

The Impact of Differentiation Model in Mathematics on Learner Achievements Obtained from the External and Internal Assessment of Knowledge

Zlatan Magajna¹, Milena Valenčič-Zuljan¹ and Amalija Žakelj²

¹*Faculty of Education, University of Ljubljana*

²*National Education Institute of the Republic of Slovenia*

Abstract

We present a study in which we consider¹ whether the learners, who had mathematics instruction in the 9th grade of primary school in heterogeneous groups, significantly differed in their mathematics achievements measured by using the external and internal assessment of knowledge from the learners in homogeneous groups. No statistically significant differences in the average achievements were found in the external assessment. On the contrary, in the internal assessment of knowledge the learners in homogenous groups on average had significantly higher achievements in comparison to the learners in heterogeneous groups. The analysis of covariance performed in homogenous and heterogeneous ability groups showed that the learners, who were placed in the lower or middle third after the initial examination in their school, in the final external examination progressed relatively better in heterogeneous groups. The learners who were placed in the upper third in their school according to their mathematics achievements showed no differences in this regard. The internal assessment of knowledge displayed that the learners who were placed in the middle or upper third in their school after the initial external examination, in the final examination progressed relatively better in

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homogenous groups. The learners who were placed in the lower third after the initial external examination in their school showed no differences in this regard.

Key words: grouping of pupils; individualization and differentiation; internal assessment of knowledge; external assessment of knowledge; mathematics.

Introduction

The grouping of learners is one of the basic pedagogical dilemmas that has always been present and dealt with following various purposes and criteria. The fundamental question is whether the pupils should be grouped according to their abilities into heterogeneous groups or according to their competencies into homogeneous groups. Grouping in homogeneous groups gives rise to several questions: When should homogenous grouping start? What are the reasons, purposes, and methods of grouping? How long should the period of dividing pupils into homogeneous groups last? Does any group have systemic advantages or disadvantages in comparison to the other(s)?

With regard to justice, quality of education and optimal development of each pupil the above questions are still current in all education systems.

Nowadays experts are concerned about differentiation and individualization, primarily about how to recognize individual differences among pupils and how to provide quality and sustainable knowledge to all of them with adequate organization and didactic approach. Hence, the social and educational roles of school are also emphasized.

The grouping of pupils, differentiation and individualization were in the focus of several studies. The survey of studies in scientific journals with impact factors showed that they were oriented to various national systems (Belgium, Germany, France, Sweden), especially those in the USA and the UK, and to their comparisons. The most frequently investigated subjects were Mathematics and L1, i.e. the subjects in which differentiation most often takes place.

In his almost classic research, Slavin (1987) surveyed seven studies on grouping the pupils in the lower grades of primary school. He found out that the assumed negative psychological effects of grouping into homogenous groups were negligible. Their results pointed to the efficiency of ability groups in the cases of appropriate adjustment of teaching methods and teaching aids to pupils' needs. The simple grouping of pupils according to their abilities without adequate adjustments had no effect regardless of the curriculum subject. Some other authors (Slavin, 1987, 1990; Gutierrez & Slavin, 1992; Žagar et al., 2003), who studied the effects of individualization and differentiation, and streaming respectively, on the primary school demonstrated that no research supported the streaming on the mentioned levels.

The advocates of grouping pupils into homogenous groups according to their abilities presuppose that homogenous groups enable the adjustment to individualized teaching, which results in optimal learning achievements and quality knowledge. The organization of teaching in homogenous groups provides more time for teacher's direct teaching and activities like responding to pupil's questions, giving clear explanations,

listening to pupils and reacting to their responses, providing information about learning objectives as a way of motivating pupils, monitoring the development of a pupil within the group, etc. Such teaching is based on cooperation between teacher and pupil, thus, communication is successful when teacher addresses a homogenous group of pupils. Large heterogeneous groups could diminish pupil's activities on tasks and results in lower achievements (Brophy & Good, 1986; Žagar et al., 2003). Whenever the instructional aspect is too narrowly emphasized, the interaction among pupils that may contribute to the educational efficiency of pupils is not adequately used; therefore, studies try to include the effects of cooperation and interpersonal interactions in diverse modes of grouping (Wilkinson & Fung, 2002). The researchers who studied different teaching methods and interactions among pupils in various ability groups (Hattie, 2002) found out that the ability group with low-achieving pupils was less active and keen to work, and taught by less qualified teachers. Hattie (2002) discussed that better qualified teachers, more stimulated environment and challenges could help pupils in this ability group to achieve more. A narrowly specified teaching aspect leaves behind the educational aspect that significantly contributes to the personality and emotional development of pupils, and further on, to higher learning efficiency (e.g. level of teacher's and pupil's expectations, pupil's self-concept, attitudes towards education and knowledge) (Agirdag et al., 2013; Belfi et al., 2012; Boaler, 1997; Chiu et al., 2008; Chmielewski et al., 2013; Ireson & Hallam, 2009; Prosen et al., 2013; Trautwein et al., 2006).

The researchers stress that the quality of teaching and the type of interactions among pupils represent the basic issue of teaching quality, even more than the composition and ability structure of a study group.

The effects of grouping have already been considered in several meta-studies (Dupriez, 2010; Hattie, 2009). Hattie (2009) made a vast meta-analysis of studies, aiming to explain the learning achievements of pupils. He included 300 examples on ability grouping that comprised several studies covering almost all curriculum subjects throughout a range of years of schooling, as well as different learning achievements. The mean effect size in homogenous groups was low ($d = 0.11$; Hattie set up the values important for understanding the mean effect size d , the value $d = 0.40$ was the threshold value, while the values under $d = 0.40$ were 'small' and the values above $d = 0.60$ were 'excellent'). The results showed that ability grouping had a very low effect on learning achievements. The effect of achievements in mathematics and reading were equally low (reading $d = 0.00$, mathematics $d = 0.02$), while the effect of attitude towards a subject was a bit higher ($d = 0.10$). The average effect of three basic ability groups in the studies was $d = 0.14$ for the group of high-achieving pupils; $d = -0.03$ for the group of average achieving pupils, and $d = 0.09$ for the group of low-achieving pupils (Hattie, 2009, p. 90).

Nowadays the grouping of pupils has been considered especially in terms of justice and equal opportunities. The project INCLUDE-ED distinguishes inclusive practices from exclusive ones, among which there are external differentiation and ability grouping. Ability grouping appears in diverse forms and is very common in the

European countries as a response to traditional mixed classes in which the teacher teaches a large group of learners coming from different cultural environments with a variety of abilities. Due to numerous differences among pupils -in schools today, besides their large number in each class, teachers hardly consider them individually and their mutual differences, so it is difficult with such teaching to achieve the stated educational and learning objectives. Ability grouping is a response to such situations and a way of adjusting teaching to their differences. It represents the adjustment of curriculum and formation of groups based on the abilities of pupils by comprising additional human resources (INCLUD-ED, 2009, p. 18). In the document of the European Commission (Commission Staff Working Document, 2006, p. 19), it is emphasized that early external differentiation, defined by the European Commission as "segregation of children according to their abilities before age thirteen" is quite common in some school systems. German and Austrian school systems are, for instance, the selective models that differentiate pupils according to their abilities starting at the age of 10 (Dupriez, 2010).

Various comparative analyses of school systems are interesting, too. Vandenberghe, Dupriez and Zachary (2001, as cited in Dupriez, 2010) analyzed achievements in mathematics tests (TIMSS 1995) and found out that in the countries with no early differentiation, the achievements of learners are less related to the socio-cultural position of family than in the countries with early differentiation.

Regarding the cognitive aspects of learning mathematics in homogeneous and heterogeneous groups, we believe that it is also important to consider pupils' achievements in relation to the assessment mode (in particular the external/internal mode). For various reasons the results from external examinations are more or less in line with those obtained with internal assessment. We measured the acquisition of knowledge with both of the assessments and, in this respect, examined the difference between homogeneous and heterogeneous groups. The fact that external and internal examinations assess different types of knowledge gave an insight into the cognitive effects of learning mathematics in homogeneous and heterogeneous groups.

Methodology

Definition of the Wider Framework and Research Problem

The program of nine-year primary school in Slovenia, introduced in the school year 1999/2000, allowed various forms of differentiated teaching. In the next decade, the legislation that regulated the models of differentiated teaching changed several times. When the research was in progress (2010/2011) schools autonomously decided whether to teach mathematics in the third triennium in heterogeneous or homogeneous ability groups. In order to avoid new unnecessary changes in external differentiation with regard to the efficiency and justice of school system, it is important to evaluate the efficiency of different models of differentiation.

In the studies mentioned in the Introduction, the conative aspects and learning efficiency were investigated. The analysis that we represent further on reveals the

relation between various models of differentiation and achievements of learners in the internal and external assessments of knowledge.

The objective of assessment of knowledge during the teaching process is to find out the progress and development of pupils in different fields (cognitive, conative, social and motor), which can be internal or external. The internal assessment of knowledge is carried out by teachers and based on oral, written or practical elements. Hence, teachers prepare questions or written tests by themselves without further information that would enable them to compare their own assessments and obtained results with those of other teachers and schools (Žakelj & Ivanuš Grmek, 2010). The internal summative assessment of knowledge is an independent step in the teaching process (Strmčnik, 2001) and, besides curricula, significantly determines what and how to teach, what and how to learn and, consequently, the quality of teaching, as well as pupils' knowledge (Marentič Požarnik, 2001).

The external assessment of knowledge is understood as an assessment by using tests prepared by educational experts and assessment experts. The main characteristics of the external assessment are: compliance with the required quality standards, equal or comparable tasks that learners should do, in addition to equal criteria of examination and administration (Bucik, 2001).

The efficiency of various models of differentiation and individualization is a complex question and refers to curricular, emotional, social, organizational and didactic aspects. In the article, we would like to relate the cognitive and organizational-didactic aspect with the types of assessment. For this purpose we shall analyze the achievements of pupils from homogenous and heterogeneous ability groups in the internal and external assessments of mathematics knowledge.

Basic Research Question

Did the pupils who were in the 9th grade of primary school in homogenous groups and the pupils who were in heterogeneous groups statistically significantly differ in their mathematics achievements measured by using the internal and external assessments of knowledge?

In the research, the following was analyzed:

- achievements of pupils in homogenous and heterogeneous ability groups in the external assessment of mathematics knowledge,
- achievements of pupils in homogenous and heterogeneous ability groups in the internal assessment of mathematics knowledge,
- progress of pupils in homogenous and heterogeneous ability groups revealed in the external and internal assessments of mathematics knowledge.

Sample

The research was conducted on 21 randomly selected Slovenian primary schools. The sample consisted of all nine graders from the selected schools, in total 887 pupils.

Nine schools in the sample (total 358 pupils) had heterogeneous ability groups, while twelve schools (total 529 pupils) had homogenous ability groups.

In the studied sample, there were no statistically significant differences in the proportion of the learners with special needs ($\chi^2=0.283$, df=1, p=0.595) between the schools with homogenous and the schools with heterogeneous ability groups. A statistically significant difference in the structure of ability groups in the schools was noticed in the case of homogenous grouping, where the proportion of the learners with special needs was not independent of the ability group ($\chi^2=60.982$, df=2, p=0.000). The same is valid for the proportion of girls in the ability group ($\chi^2=6.581$, df=2, p=0.037). In the first ability group we had a significantly higher proportion of the learners with special needs, while in the third one we had a significantly higher proportion of girls in comparison with the other two groups.

Table 1

Basic data on mathematics ability groups in the 9th grade in the studied sample of schools

	Homogenous groups			Heterogeneous groups
	Total	1 st ability group	2 nd ability group	
Schools, no.	12			9
Ability groups, no.	41	11	18	12
Ability groups, size	M = 13.27, SD = 4.77	M = 7.21, SD = 2.53	M = 15.50, SD = 3.31	M = 14.54, SD = 3.14
Pupils, %		16%	50%	34%
Pupils per school, %		12% - 25%	35% - 71%	17% - 47%
Special needs pupils, %	6.6%	26.2%	5.0%	1.0%
Gender %	boys: 49.5%, girls: 50.5%	boys: 52.4%, girls: 47.6%	boys: 53.8%, girls: 46.2%	boys: 41.1%, girls: 58.9%
				boys: 49.4%, girls: 50.6 %

Instruments

We used four instruments to determine the knowledge of mathematics in our study. The initial external examination was performed at the beginning of the school year, while the final external examination was performed at the end of the school year. The former one was prepared by the researchers along the lines of the National Assessment of Knowledge test, while for the latter one the results of the current National Assessment of Knowledge test were used.

The **initial external examination of knowledge** measured the knowledge of pupils at the beginning of the school year in the 9th grade. The objective of the initial external examination of knowledge was double:

- to determine the degree of uniformity of mathematics knowledge between the pupils from schools with homogenous ability groups and from schools with heterogeneous ability groups.
- to determine **hypothetical levels**. We introduced hypothetical levels in order to compare the achievements of pupils with different initial levels of knowledge

(which is the basic criteria for the composition of homogenous ability groups). Within each school from the sample of schools we divided the 9thgrade pupils into three equal groups called hypothetical levels with regard to their achievements in the initial external examination of knowledge (at the beginning of the school year). During the school year, we monitored the achievements of pupils in the hypothetical levels in homogenous and heterogeneous groups.

The initial examination complied in all essential traits (content and complexity of tasks, implementation, principles of scoring, moderation) with the National Assessment of Knowledge tests. It consisted of 12 tasks and was performed externally at the beginning of November 2011. All solved tasks were scored by the project collaborators. Here are two exemplary tasks from the initial external examination:

Task 6. *Nik has four 2 Euro coins, seven 50 cents coins, three 20 cents coins, two 5 cents coins, and five 1 cent coins.*

- a) What is the total value of Nik's coins?
- b) Nik wants to change his coins into paper money. What is the maximal value of paper money he can get?
- c) What is the maximal number of coins Nik can exchange for paper money?

Task 8. *The state of Arsonia has a flag: the length to width ratio of the flag is 19: 10.*

- a) Every school in the state of Arsonia has a flag that measures 1 m in width. What is the length of the flag?
- b) The Arsonians sell also smaller flags that are 28.5 cm long. What is the width of these flags?

The final external examination measured knowledge at the end of the school year in the 9th grade, after the pupils of the 9th grade had mathematics lessons in homogenous and heterogeneous ability groups, respectively. In the final external examination of knowledge, we used the National Assessment of Knowledge test, administered in schools in May 2012.

The initial and final internal examinations of knowledge simulated the essential traits of the internal assessment of knowledge. These examinations were conducted immediately before and after presenting the content of Pyramids (8 to 10 lessons taught) during the regular teaching process. All the administered tasks were open-ended, in their number and form similar to the tasks used at teacher's assessment of the knowledge of geometry. The examination tasks were equal for all pupils from the sample, the scoring scheme was provided by the researchers, but the teachers marked the tests of their own pupils.

Data Processing

Data analysis comprised the comparisons between the achievements of the learners in homogeneous and heterogeneous groups as a whole, and by the hypothetical

levels. For this purpose t-test for independent samples was used. In order to compare the acquisition of knowledge between the pupil achievements in homogeneous and heterogeneous groups, a series of analyses of covariance was performed. The assumptions of normality were controlled by considering skewness and kurtosis, which in all cases at most slightly exceeded 1 in absolute value, but the assumption of homogeneity of variances was in some cases violated (which may result in Type I statistical error).

Results and Interpretation

Achievements of Pupils in the Initial External Examination of Knowledge

As shown in Table 2, there were no statistically significant differences in the average values of the obtained scores in the initial external examination between the groups of pupils that learned mathematics in homogeneous and heterogeneous ability groups ($t = .056$, $df = 885$, $p > .05$).

Table 2

The comparison between achieved average scores in the initial external examination in mathematics of pupils from homogeneous and heterogeneous ability groups using t-test

Differentiation model	N	M	SD	t	Sig.
Homogeneous groups	529	16.41	7.873		
Heterogeneous groups	358	16.44	8.578	0.056	0.955

As explained in Methodology, pupils within each school were divided into three hypothetical levels regarding the demonstrated knowledge in the initial external examination (irrespective of the differentiation model applied in their school). In the first (second, third) hypothetical level there were all pupils that according to their achievements in the initial external examination ranked within their school into the lower (middle, upper) third. Table 3 exposes the comparison of achievements of homogenous and heterogeneous groups by different hypothetical levels. Note that the average achievements of pupils from homogeneous and pupils from heterogeneous ability groups do not statistically differ by all hypothetical levels.

Table 3

The comparison between average achievements in the initial external examination of knowledge for homogeneous and heterogeneous ability groups by hypothetical levels using t-test

Hypothetical level	Differentiation model	N	M	SD	t	Sig.
1 st hypothetical level	Homogeneous groups	189	9.26	3.699	-1.818	0.070
	Heterogeneous groups	125	8.50	3.519		
2 nd hypothetical level	Homogeneous groups	164	15.83	3.845	0.244	0.807
	Heterogeneous groups	110	15.96	4.835		
3 rd hypothetical level	Homogeneous groups	176	24.63	5.928	0.426	0.670
	Heterogeneous groups	123	24.94	6.650		

We can conclude that groups of pupils in the sample that had lessons in the homogeneous and heterogeneous groups, respectively, did not differ significantly in mathematics knowledge demonstrated in the initial external examination.

Achievements of Pupils in the Final External Examination of Knowledge

The results obtained in the final external examination of knowledge were used to compare the demonstrated knowledge of pupils that had lessons in homogeneous and heterogeneous ability groups, respectively. Since the same sample was used for the initial external examination of knowledge (beginning of the school year), the progress of pupils' knowledge between (sub)groups of pupils can be compared as well.

Table 4 shows that the average achievement of pupils in math heterogeneous groups was a bit higher than the average achievement of pupils in math homogeneous groups but the difference is not statistically significant. A comparison by hypothetical levels reveals that learners from heterogeneous groups obtained a significantly higher average score only in the 2ndhypothetical level.

Table 4

Comparison between average achievements in the final external examination of knowledge from homogeneous and heterogeneous ability groups by the hypothetical levels using t-test

Hypothetical level	Differentiation model	N	M	SD	t	Sig.
1 st hypothetical level	Homogeneous groups	186	17.40	8.662	0.642	0.522
	Heterogeneous groups	113	18.15	10.471		
2 nd hypothetical level	Homogeneous groups	158	25.18	8.581	2.175	0.031
	Heterogeneous groups	97	27.93	10.454		
3 rd hypothetical level	Homogeneous groups	163	35.66	7.871	0.986	0.325
	Heterogeneous groups	108	36.68	8.628		
Total	Homogeneous groups	529	25.74	11.398	-1.879	0.061
	Heterogeneous groups	342	27.30	12.517		

We considered also the success of each pupil in the final external examination adjusted by the achievement in the initial external examination. The analysis of covariance of the achievements in the final external examination relative to the results of the initial external examination (Table 5) demonstrates a statistically significant influence of differentiation model. We can conclude that, after the external examination of knowledge, the pupils in heterogeneous ability groups accomplished more progress in comparison to the initial external examination.

The progress of pupils by the hypothetical levels is evident from the comparison of achievements in the final external examination of knowledge adjusted for the achievements in the initial external examination of knowledge by the hypothetical levels (Table 5). In the first and second hypothetical level, a statistically significant higher average achievement is noticed in heterogeneous grouping, while in the third hypothetical level the difference is not statistically significant. Hence, the external

examination shows that the pupils who were in the lower or middle third in their school (according to the initial external examination) progressed more significantly when placed in heterogeneous groups. With the pupils from the upper third in their school, no statistically significant difference was noticed for the pupils in homogeneous and heterogeneous ability groups.

Table 5

Adjusted comparison between the achievements of pupils from homogeneous and heterogeneous ability groups (analysis of covariance)

Hypothetical level	Differentiation model	N	M	F and partial η^2	Sig.
1 st hypothetical level	Homogeneous groups	186	16.89	5.480	0.020
	Heterogeneous groups	113	18.98	0.018	
2 nd hypothetical level	Homogeneous groups	158	24.97	12.681	0.000
	Heterogeneous groups	97	28.28	0.048	
3 rd hypothetical level	Homogeneous groups	163	35.75	0.960	0.362
	Heterogeneous groups	108	36.52	0.004	
Total	Homogeneous groups	529	25.65	12.655	0.000
	Heterogeneous groups	342	27.49	0.015	

Achievements of Pupils in the Initial Internal Assessment of Knowledge

The effect of differentiation model on knowledge was studied also with regard to the internal assessment of knowledge on Pyramids. The teaching of this relatively difficult topic of space geometry was preceded by the initial internal examination of knowledge, and was followed by the final internal examination of knowledge. During the lessons on pyramids, some schools were submitted to unobtrusive observation.

Table 6

Comparison of average achievements in the initial internal examination of knowledge between the pupils from homogeneous and heterogeneous ability groups

Hypothetical level	Differentiation model	N	M	SD	t	Sig.
1 st hypothetical level	Homogeneous groups	171	10.52	5.830	-2.368	0.019
	Heterogeneous groups	108	8.71	6.437		
2 nd hypothetical level	Homogeneous groups	152	14.50	5.609	-0.821	0.413
	Heterogeneous groups	93	13.88	5.907		
3 rd hypothetical level	Homogeneous groups	169	19.22	5.859	0.145	0.885
	Heterogeneous groups	108	19.32	5.012		
Total	Homogeneous groups	492	14.74	6.807	-4.193	0.111
	Heterogeneous groups	310	13.98	7.300		

The pupils from homogeneous and heterogeneous groups achieved similar results (Table 6) in the initial internal examinations. In general, there were no significant

differences between homogeneous and heterogeneous groups, but in the hypothetical levels, a statistically significant difference was noticed only in the first hypothetical level.

Achievements of Pupils in the Final Internal Assessment of Knowledge

In the final internal examination of knowledge in all hypothetical levels, the pupils in homogeneous groups obtained higher average achievements (Table 7). The difference is statistically significant on the whole, as well as for all the hypothetical levels.

Table 7

Comparison between average achievements in the final internal examination of knowledge of the pupils from homogeneous and heterogeneous ability groups using t-test

Hypothetical level	Differentiation model	N	M	SD	t	Sig.
1 st hypothetical level	Homogeneous groups	179	8.78	6.030	-2.422	0.007
	Heterogeneous groups	106	6.87	5.208		
2 nd hypothetical level	Homogeneous groups	154	13.86	6.864	-2.837	0.005
	Heterogeneous groups	94	11.53	5.893		
3 rd hypothetical level	Homogeneous groups	167	19.04	6.222	-2.722	0.007
	Heterogeneous groups	105	16.94	6.141		
Total	Homogeneous groups	535	13.86	7.649	-4.121	0.000
	Heterogeneous groups	326	11.71	7.120		

The pupils in homogeneous groups also achieved significantly better results in the final internal examination when the adjustment for the initial internal examination of knowledge was applied. The analysis of covariance shows a significantly better acquisition of knowledge on pyramids for all the pupils in homogeneous groups. The adjustment of results for the initial internal examination of knowledge in the 1st hypothetical level displays no significant differences between the groups, but there was a statistically significant difference in the 2nd hypothetical level and in the 3rd one.

Table 8

Adjusted comparison between achievements of the pupils from homogeneous and heterogeneous ability groups in the final internal examination (analysis of covariance)

Hypothetical level	Differentiation model	N	M	F and partial η^2	Sig.
1 st hypothetical level	Homogeneous groups	164	8.45	2.811	0.095
	Heterogeneous groups	93	7.37	0.011	
2 nd hypothetical level	Homogeneous groups	146	13.72	7.626	0.006
	Heterogeneous groups	85	11.63	0.032	
3 rd hypothetical level	Homogeneous groups	161	19.02	10.325	0.001
	Heterogeneous groups	98	16.91	0.039	
Total	Homogeneous groups	504	13.72	17.460	0.000
	Heterogeneous groups	293	12.03	0.022	

The results therefore show that, in the internal examination of knowledge after the study of a chosen (rather complex) topic, the average achievement was significantly higher in homogeneous ability groups despite the fact that, at the beginning of teaching on the mentioned topic, there were no significant differences regarding average achievements. Furthermore, the achieved progress of knowledge of pupils during the teaching of the considered topic was significantly higher in homogeneous groups, especially in the 2nd and 3rd ability groups.

The summary of the presented results (Table 9) displays in which differentiation model we found statistically significantly higher achievements.

Table 9

Comparison between the achievements of pupils from schools with homogeneous ability groups and from schools with heterogeneous ability groups

	1 st hypothetical level	2 nd hypothetical level	3 rd hypothetical level	Total
Initial external examination	NS	NS	NS	NS
Final external examination	NS	Het	NS	NS
Final external examination adjusted for the initial external examination	Het	Het	NS	Het
Initial internal examination	Hom	NS	NS	NS
Final internal examination	Hom	Hom	Hom	Hom
Final internal examination adjusted for the initial internal examination	NS	Hom	Hom	Hom

Note: Het: statistically significantly higher achievement of pupils in heterogeneous groups; Hom: statistically significantly higher achievement of pupils in homogeneous groups; NS: a difference in the average achievements between homogeneous and heterogeneous groups is not statistically significant

Discussion

In the introductory section we referred to the extensive body of research on grouping learners into homogeneous and heterogeneous groups, and the effect on their achievements in mathematics. As we have pointed out, the results of these studies were not unanimous and all-encompassing; an important factor that influences the effect of the type of grouping on achievements, for example, was found to be the learners' ability. Our research, we believe, revealed the influence of the type of assessment (external or internal) on the measured level of mathematics knowledge in homogeneous and heterogeneous groups. In particular, we proposed an explanation why many teachers perceived that teaching in homogeneous groups was more effective for all learners, while external examinations showed a different picture.

The final external assessment of knowledge showed that there were no statistically significant differences in average achievements between the pupils in homogeneous and heterogeneous ability groups.

The final internal assessment of knowledge showed that the pupils in homogeneous ability groups achieved significantly better average results compared with the pupils from heterogeneous groups.

When the pupils' individual progress was measured with external examinations, a statistically significant difference in the progress between the pupils in homogeneous and heterogeneous groups was found. The pupils, ranked in the lower or middle third in their school in the initial external examination, progressed better (in the final external examination) when they were in heterogeneous ability groups. With the pupils from the upper third, no significant differences were noticed.

When the pupils' individual progress was measured with internal examinations, a statistically significant difference in the progress between the pupils in homogeneous and heterogeneous groups was found. The pupils, ranked in the middle or upper third in their school in the external initial examination, progressed better when they were placed in homogeneous ability groups. With the pupils from the lower third, no significant differences were noticed. In other words:

- the pupils in the 1st hypothetical level made better progress during the school year in heterogeneous ability groups referring to the external examination results, while the internal assessment did not display differences in progress with regard to learning a specific topic.
- the pupils in the 2nd hypothetical level made better progress in heterogeneous ability groups during the school year referring to the internal examination results, while the internal assessment displayed significantly better progress in learning a specific topic in homogeneous ability groups.
- the pupils in the 3rd hypothetical ability group progressed (according to the external examinations) equally well during the school year regardless the differentiation model, while the internal assessment showed significantly better progress in learning a specific topic in homogeneous groups.

According to our interpretation, the differences between achievements in the internal and external assessment of knowledge are related to the content of examinations. The internal assessment of knowledge in our research, like the usual internal assessment of knowledge (written tests prepared by teachers), comprised a narrow part of the learned subject, thus, very specific learning targets were assessed, pupils could predict what was going to be assessed on the basis of their previous lessons. On the other hand, the external assessment of knowledge (e.g. National Assessment of Knowledge) comprised quite a wide area of mathematics contents. The initial external examination assessed the contents from the 1st to the 8th grade, while the final external examination comprised the contents from the 1st to the 9th grade. In contrast with the internal assessment, tasks in the external assessment are related to the basic knowledge and essential objectives of the most important contents. We have to emphasize that external examinations contained also challenging problem tasks, but they were related to the basic and well-known contents.

The research, therefore, shows that pupils master some contents better in homogeneous ability groups but, with the exception of high-achievers, they often forget soon what they have learned. On the other hand, basic knowledge is better maintained and developed in heterogeneous groups, which can be especially true for low-achieving and mid-achieving pupils.

In other words, following the results of our study, low-achieving learners acquire more knowledge in heterogeneous ability groups. Even though teaching in heterogeneous groups is less adjusted to the abilities of each pupil, it offers more opportunities for implicit learning. The pupils learn simple things from high-achieving pupils, they often come across basic mathematics situations, they meet higher (but not too high) demands. In long terms, they acquire the basic knowledge that is externally assessed in our country. On the other hand, the research shows that for high-achieving pupils learning in homogeneous groups suits better when only the acquisition of mathematics knowledge is considered. While explaining the lesson, teachers can turn to learning objectives that are more complex. We may only presume that pupils master complex knowledge on long terms since external assessments (e.g. National Assessment of Knowledge) assess primarily the basic contents and general mathematical thinking.

Our research points out to some aspects (pros and cons) of external differentiation in the acquisition of mathematics knowledge. There is no unique response to the dilemma whether teaching in homogeneous groups is more effective than in heterogeneous ones when it comes to knowledge acquisition. In heterogeneous groups pupils, especially low-achieving and mid-achieving ones, acquire more basic mathematics knowledge. In homogeneous groups pupils, particularly high-achieving and mid-achieving ones, attain more complex knowledge. Regarding the results, it can be argued which knowledge to prioritize - basic or more complex. In our opinion, this is not the right dilemma. Speaking about the acquisition of quality knowledge, the relevant question is: How to improve teaching and adjust it to pupils, considering their needs and potentials, and diminish the detected deficiencies of learning in homogeneous and heterogeneous groups. Askew and Wiliam (1995) report that the grouping of pupils is efficient only if teaching methods and materials are adjusted to pupils.

Bearing in mind the acquisition of quality knowledge of all pupils, the research results indicate that the adequate adjustment of teaching to individual pupil is necessary. In fact, when we monitored teaching (Valenčič Zuljan et al., 2012), we observed that internal differentiation and individualization were hardly present. Monitoring in homogeneous groups revealed that collaborative learning, problem-teaching connected to real situations, guided discovery, and discussions were significantly more frequent in the third than in the first and second ability group (Valenčič Zuljan et al., 2012). On the other hand, in heterogeneous ability groups, only the traces of internal differentiation were observed and, in particular, high-achievers

were very rarely confronted with challenging tasks. Since the teaching of mathematics in the upper grades of our primary school occurs in groups that are not big in size, teaching could and should be better adjusted to the abilities of individual pupils.

Conclusion

The research gives us an insight into the advantages and disadvantages of different grouping concerning the acquisition of mathematics knowledge. The results show that, from the cognitive aspect of knowledge acquisition, there is no evident response with regard to the advantages of teaching in homogeneous and heterogeneous ability groups, respectively. In heterogeneous groups, pupils acquire more basic mathematics knowledge, in particular low-achieving and mid-achieving ones. In homogeneous groups, pupils attain more complex knowledge, especially high-achieving and mid-achieving ones. Concerning the results, the question is whether basic or more complex knowledge should be our priority. In our opinion, the question should be reversed. As for the acquisition of quality knowledge, the basic question is how to improve teaching in the first or second case, and adjust it to pupils considering their needs and potentials and, thus, diminish the possible disadvantages of both.

The results also indicate that teaching should be adjusted to the individual abilities of pupils regardless the type of grouping -homogeneous or heterogeneous. When it comes to the acquisition of quality knowledge, we should develop such approaches to teaching and learning that challenge all pupils and improve their level of knowledge regardless the teaching model they are included into. The mentioned objective can be realized only by well-trained teachers.

References

- Agirdag, O., Van Avermaet, P., & Van Houtte, M. (2013). School segregation and math achievement: A mixed-method study on the role of self-fulfilling prophecies. *Teachers College Record*, 115(3), 1-50.
- Askew, M., & Wiliam, D. (1995). *Recent research in mathematics education*. London: OFSTED, 5–16.
- Belfi, B., Goos, M., De Fraine, B., & Van Damme, J. (2012). The effect of class composition by gender and ability on secondary school students' school well-being and academic self-concept: A literature review. *Educational Research Review*, 7(1), 62-74. <http://dx.doi.org/10.1016/j.edurev.2011.09.002>
- Biehler, R. F., & Snowman, J. (1993). *Psychology Applied to Teaching (Seventh Edition)*. Boston: Houghton Mifflin Company.
- Boaler, J. (1997). When even the winners are losers: Evaluating the experiences of 'top set' students. *Journal of Curriculum Studies*, 29(2), 165-182. <http://dx.doi.org/10.1080/002202797184116>

- Brophy, J. E., & Good, T. L. (1986). Teacher behavior and student achievement. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (3rded.)(pp. 328–375). New York: Macmillan.
- Bucik, V. (2001). Zakaj potrebujemo kakovostno zunanje preverjanje in ocenjevanje znanja? *Sodobna pedagogika*, 52(3), 40-52.
- Chiu, D., Beru, Y., Watley, E., Wubu, S., Simson, E., Kessinger, R., & Wigfield, A. (2008). Influences of math tracking on seventh-grade students' self-beliefs and social comparisons. *The Journal of Educational Research*, 102(2), 125-136. <http://dx.doi.org/10.3200/JOER.102.2.125-136>
- Chmielewski, A. K., Dumont, H., & Trautwein, U. (2013). Tracking Effects Depend on Tracking Type An International Comparison of Students' Mathematics Self-Concept. *American Educational Research Journal*, 50(5), 925-957. <http://dx.doi.org/10.3102/0002831213489843>
- Dupriez, V. (2010). *Methods of Grouping Learners at School. Fundamentals of Educational Planning – 93*. UNESCO: International Institute for Educational Planning, Paris. Retrieved on 10th February 2011 from http://www.iiep.unesco.org/fileadmin/user_upload/Info_Services/Publications/pdf/2010/Fund_93.pdf
- European Commission. (2006). Commission staff working document. Accompanying document to the Communication from the Commission to the Council and to the European Parliament. *Efficiency and Equity in European education and training systems*. SEC(2006)1096, Brussels: European Commission.
- Gutiérrez, R., & Slavin, R. E. (1992). Achievement Effects of the Nongraded Elementary School: A Best-Evidence Synthesis. *Review of Educational Research*, 62, 333-376. <http://dx.doi.org/10.3102/00346543062004333>
- Hattie, J. A. (2002). Classroom composition and peer effects. *International Journal of Educational Research*, 37(5), 449-481. [http://dx.doi.org/10.1016/S0883-0355\(03\)00015-6](http://dx.doi.org/10.1016/S0883-0355(03)00015-6)
- Hattie, J. (2009). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement*. London: Rutledge.
- INCLUD-ED (2009). Actions for success in schools in Europe. Retrieved on 10th February 2011 from http://www.helsinki.fi/~reunamo/article/INCLUDED_actions%20for%20success.pdf
- Ireson, J., & Hallam, S. (1999). Raising standards: is ability grouping the answer?. *Oxford review of education*, 25(3), 343-358. <http://dx.doi.org/10.1080/030549899104026>
- Ireson, J., & Hallam, S. (2009). Academic self-concepts in adolescence: Relations with achievement and ability grouping in schools. *Learning and Instruction*, 19(3), 201-213. <http://dx.doi.org/10.1016/j.learninstruc.2008.04.001>
- Marentič Požarnik, B. (2001). Zunanje preverjanje, kultura učenja in kakovost (maturitetnega) znanja. *Sodobna pedagogika*, 52(3), 54–75.
- Prosen, S., Smrtnik Vitulić, H., & Poljšak-Škraban, O. (2013). Observing teachers' emotional expression in their interaction with students. *The new educational review*, 31(1), 75-85.
- Slavin, R. E. (1987). Ability grouping and student achievement in elementary schools: A best-evidence synthesis. *Review of educational research*, 57(3), 293-336. <http://dx.doi.org/10.3102/00346543057003293>

- Slavin, R. E. (1990). Achievement effects of ability grouping in secondary schools: A best-evidence synthesis. *Review of educational research*, 60(3), 471-499. <http://dx.doi.org/10.3102/00346543060003471>
- Strmčnik, F. (2001). *Didaktika. Osrednje teoretične teme*. Ljubljana: Znanstveni inštitut FF.
- Trautwein, U., Lüdtke, O., Marsh, H. W., Köller, O., & Baumert, J. (2006). Tracking, grading, and student motivation: Using group composition and status to predict self-concept and interest in ninth-grade mathematics. *Journal of Educational Psychology*, 98(4), 788-806. <http://dx.doi.org/10.1037/0022-0663.98.4.788>
- Valenčič Zuljan, Cotič, M., Felda D., Magajna, Z., & Žakelj, A. (2012). Kazalniki socialnega kapitala, kulturnega kapitala in šolske klime v napovedovanju šolske uspešnosti otrok in mladostnikov - V5-1026. Podprojekt – Diferenciacija in individualizacija. Poročilo projekta. Univerza v Ljubljani, Pedagoška fakulteta: Univerza v Ljubljani, Filozofska fakulteta; Univerza na Primorskem, Pedagoška fakulteta [2012]. 183 pp. Retrieved on 10th January 2014 from http://www.mizs.gov.si/fileadmin/mizs.gov.si/pageuploads/področje/razvoj_solstva/evalvacija/pdf/POROCILO-Kazalniki_socialnega_kulturnega_kapitala_in_solske_klime....pdf
- Wilkinson, I. A., & Fung, I. Y. (2002). Small-group composition and peer effects. *International Journal of Educational Research*, 37(5), 425-447. [http://dx.doi.org/10.1016/S0883-0355\(03\)00014-4](http://dx.doi.org/10.1016/S0883-0355(03)00014-4)
- Žagar, D., Pečjak, S., & Peklaj, C. (2003). Organizacija nivojskega pouka, stališča učencev, učiteljev in staršev do te oblike diferenciacije ter njeni učinki na učence: evalvacijkska študija: zaključno poročilo. Ljubljana: Univerza v Ljubljani, Filozofska fakulteta.
- Žakelj, A., & Ivanuš-Grmek, M. (2010). *Povezanost rezultatov pri nacionalnem preverjanju znanja s socialno-kulturnim okoljem učencev, poukom in domačimi nalogami*. Ljubljana: Zavod Republike Slovenije za šolstvo.

Zlatan Magajna

Faculty of Education, University of Ljubljana
Kongresni trg 12, 1000 Ljubljana, Slovenia
zlatan.magajna@pef.uni-lj.si

Milena Valenčič-Zuljan

Faculty of Education, University of Ljubljana
Kongresni trg 12, 1000 Ljubljana, Slovenia
milena.valencic-zuljan@guest.arnes.si

Amalija Žakelj

National Education Institute of the Republic of Slovenia
Poljanska 28, 1000 Ljubljana, Slovenia
amalija.zakelj@zrss.si

Utjecaj diferencijacijskog modela u nastavi matematike na postignuća učenika s obzirom na vanjsko i unutarnje vrednovanje znanja

Sažetak

Predstavljamo istraživanje u kojem razmatramo¹ razlikuju li se značajno učenici koji su imali nastavu matematike u devetom razredu osnovne škole u heterogenim grupama po svojim postignućima, određenim vanjskim i internim vrednovanjem znanja, od učenika u homogenim grupama. Pri vanjskom vrednovanju nisu zabilježene statistički značajne razlike u slučaju prosječnih postignuća. No, interno je vrednovanje znanja učenika u homogenim grupama u prosjeku ukazalo na statistički značajno veća postignuća u usporedbi s učenicima u heterogenim grupama. Analiza kovarijance provedena po razinama za učenike u homogenim i heterogenim grupama pokazala je da su učenici koji su, poslije inicijalnog vrednovanja u svojim školama, našli mjesto u donjoj ili srednjoj trećini, relativno bolje napredovali u heterogenim grupama pri finalnom vanjskom vrednovanju. Učenici koji su se, prema uspjehu iz matematike, našli u gornjoj trećini u svojim školama, nisu pokazali nikakve razlike u tome smislu. Interno je vrednovanje znanja pokazalo da su učenici koji su se u svojim školama, poslije inicijalnog vanjskog vrednovanja, našli u srednjoj ili gornjoj trećini pri finalnom vrednovanju, relativno bolje napredovali u homogenim grupama. Učenici koji su pripadali donjoj trećini poslije inicijalnog vanjskog vrednovanja u svojim školama, nisu pokazali nikakve razlike u tome pogledu.

Ključne riječi: podjela učenika u grupe; individualizacija i diferencijacija; unutarnje vrednovanje znanja; vanjsko vrednovanje znanja; matematika

¹Podaci koji su se koristili u radu dobiveni su iz nacionalnog istraživačkog projekta "Kazalniki socialnega kapitala, kulturnega kapitala in šolske klime v napovedovanju šolske uspešnosti otrok in mladostnikov – CRP V5-1026" (voditeljica projekta Milena Valenčič Zuljan, Pedagoški fakultet Sveučilišta u Ljubljani) podprojekt – Diferencijacija i individualizacija (2012).

Uvod

Grupiranje učenika jedna je od temeljnih pedagoških dilema koja je oduvijek prisutna i promatrana s obzirom na razne ciljeve i kriterije. Glavno je pitanje treba li učenike svrstavati u heterogene grupe prema njihovim sposobnostima ili u homogene grupe prema njihovim kompetencijama. Podjela u homogene grupe otvara nekoliko pitanja: kada treba početi svrstavati učenike u homogene grupe? Koji su razlozi, ciljevi i metode grupiranja? Koliko dugo treba trajati razdoblje podjele učenika na homogene grupe? Ima li jedna grupa sustavne prednosti ili nedostatke u usporedbi s nekom drugom (drugima)?

S obzirom na pravednost, kvalitetu obrazovanja i optimalan razvoj svakog učenika postavljena pitanja još su uvijek aktualna u svakom obrazovnom sustavu.

Danas su stručnjaci zabrinuti za diferencijaciju i individualizaciju, osobito s obzirom na to kako prepoznati individualne razlike među učenicima i kako svakome od njih omogućiti kvalitetno i održivo znanje s pomoći odgovarajuće organizacije i didaktičkog pristupa. Društvene i obrazovne uloge škole također su, dakle, naglašene.

Grupiranje učenika, razlikovanje i individualni pristup bili su u žarištu nekolicine istraživanja. Pregled istraživanja u znanstvenim časopisima s faktorima učinka pokazao je da su istraživanja usmjerena prema različitim nacionalnim sustavima (Belgija, Njemačka, Francuska, Švedska), posebno onima u SAD i UK, kao i prema njihovim usporedbama. Uglavnom su istraživani predmeti matematika i materinski jezik, u kojima najčešće dolazi do razlika među učenicima.

Slavin (1987) je analizirao sedam istraživanja na temu grupiranja učenika u nižim razredima osnovne škole u svom gotovo klasičnom istraživanju. Utvrđio je kako su pretpostavljeni negativni psihološki učinci homogenog grupiranja zanemarivi. Rezultati su pokazali da istraživanja ukazuju na učinkovitost grupa nastalih po načelu sposobnosti u slučajevima kada su nastavne metode, nastavna sredstva i pomagala na odgovarajući način prilagođeni potrebama učenika. Jednostavno grupiranje učenika prema njihovim sposobnostima bez odgovarajuće prilagodbe nema nikakvog učinka bez obzira na nastavni predmet. Neki su drugi autori (Slavin, 1987, 1990; Gutierrez i Slavin 1992; Žagar i sur., 2003), koji su proučavali učinkovitost individualnog i razlikovnog pristupa kao i podjele prema sposobnostima na primjeru osnovnog obrazovanja, pokazali da ne postoji istraživanje koje ide u prilog ovom trećem na spomenutim razinama.

Zagovornici homogene podjele učenika prema sposobnostima pretpostavljaju da homogene grupe omogućuju prilagođenu individualiziranu nastavu koja vodi optimalnim postignućima i kvalitetnom znanju. Organizacija nastave s homogenim grupama daje nastavnicima više vremena za izravnu poduku i aktivnosti kao što su odgovaranje na pitanja učenika, objašnjavanje na jasan način, slušanje učenika i reagiranje na njihove odgovore, informiranje o ciljevima učenja kao načinu motiviranja učenika, praćenje razvoja učenika unutar grupe itd. Takva se nastava temelji na suradnji između nastavnika i učenika, pa je komunikacija uspješna kada

se nastavnik obraća homogenoj grupi učenika. Velike bi heterogene grupe mogle smanjiti aktivnost učenika pri rješavanju zadataka i pritom dovesti do slabijih rezultata (Brophy i Good, 1986; Žagar i sur., 2003). Kada se god aspekt poučavanja preusko promatra, interakcija se među učenicima koja može pridonijeti njihovoj obrazovnoj učinkovitosti ne koristi dovoljno; prema tome, u istraživanja se nastoje uključiti učinci suradnje i međusobnih interakcija koji se ostvaruju različitim načinima podjele (Wilkinson i Fung, 2002). Autori koji su istraživali različito poučavanje i interakcije među učenicima podijeljenim prema različitim sposobnostima (Hattie, 2002) utvrdili su da je grupa u kojoj se nalaze slabiji učenici manje aktivna i manje spremna raditi i da ih poučavaju slabije kvalificirani nastavnici. Hattie (2002) je u raspravi istaknuo da bolje kvalificirani nastavnici, poticajnija sredina i izazovi mogu pomoći učenicima u toj skupini da postignu bolje rezultate. Usko specificirana nastava zapostavlja aspekt obrazovanja koji značajno pridonosi osobnosti i emocionalnom razvoju učenika, a zatim boljoj učinkovitosti pri učenju (npr. razina očekivanja koja imaju nastavnik i učenik, predodžba učenika o samome sebi, stavovi o obrazovanju i znanju) (Agirdag i sur., 2013; Belfi i sur., 2012; Boaler, 1997; Chiu i sur., 2008; Chmielewski i sur., 2013; Ireson & Hallam, 2009; Prosen i sur., 2013; Trautwein i sur., 2006).

Istraživači naglašavaju da kvaliteta poučavanja i vrsta interakcije među učenicima predstavljaju temeljno pitanje nastavne kvalitete, čak više nego sastav grupe i njezina struktura prema sposobnostima učenika.

Učinci podjele učenika već su bili predmetom nekolicine metaistraživanja (Dupriez, 2010; Hattie, 2009). Hattie (2009) je napravio vrlo veliku metaanalizu istraživanja da bi objasnio postignuća učenika. Obuhvatilo je 300 istraživanja o grupiranju po načelu sposobnosti koja su se odnosila na nekoliko istraživanja u gotovo svim nastavnim predmetima tijekom različitih godina školovanja, kao i na različite rezultate učenja. Srednja veličina učinka u homogenim grupama bila je niska ($d = 0.11$; Hattie je postavio vrijednosti koje su bile važne za razumijevanje srednje veličine učinka d , vrijednost $d = 0.40$ bila je vrijednost praga, vrijednosti ispod $d = 0.40$ bile su „male“, a vrijednosti iznad $d = 0.60$ „odlične“). Rezultati su pokazali da podjela učenika prema sposobnostima ima vrlo slab učinak kada su u pitanju njihova postignuća. Taj je učinak za matematiku i čitanje podjednako slab (čitanje $d = 0.00$, matematika $d = 0.02$), a učinak je stava o nastavnom predmetu nešto veći ($d = 0.10$). Prosječni učinak triju temeljnih grupa podijeljenih po sposobnostima u istraživanjima iznosio je: $d = 0.14$ za grupu vrlo uspješnih učenika; $d = -0.03$ za grupu prosječno uspješnih učenika i $d = 0.09$ za grupu slabo uspješnih učenika (Hattie, 2009 str. 90).

Danas se podjela na grupe razmatra osobito sa stajališta pravednosti i jednakih mogućnosti. Projekt INCLUDE-ED razlikuje inkluzivne od ekskluzivnih praksi među kojima su i vanjsko razlikovanje i grupiranje po sposobnostima. Grupiranje po sposobnostima pojavljuje se u raznim oblicima i vrlo je često u europskim zemljama kao odgovor na tradicionalne kombinirane razredne odjele u kojima nastavnik poučava veliku grupu učenika koji dolaze iz različitih kulturnih sredina i imaju

različite sposobnosti. Zbog brojnih razlika među učenicima u današnjima školama, osim velikog broja učenika u svakom odjelu, nastavnici se teško brinu o svakom učeniku i razlikama među njima, pa je takvim načinom rada teško postići zacrtane ciljeve obrazovanja i učenja. Podjela po sposobnostima predstavlja odgovor na takve situacije i način na koji se poučavanje prilagođava razlikama među učenicima. Riječ je o prilagodbi kurikula i formiranju grupa prema sposobnostima učenika s pomoću uključivanja dodatnih ljudskih resursa (INCLUD-ED, 2009, str. 18). U dokumentu Europske komisije (SWD dokument, 2006, str. 19) naglašava se da je rana vanjska diferencijacija koju Europska komisija definira kao „segregaciju djece prema njihovim sposobnostima do 13. godine života“ sasvim česta u nekim školskim sustavima. Njemački i austrijski školski sustavi su, primjerice, odabrani modeli koji razlikuju učenike prema njihovim sposobnostima do 10. godine života (Dupriez, 2010).

Također su zanimljive i razne usporedne analize školskih sustava. Vandenberghe, Dupriez i Zachary (2001, prema Dupriez, 2010) analizirali su postignuća na testovima matematike (TIMSS 1995) i utvrđili da su postignuća učenika u zemljama u kojima se nisu odlučili za diferencijaciju u ranoj dobi manje povezana s društveno-kulturnom pozicijom obitelji nego u zemljama s ranom diferencijacijom.

S obzirom na kognitivne aspekte ovladavanja matematikom u homogenim i heterogenim grupama uvjereni smo da je također važno razmotriti postignuća učenika u odnosu na način vrednovanja (osobito vanjski/unutarnji način). Iz različitih su razloga rezultati vanjskih provjera više ili manje usklađeni s rezultatima internih provjera. Mjerili smo usvajanje znanja s pomoću vanjske i unutarnje metode vrednovanja i u tom smo smislu istražili razliku između homogenih i heterogenih grupa. Zbog činjenice da vanjske i interne provjere vrednuju različite vrste znanja dobili smo uvid u kognitivne učinke ovladavanja matematikom u homogenim i heterogenim grupama.

Metodologija

Definiranje šireg okvira i istraživačkog problema

U slovenskim je osnovnim školama devetogodišnji program, uveden 1999./2000. školske godine, omogućio razne oblike diferencirane nastave. U sljedećem desetljeću nekoliko se puta mijenjao zakonski okvir koji je regulirao modele diferencirane nastave. Kada je provođeno ovo istraživanje (2010./2011.), škole su autonomno odlučile organizirati nastavu matematike u trećem trogodištu po grupama prema heterogenim ili homogenim sposobnostima učenika. Da bi se izbjegle nepotrebne daljnje promjene povezane s vanjskom diferencijacijom, koja uzima u obzir učinkovitost i pravednost školskog sustava, važno je vrednovati učinkovitost raznih modela diferencijacije.

U istraživanjima koja smo spomenuli u uvodu istražuju se konativni aspekti i učinkovitost učenja. Analiza koju navodimo u nastavku otkriva odnos između raznih modela diferencijacije i postignuća učenika pri internom i vanjskom vrednovanju znanja.

Cilj vrednovanja znanja tijekom nastavnog procesa jest utvrditi napredak i razvoj učenika u različitim područjima (kognitivno, konativno, društveno i motoričko), a ono može biti interno ili vanjsko. Interno vrednovanje provode nastavnici na temelju govornih, pisanih ili praktičnih elemenata. Stoga oni sami pripremaju pitanja ili pisane testove, bez dodatne informacije koja bi im omogućila usporedivost vrednovanja i rezultata vrednovanja s onima drugih nastavnika i škola (Žakelj i Ivanuš Grmek, 2010). Interno sumativno vrednovanje znanja neovisan je korak u nastavnom procesu (Strmčnik, 2001) i, osim kurikula, značajno određuje što i kako poučavati, što učenik uči i s pomoću koje metode te, samim time, kvalitetu nastave i učenikovo znanje (Marentič Požarnik, 2001).

Vanjsko se vrednovanje znanja shvaća kao vrednovanje s pomoću testova koje pripremaju obrazovni stručnjaci i stručnjaci za mjerjenje znanja. Glavne značajke vanjskog vrednovanja su: usklađenost s potrebnim standardima kvalitete, jednaki ili usporedivi zadaci koje učenici trebaju rješiti, jednaki evaluacijski i administrativni kriteriji (Bucik, 2001).

Učinkovitost različitih modela diferencijacije i individualizacije složeno je pitanje i odnosi se na kurikularne, emocionalne, društvene, organizacijske i didaktičke aspekte. U ovom bismu radu željeli povezati kognitivni i organizacijsko-didaktički aspekt s vrstama vrednovanja. S tim ćemo ciljem analizirati postignuća učenika iz matematike u homogenim i heterogenim (prema sposobnostima) grupama pri internom i vanjskom vrednovanju znanja.

Temeljno istraživačko pitanje

Razlikuju li se statistički značajno učenici koji su bili u devetom razredu osnovne škole u homogenim grupama tijekom nastave matematike od učenika koji su bili u heterogenim grupama po svojim postignućima iz tog predmeta, mjerenim internim i vanjskim vrednovanjem znanja?¹

U istraživanju analiziramo:

- postignuća učenika podijeljenih prema sposobnostima u homogene i heterogene grupe pri vanjskom vrednovanju znanja iz matematike
- postignuća učenika podijeljenih prema sposobnostima u homogene i heterogene grupe pri internom vrednovanju znanja iz matematike
- napredak učenika u tako formiranim homogenim i heterogenim grupama koji se pokazao pri vanjskom i internom vrednovanju njihova znanja iz matematike.

Uzorak

Istraživanje je provedeno na slučajno odabranom uzorku od 21 osnovne škole u Sloveniji. Sastojao se od svih učenika devetog razreda u odabranim školama, ukupno 887 učenika. Devet škola (ukupno 358 učenika) imalo je heterogene matematičke grupe, a njih 12 (ukupno 529 učenika) imalo je homogene matematičke grupe (podjela na temelju sposobnosti učenika).

U istraživanom uzorku nije bilo statistički značajne razlike u proporciji učenika s posebnim potrebama ($\chi^2=0.283$, $df=1$, $p=0.595$) između škola s homogenim i škola s heterogenim grupama. Statistički značajna razlika u strukturi grupa formiranih prema sposobnostima po školama zabilježena je pri homogenom grupiranju, pri čemu udio učenika s posebnim potrebama nije bio neovisan o grupi ($\chi^2=60.982$, $df=2$, $p = 0.000$). Isto vrijedi za proporciju učenica u grupi ($\chi^2=6.581$, $df=2$, $p=0.037$). U prvoj grupi imali smo značajno veći udio učenika s posebnim potrebama, a u trećoj smo grupi imali značajno veći udio učenica u usporedbi s drugim dvjema grupama.

Tablica 1.

Instrumenti

U istraživanju smo upotrebljavali četiri instrumenta da bismo odredili znanje iz matematike. Inicijalna vanjska provjera provedena je početkom školske godine, a ona finalna na kraju školske godine. Prvu su provjeru pripremili autori istraživanja prema Nacionalnom testu znanja, a u drugoj su se provjeri koristili rezultati aktualnog Nacionalnog testa znanja.

Inicijalna vanjska provjera znanja mjerila je znanje učenika devetog razreda na početku školske godine. Imala je dvostruki cilj:

- Odrediti ujednačenost znanja iz matematike između učenika iz škola s homogenim grupama i onih s heterogenim grupama (homogene i heterogene s obzirom na sposobnosti učenika).
- Postaviti **hipotetske razine**. Uveli smo hipotetske razine s namjerom da usporedimo postignuća učenika s različitom početnom razinom znanja (što je glavni kriterij za sastavljanje homogeno sposobnih grupa). U svakoj školi unutar uzorka podijelili smo učenike devetog razreda na tri jednakaka dijela (hipotetske razine) s obzirom na njihova postignuća u inicijalnom vanjskom vrednovanju znanja (na početku školske godine). Tijekom školske godine pratili smo postignuća učenika na hipotetskim razinama pri homogenim i heterogenim podjelama.

Početna je provjera odgovarala svim ključnim zahtjevima (sadržaj i složenost zadataka, primjena, načela ocjenjivanja, umjerenost) Nacionalnog testa znanja. Sastojala se od 12 zadataka, a provodila izvana početkom studenog 2011. Sve su riješene zadatke ocjenjivali suradnici na projektu. Ovo su dva primjera zadataka iz inicijalne vanjske provjere:

Zadatak 6. Nik ima četiri kovanice od 2 eura, sedam kovanica od 50 centi, tri kovanice od 20 centi, dvije kovanice od 5 centi i pet kovanica od 1 centa.

- a) Koliko iznosi ukupna vrijednost Nikovih kovanica?
- b) Nik želi promijeniti kovanice u papirnate novčanice. Kolika je maksimalna vrijednost papirnatih novčanica koju Nik može dobiti?
- c) Koji je maksimalan broj kovanica koje Nik može zamijeniti za papirnate novčanice?

Zadatak 8. Država Arsonia ima zastavu; omjer dužine i širine zastave iznosi 19:10.

- a) Svaka škola u državi Arsoniji ima zastavu koja je široka 1 m. Koliko je duga zastava?
- b) Stanovnici Arsonije također prodaju manje zastave koje su duge 28.5 cm. Koliko su široke te zastave?

Finalna vanjska provjera mjerila je znanje na kraju školske godine u devetom razredu nakon što su učenici tog razreda imali nastavu matematike u homogeno sposobnim i heterogeno sposobnim grupama, svatko u svojoj. U finalnoj smo vanjskoj provjeri znanja upotrijebili Nacionalni test znanja koji se provodio po školama u svibnju 2012.

Inicijalne i finalne interne provjere znanja simulirale su ključne karakteristike interne provjere znanja. Te su provjere provedene neposredno prije i poslije poduke o piramidama (sadržaj je obuhvatio 8 – 10 nastavnih sati) tijekom redovite nastave. Svi su zadaci bili otvorenog tipa, po broju i formi slični zadacima s pomoću kojih nastavnik vrednuje znanje iz geometrije. Ispitni su zadaci bili jednaki za sve učenike koji su činili uzorak, načela bodovanja predložili su autori istraživanja, ali su nastavnici ocjenivali testove svojih učenika.

Obrada podataka

Analiza podataka sastojala se od usporedbe učeničkih postignuća po grupama (homogene i heterogene), cjelovite i prema hipotetskim razinama. Za to se koristio t-test za neovisne uzorke. Da bi se usporedilo usvajanje znanja između učenika u homogenim i heterogenim grupama, proveden je niz analiza kovarijance. Kontrolirane su prepostavke normalnosti tako što su razmatrani skewness i kurtosis, što posvuda u najboljem slučaju neznatno premašuje 1 u apsolutnoj vrijednosti, iako je prepostavka o homogenosti varijanci bila narušena u nekim slučajevima (što može dovesti do statističke pogreške Tipa I).

Rezultati i tumačenje

Postignuća učenika pri inicijalnoj vanjskoj provjeri znanja

Kao što prikazuje Tablica 2, nisu zabilježene statistički značajne razlike u srednjim vrijednostima rezultata dobivenih pri inicijalnoj vanjskoj provjeri između učenika koji su učili matematiku u homogenim, odnosno heterogenim grupama ($t=0.056$, $df = 885$, $p>.05$).

Tablica 2.

Kao što je objašnjeno u metodologiji, učenici su u svakoj školi bili podijeljeni na tri hipotetske razine s obzirom na pokazano znanje pri inicijalnoj vanjskoj provjeri (bez obzira na diferencijacijski model koji se koristio u njihovoј školi). Na prvoj (drugoј, trećeј) hipotetskoj razini bilo je svih učenika, koji su prema postignućima pri inicijalnoj vanjskoj provjeri u svojoj školi rangirani u nižu (srednju, višu) trećinu.

Tablica 3 prikazuje usporedbu rezultata postignutih u homogenim i heterogenim grupama prema različitim hipotetskim razinama. Obratite pozornost na to da se prosječna postignuća učenika iz homogeno sposobnih i onih iz heterogeno sposobnih grupa statistički ne razlikuju prema svim hipotetskim razinama.

Tablica 3.

Možemo zaključiti da se učenici iz uzorka koji su imali nastavu u homogenim, odnosno heterogenim grupama nisu statistički značajno razlikovali u svom matematičkom znanju pokazanom pri inicijalnoj vanjskoj provjeri.

Postignuća učenika pri finalnoj vanjskoj provjeri znanja

Rezultati dobiveni pri finalnoj vanjskoj provjeri koristili su se za usporedbu pokazanog znanja učenika koji su imali nastavu u homogeno sposobnim, odnosno heterogenim grupama. Budući da se isti uzorak koristio pri inicijalnoj vanjskoj provjeri znanja (na početku školske godine), mogao se također uspoređivati napredak u znanju između (pod)grupa učenika.

Tablica 4 pokazuje da je prosječni uspjeh učenika koji su nastavu matematike imali u heterogenim grupama bio malo veći od prosječnog uspjeha učenika u homogenim grupama, ali ta razlika nije statistički značajna. Usporedba prema hipotetskim razinama otkriva da su učenici u heterogenim grupama postigli značajno veći uspjeh samo na drugoj hipotetskoj razini.

Tablica 4.

Razmotrili smo također uspjeh svakog učenika u fazi finalne vanjske provjere u odnosu na uspjeh u fazi inicijalne vanjske provjere. Analiza kovarijance uspjeha u fazi finalne vanjske provjere u odnosu na rezultate u fazi inicijalne vanjske provjere (Tablica 5) pokazuje statistički značajan utjecaj diferencijacijskog modela. Možemo zaključiti da su, nakon vanjske provjere znanja, učenici u heterogeno sposobnim grupama bolje napredovali u odnosu na inicijalnu vanjsku provjeru.

Tablica 5.

Napredak učenika u odnosu na hipotetsku razinu vidljiv je iz usporedbe postignuća pri finalnoj vanjskoj provjeri znanja prilagođenoj za postignuća u inicijalnoj vanjskoj provjeri znanja prema hipotetskim razinama (Tablica 5). Na prvoj i drugoj razini primjećuje se statistički značajno već prosječni uspjeh pri heterogenom grupiranju, a na trećoj hipotetskoj razini ta razlika nije statistički značajna. Prema tome, vanjska provjera pokazuje da su učenici koji su pripadali nižoj ili srednjoj trećini u svojim školama (prema inicijalnoj vanjskoj provjeri) značajno više napredovali kada ih se smjestilo u heterogene grupe. Kada je riječ o učenicima koji su pripadali višoj trećini u svojim školama, nije zabilježena statistička značajna razlika za učenike u homogeno i heterogeno sposobnim grupama.

Postignuća učenika pri inicijalnoj internoj provjeri znanja

Učinak diferencijacijskog modela na znanje također je istraživan s pomoću internog vrednovanja znanja o piramidama. Poučavanju o toj relativno teškoj temi prostorne geometrije prethodila je inicijalna interna provjera znanja, a nakon poučavanja slijedila je finalna interna provjera. Tijekom nastave o piramidama neke su škole bile podvrgnute nemametljivom promatranju.

Učenici iz homogenih i heterogenih grupa postigli su slične rezultate (Tablica 6) pri inicijalnoj internoj provjeri. Općenito gledano, nije zabilježena statistički značajna razlika između homogenih i heterogenih grupa, ali je na hipotetskim razinama statistički značajna razlika primijećena samo na onoj prvoj.

Tablica 6.

Postignuća učenika pri finalnoj internoj provjeri znanja

Pri finalnoj internoj provjeri znanja na svim hipotetskim razinama učenici u homogenim grupama postigli su bolji prosječni uspjeh (Tablica 7). Razlika je statistički značajna u cjelini kao i na svim hipotetskim razinama.

Tablica 7.

Učenici u homogenim grupama postigli su značajno bolje rezultate pri finalnoj internoj provjeri također kada se primijenila prilagodba za inicijalnu internu provjeru znanja. Analiza kovarijance pokazuje značajno veći uspjeh u znanju o piramidama za sve učenike u homogenim grupama. Prilagodba rezultata za inicijalnu internu provjeru znanja na prvoj hipotetskoj razini ne pokazuje statistički značajnu razliku između grupa, ali ona je zabilježena na drugoj i trećoj hipotetskoj razini.

Tablica 8.

Rezultati stoga pokazuju da je pri internoj provjeri znanja nakon proučavanja odabrane (prilično složene) teme prosječni uspjeh bio značajno veći u homogeno sposobnim grupama unatoč činjenici da na početku nastave o spomenutoj temi nisu postojale značajne razlike s obzirom na prosječni uspjeh. Štoviše, postignuti napredak u znanju učenika za vrijeme poučavanja spomenute teme bio je značajno veći u homogenoj grupi, osobito na drugoj i trećoj (prema sposobnostima) razini.

Sažetak rezultata (Tablica 9) pokazuje kod kojeg diferencijacijskog modela nalazimo statistički značajno bolje rezultate.

Tablica 9.

Raspis

U uvodnom smo dijelu spomenuli bogat korpus istraživanja o podjeli učenika u homogene i heterogene grupe i učinku na njihova postignuća u matematici. Kao što smo istaknuli, njihovi rezultati nisu jedinstveni i sveobuhvatni tako da su se kao

važan čimbenik povezan s učinkom načina grupiranja na postignuća, primjerice, pokazale sposobnosti učenika. Uvjereni smo da je naše istraživanje ukazalo na to kakav je utjecaj vrste provjere (vanjsko ili interno) znanja iz matematike na mjerelim razinama u homogenim i heterogenim grupama. Prije svega nudimo objašnjenje zašto mnogi nastavnici smatraju da je poučavanje u homogenim grupama učinkovitije za sve učenike, iako vanjske provjere znanja daju drugačiju sliku.

Finalna vanjska provjera znanja nije pokazala statistički značajne razlike u prosječnim postignućima učenika podijeljenima prema sposobnostima u homogene i heterogene grupe.

Finalna interna provjera znanja pokazala je da su učenici u homogenim grupama postigli statistički značajno bolje rezultate u usporedbi s učenicima u heterogenim grupama.

Kada je vanjskim vrednovanjem mjerem individualni napredak, pronađena je statistički značajna razlika između učenika u homogenim i heterogenim grupama. Učenici koji su pripadali nižoj ili srednjoj trećini u svojim školama pri inicijalnoj vanjskoj provjeri bolje su napreovali (pri finalnoj vanjskoj provjeri) kada su bili u heterogenim grupama. U slučaju gornje trećine učenika nisu zabilježene statistički značajne razlike.

Kada je internim vrednovanjem mjerem individualni napredak, pronađena je statistički značajna razlika između učenika u homogenim i heterogenim grupama. Učenici koji su bili rangirani u srednjoj ili gornjoj trećini u svojim školama pri inicijalnoj vanjskoj provjeri bolje su napreovali kada su bili smješteni u homogene grupe. U slučaju učenika iz donje trećine nisu zabilježene statistički značajne razlike. Drugim riječima:

- Učenici na prvoj hipotetskoj razini bolje su napreovali u toku školske godine u heterogenim grupama u odnosu na rezultate vanjske provjere znanja, a interno vrednovanje nije pokazalo razlike u napretku pri usvajanju određene teme.
- Učenici na drugoj hipotetskoj razini bolje su napreovali u toku školske godine u heterogenim grupama u odnosu na rezultate interne provjere znanja, a interno je vrednovanje pokazalo značajno bolji napredak pri usvajanju određene teme u homogenim grupama.
- Učenici na trećoj hipotetskoj razini napreovali su podjednako dobro u toku školske godine (prema rezultatima vanjskog vrednovanja) bez obzira na model podjele, a interno je vrednovanje pokazalo značajno bolji napredak pri usvajanju određene teme u homogenim grupama.

Prema našem tumačenju, razlike u postignućima pri internoj i vanjskoj provjeri znanja povezane su sa sadržajem. Interna provjera u našem istraživanju, kao što je za nju uobičajeno (pisani testovi koje priprema nastavnik), sastojala se od manjeg dijela koji se odnosio na prethodno usvojenu temu, pa su tako vrednovani vrlo specifični ciljevi učenja; učenici su mogli predvidjeti što će se vrednovati polazeći od održanih

nastavnih sati. Međutim, vanjska je provjera (npr. Nacionalni test znanja) vrlo široko obuhvatila matematički sadržaj. Inicijalna je vanjska provjera vrednovala sadržaj za učenike od prvog do osmog razreda, a finalna je vanjska provjera vrednovala sadržaj za učenike od prvog do devetog razreda. Suprotno internoj provjeri zadaci koji su se koristili pri vanjskoj provjeri odnosili su se na temeljno znanje i ključne ciljeve najvažnijeg nastavnog sadržaja. Moramo istaći da je vanjsko vrednovanje također obuhvaćalo izazovne problemske zadatke, ali oni su se odnosili na temeljni i dobro poznati sadržaj.

Istraživanje, dakle, pokazuje da učenici bolje ovladavaju određenim nastavnim sadržajem u homogenim grupama, ali, s iznimkom onih s velikim uspjehom, često brzo zaboravljaju ono što su naučili. No, temeljno se znanje bolje zadržava i razvija u heterogenim grupama, što može biti osobito točno kada govorimo o učenicima koji postižu slab ili osrednji uspjeh.

Drugim riječima, prateći rezultate našeg istraživanja, učenici sa slabim uspjehom usvajaju više znanja u heterogenim grupama. Iako je poučavanje u heterogenim grupama manje prilagođeno sposobnostima individualnog učenika, ono daje više mogućnosti za implicitno učenje. Slabiji učenici uče jednostavne stvari od vrlo uspješnih učenika, često nailaze na osnovne matematičke situacije, susreću se s većim (ali ne prevelikim) zahtjevima. Dugoročno usvajaju osnovno znanje koje se u našoj zemlji vrednuje vanjskim provjerama. S druge pak strane, naše istraživanje pokazuje da vrlo uspješnim učenicima homogene grupe bolje odgovaraju samo kada je u pitanju nastava matematike. Dok daju objašnjenja učenicima, nastavnici se mogu usmjeriti prema složenijim ciljevima učenja. Možemo jedino pretpostaviti da učenici dugoročno ovladavaju složenijim znanjem jer vanjska provjera (npr. Nacionalni test znanja) vrednuje, prije svega, osnovni nastavni sadržaj i temeljno matematičko razmišljanje.

Naše je istraživanje ukazalo na nekoliko aspekata (za i protiv) vanjske podjele kada je u pitanju usvajanje znanja iz matematike. Ne postoji jedinstven odgovor na dilemu je li poučavanje u homogenim grupama učinkovitije od poučavanja u heterogenim grupama s obzirom na usvajanje znanja. U heterogenim grupama učenici usvajaju osnovnije matematičko znanje, što se osobito odnosi na učenike sa slabim i prosječnim uspjehom. U homogenim grupama učenici, osobito vrlo uspješni i srednje uspješni, usvajaju složenije znanje. S obzirom na rezultate može se raspravljati o tome treba li prednost dati temeljnom ili složenijem znanju. Smatramo da to nije prava dvojba. Kada je u pitanju usvajanje kvalitetnog znanja, pravo je pitanje: kako unaprijediti nastavu i prilagoditi je učenicima, uzimajući u obzir njihove potrebe i potencijale, te smanjiti otkrivene nedostatke učenja u homogenim i heterogenim grupama. Askew i Wiliam (1995) izvještavaju da je grupiranje učenika učinkovito jedino ako se nastavne metode i materijali prilagode učenicima.

Imajući na umu usvajanje kvalitetnog znanja svih učenika, rezultati istraživanja pokazuju da je odgovarajuća prilagodba svakom učeniku u nastavi potrebna. Zapravo, promatrajući nastavu (Valenčič Zuljan i sur., 2012), primjetili smo da interna

diferencijacija i individualizacija jedva postoje. Promatranje homogenih grupa otkrilo je da su suradničko učenje, problemska nastava povezana s realnim situacijama, vođeno otkrivanje i raspravljanje značajno češći u trećoj nego u prvoj i drugoj grupi (podijeljene prema sposobnostima učenika) (*Ibid.*). S druge strane, u heterogenim su grupama primjećeni samo tragovi interne diferencijacije, a posebno vrlo uspješni učenici bili su rijetko suočeni s izazovnim zadacima. Budući da se nastava matematike u višim razredima osnovne škole u nas realizira u grupama koje nisu velike, ona bi se mogla i trebala prilagoditi sposobnostima individualnih učenika.

Zaključak

Istraživanje nam daje uvid u prednosti i nedostatke različitog grupiranja pri usvajanju matematičkog znanja. Rezultati pokazuju da, s kognitivnog aspekta učenja, nema jasnog odgovora o prednostima odvojenog poučavanja u heterogenim i homogenim grupama. U heterogenim grupama učenici, osobito oni sa slabim i prosječnim uspjehom, usvajaju osnovnije matematičko znanje. U homogenim grupama učenici, osobito vrlo uspješni i prosječno uspješni, usvajaju složenije znanje. S obzirom na rezultate postavlja se pitanje treba li nam biti prioritet osnovno ili složenije znanje. Smatramo da pitanje treba obrnuto postaviti. Uzmemo li u obzir usvajanje kvalitetnog znanja tada temeljno pitanje glasi kako unaprijediti nastavu u prvom ili drugom slučaju te je prilagoditi učeničkim potrebama i potencijalima i tako smanjiti moguće nedostatke u oba slučaja.

Rezultati također zahtijevaju da se nastava obvezno prilagodi individualnim sposobnostima učenika bez obzira na podjelu (homogene ili heterogene grupe). Kada je u pitanju usvajanje kvalitetnog znanja, takve pristupe poučavanju i učenju treba razvijati kako bi predstavljali izazov svim učenicima i povećavali im razinu znanja, neovisno o nastavnom modelu u koji su uključeni. Spomenuti cilj mogu ostvariti samo dobro osposobljeni nastavnici.