Noise and Sensitivity Improvement using SC Filters

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The discrete time switched capacitor (SC) based filters have number of advantages over the classical continuous time active RC based filters, the one most important being reduced circuit silicon area, allowing SC filters to be integrated to a single monolithic integrated circuit (IC). In this paper we have designed fifth order Chebyshev low pass SC filter, with a cut-off frequency \( f_c = 3.4 \text{ kHz} \), and pass-band ripple \( \alpha_{\text{max}} = -0.5 \text{ dB} \), and we have compared its time and frequency performance with a performance of the active RC based filter. The both SC and active RC filters are realized as a cascade of two second order sections and one first order section. All filter analysis has been performed using MATLAB and SPICE program packages where SC based filter has shown significant noise and sensitivity improvement, when compared with active RC based filter.

Key words: Low-pass filter, Filter design, Switched capacitor filter, Noise, Sensitivity

1 INTRODUCTION

The main motivation behind switched capacitor (SC) based circuits is to replace a resistor with a capacitor and two electric switches (usually MOS transistors) driven by the two phase non-overlapping clock [1, 2]. The time constant \( \tau \) of such circuit is:

\[
\tau = RC = \frac{1}{C_R f_{CL}},
\]

where \( f_{CL} \) is the clock frequency. Even though SC circuits have increased number of components, the overall size of the circuit is reduced since small valued capacitors can be used for acquiring desired \( C/C_R \) ratio, thus allowing the integration of the circuit into a single monolithic integrated circuit (IC). Furthermore, the \( C/C_R \) ratio can be realized with great accuracy, even up to 0.1%, while \( RC \) error can range from 20% up to 50%. In addition, since \( \tau \) is dependent of \( f_{CL} \) (as shown by (1)), the cut-off frequency of SC based filters can be regulated simply by varying the clock frequency.

In this paper, we have designed switched capacitor fifth order Chebyshev low pass filter realized as a cascade of two second order sections and one first order section. Sections are based on the Sallen-Key topology, with a cut-off frequency \( f_c = 3.4 \text{ kHz} \) and a pass-band ripple \( \alpha_{\text{max}} = -0.5 \text{ dB} \). Through the paper we have shown significant improvement of the SC filter over the active RC filter in terms of noise and sensitivity reduction.

The paper is organized as follows. In the Section 2 we have designed and analyzed the continuous time active RC filter, while in the Section 3 we did the same with discrete time SC filter. The analysis was conducted in the frequency domain, and furthermore we investigated and compared noise and sensitivity of both filter designs.

2 CONTINUOUS TIME ACTIVE RC FILTER

The continuous time transfer function of the active RC low pass Chebyshev fifth order filter with a cut-off frequency \( f_c = 3.4 \text{ kHz} \) and a pass-band ripple
\[ H(s) = H_1(s) \cdot H_2(s) \cdot H_3(s), \] (2a)
\[ H_1(s) = \frac{4.727 \cdot 10^8}{s^2 + 47848 + 4.727 \cdot 10^8}, \] (2b)
\[ H_2(s) = \frac{2.176 \cdot 10^8}{s^2 + 1.252 \cdot 10^4 s + 2.176 \cdot 10^8}, \] (2c)
\[ H_3(s) = \frac{7740}{s + 7740}. \] (2d)

The Sallen-Key topology of low pass filter is given in Fig. 1. The filter is realized as a cascade of two second order sections and one first order section. The transfer function of the given circuit is:
\[ H(s) = \prod_{i=1,2} H_i(s) \cdot H_3(s), \] (3a)
\[ H_i(s) = \frac{1 + \frac{G_{2,i}}{C_{1,i}} \frac{G_{1,i}G_{2,i}}{C_{1,i}C_{2,i}}} {s^2 + s \left( \frac{G_{1,i} + G_{2,i}}{C_{1,i}} \right) + \frac{G_{1,i}G_{2,i}}{C_{1,i}C_{2,i}}}, \] (3b)
\[ H_3(s) = \frac{1}{R_{1,3}C_{1,3}}. \] (3c)

Element values have been calculated by comparing equations (2a) - (2d) with (3a) - (3c) with \( C_{1,1} = C_{2,1} \) and \( R_{1,1} = R_{2,2} \), and are given in Table 1.

Frequency analysis of the filter is shown in Fig. 2 and 3. Calculated values are obtained from the filter transfer function (2a) - (2d), while simulated values are obtained by simulating the circuit shown in Fig. 1 with element values shown in the Table 1. The difference between the characteristics is minimal, as can be seen in zoomed inset of Fig. 2.

The equivalent circuit of the active RC filter for the calculation of the voltage noise spectral density is shown in Fig. 4. Current noise sources: \( I_{n,1,i} = \frac{1.6V}{\sqrt{Hz}} \) were added parallel to the resistors, while voltage noise sources \( (E_{n,1,i} = 15 \, nV/\sqrt{Hz} \) for LT1055) were added at input of the operational amplifiers. The operational amplifier input noise currents were neglected since their values are significantly smaller \( (I_n = 1.8 \, fA/\sqrt{Hz} \) for LT1055). The total voltage noise spectral density is calculated as [3]:
\[ V_n^2(\omega) = \sum_{k=1}^m |T_{1,k}(j\omega)|^2 (I_{n,k})^2 + \sum_{l=1}^n |T_{L,l}(j\omega)|^2 (E_{n,l})^2, \] (4)
Table 2: Section specific voltage noise transfer functions of the continuous time active RC filter.

<table>
<thead>
<tr>
<th>Noise source</th>
<th>Transfer function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{\text{in}} ) ( T_{\text{i},k} ), ( i = 1,2 )</td>
<td>( R_{11} ), ( R_{12} ), ( R_{21} ), ( R_{22} ) ( C_1 ), ( C_2 ) ( s ) ( (C_1, C_2, R_{11}, R_{12}, R_{21}, R_{22}, \ldots) ) ( + ) ( \ldots )</td>
</tr>
<tr>
<td>( U_{\text{in}} ) ( T_{\text{i},k} ), ( i = 1,2 )</td>
<td>( R_{11} ), ( R_{12} ), ( R_{21} ), ( R_{22} ) ( C_1 ), ( C_2 ) ( s ) ( (C_1, C_2, R_{11}, R_{12}, R_{21}, R_{22}, \ldots) ) ( + ) ( \ldots )</td>
</tr>
<tr>
<td>( U_{\text{out}} ) ( T_{\text{out}} ), ( i = 1 )</td>
<td>( R_{11} ), ( R_{12} ), ( R_{21} ), ( R_{22} ) ( C_1 ), ( C_2 ) ( s ) ( (C_1, C_2, R_{11}, R_{12}, R_{21}, R_{22}, \ldots) ) ( + ) ( \ldots )</td>
</tr>
</tbody>
</table>

\[ E_n^{\text{cf}} = \int_{-\omega}^{\omega} V_n^2(\omega) d\omega. \] (5)

In order to investigate filters sensitivity we calculated Schoeffler sensitivity as [4]:

\[ I_S^2(\omega) = \sum_i (S_i^2(\omega)) \] (6)

\[ S_i^2(\omega) = \frac{d[H(\omega)]}{dx_i} \frac{|H(\omega)|}{|H^*(\omega)|} \] (7)

where:

\[ S_i^2(\omega) = \frac{d[H(\omega)]}{dx_i} \frac{|H(\omega)|}{|H^*(\omega)|} \] (7)

for each passive filter element \( x_i \) and the result is presented in Fig. 6. The multi-parameter sensitivity is defined by:

\[ M = \int_{\omega_1}^{\omega_2} I_s^2(\omega) d\omega. \] (8)

The Monte Carlo analysis of filter has been conducted over 100 passes with 1% element tolerance, and is shown in Fig. 7. The most significant contributions to the overall noise and sensitivity figures are presented in Figs. 8 and 9, respectively.

In order to decrease both the noise and the sensitivity, optimization algorithm described in [5] has been applied, and operational amplifier LT1055 has been replaced with low noise LT1007 operational amplifier (\( E_n = 2.5 \text{nV}/\sqrt{\text{Hz}} \)). The newly calculated filter element values are shown in Table 3, while voltage noise spectral density and Schoeffler sensitivity are shown in Fig. 10 and 11, respectively. The noise RMS value is 140% smaller, while multiparameter sensitivity measure is 15% smaller, when compared to non-optimized filter values, shown in Fig. 5 and 6. Note that significant decrease of noise and sensitivity is obtained in filter pass-band.

3 DISCRETE TIME SC FILTER

The discrete time transfer function is obtained by applying forward Euler transformation (\( s = \frac{z-1}{Ts} \)), where \( T_S \) is the sampling time) to the continuous time transfer function, defined by (2a) - (2d), with sampling frequency...
Fig. 5: Voltage noise spectral density of the continuous time active RC filter: simulated (full) and calculated (dashed). Noise RMS value is $(E_n)_{rms} = 17.595 \, \mu V$ ($f_1 = 0.1 \cdot f_c$, $f_2 = 10 \cdot f_c$).

Fig. 6: Schoeffler sensitivity of the continuous time active RC filter. The multi-parameter sensitivity is $M = 8.0286 \cdot 10^5$ ($f_1 = 0.1 \cdot f_c$, $f_2 = 10 \cdot f_c$).

Fig. 7: Monte Carlo analysis over 100 passes with 1% element tolerance of the continuous time active RC filter. Magnitude values between $f_c/100$ and $f_c$ are zoomed in to a range from $-3 \, \text{dB}$ to $3 \, \text{dB}$.

Table 3: Element values of the optimized continuous time active RC filter shown in Fig. 1.

<table>
<thead>
<tr>
<th>Element</th>
<th>1st section</th>
<th>2nd section</th>
<th>3rd section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{1,1}$</td>
<td>12.2 kΩ</td>
<td>14.0 kΩ</td>
<td>-</td>
</tr>
<tr>
<td>$R_{1,2}$</td>
<td>20.9 kΩ</td>
<td>16.2 kΩ</td>
<td>-</td>
</tr>
<tr>
<td>$R_{2,1}$</td>
<td>-</td>
<td>-</td>
<td>12.9 kΩ</td>
</tr>
<tr>
<td>$R_{2,2}$</td>
<td>7.7 kΩ</td>
<td>7.5 kΩ</td>
<td>-</td>
</tr>
<tr>
<td>$R_{3,1}$</td>
<td>8.0 kΩ</td>
<td>5.4 kΩ</td>
<td>-</td>
</tr>
<tr>
<td>$R_{4,1}$</td>
<td>4.7 kΩ</td>
<td>4.7 kΩ</td>
<td>-</td>
</tr>
<tr>
<td>$C_{1,1}$</td>
<td>10 nF</td>
<td>10 nF</td>
<td>10 nF</td>
</tr>
<tr>
<td>$C_{2,1}$</td>
<td>3.6 nF</td>
<td>8.2 nF</td>
<td>-</td>
</tr>
</tbody>
</table>
\[ f_s = 900f_c: \]
\[ H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z), \]
\[ H_1(z) = \frac{5.048 \cdot 10^{-5}}{z^2 - 1.998z + 0.999}, \]
\[ H_2(z) = \frac{2.324 \cdot 10^{-5}}{z^2 - 1.996z + 0.996}, \]
\[ H_3(z) = \frac{0.003}{z - 0.998}. \]

Figure 12 shows the electric circuit of the discrete time SC filter based on Sallen-Key topology [1]. The transfer function of the given circuit is:

\[ H(z) = \prod_{i=1}^{2} \frac{n_{1,i}}{d_{1,i}z^2 - d_{2,i}z + d_{3,i}}, \]
\[ n_{1,i} = C_{R1,i}C_{R2,i}(C_{1,i} + C_{g,i}), \]
\[ d_{1,i} = C_{2,i}[(C_{1,i} + C_{g,i})^2 + C_{R1,i}(C_{1,i} + C_{g,i})] + + C_{R2,i}[(C_{1,i} + C_{g,i})^2 + C_{R2,i}(C_{1,i} + C_{g,i})] + + C_{g,i} + C_{R1,i} + C_{2,i}(C_{1,i} + C_{g,i}) + + C_{R1,i} + C_{R2,i}(C_{1,i} + C_{g,i}), \]
\[ d_{2,i} = C_{2,i}[2(C_{1,i} + C_{g,i})^2 + C_{R1,i}(C_{1,i} + C_{g,i})] + + C_{R2,i}[(C_{1,i} + C_{g,i})^2 + C_{R2,i}(C_{1,i} + C_{g,i})] + + C_{g,i} + C_{R1,i} + C_{1,i}(C_{1,i} + C_{g,i}) + + C_{2,i}(C_{1,i} + C_{g,i} + C_{R1,i}), \]
\[ d_{3,i} = n_{1,i} - d_{1,i} + d_{2,i} = C_{2,i}(C_{1,i} + C_{g,i})^2 + + C_{1,i}C_{R2,i}(C_{1,i} + C_{g,i}). \]

By comparing (9a) - (9d) with (10a) - (10e) the element values of the circuit have been calculated and are shown in Table 4.

Frequency analysis of the filter is shown in Figs. 13 and 14. The values were calculated from a discrete time transfer function of the SC filter (9a) - (9d). The Z-transformation hasn’t affected desired continues time designed filter specifications, as can be observed by comparing them with Fig. 2 and 3.

Equivalent circuit for the calculation of the voltage noise spectral density is based on [6], and calculated voltage noise spectral density is shown in Fig. 15. The noise RMS value, defined by (5), is over 5 times lower, when compared with continues time active RC filter, shown in Fig. 5 and over 2 times lower, when compared with optimized active RC filter, shown in Fig. 10.

Figure 16 shows Schoeffler sensitivity of the SC filter calculated by (6), while Fig. 17 shows the results of Monte Carlo analysis over 100 pases with 1% element tolerance. The multi-parameter sensitivity, obtained from (8)
Fig. 13: Transfer function of the discrete time SC filter. Magnitude values between \( f_c / 100 \) and \( f_c \) are zoomed in to a range from \(-0.5\, \text{dB}\) to \(0.3\, \text{dB}\).

Fig. 14: Group delay of the discrete time SC filter. The values have been calculated as: \( \text{GD}(\omega) = -\frac{d\varphi}{d\omega} \), where \( \varphi \) is a filter phase response.

Table 4: Element values of the discrete time SC filter shown in Fig. 12.

<table>
<thead>
<tr>
<th></th>
<th>1st section</th>
<th>2nd section</th>
<th>3rd section</th>
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<tbody>
<tr>
<td>( C_{R1,i} )</td>
<td>121.1 pF</td>
<td>1.0 nF</td>
<td>560 pF</td>
</tr>
<tr>
<td>( C_{R2,i} )</td>
<td>100 pF</td>
<td>68 pF</td>
<td>-</td>
</tr>
<tr>
<td>( C_{q,i} )</td>
<td>100 pF</td>
<td>10 nF</td>
<td>-</td>
</tr>
<tr>
<td>( C_{1,i} )</td>
<td>150 nF</td>
<td>270 nF</td>
<td>220.8 nF</td>
</tr>
<tr>
<td>( C_{2,i} )</td>
<td>1.5 nF</td>
<td>10.5 nF</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 15: Voltage noise spectral density of the discrete time SC filter. Noise RMS value is \( (E_n)_{ef} = 3.129 \, \mu \text{V} \) \((f_1 = 0.1 \cdot f_c, f_2 = 10 \cdot f_c)\).

Fig. 16: Schoeffler sensitivity of the discrete time SC filter. The multi-parameter sensitivity is \( M = 6.7563 \cdot 10^5 \) \((f_1 = 0.1 \cdot f_c, f_2 = 10 \cdot f_c)\).

Fig. 17: Monte Carlo analysis over 100 pases with 1\% element tolerance of the discrete time SC filter. Magnitude values between \( f_c / 100 \) and \( f_c \) are zoomed in to a range from \(-1\, \text{dB}\) to \(1\, \text{dB}\).
is 20% smaller, and spread off the Monte Carlo analysis is significantly decreased, when compared with values obtained from continuous time active RC filter, shown in Fig. 6 and 7. The comparison of voltage noise spectral density and Schoeffler sensitivity throughout all discussed filter designs are shown in Figure 18 and 19, respectively.

**Fig. 18**: Voltage noise spectral density of: active RC filter (dashed), optimized active RC filter (dot-dashed), and SC filter (full).

**Fig. 19**: Schoeffler sensitivity of: active RC filter (dashed), optimized active RC filter (dot-dashed), and SC filter (full).

4 CONCLUSION

In this paper we have shown the advantages of discrete time switched capacitor (SC) based filter over the continuous time active RC based filter by comparing their frequency performances. The filters have been designed as Chebyshev fifth order low pass filters, with a cut-off frequency \(f_c = 3.4 \text{kHz}\) and pass-band ripple \(\delta_{\text{max}} = -0.5 \text{dB}\), and were realized as a cascade of two second order sections and one first order section, based on Sallen-Key topology. The analysis of the SC filter showed both noise and sensitivity reduction, while retaining desired frequency specifications.

Further improvement of the presented filter design can be implemented by using continuous OTA-C topology, however, this approach has its own drawbacks. Alternatively, more complex filter structure with feedbacks instead of used cascade filter structure can be used.

REFERENCES


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