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DETERMINATION OF THE BENDING MOMENT – CURVATURE RELATIONSHIP FOR REINFORCED CONCRETE HOLLOW SECTION BRIDGE COLUMNS

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Subject review
The paper deals with accurate determination of the relationship between bending moment and curvature of hollow rectangular bridge columns. With the calculation procedure to determine accurate curve of relationship, the method of determining the idealized diagram typically used in practice is also shown. The aim of this work is to point to a reserve in load capacity of an element when comparing the results of an accurate curve and the idealized diagram, and to analyse the impact of various calculation situations on the curve shape. The conclusion is that for practical use it is sufficient to use the idealized diagram, but for detailed scientific analysis the exact relationship between bending moment and curvature must be determined.

Keywords: *bending moment; bridge columns; curvature; hollow section*

Iznalaženje odnosa momenta savijanja – zakrivljenosti armiranobetonskih stupova šupljeg presjeka na mostovima

Pregledni članak
Predmet rada je točno određivanje odnosa momenta savijanja i zakrivljenosti šupljih pravokutnih stupova mostova. Uz postupak proračuna točne krivulje odnosa prikazan je i način određivanja idealiziranog dijagrama obično korištenog u praksi. Cilj rada je ukazati na zalihu nosivosti elementa u usporedbi točne krivulje i idealiziranog dijagrama te analizirati utjecaj različitih proračunskih situacija na oblik krivulje. Zaključak je da je za praktičnu uporabu dovoljno koristiti idealizirani dijagram dok se za detaljne znanstvene analize mora određivati točan odnos momenta savijanja i zakrivljenosti.

Ključneriječi: *moment savijanja; stupovi mostova; šuplji presjek; zakrivljenost*

1 Introduction

In seismically active areas, ductility of structures is an important parameter for structural design because the modern dimensioning of structures resistant to seismic action is based on absorption of energy and its dissipation through targeted plastic deformation in order to preserve the majority of the structure in an event of a major earthquake.

Accurate determination of the bending moment and section curvature relationship curve of reinforced concrete sections is a reliable indicator of the load capacity of structures subjected to seismic load [1]. To assess the bearing capacity of bridge columns under seismic load, according to the current European standards [2÷5] nonlinear methods like the *Pushover-Analysis* and similar are used. In order to achieve a more accurate simulation of the real structural behaviour, designers need accurate data entry (input) required for the calculation, in this case the bending moment and section curvature relationship curve ($M-\phi$ diagram). Sectional curvature is also used when calculating the rotational capacity of plastic hinges [6], etc.

For a known form of element's cross section and material characteristics it is possible to calculate the diagram of bending moment – curvature relationship. Designers typically use the idealized $M-\phi$ diagram, which is described with three characteristic points that is, three states in which a cross section can be found by gradually increasing the load (Fig. 1).

The first characteristic point is reaching of the tensile strength of concrete at the edge of section in tension. After reaching the tensile strength a crack in concrete appears and spreads with the increase of tension load, which automatically decreases the cross section's moment of inertia. The next characteristic point is the moment of occurrence of "flow" in the tensile reinforcement. The third point is the point of collapse, which could be a collapse of tensile reinforcement, compressive failure of concrete and very rare, though possible, the simultaneous collapse of both materials.

Idealized diagram is obtained by taking the relationship between the above mentioned characteristic points as linear (trilinear diagram). More accurate form of the diagram can be determined by gradually increasing bending moment and calculating the corresponding curvature or gradually increasing curvature and calculating the corresponding bending moment.

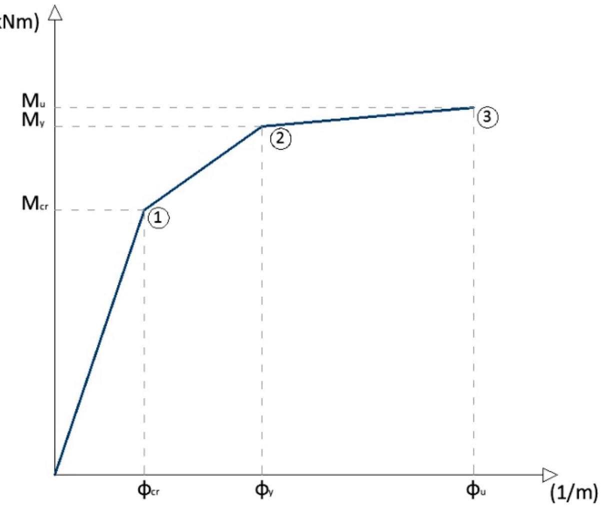

Figure 1 Idealized $M-\phi$ diagram

Due to the fact that reinforced concrete is a composite of two materials with different characteristics (Fig. 2), that designers work with sections of different forms that are subjected to cracking, changing of the effective height of elements, etc., and that we have different combinations of bending moments and compressive forces, it can be concluded that the exact determination of the $M-\phi$ curve is a rather complex problem and that there is no practical and simple expression for its computation.

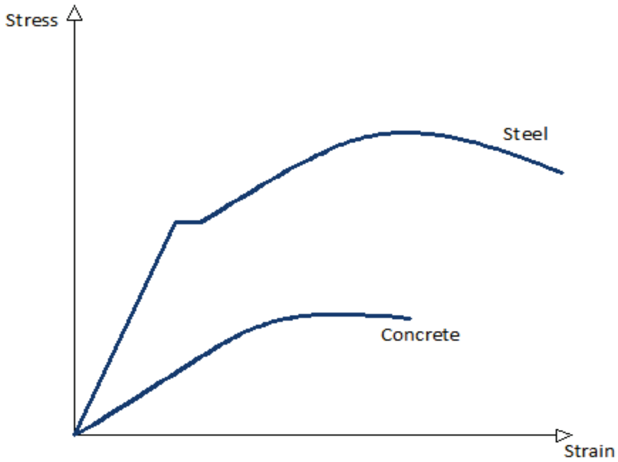

Figure 2 Stress-strain diagrams for concrete and steel

Expressions for calculating the $M–\phi$ curve, given below, refer to the reinforced concrete columns with rectangular hollow sections typical for bridges whose superstructure is high above the ground.

General rules and assumptions regarding calculation of reinforced concrete elements and design of reinforcement were based on the work done in [7÷9], based on European standards and Croatian tradition and experience in bridge construction and design. The calculation procedure proposed in the paper is suitable for modification to other standards, which will be commented later in the paper. Geometrical relations described in the paper are generally applicable.

2 Previous research

Through the advancement of science and seismic construction design, this topic has been becoming more and more interesting for authors as the basis of scientific research. So far, a lot of work on a similar topic and with very "colourful" results has been done, but most with the same or close to, general conclusions about the behaviour of elements subjected to bending or pressure and bending. Some of the better known studies are discussed below.

The conclusion of one of the first studies dating back to the 1964 on this subject [10], which is still accurate today, is that it would be most practical to have an expression which would include relationship between compressive strength, bending moment and curvature, but because of the properties of reinforced concrete as a composite material and its ways of response to external stimuli for full bearing capacity, this is a very difficult task. The authors of this study in their research have analysed the impact of various input data such as the thickness of the protective layer of reinforcement, the impact of changes in compressive force, share of reinforcement, etc. on solid rectangular sections. An excellent input parameter that the authors of this study took into account is not the maximum compressive force that section can handle but compressive force at which the loaded element loses stability.

By far the most detailed theoretical development of the bending moment and curvature relationship has been created by a group of authors headed by Professor S. Chandrasekaran from the Indian Banaras University [1], also on the problem of bending of solid rectangular cross sections. The work analyses the effects of different input data on $M–\phi$ diagram's shapes and different structure collapse scenarios. The authors have attached with the theoretical background a program that is very simple to use, which after entering basic parameters calculates moment and curvature relationship. The problem lies in the fact that there is no connection between the theoretical background presented in the paper and the program that is used to obtain the results and because of that potential users cannot be sure of the accuracy of the information received?! The comment of the authors of this article is that the proposed method of calculating is very complicated and even enters the sphere of complex numbers, and that for this reason the ability of applying it in everyday practice is questionable but it can be said that the concept of the book [1] and the results obtained serve as a conceptual basis for research in this article.

In a very interesting paper [11] the authors analysed behaviour of elements with the occurrence of cracks due to bending. The impact of cracks on the $M–\phi$ diagram, behaviour in between the cracks as well as the phenomenon of "reinforcement" of reinforcing steel has been analysed. The disadvantage of this study is that the authors used the stress-strain diagram of concrete as bilinear in disregard of reduction of the elastic modulus by increasing pressure load, which automatically tends to decrease the accuracy of the final result.

Croatian authors [12] have produced a similar calculation of bending of beams with solid rectangular and "T" intersection. Interesting feature of this study is that the authors have shown a way of getting the "correct" diagram in two ways: by gradually increasing the bending moment and by gradually increasing curvature of the cross section. From here other values required to obtain the diagrams are calculated. In this paper the intention to accurately display the section's behaviour after exceeding the tensile strength of concrete is visible.

It is evident from the described text that the individual researchers based themselves only on certain types of sections or loads due to the high complexity of this type of calculation per se and because it regards a composite material consisting of two materials with completely different properties. The conclusions of previous studies, which relate to the general behaviour of elements, are that increase of the share of reinforcement reduces sections ductility, the greater strength of the material results in increased bearing capacity... The problem arises in the fact that each of the authors in their study provides different results and different relations between bending moment and curvature. So far the point of reaching the tensile strength of concrete with start of cracks spreading in the cross-section, and the question of the impact of stability of the element on actual behaviour compared with numerical methods is insufficiently researched, which leaves room for improvement in this area.

One of the questions raised by the analysis of previous research is also how to find out which research is really accurate and suitable for everyday use. Comment of the authors of this article is that all previous studies are correct to the extent that they meet their basic assumptions and logic of the calculation. The difference is in how much influence and input data has been taken into account when forming the calculations themselves, how the possible experimental research has been analysed and how the results were presented. It is very difficult to gather, until now proposed, calculation procedures and subject them to comparison with "live" results, the results obtained by extensive experimental studies on real samples. The reason may lie in the fact that all of the above authors came to the conclusion that for everyday practice it is accurate enough to use an idealized diagram which is a counter-argument in search of financing expensive testing that this problem requires so that we can verify and calibrate any expression obtained by numerical methods.

Expected scientific contribution of this study compared with previous works is:

1) A new concept of exact calculation of $M-\phi$ diagram, using the existing terms, which takes into account the influence of compressive force, variation of the concrete's modulus of elasticity and changes in geometric characteristics of the cross section due to the appearance of cracks. Not taking into account all these parameters was a major drawback of previous research and had effect on the accuracy of their results.
2) Understandable and easily usable algorithm for writing a computer program with focus on automated calculation of $M-\phi$ diagram in which after entering the required input data, results are obtained without any further action by the user.
3) Clearly diagrammatically shown and proper interpretation of the research results, which represent the real behaviour of processed elements.

3 Analytical expressions for calculation of the bending moment - curvature relationship of hollow section concrete bridge columns.

In preliminary dimensioning of columns, for initial geometric size of cross sections designers can use the following values [cm] depending on whether the columns are articulated (Tab. 1) or individual (Tab. 2):

Table 1 Recommended dimensions of hollow articulated columns

Height	*B*	*H*	*D*
< 50 m	180 ÷ 250	200 ÷ 220	30 ÷ 40
≥ 50 m	250 ÷ 350	220 ÷ 250	40 ÷ 60

Table 2 Recommended dimensions of hollow single columns

Height	*B*	*H*	*D*
< 50 m	400 ÷ 500	200 ÷ 300	30 ÷ 40
≥ 50 m	500 ÷ 800	300 ÷ 500	40 ÷ 60

The real $M-\phi$ diagram will be determined with the exact value of the relation of bending moment and curvature between the characteristic points (Fig. 1). The diagram area from 0 to the point where concrete tensile strength is reached can be taken as linear in reference to linear part of stress-strain diagram (Fig. 2), and the rest of diagram must be accurately calculated.

To calculate curvature, it is necessary to calculate the geometrical characteristics of the cross section and the critical moment at which it reaches the observed situation with active compressive force. For the example of $M-\phi$ diagram calculation the selected hollow section is as follows: width B = 400 cm, height H = 200 cm and thickness D = 30 cm of which concrete cover is d = 3 cm (Fig. 3), material characteristics: concrete f_{ck} = 40 N/mm^2, reinforcement f_y = 500 N/mm^2 and under compressive force of 2000 kN. Reinforcement of the ridge was ignored because simplification of the calculation process, resulting in a reserve of bearing capacity compared to the results obtained.

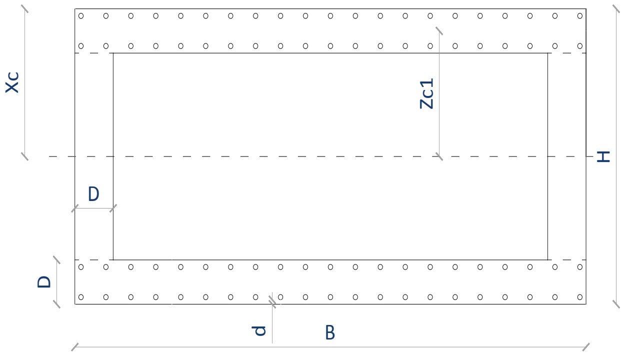

Figure 3 Columns hollow section

3.1 Reaching the tensile strength of concrete

The cross section is loaded with compressive force and bending moment in the amount necessary to reach the tensile strength of concrete.

- **curvature** [13] is calculated using the expression:

$$\phi_1 = \frac{M_{cr}}{E_{cm} \times I}, \tag{1}$$

where: ϕ – curvature; E_{cm} - secant modulus of elasticity of concrete; I - second moment of area of concrete cross section; M_{cr} - critical bending moment at which the tensile strength of concrete is reached.

- position of **neutral axis** is calculated from the relationships in stress diagram (Fig. 4)

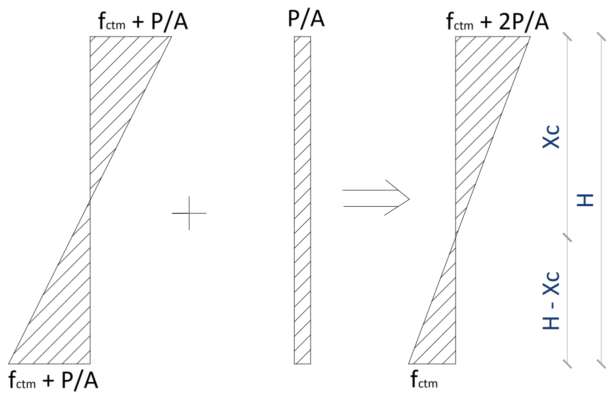

Figure 4 Strain diagram at reaching tensile strength

$$x_c = \frac{(f_{\mathrm{ctm}} + 2P/A) \times H}{2 f_{\mathrm{ctm}} + 2P/A}, \tag{2}$$

where: x_c - position of the neutral axis at the moment of reaching the tensile strength; f_{ctm} - mean value of axial tensile strength of concrete; P - compressive force; A - section area; H - section height.

- cross sections moment of inertia for the overall height H is calculated as:

$$\begin{aligned} I_{\mathrm{i}} = {} & I_{\mathrm{c}} + A_{\mathrm{c}} \times (H/2 - t_i)^2 + (\alpha_{\mathrm{ep}} - 1) \times [I_{\mathrm{sc1}} \times n_{\mathrm{sc1}} + \\ & + A_{\mathrm{sc1}} \times n_{\mathrm{sc1}} \times (t_i - d - r_{\mathrm{sc1}})^2 + I_{\mathrm{sc2}} \times n_{\mathrm{sc2}} + \\ & + A_{\mathrm{sc2}} \times n_{\mathrm{sc2}} \times (t_i - D + d + r_{\mathrm{sc2}})^2 + I_{\mathrm{st1}} \times n_{\mathrm{st1}} + \\ & + A_{\mathrm{st1}} \times n_{\mathrm{st1}} \times (H - t_i - d - r_{\mathrm{st1}})^2 + I_{\mathrm{st2}} \times n_{\mathrm{st2}} + \\ & + A_{\mathrm{st2}} \times n_{\mathrm{st2}} \times (H - t_i - D + d - r_{\mathrm{st2}})^2], \end{aligned} \tag{3}$$

where for the selected geometric section properties and materials applies: I_{i} - moment of inertia of the cross section taking into account the reinforcement moment of inertia; A_{c} - area of concrete section without excluding the area of the reinforcement; t_{i} - section centroid; α_{ep} - steel and concrete modulus of elasticity relationship; I_{scn} - moment of inertia of the reinforcement in the n^{th} row of the cross section compression zone; n_{scn} - number of reinforcing bars in the n^{th} row of the cross section compression zone; A_{scn} - area of reinforcement bars in the n^{th} row of the cross section compression zone; r_{scn} - radius of reinforcement bars in the n^{th} row of the cross section compression zone; I_{stn} - moment of inertia of the reinforcement in the n^{th} row of the cross section tension zone; n_{stn} - number of reinforcing bars in the n^{th} row of the cross section tension zone; A_{stn} - area of reinforcement bars in the n^{th} row of the cross section tension zone; r_{stn} - radius of reinforcement bars in the n^{th} row of the cross section tension zone.

- the sections centroid is calculated as follows:

$$\begin{aligned} t_{\mathrm{i}} = {} & [A_{\mathrm{c}} \times H/2 + (\alpha_{\mathrm{ep}} - 1) \times A_{\mathrm{sc1}} \times (d + r_{\mathrm{sc1}}) + \\ & + A_{\mathrm{sc2}} \times (D - d - r_{\mathrm{sc2}}) + A_{\mathrm{st1}} \times (H - d - r_{\mathrm{st1}}) + \\ & + A_{\mathrm{st2}} \times (H - D + d - r_{\mathrm{st2}})] / A_{\mathrm{i}}, \end{aligned} \tag{4}$$

where: A_{i} - area of the section taking into account the area of reinforcement.

- bending moment at which the effect of compressive force reaches the tensile strength of concrete was calculated as follows:

$$M_{\mathrm{cr1}} = \frac{\sigma_{\mathrm{ct}} \times I_{\mathrm{i}}}{H - x_{\mathrm{c}}}, \tag{5}$$

With this we obtain the first characteristic point of M–ϕ diagram (M_{cr1}; ϕ_1), i.e., the bending moment and curvature relationship at reaching the tensile strength of concrete. Flowchart diagram of calculation steps is shown in Fig. 5.

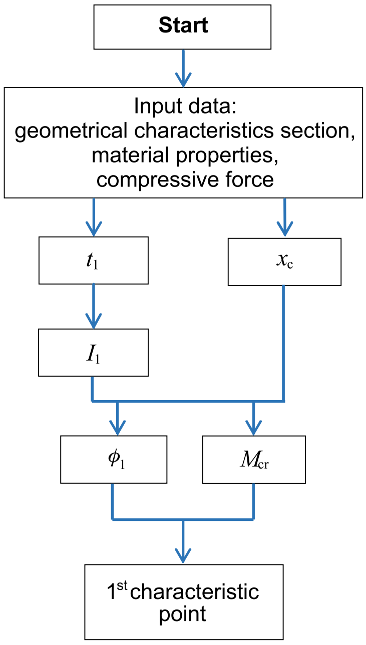

Figure 5 Calculation steps flowchart diagram

3.2 Reaching of reinforcements yield point

If we continue to increase the bending moment above the critical moment at which tensile strength of concrete is reached the appearance of crack occurs and the crack expands over the cross section which results in reduced section height, and thus the reduction of section's moment of inertia (Fig. 6).

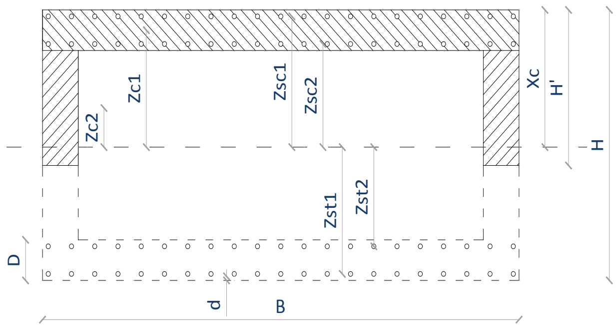

Figure 6 Cross section after crack propagation

By increasing the load further, the point at which yielding of reinforcement in tension occurs is reached and the belonging deformation is:

$$\varepsilon_{\mathrm{s0}} = \frac{f_{\mathrm{sd}}}{E_{\mathrm{s}}}, \tag{6}$$

The curvature is calculated from relationship of the position of neutral axis and deformation of concrete - Eq. (7). Reason for this is that the deformation of concrete being the value to be varied in a spread sheet in order to obtain exact position of the neutral axis, which will be explained later in the paper.

$$\phi_2 = \frac{\varepsilon_{\mathrm{cc}}}{x_{\mathrm{c}}}, \tag{7}$$

- where ε_{cc} is the deformation of concrete at edge of the section in compression as result of compressive force and bending moment

- **neutral axis'** (at this point of calculation we can use an expression from other norms and standards as [6, 14]) position is calculated as:

$$x_c = \frac{(\varepsilon_{ccM} + \varepsilon_P) \times (H - d - r)}{\varepsilon_{st1} + \varepsilon_{ccM} + \varepsilon_P}, \tag{8}$$

where: ε_{ccM} - concrete deformation at the edge in compression as a result of the bending moment; ε_{ccP} - concrete deformation at the edge in compression as a result of the compressive force; ε_{st1} - deformation of reinforcement in tension at the moment of yield occurrence - ε_{s0}.

The value of concretes compressive deformation at the edge of the section as a result of the bending moment (ε_{ccM}) is calculated by iterating deformation values until balance of inner forces in the cross section applies (Fig. 7):

$$0 = F_{st1} + F_{st2} - F_{sc1} - F_{sc2} - F_{cc}, \tag{9}$$

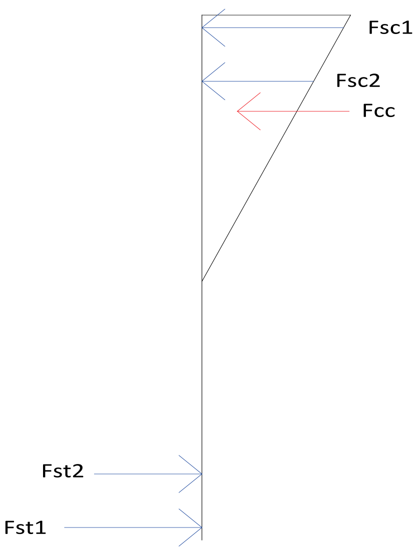

Figure 7 Horizontal forces balance

- if the neutral axis is in the web of the section, force in concrete (F_{cc}) is calculated as:

$$F_{cc} = 0{,}85 \times f_{cd} \times \alpha_v \times [B \times D - A_{sc1} - - A_{sc2} + 2 \times D \times (X_c - D)], \tag{10}$$

- if the neutral axis is in the flange of the section, force in concrete (F_{cc}) is calculated as:

$$F_{cc} = 0{,}85 \times f_{cd} \times \alpha_v \times (B \times X_c - A_{sc1} - A_{sc2}), \tag{11}$$

where for $\varepsilon_{cc} = \varepsilon_{ccM} + \varepsilon_{ccP} < 0{,}002$.

$$\alpha_v = \frac{1000 \times (\varepsilon_{ccM} + \varepsilon_P)}{12} \times [6 - 1000 \times (\varepsilon_{ccM} + \varepsilon_P)], \tag{12}$$

where for $\varepsilon_{cc} = \varepsilon_{ccM} + \varepsilon_{ccP} \geq 0{,}002$.

$$\alpha_v = \frac{3000 \times (\varepsilon_{ccM} + \varepsilon_P) - 2}{3000 \times (\varepsilon_{ccM} + \varepsilon_P)}, \tag{13}$$

- forces in reinforcement are as follows:

$$F_{sc1} = A_{sc1} \times \varepsilon_{sc1} \times E_s, \tag{14}$$

$$F_{sc2} = A_{sc2} \times \varepsilon_{sc2} \times E_s, \tag{15}$$

$$F_{st1} = A_{st1} \times \varepsilon_{st1} \times E_s, \tag{16}$$

$$F_{st2} = A_{st2} \times \varepsilon_{st2} \times E_s, \tag{17}$$

- section deformation as result of compressive force action is:

$$\varepsilon_P = \frac{P}{A \times E_{cm}}. \tag{18}$$

To calculate the critical bending moment, at which the yield of reinforcement in strain occurs, for our case, we have to calculate the effective height and the exact moment of inertia of the cross section after the crack propagation.

- effective height of the section is:

$$H' = \frac{x_c \times (\varepsilon_{cc} + \varepsilon_{ct})}{\varepsilon_{cc}}. \tag{19}$$

Concrete's modulus of elasticity decreases with pressure load increasing and if this fact is ignored, we cannot be sure that the results obtained reflect the actual behaviour of the element. So in order to get accurate strain in concrete the following terms are used:

- if $\varepsilon_{cc} < \varepsilon_{c0} = 0{,}002$ then the concrete strain at compressive edge of the cross section is calculated as:

$$\sigma_{cc} = f_{cd} \times \left(2 \times \frac{\varepsilon_{cc}}{\varepsilon_{c0}} - \left(\frac{\varepsilon_{cc}}{\varepsilon_{c0}} \right)^2 \right), \tag{20}$$

- if $\varepsilon_{cc} > \varepsilon_{c0} = 0{,}002$ then the concrete strain at compressive edge of the cross section is calculated as:

$$\sigma_{cc} = f_{cd} \times \left(1 - 0{,}2 \times \frac{\varepsilon_{cc} - \varepsilon_{c0}}{\varepsilon_{cu} - \varepsilon_{c0}} \right), \tag{21}$$

Geometrical characteristics of the effective cross section are calculated according to the following expressions:

- if the neutral axis is in the sections web i.e. $x_c > D$, geometrical characteristic of the section are calculated as:

$$A_i = B \times D + 2 \times (X_c - D) \times D + (\alpha_{ep} - 1) \times A_s, \tag{22}$$

$$t_i = [(B\times D\times D/2 + 2\times(x_c - D)\times D\times \times(D+(x_c - D)/2) + (\alpha_{ep}-1)\times A_{sc1}\times(d+r) + + A_{sc2}\times(D-d-r) + A_{st1}\times(H-d-r) + + A_{st2}\times(H-D+d+r)]/A_i\,, \tag{23}$$

$$I_i = \frac{B\times D^3}{12}\times B\times D\times(t_i - D/2)^2 + \frac{D\times(H'-D)^3}{12} + + D\times(H'-D)\times((H'-D)/2 - t_i)^2 + (\alpha_{ep}-1)\times \times[\frac{(2r)^4}{64}\times n_{sc1} + A_{sc1}\times(t_i - d - r)^2 + \frac{(2r)^4}{64}\times n_{sc2} + + A_{sc2}\times(t_i - D + d + r)^2 + \frac{(2r)^4}{64}\times n_{st1} + + A_{st1}\times(H - t_i - d - r)^2 + \frac{(2r)^4}{64}\times n_{st2} + + A_{st2}\times(H - t_i - D + d + r)^2], \tag{24}$$

- if the neutral axis is in the sections web i.e. $x_c \le D$, geometrical characteristic of the section are calculated as:

$$A_i = x_c\times B + (\alpha_{ep}-1)\times A_s\,, \tag{25}$$

$$t_i = [(x_c\times B\times x_c/2 + (\alpha_{ep}-1)\times A_{sc1}\times(d+r) + + A_{sc2}\times(D-d-r) + A_{st1}\times(H-d-r) + + A_{st2}\times(H-D+d+r)]/A_i\,, \tag{26}$$

$$I_i = \frac{B\times x_c^3}{12}\times B\times x_c\times(t_i - x_c/2)^2 + (\alpha_{ep}-1)\times \times[\frac{(2r)^4}{64}\times n_{sc1} + A_{sc1}\times(t_i - d - r)^2 + \frac{(2r)^4}{64}\times n_{sc2} + + A_{sc2}\times(t_i - D + d + r)^2 + \frac{(2r)^4}{64}\times n_{st1} + + A_{st1}\times(H - t_i - d - r)^2 + \frac{(2r)^4}{64}\times n_{st2} + + A_{st2}\times(H - t_i - D + d + r)^2], \tag{27}$$

- bending moment at which flow of the tensile reinforcement appears can be calculated in several ways. In the paper the following expression was selected:

$$M_{cr2} = \frac{\sigma_{cc}\times I_i}{x_c}. \tag{28}$$

With this we get the first characteristic point M–ϕ diagram (M_{cr2}; ϕ_2), i.e., the bending moment and curvature at reaching flow of reinforcement. Flowchart diagram of calculation steps is shown in Fig. 8.

As the diagram being non-linear after the first characteristic point, in order to get its exact form we need to calculate moment and curvature relationship in as many points as depending on how precise we want to be.

To accurately define the form of the diagram we determine the step of increase in curvature and deformation from the first to the second characteristic point from which we calculate other values required to obtain the corresponding bending moment.

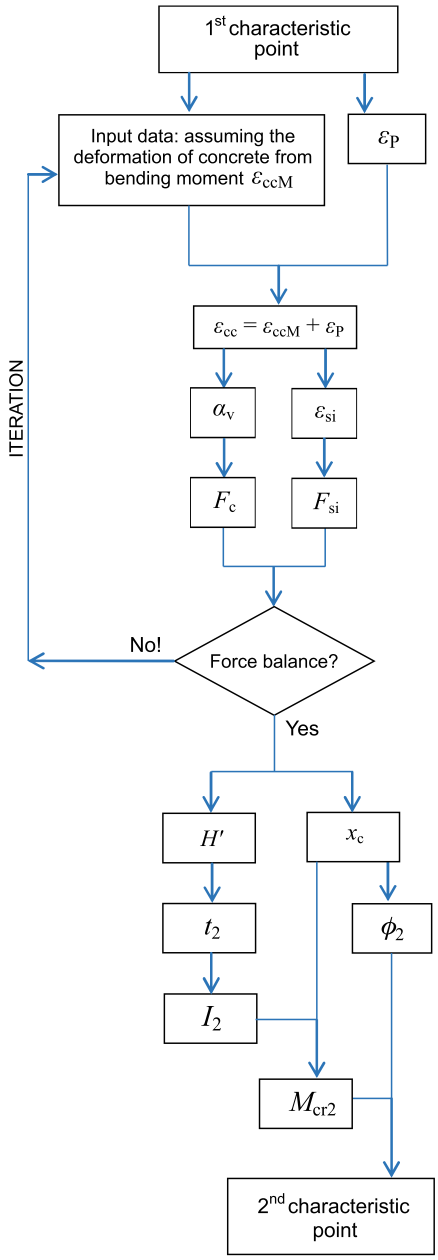

Figure 8 Calculation steps flowchart diagram

3.3 Construction collapse

Last point of M–ϕ diagram is the point where construction loses its bearing capacity. The structure collapse may occur through concrete in compression, reinforcement in strain or even but very rarely over both materials in the same moment, as mentioned before. In order to know which form of collapse is characteristic for the case that is considered, we carry out two calculations as described in Section 3.2. and Fig. 8. If we assume the collapse of concrete, we calculate as the deformation of concrete is equal to the maximum permitted, i.e. ε_{cu}= 0,0035 and vary the deformation of reinforcement in tension until we achieve balance of inner forces in the cross section. In the second calculation, a collapse through reinforcement in tension is assumed. We calculate with the deformation being ε_{su} = 0,0500 and vary the value of deformation of concrete on the edge of section in compression until the balance of horizontal inner forces in the cross section is achieved. Relevant is the case where the collapse occurs at lower force and bending moment.

Thus the third characteristic point of $M\!-\!\phi$ diagram is obtained (M_{cr3}; ϕ_3) i.e., the bending moment and the curvature in the moment of the structure losing its bearing capacity. The exact form of the diagram is determined as in the section 3.2 by incrementally increasing curvature and deformation, and accordingly calculating other values required to obtain the corresponding bending moment. In this way we find the maximum bending moment that a cross section can handle at the corresponding curvature.

In Fig. 9 the dashed curve represents the actual $M\!-\!\phi$ diagram, calculated with the above proposed procedure and the continuous curve represents the idealized diagram (used in everyday practice) for the cross section and conditions in Fig. 3. We notice a significant reserve in bearing capacity when comparing the real and idealized diagrams.

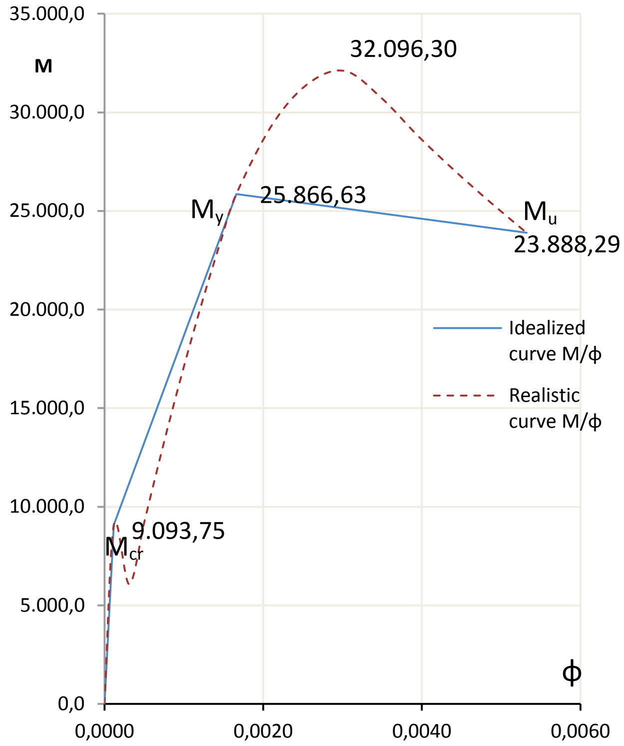

Figure 9 Bending moment - curvature relationship diagram

3.4 Comparison of the impact of the basic parameters of the cross-section and material to the curve form

Besides the calculation of a realistic $M\!-\!\phi$ diagram the aim of this research was to discover how various inputs affect the shape of $M\!-\!\phi$ diagram. Different amounts of reinforcement, geometric characteristics of the section, material strength and levels of compressive force were varied through the proposed calculation procedure. Results were obtained as follows. The dashed curve in the figures below represents the realistic $M\!-\!\phi$ curve in Fig. 9. Every curve in the diagrams below is obtained in the calculation principle described in this paper.

Fig. 10 shows the $M\!-\!\phi$ diagrams for different amounts of reinforcement in the cross section, symmetrically arranged in tensile and compression zone, (cross section: 400/200/30 cm, material: f_{ck} = 40 N/mm^2, f_y = 500 N/mm^2). The share of reinforcement in the analysed cross section ranges from 1 to 4 %, as recommended in standards.

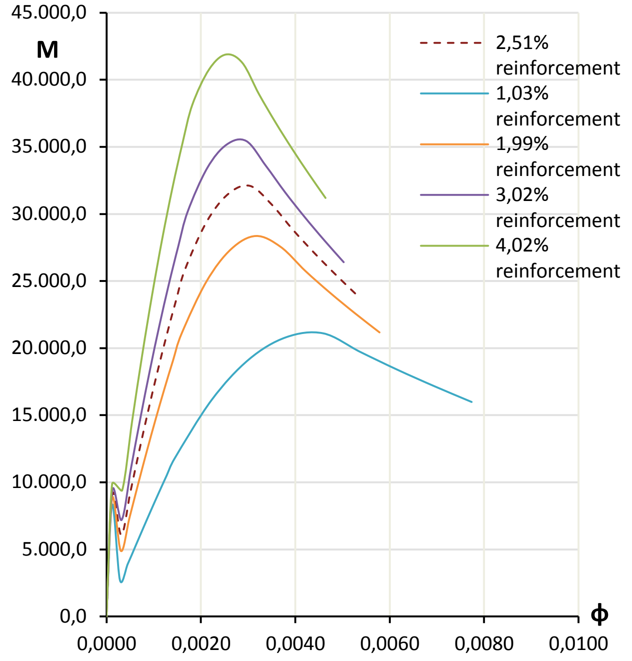

Figure 10 Bending moment - curvature relationship diagram for different amounts of reinforcement

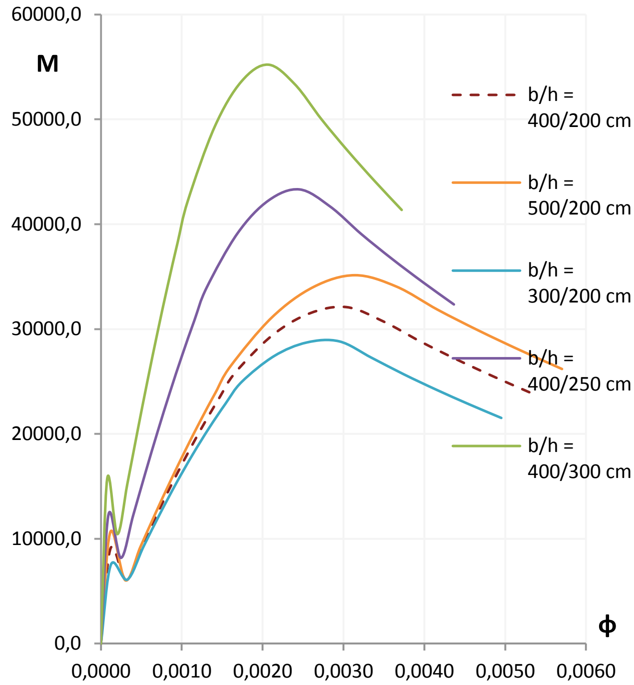

Figure 11 Bending moment - curvature relationship diagram for different geometrical characteristic of section

On the diagram (Fig. 10) is evident that reducing the amount of reinforcement decreases total bearing capacity which was to be expected, but unexpectedly increases ductility of the section and although we have less reinforcement section withstands greater deformation before construction collapse.

Fig. 11 shows the $M\!-\!\phi$ diagrams for geometrical characteristics of a rectangular hollow section with an equal amount of reinforcement distributed symmetrically in tensile and compression zone, with the same material properties for all cases (f_{ck} = 40 N/mm^2, f_y = 500 N/mm^2). Total area of reinforcement for all sections is 0,0814m^2, while the share varies from 2,1to 3,1 % depending on the section size.

The results were as expected. Increasing the size of section is increasing the capacity i.e., section can "handle" higher bending moments.

A specific comparison of two sections (400/300 and 500/200 cm) is that of equal area one with the larger bearing capacity is one with smaller width and larger height (higher moment of inertia of the section), while the section with greater width and lower height may "suffer" greater curvature before the construction collapses.

M−ϕ diagrams (Fig. 12) for different characteristic values of concrete strength are shown for sections with equal geometric characteristics (400/200/30cm) and constant amount of reinforcement (2,51 %).

Increasing characteristic compressive strength of concrete increases total capacity of the cross section and the same section can be submitted prior to the collapse to more curvature as it was to be expected.

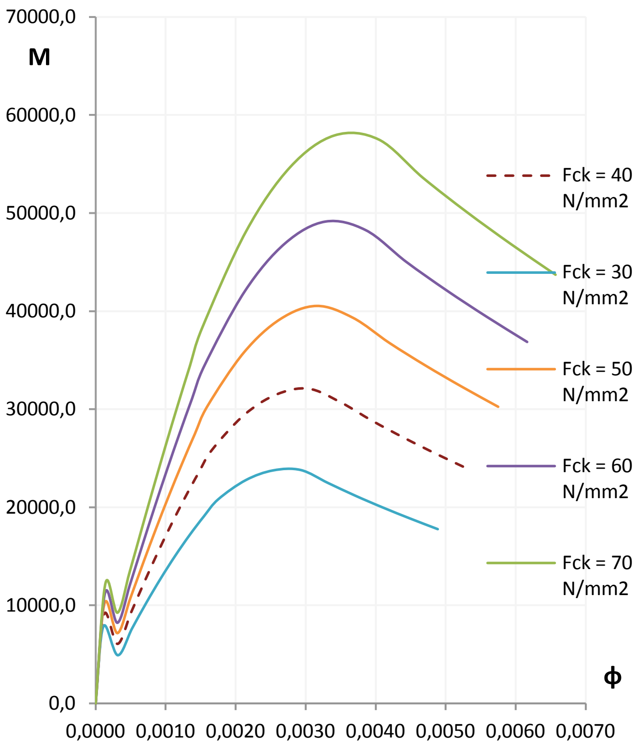

Figure 12 Bending moment - curvature relationship diagram for different concrete characteristic strengths

Analysed were also M−ϕ diagrams for different amounts of compressive forces along the same geometric sectional characteristics and the same material in all cases (400/200/30 cm, f_{ck} = 40 N/mm^2, f_y = 500 N/mm^2, 2,51 % armature). Results are shown in Fig. 13.

It is evident that reaching the tensile strength of concrete (1. characteristic point of M−ϕ diagram) occurs at higher bending moment as it is expected.

Slight differences of the maximum bending moment and bending moment and curvature at collapse are characteristic for the observed different cases.

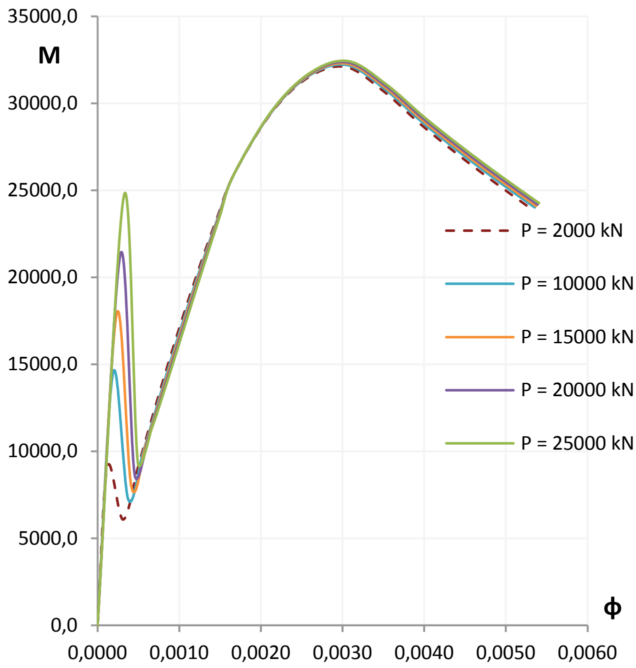

Figure 13 Bending moment - curvature relationship diagram for different amounts of compressive force

4 Conclusion

The paper discusses current practice of determining bending moment – curvature relationship diagrams and after all shown we can freely conclude that an idealized diagram is accurate only in 3 points, while in other parts, we find drastic deviations from the actual behaviour of the cross section. Both the lack and reserve of capacity in comparison of the real and the idealized diagram in certain segments are possible. Also very interesting results how various inputs impact the form of M−ϕ diagram have been obtained.

The conclusion of the study is that for practical use on simple structures idealized diagrams are sufficient while for detailed scientific analysis they cannot and should not be the basis for any serious calculation, but it must be resorted to the exact calculation of the relationship of bending moment and curvature of the section of observed elements.

Contribution of this scientific research are expressions for accurate determination of the M−ϕ diagram that can be, as shown in the article, written in an algorithm and used in everyday practice. Also the presented diagrams can serve as an example to engineers of what and how affects the shape of the curve, and how to vary the input data to get the required results in a simple manner.

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