The Concept of the *Infiniti mysteria* in Bošković's Geometrical Investigations

IVICA MARTINOVIĆ
Institut za filozofiju, Zagreb

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Summary

When Bošković first mentioned the concept 'the mysteries of the infinite' (*Infiniti mysteria*) in his treatise *De maris aestu* (1747), he asserted that it was necessary to include the mysteries of the infinite into the investigation of geometric transformations. At that time, on the basis of the demonstration in his early treatise *De natura et usu infinitorum et infinite parvorum* (1741), he already had some experience in disputing the actual infinite in geometry. Therefore he based his study of the mysteries of the infinite on the philosophical assumption of the existence of the infinite.

While forming the theory of geometric transformations in his treatise *De transformatione locorum geometricorum* (1754), he gave a large meaning to this concept: all the manifestations of the potential and actual infinite. Only with his treatise *De continuitatis lege* (1754) did he start to make a strict distinction between mystery and absurdity in the understanding of the geometric infinite, and from that time on he recognized the mysteries of the infinite only in those geometric quantities and transformations in which the potential infinite occurs, on condition that the principle of continuity was preserved.

Two confirmations of Bošković's understanding of the *Infiniti mysteria* in this specific way are to be found in his correspondence in the 1760s: his valuable epistolary treatise written in Constantinople from 20 December 1760 to 26 February 1761 for the young Giovan Stefano Conti, and Bošković's exhaustive reply to the Swiss scholar Le Sage of 8 May 1765.

On the contrary, absurdity always follows from the assumption of the actual infinite, and it is ascertained during the process in which the structure of bijection and relationship 'part-whole' are used, that is, both aspects which strongly mark Bernard Bolzano's paradoxical conception of the relationship between infinite sets, and Richard Dedekind's mathematical definition of the infinite system.

In his model for ascertaining absurdity, Bošković always uses the relations between *geometric* quantities as representatives of the relationships between infinite quantities. The turning point which was prepared by Bolzano in his *Paradoxien des Unendlichen* (1851), and achieved by Dedekind and Georg Cantor, took place in another mathematical field, namely, in the *set* approach to the *real numbers*. These two points, the use of the same mathematical contents, such as the structure of bijection and the relationship 'part-whole,' on the one hand, and the difference between the Euclidean geometric approach and the set approach on the other hand, determine the place of Ruđer Bošković in the historical process of forming the exact, mathematical definition of the infinite.

Key words: Ruđer Bošković, Bernard Bolzano; geometry, theory of geometrical transformations, the actual infinite, the potential infinite, *Infiniti mysteria*

1. Within a research into the transformations of geometric loci

Ruđer Bošković distinguished two stages when he referred to the development of his geometrical views. The first stage he described as his stay "in the very vestibule of geometry" (*in ipso nimirum Geometriae vestibulo*),[1] in which he had not yet shaped his attitude towards the principle of continuity and laid the foundations of his theory of forces, and that was before 1745. In this period, among other things, he was working on a textbook on planimetry and stereometry, published in 1752 in the first volume of his *Elementa universae matheseos*. Even though with regard to his geometric expositions in the first volume he could speak about his being in the vestibule of geometry, mostly because he expounded topics from Euclid's *Elements*,[2] with the third volume of his *Elementa* he stepped into the realm of "geometry which never operates by leap" (*Geometria, quae nihil usquam operatur per saltum*).[3] Since most of this volume, as I have already pointed out,[4] was prepared as early as 1747, it

[1] Rogerius Josephus Boscovich, *De continuitatis lege et ejus consectariis pertinentibus ad prima materiae elementa eorumque vires* (Romae: Ex Typographia Generosi Salomoni, 1754), n. 12, p. 7: "<...> imaginem, qua in primo tomo nostrorum Elementorum, in ipso nimirum Geometriae vestibulo, usi olim sumus, cum in hanc nostram theoriam nondum incidissemus, <...>".

[2] Bošković organized his geometry differently than Euclid, and therefore at the end of his exposition on planimetry, he added an index of propositions quoted from the first three books of Euclid's *Elements*, while an index of propositions from the eleventh and twelfth book of Euclid's *Elements* was added at the end of his stereometry. Cf. Rogerius Josephus Boscovich, *Elementorum universae matheseos tomus I.* (Romae: Typis Generosi Salomoni, 1752), pp. 65–66 and 175.

[3] Rogerius Josephus Boscovich, "Auctoris praefatio," in Boscovich, *Elementorum universae matheseos tomus III.* (Romae: Typis Generosi Salomoni, 1754), pp. III–XXVI, on pp. XVIII–XIX.

[4] Ivica Martinović, "Theories and inter-theory relations in Bošković," *International studies in the philosophy of science* 4 (1990), pp. 247–262, on pp. 251–252.

therefore belongs to the mature period of Bošković's geometric investigations, i.e. to the period of his research into continuity and infinity in geometry.

In the third volume of his mathematical textbook Bošković systematically expounded his *Sectionum conicarum elementa* (*Elements of Conic Sections*). Here, among others things, he elaborated the transformations of conic sections one into another and, in particular, into a circle, as well as the degenerations of conic sections into straight lines and into a point.[5] He listed all cases in which these curves are connected with the infinite, and in which the continuity of the curve is being still preserved. Here is Bošković's catalogue of the geometric cases from his treatise *De transformatione locorum geometricorum*:

(1) multiple points;
(2) the transit through zero and the infinite viewed from two standpoints: approaching to these values and returning from them;
(3) the points studied in relation to the states in which they are or in which they persevere when they tend towards zero or the infinite: the points are real whether they are visible in some places or concealed by the infinite, as if the points are hidden even when they fall into the imaginary;
(4) the lines, which by themselves are terminating, studied with regard to direction: perseverance of the line in the same direction, or change of direction, or impossibility to ascertain the direction;
(5) annihilation and evanescence of lines;
(6) the lines considered with regard to their relation to the infinite: prolongation in the infinite, or circulation through the infinite, or a certain extension which seems to be greater than infinite extension.[6]

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[5] Rogerius Josephus Boscovich, "Sectionum conicarum elementa," in Boscovich, *Elementorum universae matheseos tomus III.*, nn. 1–672, pp. 1–296, in n. 110, p. 36: "Atque hoc quidem pacto conicae sectiones in se invicem transformantur vel in circulum. Possunt autem et ad rectas lineas et ad punctum ita accedere, ut demum in eas desinant."

[6] Rogerius Josephus Boscovich, "De transformatione locorum geometricorum, ubi de continuitatis lege, ac de quibusdam Infiniti mysteriis," in Boscovich, *Elementorum universae matheseos tomus III.*, nn. 673–886, pp. 297–468, on p. 367, n. 759:

"Porro in hujusmodi transformationibus Sectionum Conicarum aliarum in alias habentur punctorum multiplices et transitus per nihilum ac per infinitum, et regressus inde: ipsi autem appulsus ad infinitum vel nihilum saepe puncta retinent in statu reali, vel alicubi conspicua vel infinito obruta, ibique velut delitescentia, quandoque etiam ad imaginarietatem deturbant, adeoque linearum, quae ipsis terminantur, habetur iam perseverantia in eadem directione, iam directionis mutatio, iam impossibilitas, et saepe annihilatio ac evanescentia, saepe productio in infinitum, saepe etiam circuitas quidam per infinitum, et quaedam veluti plusquam infinita extensio."

It was this final form of the existence of curve in the infinite that Charles Taylor explicitly mentioned, though incorrectly located in Bošković's work, only to interpret in an encyclopaedic →

The multiform behaviour of conic sections in the infinite, or within infinite processes, induced Bošković to tackle the question of geometric transformations in general. He wrote the treatise *De transformatione locorum geometricorum, ubi de continuitate ac de quibusdam Infiniti mysteriis* (*On the transformation of geometric loci, in which the law of continuity and some mysteries of the infinite are discussed*) and published it together with *Sectionum conicarum elementa* (*Elements of Conic Sections*) in the same third volume of his mathematical textbook, at the beginning of 1754.[7] In doing so, he was convinced that the study of the flow of the curves most often used in geometry and calculus will help

> "to elucidate some mysteries of the infinite and provide a more intimate understanding of the law of geometric continuity".[8]

In this way Bošković once again drew attention to the two key concepts of his geometrical research in the period 1753–1754: mysteries of the infinite and the law of continuity.

However, while the importance of the law of continuity for Bošković's contribution to mathematics and natural philosophy has been emphasized and vastly explored,[6] the concept of the mysteries of the infinite in Bošković's work is still an open question, understudied to date. What does it actually mean to talk about a mystery in geometry? Is there any sense in talking about the infinite in geometry as a mystery and when? Is it at all possible to follow and grasp Bošković's reasoning within the theory of geometric loci if it is not preceded by an understanding of what the 'mysteries of the infinite' are to Bošković?

2. The first concept of Infiniti mysteria

Bošković himself made a considerable effort to understand the manifestations of the infinite in geometry which he termed *Infiniti mysteria*. He did not determine the concept of the mystery of the infinite at once, but rather developed it in a series of his papers. He mentioned it as early as 1747, in his treatise *De*

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entry that Bošković comprehended geometrical continuity, on this see: C.[harles] T.[aylor], "Geometrical continuity," in *The Encyclopaedia Britannica*, eleventh edition, Vol. 4 (Cambridge: At the University Press, 1910), pp. 674a–675a, on p. 674b. Taylor included Bošković among the foremost scholars Kepler, Briggs, Desargues, Leibniz, Newton and Poncelet.

[7] For exact data on the completion of the treatise "De transformatione locorum geometricorum," and with it on the third volume of his textbook in mathematics, see: Ivica Martinović, "Pretpostavke za razumijevanje geneze Boškovićevih ideja o neprekinutosti i beskonačnosti: kronologija radova, povijesna samosvijest, tematske odrednice," *Vrela i prinosi* 16 (1986), pp. 3–22, on pp. 9–10.

[8] Boscovich, "De transformatione locorum geometricorum," n. 692, p. 312: "ad quaedam infiniti mysteria evolvenda et cognoscendam intimius continuitatis geometricae legem, <...>".

maris aestu (*On tides*), announcing the content of his theory of conic sections, i.e. *Sectionum conicarum elementa*:

> "Here, among many other things, we will reveal the amazing properties, marvellous transformation, and nexus of geometric loci, as well as the arcana of the infinite which are surely necessary, if the infinite is admitted, and which considerably exceed all human power of comprehension."[9]

The concept of the mysteries of the infinite is considered here in close relation to the properties, transformation and connexion of geometric loci. Moreover, the correct understanding of this concept enables a unique observation of conic sections with the perseverance of continuity:

> "In this way, a parabola, as well as each of the two branches of hyperbola do not differ from a certain unique and continuous ellipse, if certain mysteries of the infinite are properly understood and applied."[10]

On this occasion, when first mentioned, the mysteries of the infinite were referred to in Latin both as *Infiniti mysteria* and *Infiniti arcana*, and Bošković explicitly points out a condition that allows the reasoning on the mysteries of the infinite in geometry: if the infinite be admitted as such.[11]

Bošković's announcement from 1747 resulted in a treatise *De transformatione locorum geometricorum*, completed by the end of 1753. Therefore, it is obvious why in this treatise Bošković frequently mentions the mysteries of the infinite, practically whenever he refers to his approach to the transformations of geometric loci. He does so by using the concept *mysteria* which, in terms of meaning and usage, he now distinguishes from the concept *arcana*. Bošković uses the term *arcana* when speaking metaphorically, as about his treatise, for example:

> "It brings light, and stretches like a wonderful way before the one who is ready to penetrate into the arcana that are intimate to geometry."[12]

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[9] Rogerius Josephus Boscovich, *De maris aestu* (Romae: Ex Typographia Komarek in Viâ Cursus, 1747), n. 90, p. 45: "et Infiniti arcana omnino necessaria, si Infinitum admittatur, at omnem humanum captum longe excedentia proferemus."

[10] Boscovich, *De maris aestu*, n. 90, p. 45: "quo pacto et parabola et quidem etiam uterque hyperbolae ramus ab unica quadam et continua ellipsi non discrepant, si quaedam Infiniti mysteria rite intelligantur et applicentur."

[11] Boscovich, *De maris aestu*, n. 90, p. 45: "si Infinitum admittatur" i "et si Infinitum admitti possit".

[12] Boscovich, "Auctoris praefatio," in Boscovich, *Elementorum universae matheseos tomus III.*, p. XVIII: "at ea [= dissertatio autem ipsa] in Geometriae arcana intimiora irrumpere meditanti facem praeferet et viam sternet mirum in modum."

On the other hand, he employs the concept of *Infiniti mysteria* as a technical term including diverse behaviours of geometric quantities, in which indefiniteness or infinity of geometric product is manifest in whatever form, and this product is not observable in the manner as it usually happens with the finite and definite geometric products. At the same time, the term includes certain distinctions. Here are Bošković's characteristic views:

> "Yet in order to preserve this continuity, there often occurs a certain progress in infinity and a certain transit through the infinite which involve something that cannot be called better by its own name, or by any other name, rather than by certain mysteries of the infinite which grow forth to such an extent that, apparently, they become finally reduced to mere absurdities."[13]

> "Occurring many times are certain mysteries of the infinite which grow forth to such an extent that they finally convince us of the impossibility of the extended infinite and lead us towards the theory of indefinite [quantities], whether they are indefinitely small or indefinitely great, the theory of which will be dealt with in another work."[14]

The concept of 'the mysteries of the infinite' includes a limit process towards the infinite and zero as a manifestation of the *potential* infinite, since it is a topic discussed by the "theory of indefinite quantities," a term Bošković obviously used for calculus under the influence of Leibniz. This concept equally conveys the impossibility of existence of the *actual* geometric infinite, which, according to Bošković, is manifested as absurdity (*absurdum*). The mystery of the infinite may or need not grow into absurdity, depending on whether, by analogy with finite quantities, a certain property, relationship or behaviour is no longer valid, or is still valid. In this way, the semantic field of the concept *Infiniti mysteria* was determined in the treatise *De transformatione locorum geometricorum*.

On the basis of Bošković's views from the end of 1753, one may conclude the following: the concept of 'the mysteries of the infinite' emerges as a concept superior to all other concepts related to the manifestations of the potential and actual infinite. The kind of manifestation and its character should be recognized in every particular case. An appropriate example of the manifestation of

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[13] Boscovich, "Auctoris praefatio," p. XIX: "Sed in ejusmodi [= geometrica] continuitate servanda occurrunt saepe quidam progressus in infinitum et quidam transitus per infinitum, qui secum trahunt quaedam, quae haud suo, an alio melius nomine appellari possint, quam mysteriorum quorundam infiniti, quae tamen eo excrescunt, ut in vera demum absurda videantur recidere."

[14] Boscovich, "De transformatione locorum geometricorum," n. 759, p. 368: "Occurrent autem identidem quaedam etiam infiniti mysteria, quae eo usque excrescent, ut infiniti extensi impossibilitatem demum suadeant, ac ad indefinitorum, sive indefinite parva sint, sive indefinite magna, theoriam, quam alio opere pertractabimus, nos deducent."

the infinite is provided by the relationships between infinite quantities, which Bošković had already discussed in his treatise *De natura et usu infinitorum et infinite parvorum* (1741) when, at the beginning of his mathematical career, he tried to elucidate the fundamental notions of calculus.

By 1741 Bošković already understood the infinitely small and infinitely great quantity in the same manner as the founders of calculus Newton and Leibniz, that is, as a manifestation of the potential infinite. In this respect he recurrently pointed to the absurdity of the notion and impossibility of the existence of the actual infinite. He came forward with a special proof which, in the opinion of Vladimir Varićak, resembles the later paradoxes reasoned by Bernard Bolzano, Bertrand Russell and Gerhard Hessenberg, founded on the comparison of angles when considered as parts of the plane. Here is Bošković's proof in full:

> "11. That the apsolute infinite or infinitely great quantity in extension cannot exist nor without danger be conceived, we are demonstrating in this manner:
>
> Let ABC be a whatever angle with sides, prolonged in the infinite if possible, and let it be bisected by line BD, also prolonged in the infinite. Given that the angles CBD and DBA placed one upon another are congruent, it is evident that the infinite surfaces comprised by the sides are equal.
>
> From a whatever point C of the side CB a line CD is drawn parallel to the side BA and cuts BD at D and is extended until DE is twice greater than DC. Also drawn is BE, perceived also to be prolonged in the infinite. May the lines *cde* which are infinite by number be parallel to CDE.
>
> Given that the triangles CBd and DBe always relate as bases cd and de, it is evident that the surface dDEe will always be twice greater than the surface cCDd. That is why the sums of these surfaces will stand in equal ratio, that is, an infinite surface lying between the sides BE and BD will be twice greater than the infinite surface CBD, even twice greater than the surface DBA, [which means]:
>
> The part will be double the whole, which is absurd."[15]

[15] [Rogerius Josephus Boscovich], *De natura et usu infinitorum et infinite parvorum* (Romae: Ex Typographia Komarek in Via Cursus, 1741), p. 7, n. 11, Fig. 3:

"Infinitum autem absolutum sive infinite magnum in extensione nullum esse posse nec tuto concipi sic demonstramus:

Sit angulus quicunque ABC, lateribus, si fieri potest, in infinitum productis, quem secet bifariam recta BD in infinitum pariter producta. Quoniam anguli CBD, DBA superimpositi congruerent, patet areas infinitas lateribus comprehensa aequales esse.

Ex quocunque puncto C lateris CB ducatur CD paralella ipsi Ba occurens rectae BD in D, producaturque donec sit DE ipsius DC dupla, ducaturque BE, quae pariter in infinitum produci intelligatur. Sint autem infinitae numero *cde* ipsi CDE parallelae.

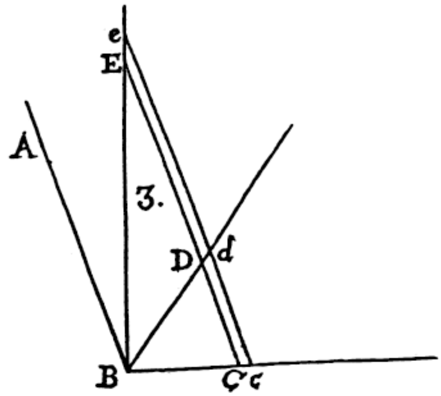

Figure 1. Bošković's first paradox. [Rogerius Josephus Boscovich], *De natura et usu infinitorum et infinite parvorum* (Romae: Ex Typographia Komarek in Via Cursus, 1741), Fig. 3.

Bošković's proof "that the actual infinite cannot exist nor without danger be conceived" calls for several remarks. First of all, the statement refers to the infinite which is manifested in extension. Among all the extended quantities Bošković decides to choose the surface, and the measure for determining surface is the angle. Why? According to Euclid's seventh axiom, congruence establishes the equality of angles, and from the equality of angles follows the equality of surfaces comprised by the sides of these angles. All the employed objects are geometrical.

The actual infinite is consistently used in the presuppositions, and also appears in the demonstration procedure. But the character of the actual infinite does not remain the same. In Bošković's formulation that all sides "are prolonged in the infinite, if it can be," they are understood as actually infinite *by quantity*. In this way, actually infinite are: the initial angle ABC, its bisector Bd and mutually equal surfaces determined by angles CBD and DBA. Finally, actually infinite is also the (straight) line BE, essential for construction.

Bošković's starts his proof with the triangles CBD and DBE. They are equal in height, their bases standing in the ratio 1 : 2. Therefore, their surfaces stand in the ratio 1: 2. When Bošković draws lines *cde* parallel to *CDE*, he assumes that the lines *cde* are actually infinite by number (*numero*). Therefore, in the

Quoniam triangula cBd, dBe sunt semper ut bases cd, de, patet fore semeper aream dDEe duplam areae cCDd. Quare et omnium illarum arearum summae in eadem ratione erunt. Nimirum area infinita interiacens lateribus BE, BD dupla erit areae infinitae CBD, adeoque etiam dupla areae DBA, sive pars dupla totius, quod est absurdum."

construction there appears the actual infinite but of a different kind than in the presuppositions. According to Bošković's construction, for each parallel cde it is easy to establish that the surfaces of the respective trapezoids cCDd and dDEe stand in the already established ratio 1: 2. Bošković then studies the sums of these surfaces, i.e. the infinite sums which correspond to the actually infinite surfaces determined by the angles CBD and DBA, and by analogy deduces that the sums stand in the ratio 1 : 2. The sum of the surfaces is here understood actually, and not as a series of partial sums whose limit we are trying to establish. For reasons of methodology, Bošković once again insists on the actual infinite, although familiar with the theory of infinite orders, at least with what of it mirrored in the works of Gregoire de St. Vincent and Andre Tacquet. Apart from studying *Opus geometricum* of the first mathematician, the work significant not only for its discussion of the conic sections and the method of exhaustion but also for the analysis of the convergence of the infinite series, at the beginning of this treatise he defends Tacquet's method of deduction in geometry.[16]

Bošković's proof ends with a statement that an infinite surface determined by angle DBE is twice greater than just as infinite surface CBD, and the surface DBA, the latter's equal. This means that the part is double the whole, which directly contradicts Euclid's eighth axiom: "The whole is greater than a part."[17] No doubt, the actual infinite expressed by quantity and by number results in a contradiction by which a part is greater than the whole. It took more than a century for this paradox from 1741 to give way to a historic feedback: this very paradox contributed to the mathematical understanding of the infinite. In the new circumstances, Bolzano, Cantor and Dedekind in their seminal works from the second half of the nineteenth century were to approach the problem by examining if the whole could be equivalent to its part. Finally, Dedekind employed this property *to define the actual infinite mathematically*.

By using these examples from Bošković's early treatise *De natura et usu infinitorum et infinite parvorum,* among the manifestations of the infinite it is possible to establish the basic distinction:

If the relationship between infinites is understood as a relationship between potential infinites, it is mathematically expressed by means of an order of infinite quantity–in this particular case, as an order of infinitely small quantity–then

[16] [Boscovich], *De natura et usu infinitorum et infinite parvorum*, n. 5, pp. 4–5.

[17] *The Thirteen Books of Euclid's Elements*, Vol. I, translated from the text of Heiberg with introduction and commentary by Sir Thomas L. Heath, second edition revised with additions (New York: Dover, 1956), p. 155. Cf. Vladimir Varićak, "Matematički rad Boškovićev: Dio I.," *Rad JAZU* 181 (1910), pp. 75–208, on p. 81.

it is explained that there is no logical contradiction, and finally the considered relationship is accepted in mathematics.[18] In addition, the introduction of infinitely small and infinitely great quantities of the first, second and n-th order was the usual procedure since the very beginnings of calculus.

On the contrary, if the relationship between the infinites is understood as a relationship between actual infinites, it is recognized as a request for an infinite greater than the absolute infinite, that is, a request for transfiniteness. Then it is ascertained that this is an instance of logical contradiction – this is Bošković's first geometric paradox from 1741 – and finally the considered relationship is denied.[19]

Bošković also states that the nature of the infinite requires the infinite *simplicity*, although *infinite parts* of increasing quantities are found in the examples from Bošković's theory of conic sections.[20] Evidently, the existence of parts, even if they are infinite, is not in harmony with the request of simplicity appropriate to the infinite. This, no doubt, is a philosophico-theological argument, in the light of which at the end of the preface to the *geometric* treatise Bošković contemplates:

> "Omnipotent Divine Majesty, immune from every composition, connecting the immense simplicity with the infinite."[21]

In this manner Bošković's penetration into the mysteries of the infinite in geometry reveals its *philosophico-theological* fundament, but also casts more light on his basic *geometric* dilemma: Should the behaviour in the geometric

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[18] See the exposition on the orders of infinitely small quantities in [Rogerius Josephus Boscovich], *De natura et usu infinitorum et infinite parvorum* (Romae: Ex Typographia Komarek in Via Cursus, 1741), nn. 13–15, pp. 7–8.

[19] See the exposition of Bošković's first geometrical paradox in: Boscovich, *De natura et usu infinitorum et infinite parvorum*, n. 11, p. 7, and his view on transfinitedness in n. 19, pp. 9–10, notably in the statement on p. 9:

"Notandum autem est hic iterum infinitum absolutum admitti non posse."

Cf. Varićak, "Matematički rad Boškovićev: Dio I.," pp. 79–81; Željko Marković, *Ruđe Bošković*, dio prvi (Zagreb: JAZU, 1968), pp. 96–97; Ivica Martinović, *Problem neprekinutosti i beskonačnosti kod Ruđera Boškovića* (Dubrovnik: Interuniverzitetski centar za postdiplomske studije Sveučilišta u Zagrebu, 1984), M. Sc. thesis, pp. 79–85; Žarko Dadić, *Ruđer Bošković* (Zagreb: Školska knjiga, 1987), pp. 78–80.

[20] Boscovich, "Auctoris praefatio," p. XXV: "ut infiniti ipsius natura simplicitatem infinitam requirat, quae cum infinitis partibus ab omni quantitatum excrescentium genere requisita conjungi omnino non potest; <…>".

[21] Boscovich, "Auctoris praefatio," p. XXV: "ad ipsam illam Dei O.[mnipotentem] M.[aiestatem], immunem ab omni compositione simplicitatem immensam cum infinitate conjunctam contemplandam".

infinite be postulated in the simplest possible way, or should complex forms be admitted? Which criterion or property should be used to establish the simplicity in geometry?

Bošković used a specific language when writing about the mysteries of the infinite, so that the subject is easily recognized even if the mysteries of the infinite are not explicitly mentioned. This is the *veluti*-language or the *as if*-language, the language of comparisons, which by means of expressions such as *quoddam*, *veluti*, *ut nusquam jam sit* and the like corresponds with the undifferentiated and the undetermined that are characteristic of Bošković's understanding of the mysteries of the infinite in geometry. In 1755, Bošković composed annotation (*adnotatio*) about this language that accompanies the verses of Book I of the didactic poem *Philosophia recentior* by Benedikt Stay:

> "[Our poet] has mentioned that there are many other movements of the soul which elude the notion, so often expressed by *quoddam*, *veluti*, *quasi* and similar other terms."[22]

And this exactly were the movements of Bošković's soul when he pondered over the mysteries of the infinite. This note, therefore, conceals in itself Bošković's postponed and indirect admission that, while speaking about the mysteries of the infinite in 1753, he avoided the exact definition of a geometric concept.

3. A clear distinction between mystery and absurdity

However, in the works written after the publication of *De transformatione locorum geometricorum* at the very beginning of 1754, Bošković is in search of a more precise definition of the concept of *Infiniti mysteria.* His efforts primarily concerned the cases he had noted earlier, in which the mysteries of the infinite were reduced to absurdity. By studying these cases Bošković tried to explain when exactly the mysteries of the infinite turned into absurdity and thus make a clear distinction as to when exactly the behaviour of geometric curves in the infinite remained a mystery, and when it became an absurdity.

That is why in his next treatise *De continuitatis lege* he again refers to "the most obvious absurdity" (*absurdum manifestissimum*) that he had dealt with in

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[22] Rogerius Josephus Boscovich, "Adnotationes," in *Philosophiae recentioris a Benedicto Stay versibus traditae libri X ... cum adnotationibus, et supplementis P. Rogerii Josephi Boscovich ... tomus I.* (Romae: Typis, et sumptibus Nicolai, et Marci Palearini, 1755), p. 14, a. 2: "Innuit autem, esse plures alios animi motus, qui nomine careant, quos quidem saepe exprimimus per illa *quoddam*, *veluti*, *quasi* et alia ejusmodi." Italicized by Bošković.

De transformatione locorum geometricorum.[23] It is an absurdity which follows "from the infinite extension of a parabola" (*ex infinita Parabolae extensione*),[24] if the parabola MVN and its tangent AB in the vertex V are considered (Fig. 2). The size of aperture or hiatus of the parabola is estimated by two independent reasonings.

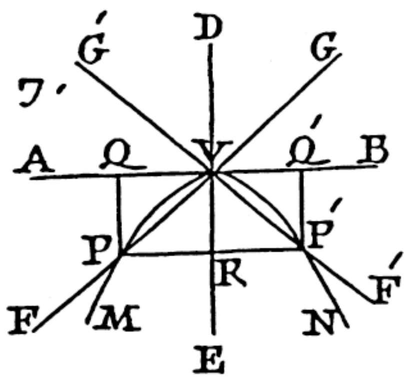

Figure. 2. Bošković's estimate of an infinitely great aperture of a parabola. Boscovich, *De continuitatis lege* (1754), Fig. 7. Courtesy of the State Archives (Državni arhiv) in Dubrovnik.

In the first estimate, the vertex V of the parabola is considered the centre of an absolutely infinite circle. In that case, the tangent AB, which according to Bošković's idea is extended on both sides in the infinite, is the diameter of this circle. This means, by analogy with the expression valid for every finite circle $c = 2r\pi$, that the tangent AB is greater than a quarter of the circumference of this infinite circle:

$2r > 2r \,.\, (\pi/4)$,

$AB > c/4$.

[23] See the exposition and explanation of the paradox in Boscovich, "De transformatione locorum geometricorum," n. 882, pp. 461–463, and n. 855, pp. 465–466; Boscovich, *De continuitatis lege*, nn. 86–87, pp. 38–39.

Cf. Ruđer Bošković, *O zakonu kontinuiteta i njegovim posledicama u odnosu na osnovne elemente materije i njihove sile* (Beograd: Matematički institut SANU, 1975), translated from Latin by Darinka Nevenić-Grabovac, nn. 86–87, pp. 50–51; Rogerius Iosephus Boscovich / Ruđer Josip Bošković, *De continuitatis lege / O zakonu neprekinutosti*, uvod, kritičko izdanje latinskoga teksta, prijevod na hrvatski, komentar, dodaci i kazala Josip Talanga (Zagreb: Školska knjiga 1996), nn. 86–87, pp. 86–89.

See *absurdum manifestissimum* in Boscovich, *De continuitatis lege*, n. 87, p. 39.

[24] Boscovich, *De continuitatis lege*, n. 86, p. 38.

And, since a perpendicular from any point Q of the tangent AB always intersects the parabola MVN at a point P, it is justifiable to consider equal the size of the aperture of the parabola and the size of the whole infinite tangent AB. Therefore, the aperture of the parabola MN is greater than the quarter of the circumference of the imaginary infinite circle:

a(MVN) > c/4.

In the second estimate, one immediately notes that the straight line FG, passing through the vertex V of the parabola, always intersects the parabola in point P, and thus passes out of its aperture. In doing so, a new starting-point is chosen, according to which the parabola MVN encompasses, with its branches, only a part of the circumference of the infinite circle with the centre in point V. The question is as to how great this part is in comparison to the whole circumference, that is, what the ratio between the considered quantities is. Without detailed argumentation Bošković draws a direct conclusion: the ratio is smaller than any definite ratio. And this means that the aperture of the parabola is an infinitely smaller quantity than the circumference of the infinite circle:

a(MVN) << c.

One may rightly assume that Bošković reached this conclusion by studying the inverse ratio. Supposing that the straight lines drawn from the vertex of the parabola encompass always more than the aperture of the parabola, it is plausible to assert that the ratio between the circumference of the infinite circle and its part encompassed by the branches of the parabola is greater than any definite ratio. In his comprehensive historical and scientific commentary on the treatise *De continuitatis lege*, E. Stipanić justified Bošković's intuitive finding with the help of analytic expression and limit process.[25] This confirms once again that Bošković understood infinitesimal quantities correctly, although in the rudimentary form of Newton's method of prime and ultimate ratios.

According to Bošković's first estimate, the aperture of the parabola is greater than a quarter of the circumference of the infinite circle, and according to the second estimate, the aperture of the parabola is infinitely smaller than the circumference of this circle. This is a contradiction which Bošković indicated as "the most obvious absurdity" (*absurdum manifestissimum*). Moreover, Bošković's basic model for the ascertainment of the absurdity is presented here. When should one talk about absurdity in the estimation of an infinite quantity? First, an infinite quantity which will be compared with the infinite quantity

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[25] Ernest Stipanić, "Naučni i istorijski komentar [Boškovićeve rasprave *De continuitatis lege*]", in Bošković, *O zakonu kontinuiteta*, pp. 93–158, on pp. 133–134.

under consideration ought to be singled out. Then, two independent reasoning processes ought to be conceived and carried out, from which will follow two contradictory estimates of the infinite quantity. To the paradox thus established Bošković refers as an absurdity.

In his treatise *De continuitatis lege*, Bošković followed the same procedures in his approach to some other absurdities that are connected with infinite quantities in geometry. They were also constructed according to the same model, but his earliest paradox from *De natura et usu infinitorum et infinite parvorum* stands out among them, since it is characterized by a special form of contradiction: a part is greater than the whole, and in this particular case "a part is double the whole" (*pars dupla totius*).[26] This statement directly contradicts Euclid's eighth axiom: "The whole is greater than a part."[27] When a geometric proof ends in such a contradiction, Bošković need not make an additional effort to elucidate the contradiction. That is why he readily employs this model in many of his works.[28]

Bošković accompanied his considerations on the role of absurdity in geometry with a number of far-reaching remarks. The first of them concerns the relationship between the mysteries of the infinite and the absurdity. Having ascertained the contradiction according to the above mentioned model, Bošković concluded: "Indeed, it is a mystery no more, but absurdity."[29] By this distinction, he excluded all the behaviour of geometric quantities which in the infinite is reduced to an absurdity from the semantic field of the concept of the mysteries of the infinite (*Infiniti mysteria*). In addition, he draws attention to the source of absurdity. Absurdity follows "from that supposition of the absolute infini-

[26] Boscovich, *De continuitatis lege*, n. 89, p. 39.

[27] Euclid, *The Elements*, Vol. I, translated by Heath, p. 155.

[28] For instance, in Boscovich, *De natura et usu infinitorum et infinite parvorum*, n. 11, p. 7: "sive pars dupla totius, quod est absurdum"; also quoted in Boscovich, "Appendix ad Metaphysicam pertinens de anima et Deo," in Boscovich, *Theoria philosophiae naturalis*, n. 546, p. 257, nota (t): "nimirum pars dupla totius, quod est absurdum";

Boscovich, "De transformatione locorum geometricorum" (1754), n. 883, p. 463: "in dissertatione *de natura et usu infinitorum et infinite parvorum*, ubi ostendimus, admisso infinito absoluto in extensione, partem obvenire aequalem, immo etiam majorem toto.";

[Rogerius Josephus Boscovich], *De lege virium in natura existentium* (Romae: Typis Joannis Generosi Salomoni, 1755), n. 66, p. 26: "unde concluditur esse partem majorem toto, maximum nimirum absurdum"; also quoted in Boscovich, "Contra vires in minimus distantiis attractivas et excrescentes in infinitum.", supplementum IV. in Boscovich, *Theoria philosophiae naturalis*, n. 84, p. 291, with the marginal subtitle "Partem fore majorem toto."

[29] Boscovich, *De continuitatis lege*, n. 88, p. 39: "Id sane iam non mysterium quoddam est, sed absurdum."

te" (*ex illa absoluti infiniti suppositione*).[30] These ideas supported Bošković's fundamental views in natural philosophy:

> "Hence, in my opinion, the extended quantity, both infinitely great and infinitely small, existing actually and determined in itself, is wholly impossible."[31]

On no account did Bošković admit the absurdities into his understanding of nature, particularly not into his theory of forces.

According to Bošković's opinion presented in his treatise *De continuitatis lege*, geometry is a case for itself. In geometry it is possible to investigate space as an actual infinite (*spatium ut actu infinitum*), the actual prolongation of a line in the infinite, and, finally, the connections of these lines in the infinite as well as the mysteries of the infinite which follow therefrom.[32] Bošković expects the study of geometric quantities in the infinite to produce a feedback, that is, lead to a better understanding of the nature of the finite continuous quantities.[33]

What, after all, is the meaning of Bošković's notion of *Infiniti mysteria*? From Bošković's remarks in his treatise *De continuitatis lege* it follows that this notion does not denote the manifestation of the actual infinite, but refers only to those geometric quantities and transformations in which the potential infinite is manifested. In addition, preservation of the principle of continuity is required in geometry, because only in this way it is possible to grasp Bošković's presumption by which the study of the mysteries of the infinite will further the understanding of continuity in the finite quantities.

4. Additional explanations in Bošković's correspondence

Is there any direct confirmation that Bošković understood *Infiniti mysteria* in this particular way? There is, in Bošković's correspondence of the 1760s. A clear statement in favour of the distinction between mystery and absurd is comprised in Bošković's letter to Giovan Stefano Conti, written in Pera di Constantinopoli in the period from 20 December 1761 to 26 February 1762, during his long recuperation from an unknown fever. One of the topics in this valuable

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[30] Boscovich, *De continuitatis lege*, n. 90, p. 40.

[31] Boscovich, *De continuitatis lege*, n. 91, p. 40: "Hinc nobis extensum, et infinite parvum et infinite magnum actu existens, et in se determinatum est prorsus impossibilis, <...>".

[32] Boscovich, *De continuitatis lege*, n. 91, p. 40: " In Geometria autem, quae spatium ut actu infinitum considerat, lineae considerantur tanquam actu in infinitum productae, ex qua deinde productione omnes illi nexus in ipso infinito et mysteria, quae persequi caepimus, consequuntur."

[33] Boscovich, *De continuitatis lege*, n. 91, p. 40: "<...>, quae tamen ad intelligendam melius continuae extensionis naturam, ubi de finitis quantitatibus agitur, adhuc conducunt."

epistolary treatise, the importance of which I have already emphasized while establishing the chronology of Bošković's writings on continuity and infinity,[34] were the difficulties in understanding a certain thing. "To exclude a certain thing from consideration," Bošković begins his exposition, "it is necessary to have positive and convincing proof of its impossibility, a contradiction to which it would lead by direct reasoning."[35] This view he also consistently applied to the concept with which Conti struggled, and that was an untransparent concept of the *misterii dell'infinito*:

> "Whenever and from whatever side the infinite or, as I call it on similar occasions, a sequence of finite terms continuing in infinity should appear, our mind, too limited and finite, is downcast, and our ideas are too weak to understand the infinite clearly. That is why I name them the *mysteries of the infinite* and distinguish them from the absurdities which I discover again in the actual extension, such as the extension of the absolutely infinite line. The absurdities make me consider the thing impossible, and the mysteries, the difficulties in understanding, the clouds that veil our imagination, make me think only of the weakness of our mind."[36]

This passage distinctly confirms that at the turn of 1762 Bošković thought about the manifestations of the potential infinite while discussing the mysteries of the infinite. In fact, he writes only about the infinite, yet has the potential infinite in mind, because it is the infinite of this kind that actualizes in the example to which he refers: an infinite sequence which is not actually realized in its totality, but rather viewed in the process of infinite continuation with the help of its finite terms.

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[34] Cf. Martinović, "Pretpostavke za razumijevanje geneze Boškovićevih ideja o neprekinutosti i beskonačnosti," *Vrela i prinosi* 16 (1986), p. 16.

[35] Ruđer Bošković to Giovan Stefano Conti, Pera di Constantinopoli, 26 February 1762, filed in *Fondo Ruggero G. Boscovich*, cartella 196, fascicolo 1, in Archivio dell'Osservatorio astronomico di Brera, Milano; catalogued in *Carteggio Boscovich. Estratto da: Catalogo della corrispondenza degli Astronomi di Brera 1726–1799*, a cura di Agnese Mandrino, Guido Tagliaferri e Pasquale Tucci (Milano: Università degli Studi, 1986), as n. 213, p. 11; published in Ruggiero Giuseppe Boscovich, *Lettere a Giovan Stefano Conti*, a cura di Gino Arrighi (Firenze: Leo S. Olschki, 1980), pp. 46–85, on p. 49:

"Conviene avere una pruova positiva, e convincente della impossibilità di essa, una contradizione, a cui essa conduca con un ragionamento diretto, per escluderla."

[36] Boscovich, *Lettere a Giovan Stefano Conti*, p. 50:

"Dovunque l'infinito, o come io in somiglianti occasioni lo chiamo, serie di termini finiti continuata in infinito, entra per qualunque verso si sia, la nostra mente troppo limitata, e finita si perde, e le idee nostre son troppo deboli per concepirlo con chiarezza. Per cio questi io li chiamo *misterii dell'infinito*, e li distinguo dagli assurdi, quali ritrovo in una estensione attuale come di linea assolutamente infinita. Gli assurdi mi fanno credere la cosa impossibile; i misterj, le difficoltà di concepire, le nuvole, che offuscano la nostra immaginazione, mi fanno solamente pensare alla debolezza della nostra mente." Underlined in the autograph.

Bošković's consideration of the mysteries of the infinite draws attention to the weakness of the human mind. Contrary to the expected, argumentation of this kind is not a peculiarity of Bošković's mathematical manuscript. The argument *debilitas humani ingenii* was used by Cristoph Clavius, Bošković's most renowned predecessor at the chair of mathematics at the Collegium Romanum (1564–1571, 1576–1584, 1587–1595?). In 1612 Clavius applied it to the rule of the multiplication of two negative numbers, therefore, to an utterly simple algebraic rule:

> "it appears that the weakness of human nature should be accused of the incapacity to understand the manner in which this can be true."[37]

Three years later, prompted by the correspondence with the Swiss scholar Georg-Loius Le Sage, in even greater detail Bošković discussed the relationship between infinity and continuity, more precisely, the nexus between absurdity and leap. On the basis of his study of conic sections, in a letter of 22 April 1765 Le Sage warned Bošković that by the rotation of the section plane a series of ellipses turns into a parabola, which is a leap that nullifies the law of geometric continuity. In his exhaustive reply of 8 May 1765, Bošković elaborated his view on the problem as follows:

> "As a solution to such a difficulty, it would suffice to mention what I have demonstrated on many places, including the dissertation *De transformatione locorum geometricorum*: an absolutely infinite extension involves not only mysteries, but also absurdities. At the very end of the dissertation I also revealed the peculiar source of that thing –supreme simplicity, which the nature of the infinite requires fighting with the composition of parts, which the extension requires even more so as it further increases and approaches the infinite. That is why in the extension, as in the number of all the coexisting things, I do not admit the actual infinite, but finite magnitudes, finite numbers, but among the possible things (I admit) a series of finite terms, the series of which continues in the infinite. That [i.e. the potential infinite] I allow also in geometry, in which never will there be any leap, nor any absurdity. The absurdities and leaps follow from the actual infinite. On the same place I warned about the absurdity which arises from the area of the bisected angle, when this area is produced in the infinite, and from the

[37] Cf. Hermann Weyl, *Philosophie der Mathematik und Naturwissenschaft* (München/Wien: R. Oldenbourg, 1966), in the chapter "Zahl und Kontinuum. Das Unendliche", on p. 50, note 4: "debilitas humani ingenii accusanda (videtur), quod capere non potest, quo pacto id verum esse possit."; Felix Kaufmann, *The Infinite in Mathematics: Logicomathematical Writings*, edited by Brian McGuinness (Dordrecht: D. Reidel Publishing Company, 1978), in the chapter "Negative numbers, fractions and irrational numbers," on p. 109, note 4.

parabola extended in the infinite in such manner that it is conceived coexisting all simultaneously without any limit."[38]

Bošković's explanation of the infinite in his letter to Le Sage in 1765, includes three approaches. According to the first, absurdity and leap are the results of the same process, but viewed from two different aspects. If the infinite process is considered in its mutual relationship with the continuity, this means that a leap must not be allowed in geometry. If the same infinite process is considered from the logical point of view, absurdity must not be allowed. Both absurdity and leap follow from the actual infinite as an infinite which, in accordance with Bošković's distinction, includes "not only mysteries but also absurdities".

The second approach refers to numerically expressed multitude that exists and coexists in geometry and arithmetic. The multitude that exists and coexists actually can only be finite. But the multitude that manifests its existence potentially i.e. in the possible things (*in possibilibus*), can be also infinite.

Lastly, the third approach focuses on the tool that a mathematician can use in the explanation of the infinite quantity or infinite process. Bošković points to the "composition of parts" (*compositio partium*), and with it to continuity

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[38] Ruđer Bošković to Georges-Louis Le Sage, Milan, 18 May 1765, filed in the manuscript bundle: Georges-Louis Le Sage, *Ma Correspondance avec le Roger Joseph Boscovich, jésuite, membre de la Société Royale de Londres, Correspondant de l'Académie Royale des Sciences de Paris (par Mr. de Mairan) commencée le 20 septembre 1763*, Bibliothèque Publique et Universitaire, Genève; published in Vladimir Varićak, "Nekoliko pisama Boškovićevih," *Rad JAZU* 241 (1931), pp. 207–228, on pp. 213–223, and quotation on pp. 218–219:

"Pro eiusmodi difficultatis solutione satis est notare illud, quod ego quidem pluribus in locis demonstravi, et in illa ipsa dissertatione de transformatione locorum geometricorum, extensionem absolute infinitam involvere non mysteria tantummodo, sed etiam absurda, cujus rei et fontem indicavi praecipuum in ipso dissertationis fine, simplicitatem summam, quam infiniti natura requirit pugnantem prorsus cum illa partium compositione, quam extensio eo magis requirit, quo magis augetur, et ad infinitum accedit. Quamobrem in extensione, ut et in numero eorum omnium, quae coexistunt ego quidem nullum admitto actuale infinitum, sed finitas magnitudines, finitos numeros, et in possibilibus seriem terminorum finitorum continuatam in infinitum. Eam et in Geometria agnosco, in qua nullus unquam saltus habebitur, nec absurdum ullum. Absurda et saltus provenient ex actuali infinito. Ibidem ostendi absurdum, quod provenit ex area anguli bifariam secti in infinitum producta, et ex parabola ipsa ita producta in infinitum, ut concipiatur coexistens tota simul sine ullo limite."

The letter was discussed in Pierre Costabel, "La correspondance Le Sage – Boscovich," in *Atti del Convegno internazionale celebrativo del 250° anniversario della nascita di R. G. Boscovich e del 200° anniversario della fondazione dell'Osservatorio di Brera, Milano – Merate 6 8 Ottobre 1962* (Milano: Istituto Italiano per la storia della tecnica, 1963), pp. 205–216, on pp. 212–215, but Costabel was not familiar with the fact that the letter had been published in 1931.

as the highest degree of excellence in the composition of parts of the same kind according to Aristotle. Whenever in understanding the infinite process contradiction is avoided, yet difficulties are encountered, Bošković insists that the continuity in this process be maintained or preserved. Thus, the principle of continuity becomes the only reliable tool for the explanation of uncontradictory difficulties in understanding the geometrical infinite.

5. The meaning of Bolzano's remark on Bošković's Infiniti mysteria

Bošković's concept *Infiniti mysteria* did not pass unnoticed. After Le Sage, it prompted Bernard Bolzano's comment included in his *Paradoxien des Unendlichen*, completed in the period 1847–1848, therefore, shortly before his death, and published posthumously by Fr. Přihonský in 1851. Bolzano's entire remark reads as follows:

> "A single remark that the line produced only on one side in the infinite is not for that reason on that very side a limited line, that so little can also be said about the limit point of this line as, for example, about the apex of a sphere, or about the curving of a straight line, or a single point, or a collision point of two bodies moving equally —this unique remark, I say, accomplishes for most paradoxes (*mysteria infiniti*) which Bošković exposed in his *Dissertatio de transformatione locorum geometricorum* (appended to the third volume of his *Elementa universae matheseos*, published in Rome in 1754) to be shown in their nullity."[39]

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[39] Bernard Bolzano, *Paradoxien des Unendlichen*, herausgegeben aus dem schriftlichen Nachlasse des Verfassers von Fr. Přihonský (Leipzig: Reclam, 1851); reprint: mit Einleitung und Anmerkungen herausgegeben von Bob van Rootselaar (Hamburg: Felix Meiner, 1975), n. 44, pp. 86–87:

"Die einzige Bemerkung, daβ eine, auch nur nach einer Seite hin in das Unendliche hinaus gezogene Linie eben deshalb keine nach dieser Seite hin begrenzte Linie sei, daβ also auch von einem Grenzpunkte derselben so wenig gesprochen werden könne, wie etwa von der Spitze einer Kugel oder der Krümmung einer Geraden oder eines einzelnen Punktes, oder dem Punkte des Zusammenstoßes zweier Gleichlaufenden – diese einzige Bemerkung, sage ich, reicht hin, um die meisten Paradoxien (*mysteria infiniti*) die B o s c o w i c h in s. *Diss. de transformatione locorum geometricorum* (angehängt s. *Elem. univ. Matheseos T. III. Romae* 1754) vorgebracht hat, in ihrer Nichtigkeit zu zeigen."

Italicized by Bolzano.

The quotation undoubtedly belongs to the final redaction of Bolzano's work, since in the manuscript *Vorarbeiten zu den "Paradoxien des Unendlichen"* (1844) Bolzano twice refers to Schultz's calculation of the volume of the whole infinite space, which prompted him to comment on Bošković's *Infiniti mysteria*, yet he fails to mention Bošković's treatise *De transformatione*

With this remark, Bolzano evidently did not reject the true content and application of Bošković's concept of *Infiniti mysteria*, because, had that been the case, he would have examined and evaluated:

(1) its connection with the concept of geometric transformation;
(2) its harmony with the principle of continuity;
(3) the model for ascertaining absurdity in the understanding of the infinite.

On the contrary, his remark aimed to invalidate any discussion about the actual infinite in geometry.

What was Bolzano's objection aimed at? First of all, it should be noted that Bolzano made this remark while considering Johann Schulz's attempt at calculating the volume of the whole infinite space. Schultz assumes that from any given point *a*, straight lines can be produced in any direction in the infinite, and that every point *m* of the space lies on one and only one straight line drawn from the point *a*.

Given that these straight lines are radii, that is, the lines terminated on both sides, Schulz concludes that the whole of space can be recognized as a globe with the radius of the infinite length ∞, and the volume of $4/3\ \pi\ \infty^3$.[40] Judging Schulz's calculation of the volume of the whole space, Bolzano remarked that Schulz had made an expected error by regarding the half-lines as radii, that is, as terminated lines. [41]

The assumptions of this kind Bolzano may have frequently encountered in Bošković's treatise *De transformatione locorom geometricorum*, e.g. in the above explicated example of absurdity which follows from the infinite extension of the parabola. But in all these cases, Bošković presumed, although not always explicitly formulated, that a contradiction, i.e. absurdity always follows from the assumption of the absolute or actual infinite. It would suffice to mention

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locorum geometricorum. See Bernard Bolzano, *Philosophische Tagebücher 1827–1844*, zweiter Teil, hrsg. von Jan Berg, in *Bernard Bolzano-Gesamtausgabe*, hrsg. von Eduard Winter, Jan Berg, Friedrich Kambartel, Jaromír Loužil, und Bob van Rootselaar, Reihe II B, Band 18 (Stuttgart-Bad Cannstatt: Friedrich Fromann, 1979), p. 84 and 98.

[40] Johann Schultz, *Versuch einer genauen Theorie des Unendlichen. Erster Theil. Vom Unendlichgroßen und der Meßkunst desselben* (Königsberg/Leipzig: Hartung, 1788), p. 320; Bolzano, *Paradoxien des Unendlichen*, n. 44, pp. 85–86.

Cf. Gert Schubring, "Ansätze zur Begründung theoretischer Terme in der Mathematik: Die Theorie des Unendlichen bei Johann Schultz," *Historia Mathematica* 9 (1982), pp. 441–484, on p. 466 and 476.

[41] Bolzano, *Paradoxien des Unendlichen*, n. 44, p. 86: "Gefehlt und ganz offenbar gefehlt hat Schulz nur darin, daβ er die Geraden, die aus dem Punkte *a* nach allen Richtungen ins Unbegrenzte hinaus gezogen sein müssen, wenn jeder Punkt des Raumes in irgendeiner derselben gelegen sein soll, dennoch als Halbmesser, somit als beiderseits begrenzte Linien annahm."

that in his early mathematical treatise *De natura et usu infinitorum et infinite parvorum* Bošković asserted:

> "that the apsolute infinite or infinitely great quantity in extension cannot exist nor without danger be conceived."[42]

He repeated the same assertion about the origin of absurdity in geometry in his works published after the treatise *De transformatione locorum geometricorum*, namely in *De continuitatis lege*, in the appendix *De anima et Deo* to *Theoria philosophiae naturalis*,[43] and in the above mentioned letters to Giovan Stefano Conti and Georges-Louis Le Sage. This means that Bolzano did not perceive Bošković as a like-minded mathemathician.

In addition, Bolzano reproached Schulz for not having overcome the bias in his understanding of the infinite, which he, by his own admission, refuted in *Paradoxien des Unendlichen*, in paragraph 21 and the following ones.[44] These paragraphs deal with the most important yet paradoxical property, which describes the relationship between two infinite sets:

> "Namely, I assert that two sets, both of which are infinite, may stand mutually in such a relationship
> that, *on the one hand*, it is possible for each thing belonging to one set to be successfully paired with a thing from the other set in such a way that not a single thing from either sets is left unpaired, and also, that not a single thing is found in two or more pairs,
> while, *on the other hand*, it is still possible that one of these sets includes the other one as its mere *part*, so that the multitudes they present, if we consider all the things of the same sets at once, that is, as unities, (multitudes) mutually have *most diverse relationships*."[45]

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[42] Boscovich, *De natura et usu infinitorum et infinite parvorum* (1741), n. 11, p. 7: "Infinitum autem absolutum sive infinite magnum in extensione nullum esse posse nec tuto concipi sic demonstramus." Cf. note 15.

[43] Boscovich, *De continuitatis lege*, n. 90, p. 40: "Absurdum ipsum totum oritur ex illa absoluti infiniti suppositione."; Boscovich, "Appendix ad Metaphysicam pertinens de anima et Deo," in Boscovich, *Theoria philosophiae naturalis*, n. 546, p. 257, nota (t): "suppositio infiniti absoluti, quae contradictionem involvit."

[44] Although Bolzano, *Paradoxien des Unendlichen*, on p. 86, refers to paragraph 21 and the following ones, of particular philosophical and mathematical significance are the paragraphs 19–21. Cf. Bolzano, *Paradoxien des Unendlichen*, pp. 26–31.

[45] Bolzano, *Paradoxien des Unendlichen*, n. 20, p. 28: "Ich behaupte nämlich: zwei Mengen, die beide unendlich sind, können in einem solchen Verhältnisse zueinander stehen, daβ es *einerseits* möglich ist, jedes der einen Menge gehörige Ding mit einem der anderen zu einem Paare zu verbinden mit dem Erfolge, daβ kein einziges Ding in beiden Mengen ohne Verbindung zu einem Paare bleibt, und auch kein einziges in zwei oder mehreren Paaren vorkommt; und dabei ist es doch *andererseits* möglich, daβ die eine dieser Mengen die andere als einen bloβen *Teil* in sich

According to Bolzano's basic idea, between two infinite sets it is possible to establish, in terms of contemporary mathematics, bijection as a specific criterion for the equality of sets. At the same time, it is possible to establish a relationship of inequality between these two infinite sets, concerning the number or the multitude of their elements, so that one infinite set can be a proper part of the other. Bolzano's example of such a relationship between two infinite sets is the linear mapping of the interval [0.5] into the interval [0.12] shown by the equation

$$5y = 12x.$$

This example contains both aspects of Bolzano's paradoxical explanation of the relationship between the infinite sets: the relationship 'part-whole', and the structure of bijection.

Bolzano's objection to Schultz implied that the latter had not really understood the apory in the relationship between the infinite sets. Might this objection also partly refer to Bošković, even though he was not explicitly mentioned? The answer is: definitely not, as Bošković's estimate of the size of the aperture of a parabola confirms it. Bošković implicitly accepts bijection as a criterion for the equality between two infinite sets. It occurs when he assigns any point Q of the tangent AB to the point P at which the parabola is intersected by a perpendicular to AB drawn from the point Q. On the basis of this assignment, he considers equal the size of the aperture of the parabola and the size of the tangent AB, i.e. the size of the diameter of the infinite circle with the centre in point V.[46] In 1754 Bošković did not apprehend the structure of bijection in its strict mathematical form, which was introduced by Bolzano in his *Paradoxien des Unedlichen* a hundred years after Bošković had worked on it. Nevertheless, the relationship between the part and the whole was an unavoidable constant in Bošković's investigations of the problem of the infinite. As I have already shown in a number of examples, Bošković explicitly and systematically used this relationship (between the part and the whole) in disputing the absolute or actual infinite in mathematics, starting from 1741 until 1765.[47] Therefore, with regard to the apory arising from the use of the infinite in mathematics, Bošković and Bolzano are likeminded.

faβt, so daβ die Vielheiten, welche sie vorstellen, wenn wir die Dinge derselben alle als gleich, d. h. als Einheiten betrachten, die *mannigfaltigsten Verhältnisse* zueinander haben."

[46] Boscovich, "De transformatione locorum geometricorum", n. 882, p. 462: "Quare hiatus ille idem tantumdem extenditur, quantum ipsa circuli infiniti diameter, cui proinde aequalis erit."; Boscovich, *De continuitatis lege*, n. 87, p. 39: " <…>, patet, hiatum Parabolae ipsius MN aequari toti tangenti infinitae AB, <...>".

[47] Cf. quotations in the note 28.

Why did Bolzano fail to notice the fundamental characteristic of Bošković's approach? Indeed, one of the reasons was that Bošković's arguments were scaterred in a series of treatises and letters written between 1741 and 1765. With some of these sources Bolzano was not familiar, moreover, the correspondence was not even published. It is certain that Bolzano was familiar with only two of them: *De transformatione locorum geometricorum* and *Theoria philosophiae naturalis*, the latter as early as 1815, as concluded by Bob van Rootselaar on the basis of Bolzano's manuscript *Miscellanea Mathematica*.[48] In addition, Jan Berg warned that Bolzano's private library housed the second Vienna edition of Bošković's masterpiece *Philosophiae naturalis theoria* from 1759.[49] The other reason rests in the difference between Bošković's and Bolzano's motives for the research into the infinite. Although guided by philosophical reasons, Bošković's refutation of the absolute infinite in extension and geometry in the middle of the eighteenth century mainly serves a two-fold purpose:

(1) advancement of calculus as a new mathematical method applicable to undetermined, potentially infinite quantities;
(2) determining the role of the principle of continuity in the systematic study of transformations of geometric loci.

By contrast, Bolzano's approach refers to mathematical investigation of the properties of infinite sets, as confirmed by the regular use of the subsequently generally accepted term *Menge*. Therefore, Bolzano's approach belongs to that development stage which different mathematical entities reduced to sets and mappings as basic mathematical entities, and finally, led to the set theory.

However, despite the difference in approach, Bošković and Bolzano described the relationship between infinite sets by means of (1) the relationship 'part-whole', and (2) the structure of bijection, that is, by the two insights that Richard Dedekind included in his definition of the infinite system in his famous article *Was sind und was sollen die Zahlen?* (1888).

Dedekind's definition required a considerable mental effort:

(1) the intuitive description of the notion of system (n. 2);
(2) the definition of the proper (right) part of the system (n. 6);
(3) the definition of the conformal mapping, that is, by contemporary sense the bijective mapping (n. 26);

[48] Bob van Rootselaar, "Anmerkungen," in Bolzano, *Paradoxien des Unendlichen*, pp. 133–149, in Anm. zu n. 45, p. 145: "Die *Theoria* von Boscovich war Bolzano schon seit 1815 bekannt (vgl. MM S. 1204)."

[49] See the bibliographical note in Bolzano, *Philosophische Tagebücher 1827–1844*, zweiter Teil, p. 123.

(4) the definition of the conformal systems, that is, the equipolent sets in terms of contemporary mathematics (n. 32).

Only then did Dedekind pronounce his basic definition:

"*Pronouncement*. The system S is called *infinite* if it is conformal to its proper (right) part; if this is not the case, S is (called) a *finite* system."[50]

The property he used for the definition had already been used by Bolzano and Georg Cantor, as Dedekind pointed out in the preface to the second edition of his article published in 1893. "But none of the above mentioned writers," he wrote referring to Bolzano and G. Cantor, but not to Bošković, "tried to raise*elevate this property to the definition of the infinite, and, on this fundament, build a science of numbers in a strictly logical manner, <...>"[51] Indeed, Dedekind was right. Bolzano's and Cantor's views, as well as those of Bošković and Schultz that preceded them, belonged to the historical development of the paradoxical relationship between infinite sets which Dedekind, for the first time in the history of mathematics, incorporated into the mathematical definition of the infinite set.

6. Towards the exact definition of the infinite

Bošković's concept of *Infiniti mysteria* should, therefore, be assessed from the perspective of the historical development that started with Bošković and ended with Dedekind. When Bošković for the first time mentioned this notion in his treatise *De maris aestu* (1747), he asserted that in the investigation of geometric transformations it was necessary to explain the mysteries of the infinite. At that time, on the basis of demonstrations in his treatise *De natura et usu infinitorum et infinite parvorum* (1741), he already had some experience in disputing the actual infinite in geometry, so that he based the study of the mysteries

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[50] Richard Dedekind, "Was sind und was sollen die Zahlen?," zehnte Auflage, in Richard Dedekind, *Was sind und was sollen die Zahlen? Stetigkeit und Irrationale Zahlen* (Braunschweig: Friedr. Vieweg & Sohn, 1965), pp. III–XI, 1–47, in n. 64, p. 13:

"*Erklärung*. Ein System S heiβt *unendlich*, wenn es einem echtem Teile seiner selbst ähnlich ist (32); im entgegengesetzten Falle heiβt S ein *endliches* System."

Cf. account of Dedekind's contribution in Walter Purkert und Hans Joachim Ilgauds, *Georg Cantor 1845–1918* (Basel: Birkhäuser, 1987), pp. 136–138.

[51] Richard Dedekind, "Vorwort zur zweiten Auflage," dated 24 August 1893, in Dedekind, "Was sind und was sollen die Zahlen?," pp. IX–XI, on pp. IX–X:

"Aber keiner der genannten Schriftsteller [G. Cantor (*Ein Beitrag zur Mannigfaltigkeitslehre*, Crelle's Journal, Bd. 84; 1878) und Bolzano (*Paradoxien des Unendlichen*, § 20; 1851] hat den Versuch gemacht, diese Eigenschaft zur Definition des Unendlichen zu erheben und auf dieser Grundlage die Wissenschaft von den Zahlen streng logisch aufzubauen, <…>".

of the infinite on the philosophical assumption of the existence of the infinite. While constructing the theory of geometric transformations in his treatise *De transformatione locorum geometricorum*, he avoided defining the mysteries of the infinite in terms of Euclidean tradition, but he gave a large meaning to this notion: all the manifestations of the potential and actual infinite. Only with his treatise *De continuitatis lege* did he start to make a strict distinction between mystery and absurdity in the understanding of the geometric infinite, and from that time he recognized the mysteries of the infinite only in those geometrical quantities and transformations in which the potential infinite was manifest, on condition that the principle of continuity was preserved.

On the contrary, absurdity always follows from the assumption of the actual infinite, and it is ascertained during the proof in which both the structure of bijection and the relationship 'part-whole' are used, that is, both aspects that essentially mark Bolzano's paradoxical conception of the relationship between infinite sets, and Dedekind's mathematical definition of the infinite system. In his model for ascertaining absurdity, Bošković always uses the relationships between *geometric* quantities as representatives of the relationships between infinite quantities. The turning point prepared by Bolzano and achieved by Dedekind and Georg Cantor, took place in another mathematical field, namely, in the *set* approach to the real numbers. These two points, the use of the same mathematical contents, such as the structure of bijection and the relationship 'part-whole', on the one hand, and the difference between the Euclidean geometric approach and the set approach, on the other hand, determine the place of Ruđer Bošković on the historical path towards the exact, mathematical definition of the infinite.

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Pojam *Infiniti mysteria* u Boškovićevim geometrijskim istraživanjima

Sažetak

Ruđer Bošković prvi je put spomenuo pojam 'tajne beskonačnine' (*Infiniti mysteria*) kad je u raspravi *De maris aestu* (1747) ustvrdio kako je pri istraživanju geometrijskih transformacija potrebno protumačiti upravo tajne beskonačnine. Već je tada, na osnovi dokazā provedenih u raspravi *De natura et usu infinitorum et infinite parvorum* (1741), imao iskustvo osporavanja aktualne beskonačnine u geometriji, pa je proučavanje tajnā bekonačnine uvjetovao filozofskom pretpostavkom o postojanju beskonačnine.

Dok je u raspravi *De transformatione locorum geometricorum* (1754) izgrađivao teoriju geometrijskih transformacija, Bošković je izbjegavao definirati 'tajne beskonačnine' u duhu euklidske tradicije, ali je odredio široko značenjsko polje tog pojma: sva očitovanja potencijalne i aktualne beskonačnine. Tek je od rasprave *De continuitatis lege* (1754) počeo strogo lučiti tajnu i apsurd u razumijevanju geometrijske beskonačnine, pa je otada 'tajnu beskonačnine' prepoznavao samo u onim geometrijskim veličinama i transformacijama u kojima se očituje potencijalna beskonačnina – uz uvjet da pritom vrijedi princip neprekinutosti.

Dvije potvrde Boškovićeva razumijevanja pojma *Infiniti mysteria* mogu se pronaći i u njegovoj korespondenciji tijekom 1760-ih: dragocjeno pismo rasprava što ga je od 20. prosinca 1760. do 26. veljače 1761. pisao u Carigradu mladom Giovanu Stefanu Contiju odlikovalo se jasnim govorom u prilog razlikovanju tajna-apsurd, a iscrpan Boškovićev odgovor švicarskom učenjaku Georges-Louisu Le Sageu s nadnevkom 8. svibnja 1765. razmatrao je povezanost između beskonačnine i neprekidnine, odnosno povezanost apsurda i skoka.

Naprotiv, apsurd uvijek slijedi iz pretpostavke o aktualnoj beskonačnini, a ustanovljuje se tijekom dokaznoga postupka u kojem se upotrebljavaju i struktura bijektivnog preslikavanja i sklop 'dio – cjelina', dakle oba aspekta koji bitno obilježavaju Bolzanovu paradoksalnu zamisao o odnosu između beskonačnih skupova, a potom Dedekindovu matematičku definiciju beskonačnog sistema.

U svom modelu za utvrđivanje apsurda Bošković se redovito koristi odnosima među *geometrijskim* veličinama kao reprezentantima odnosā među bekonačnim veličinama. A prekretnica koju je Bolzano pripremio u svojim *Paradoxien des Unendlichen* (1851), a izveli je Dedekind i Georg Cantor, zbila se u drugom matematičkom području: unutar *skupovnog* proučavanja realnih *brojeva*. Ta dva momenta, s jedne strane – uporaba istih matematičkih sadržaja, kao što su struktura bijektivnog preslikavanja i odnos 'dio – cjelina', a s druge – bitna razlika između euklidskoga geometrijskog i skupovnog pristupa, određuju mjesto Ruđera Boškovića na povijesnom putu prema egzaktnomu, matematičkom određenju beskonačnine.

Ključne riječi: Ruđer Bošković, Bernard Bolzano; geometrija, teorija geometrijskih transformacija, aktualna beskonačnina, potencijalna beskonačnina, *Infiniti mysteria,* neprekinutost