 [21]. The layout of the measuring points is shown in Fig. 9. This type of strain gauges setup makes measuring easier and does not require knowledge of the direction of the principal stresses propagation.

The strain gauges placed on spherical tank are shown in Fig. 10.
from the results obtained analytically and by using the FEM model in relation to the results obtained experimentally are presented in Tab. 5.

Stress values obtained by experiments for some measuring sites are higher than analytical or FEM calculated stress, while for other sites they have lower value as shown in Tabs. 2 and 4. This disagreement is proof that experimental verification is necessary in order to be certain that proposed design will meet all safety requirements.

Table 5 Percent deviation of equivalent stresses obtained analytically and by FEM model in relation to the results obtained experimentally for the test pressure

<table>
<thead>
<tr>
<th>Measuring site</th>
<th>Deviation of results obtained analytically / %</th>
<th>Deviation of results obtained by using FEM / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM1</td>
<td>8.9</td>
<td>6.7</td>
</tr>
<tr>
<td>MM2</td>
<td>8.2</td>
<td>4.8</td>
</tr>
<tr>
<td>MM3</td>
<td>6.7</td>
<td>5.7</td>
</tr>
<tr>
<td>MM4</td>
<td>-</td>
<td>8.1</td>
</tr>
<tr>
<td>MM5</td>
<td>1.9</td>
<td>6.0</td>
</tr>
<tr>
<td>MM6</td>
<td>12.6</td>
<td>0.2</td>
</tr>
<tr>
<td>MM7</td>
<td>11.2</td>
<td>0.8</td>
</tr>
</tbody>
</table>

From Tabs. 3 and 5 it can be clearly seen that FEM results are more accurate than analytical results in comparison to values obtained by experiments. FEM analysis is also much cheaper than construction and testing of prototype, it can identify critical areas and, if needed, modifications of design are easily and quickly conducted, so when prototype is constructed experiments are used to verify design and no further modifications on tank design are needed. In the case of analysed spherical tank the equivalent values of stresses at the point MM4 exceeded the values of yield stress, but not across the whole section (Fig. 7). The areas of local plastic deformation associated with stress concentrations are sufficiently small so there is no significant permanent deformation when the load is removed. Stress concentration predicted by FEM was also registered by experimental testing. Tank design was proved to be reliable, namely, because the measured values of equivalent stresses are, after eight months of exploitation (Tab. 2 and Tab. 4), identical to the original values after the installation of the spherical tank and its putting into operation.

4 Conclusion

The research carried out showed a high level of correspondence of the results obtained analytically and by FEM model with the results of experimental testing of spherical tanks. This correspondence of the results allows analytical expressions to be used for dimensioning spherical tanks. It is particularly important because, in a short period of time, in the initial design phase, the basic dimensions of the spherical tank can be obtained without carrying out an experiment and without FEM modeling. After the initial design phase, when all tank dimensions are known, more accurate FEM analysis is used to identify areas of high stress concentration. In the case of analyzed tank for pressure of 2,5 MPa equivalent stress exceeds yield stress at the points of tank connection with
the supports. Since this high stress is concentrated in small region of connection area, associated plastic deformations are sufficiently small, so we draw a conclusion that this value of stress is not critical and that construction of spherical tank can proceed. After construction of spherical tank FEM results are verified with experiments. Experiments were repeated after 8 months confirming validity of the FEM conclusion that it is not necessary to reinforce the spherical tank at the points of its connection with the supports despite the fact that minor plastic deformation occurs. Methodology presented in this paper which utilizes advantages of analytical, numerical and experimental procedures, ensures fast design time, optimum dimensioning, and most importantly, safety within envisioned operating conditions.

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5 References


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