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Optimization of RFID Tags Coil's System Stability under Delayed Electromagnetic Interferences

Regular Paper

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Abstract This article discusses the very crucial subject of RFID TAG's stability. RFID equivalent circuits of a label can be represented as Parallel circuits of Capacitance (Cpl), Resistance (Rpl), and Inductance (Lpc). We define V(t) as the voltage that develops on the RFID label therefore making dV(t)/dt the voltage-time derivative. Due to electromagnetic interference, there are different time delays with respect to RFID label voltages and voltage time derivatives. We define $V_1(t)$ as V(t) and $V_2(t)$ as dV(t)/dt. The delayed voltage and voltage derivative are $V_1(t-\tau_1)$ and $V_2(t-\tau_2)$ respectively ($\tau_1 \neq \tau_2$). The RFID equivalent circuit can be represented as a delayed differential equation that depends on variable parameters and delays. The investigation of RFID's differential equation is based on bifurcation theory [1], which is the study of possible changes in the structure of the orbits of a delayed differential equation as a function of variable parameters. This article first illustrates certain observations and analyzes local bifurcations of an appropriate arbitrary scalar delayed differential equation [2]. RFID label stability analysis is done under different time delays with respect to label voltage and voltage derivative. Additional analysis of the bifurcations of RFID's delayed differential equation on the circle. Bifurcation behavior of specific delayed differential equations can be condensed into bifurcation diagrams. This serves to optimize dimensional parameters analysis of RFID TAGs under electromagnetic interferences to get ideal performances.

Keywords RFID, stability, locus

1. Introduction

This article discusses a very critical and useful subject of passive RFID TAGs system stability analysis under electromagnetic interferences. RFID TAG system has two main variables—TAG-voltage and TAG-voltage derivative with respect to time which may be subject to delay as a result of electromagnetic interferences. We define τ_1 as time delay respect to TAG's voltage and τ_2 as time delay respect to TAG's voltage derivative. RFID Equivalent circuits of a Label can be represented as parallel circuits of Capacitance (Cpl), Resistance (Rpl), and Inductance (Lpc). Our RFID TAG system delay differential and delay differential model can be utilized for analysis of the dynamics of delay differential equations. Incorporation of a time delay is often necessary during some stage. It is often difficult to analytically study models with delay dependent parameters, even if only a single discrete delay is present. Practical guidelines exist that combine graphical information with analytical work to effectively study the local stability of models involving delay dependent parameters. The stability of a given steady state is determined simply through the use of the graphs of a function of τ_1, τ_2, which can be expressed distinctly and thus can be easily depicted by Matlab and other popular software--we need only look at one such function and locate the zeros. This function often has only two zeros, providing thresholds for stability switches. As time delay increases, the stability fluctuates. We emphasize the local stability aspects of certain models with delay-dependent parameters. In addition there is a general geometric criterion that, theoretically speaking, may be applied to models that include many delays or even distributed delays. The simplest case is that of a first order characteristic equation which provides more user-friendly geometric and analytic criteria for stability switches. The analytical criteria provided for the first and second order cases can be used to obtain insightful analytical statements and can be helpful for conducting simulations.

2. RFID Equivalent Circuit and Representation of Delay Differential Equations

RFID TAG can be represented as a parallel Equivalent Circuit of Capacitor and Resistor in Parallel. For example, see NXP/PHILIPS ICODE IC Parallel equivalent circuit and simplified complete equivalent circuit of the label (L1 is the antenna inductance) [6].

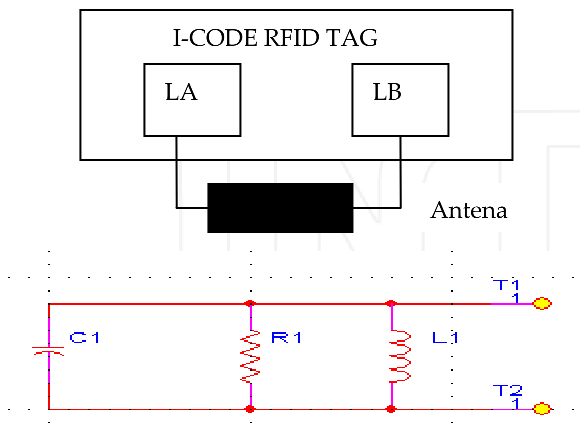

Figure 1. NXP/PHILIPS ICODE IC Parallel equivalent circuit and simplified complete equivalent circuit of the label (L1 represents the antenna inductance).

$\frac{1}{R1}\cdot\frac{dV}{dt}+C1\cdot\frac{d^2V}{dt^2}+\frac{1}{L1}\cdot V=0$ This gives us the differential equation of RFID TAG sys which describes the evolution of the sys in continuous time. V = V(t). Now I define the following Variable settings: $V2=\frac{dV1}{dt}=\frac{dV}{dt}, V1=V$. The dynamic equation system:

$$\frac{dV1}{dt}=V2\ ,\ \frac{dV2}{dt}=-\frac{1}{C1*R1}*V2-\frac{1}{C1*L1}*V1 \qquad (1)$$

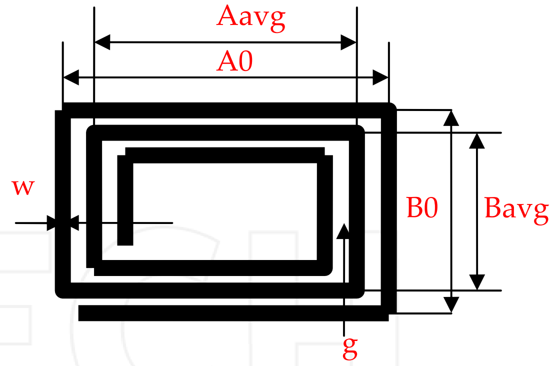

Figure 2. RFID's coil dimensional parameters

$$\begin{aligned} d &= 2\cdot(t+w)/\Pi \\ Aavg &= a0 - Nc\cdot(g+w) \\ Bavg &= b0 - Nc\cdot(g+w) \end{aligned} \qquad (2)$$

a0, b0 – Overall dimensions of the coil. Aavg, Bavg – Average dimensions of the coil. t –Track thickness, w – Track width, g –Gap between tracks. Nc -Number of turns, d – Equivalent diameter of the track. Average coil area ; - Ac=Aavg * Bavg. Integration of all of these parameters gives the equations for inductance calculation :

$$X1 = Aavg*\ln\left(\frac{2*Aavg*Bavg}{d*(Aavg+\sqrt{Aavg^2+Bavg^2})}\right) \qquad (3)$$

$$X2 = Bavg*\ln\left(\frac{2*Aavg*Bavg}{d*(Bavg+\sqrt{Aavg^2+Bavg^2})}\right) \qquad (4)$$

$$X3 = 2*\left[Aavg+Bavg-\sqrt{\left[Aavg^2+Bavg^2\right]}\right] \qquad (5)$$

$$X4 = (Aavg+Bavg)/4 \qquad (6)$$

The RFID's coil calculation inductance is

$$Lcalc = \left[\frac{\mu 0}{\pi}\cdot[X1+X2-X3+X4]\cdot Nc^p\right],\ \text{L1 = Lcalc} \qquad (7)$$

Definition of limits, Estimations: Track thickness t, Al and Cu coils (t > 30um).

$$\begin{bmatrix} \frac{\mathbf{dV1}}{\mathbf{dt}} \\ \frac{\mathbf{dV2}}{\mathbf{dt}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \{-\frac{1}{C1\cdot\left[\frac{\mu 0}{\pi}*[X1+X2-X3+X4]\cdot Nc^{p}\right]}\} & \{-\frac{1}{C1\cdot R1}\} \end{bmatrix} \cdot \begin{bmatrix} V1 \\ V2 \end{bmatrix} \quad (8)$$

Due to electromagnetic interferences we get RFID TAG's voltage and voltage derivative with delays τ_1 and τ_2 respectively $V_1(t) \rightarrow V_1(t-\tau_1)$; $V_2(t) \rightarrow V_2(t-\tau_1)$. We consider no delay effect on dV_1/dt and dV_2/dt. The RFID TAG's differential equations under the effects of electromagnetic interferences (we consider electromagnetic interferences (delay terms) influence only RFID TAG voltage $V_1(t)$ and the voltage derivative $V_2(t)$ with respect to time. There is no influence on $dV_1(t)/dt$ and $dV_2(t)/dt$) :

$$\frac{\mathbf{dV1}}{\mathbf{dt}} = V2(t-\tau_2) \quad (9)$$

$$\frac{\mathbf{dV2}}{\mathbf{dt}} = \{-\frac{1}{C1\cdot\left[\frac{\mu 0}{\pi}\cdot[X1+X2-X3+X4]*Nc^{p}\right]}\}\cdot V1(t-\tau_1) - \frac{1}{C1\cdot R1}\cdot V2(t-\tau_2)$$

To find the Equilibrium points (fixed points) of the RFID TAG system is by $\lim_{t\to\infty} V_1(t-\tau_1) = V_1(t)$;

$$\lim_{t\to\infty} V_2(t-\tau_2) = V_2(t); \frac{dV_1(t)}{dt} = 0 ; \frac{dV_2(t)}{dt} = 0 \quad (10)$$

$$\forall\ t \gg \tau_1 ; t \gg \tau_2 \ \exists\ (t-\tau_1) \approx t ; (t-\tau_2) \approx t , t \rightarrow \infty$$

We get two equations and the only fixed point is: $E^{(0)}(V_1^{(0)}, V_2^{(0)}) = (0,0)$. (11)

<u>Stability analysis</u>: The standard local stability analysis about any one of the equilibrium points of RFID TAG system consists of adding arbitrarily small increments of exponential form $[v_1\ v_2]\cdot e^{\lambda\cdot t}$ to coordinates [V_1 V_2], and retaining the first order terms in v_1, v_2. The system of two homogeneous equations leads to a polynomial characteristics equation in the eigenvalues λ. The polynomial characteristics equations accept by set the following voltage and voltage derivative with respect to time into two RFID TAG system equations.

The RFID TAG system fixed values with arbitrarily small increments of exponential form $[v_1\ v_2]\cdot e^{\lambda\cdot t}$ are : i=0 (first fixed point), i=1 (second fixed point), i=2 (third fixed point).

$$V_1(t) = V_1^{(i)} + v_1\cdot e^{\lambda\cdot t} ; V_2(t) = V_2^{(i)} + v_2\cdot e^{\lambda\cdot t}$$
$$V_1(t-\tau_1) = V_1^{(i)} + v_1\cdot e^{\lambda\cdot(t-\tau_1)} \quad (12)$$
$$V_2(t-\tau_2) = V_2^{(i)} + v_2\cdot e^{\lambda\cdot(t-\tau_2)}$$
$$\forall\ i=0,1,2$$

We choose the above expressions for our $V_1(t)$, $V_2(t)$ as small displacement [v_1 v_2] from the system's fixed points at time t=0.

$$V_1(t=0) = V_1^{(i)} + v_1 ; V_2(t=0) = V_2^{(i)} + v_2 \quad (13)$$

When $\lambda < 0$, $t > 0$ the selected fixed point is stable. however when $\lambda > 0$, $t > 0$ is unstable. Our system tends towards the selected fixed point exponentially for $\lambda < 0$, $t > 0$ and otherwise will deviate exponentially from the selected fixed point. λ is the eigenvalue parameter which establishes whether the fixed point is stable or unstable. In addition, the absolute value ($|\lambda|$) establishes the speed of flow toward or away from the selected fixed point [1] [2].

	$\lambda<0$	$\lambda>0$
t=0	$V_1(t=0)= V_1^{(i)}+v_1$ $V_2(t=0)= V_2^{(i)}+v_2$	$V_1(t=0)= V_1^{(i)}+v_1$ $V_2(t=0)= V_2^{(i)}+v_2$
t>0	$V_1(t)= V_1^{(i)}+v_1e^{-\|\lambda\|\cdot t}$ $V_2(t)= V_2^{(i)}+v_2e^{-\|\lambda\|\cdot t}$	$V_1(t)= V_1^{(i)}+v_1e^{\|\lambda\|\cdot t}$ $V_2(t)= V_2^{(i)}+v_2e^{\|\lambda\|\cdot t}$
t<0	$V_1(t\to\infty)= V_1^{(i)}$ $V_2(t\to\infty)= V_2^{(i)}$	$V_1(t\to\infty, \lambda>0) \sim v_1e^{\|\lambda\|\cdot t}$ $V_2(t\to\infty, \lambda>0) \sim v_2e^{\|\lambda\|\cdot t}$

The speeds of flow toward or away from the selected fixed point for RFID TAG system voltage and voltage derivative respect to time are (14)

$$\frac{dV_1(t)}{dt} = \lim_{\Delta t\to 0}\frac{V_1(t+\Delta t)-V_1(t)}{\Delta t}$$
$$= \lim_{\Delta t\to 0}\frac{V_1^{(i)}+v_1\cdot e^{\lambda\cdot(t+\Delta t)}-[V_1^{(i)}+v_1\cdot e^{\lambda\cdot t}]}{\Delta t} \quad (15)$$
$$= \lim_{\Delta t\to 0}\frac{v_1\cdot e^{\lambda\cdot t}\cdot[e^{\lambda\cdot\Delta t}-1]}{\Delta t}\xrightarrow{e^{\lambda\cdot\Delta t}\approx 1+\lambda\cdot\Delta t}$$
$$\lim_{\Delta t\to 0}\frac{v_1\cdot e^{\lambda\cdot t}\cdot[1+\lambda\cdot\Delta t-1]}{\Delta t} = \lambda\cdot v_1\cdot e^{\lambda\cdot t}$$

$$\frac{dV_2(t)}{dt} = \lim_{\Delta t\to 0}\frac{V_2(t+\Delta t)-V_2(t)}{\Delta t}$$
$$= \lim_{\Delta t\to 0}\frac{V_2^{(i)}+v_2\cdot e^{\lambda\cdot(t+\Delta t)}-[V_2^{(i)}+v_2\cdot e^{\lambda\cdot t}]}{\Delta t}$$
$$= \lim_{\Delta t\to 0}\frac{v_2\cdot e^{\lambda\cdot t}\cdot[e^{\lambda\cdot\Delta t}-1]}{\Delta t}\xrightarrow{e^{\lambda\cdot\Delta t}\approx 1+\lambda\cdot\Delta t}$$
$$\lim_{\Delta t\to 0}\frac{v_2\cdot e^{\lambda\cdot t}\cdot[1+\lambda\cdot\Delta t-1]}{\Delta t} = \lambda\cdot v_2\cdot e^{\lambda\cdot t}$$

and the time derivative of the above equations:

$$\frac{dV_1(t)}{dt} = v_1 \cdot \lambda \cdot e^{\lambda \cdot t} \; ; \; \frac{dV_2(t)}{dt} = v_2 \cdot \lambda \cdot e^{\lambda \cdot t} \quad (16)$$

$$\frac{dV_1(t-\tau_1)}{dt} = v_1 \cdot \lambda \cdot e^{\lambda\cdot(t-\tau_1)} = v_1 \cdot \lambda \cdot e^{\lambda \cdot t} \cdot e^{-\tau_1 \cdot \lambda}$$
$$\frac{dV_2(t-\tau_2)}{dt} = v_2 \cdot \lambda \cdot e^{\lambda\cdot(t-\tau_2)} = v_2 \cdot \lambda \cdot e^{\lambda \cdot t} \cdot e^{-\tau_2 \cdot \lambda} \quad (17)$$

First we take the RFID TAG's voltage (V_1) differential equation: $\frac{dV_1}{dt} = V_2$ adding arbitrarily small increments in exponential form $[v_1 \; v_2] \cdot e^{\lambda \cdot t}$ to the coordinates [V_1 V_2] and retaining the first order terms in v_1, v_2.

$$\lambda \cdot v_1 \cdot e^{\lambda \cdot t} = V_2^{(i)} + v_2 \cdot e^{\lambda \cdot t} \; ;$$
$$V_2^{(i=0)} = 0 \; ; \; \lambda_1 = \frac{v_2}{v_1} \approx 1 > 0 \quad (18)$$

Second we take the RFID TAG's voltage (V_2) differential equation: $\frac{dV_1}{dt} = V_2$ and adding arbitrarily small increments in exponential form $[v_1 \; v_2] \cdot e^{\lambda \cdot t}$ to the coordinates [V_1 V_2] and retaining the first order terms in v_1, v_2 . (19)

$$\frac{\mathbf{dV2}}{\mathbf{dt}} = \{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] * Nc^{p}\right]}\} \cdot V1(t-\tau_1) - \frac{1}{C1 \cdot R1} \cdot V2(t-\tau_2) \quad (20)$$

$$\lambda \cdot v_2 \cdot e^{\lambda \cdot t} = \{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]}\} \cdot (V_1^{(i)} + v_1 \cdot e^{\lambda \cdot t}) - \frac{1}{C1 \cdot R1} \cdot (V_2^{(i)} + v_2 \cdot e^{\lambda \cdot t})$$

$$V_1^{(i=0)} = V_2^{(i=0)} = 0 \text{ then } \frac{v_1}{v_2} \approx 1 \; ;$$

$$\lambda_2 = -\frac{1}{C1} \cdot [\frac{1}{\left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]} + \frac{1}{R1}] \quad (21)$$

If

$$\frac{1}{\left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]} + \frac{1}{R1} > 0$$

then we have saddle fixed point otherwise it is an unstable node (both eigenvalues are positive). We define (22)

$$V_1(t-\tau_1) = V_1^{(i)} + v_1 \cdot e^{\lambda\cdot(t-\tau_1)} \; ;$$
$$V_2(t-\tau_2) = V_2^{(i)} + v_2 \cdot e^{\lambda\cdot(t-\tau_2)}$$

which gives us two delayed differential equations adding arbitrarily small increments of exponential form $[v_1 \; v_2] \cdot e^{\lambda \cdot t}$ to the coordinates [V_1 V_2].

$$v_1 \cdot \lambda \cdot e^{\lambda \cdot t} = V_2^{(i)} + v_2 \cdot e^{\lambda\cdot(t-\tau_2)} \; ;$$
$$V_2^{(i=0)} = 0 \Rightarrow v_1 \cdot \lambda \cdot e^{\lambda \cdot t} = v_2 \cdot e^{\lambda\cdot(t-\tau_2)} \quad (23)$$

$$\lambda \cdot v_2 \cdot e^{\lambda \cdot t} = \{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]}\} \cdot V_1^{(i)} - \frac{1}{C1 \cdot R1} \cdot V_2^{(i)}$$

$$\{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]}\} \cdot v_1 \cdot e^{\lambda\cdot(t-\tau_1)} - \frac{1}{C1 \cdot R1} \cdot v_2 \cdot e^{\lambda\cdot(t-\tau_2)}$$

In the equilibrium fixed point $V_1^{(i=0)} = V_2^{(i=0)} = 0$ and

$$\{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]}\} \cdot V_1^{(i)} - \frac{1}{C1 \cdot R1} \cdot V_2^{(i)} = 0 \quad (24)$$

Then we get

$$\lambda \cdot v_2 \cdot e^{\lambda \cdot t} = \{-\frac{1}{C1 \cdot \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right]}\} \cdot v_1 \cdot e^{\lambda\cdot(t-\tau_1)} - \frac{1}{C1 \cdot R1} \cdot v_2 \cdot e^{\lambda\cdot(t-\tau_2)} \quad (25)$$

We define

$$f_{\#}(X1, X2, etc...) = \left[\frac{\mu 0}{\pi} \cdot [X1+X2-X3+X4] \cdot Nc^{p}\right] \quad (26)$$

The small Jacobian increments of our RFID TAG system

$$\begin{bmatrix} -\lambda & e^{-\lambda \cdot \tau_2} \\ -\frac{1}{C1 \cdot f_{\#}} \cdot e^{-\lambda \cdot \tau_1} & -\frac{1}{C1 \cdot R1} \cdot e^{-\lambda \cdot \tau_2} - \lambda \end{bmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad (27)$$

$$A-\lambda\cdot I=\begin{bmatrix} -\lambda & e^{-\lambda\cdot\tau_2} \\ -\frac{1}{C1\cdot f_{\#}}\cdot e^{-\lambda\cdot\tau_1} & -\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau_2}-\lambda \end{bmatrix} \quad (28)$$

$$\det|A-\lambda\cdot I|=0 \quad (29)$$

$$D(\lambda,\tau_1,\tau_2)=\lambda^2+\lambda\cdot\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau_2}+\frac{1}{C1\cdot f_{\#}}\cdot e^{-\lambda\cdot(\tau_1+\tau_2)} \quad (30)$$

We have three stability analysis cases: $\tau_1=\tau$; $\tau_2=0$ or $\tau_2=\tau$; $\tau_1=0$ or $\tau_1=\tau_2=\tau$ otherwise $\tau_1\neq\tau_2$.

Just as in all of the above stability analysis cases, we need to identify characteristic equations. We study the occurrence of any possible stability switching resulting from the increase in value of the time delay τ for the general characteristic equation $D(\lambda,\tau)$.

$$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)\cdot e^{-\lambda\tau} \quad (31)$$

The expression for $P_n(\lambda,\tau)$ is

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\cdot\lambda^k=P_0(\tau)+P_1(\tau)\cdot\lambda+P_2(\tau)\cdot\lambda^2+P_3(\tau)\cdot\lambda^3+\dots\dots \quad (32)$$

The expression for $Q_m(\lambda,\tau)$ is

$$Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\cdot\lambda^k=q_0(\tau)+q_1(\tau)\cdot\lambda+q_2(\tau)\cdot\lambda^2+\dots\dots \quad (33)$$

3. RFID Tag System Second Order

CHARACTERISTIC EQUATION $\tau_1=\tau$; $\tau_2=0$

The first case we analyze involves a delay in RFID Label voltage with no delay in voltage time derivative [4] [5].

$$D(\lambda,\tau_1=\tau,\tau_2=0)=\lambda^2+\lambda\cdot\frac{1}{C1\cdot R1}+\frac{1}{C1\cdot f_{\#}}\cdot e^{-\lambda\cdot\tau_1} \quad (34)$$

$$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)\cdot e^{-\lambda\tau} \quad (35)$$

The expression for $P_n(\lambda,\tau)$ is

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\cdot\lambda^k=P_0(\tau)+P_1(\tau)\cdot\lambda+P_2(\tau)\cdot\lambda^2 \quad (36)$$

$$=\lambda^2+\lambda\cdot\frac{1}{C1\cdot R1}\ ;\ P_2(\tau)=1\ ;\ P_1(\tau)=\frac{1}{C1\cdot R1}\ ;\ P_0(\tau)=0$$

The expression for $Q_m(\lambda,\tau)$ is

$$Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\cdot\lambda^k=q_0(\tau)=\frac{1}{C1\cdot f_{\#}} \quad (37)$$

Our RFID system second order characteristic equation is :

$$D(\lambda,\tau)=\lambda^2+a(\tau)\cdot\lambda+b(\tau)\cdot\lambda\cdot e^{-\lambda\cdot\tau}+c(\tau)+d(\tau)\cdot e^{-\lambda\cdot\tau} \quad (38)$$

Then

$$a(\tau)=\frac{1}{C_1\cdot R_1}\ ;\ \mathrm{b}(\tau)=0\ ;\ \mathrm{c}(\tau)=0\ ;\ \mathrm{d}(\tau)=\frac{1}{C_1\cdot f_{\#}} \quad (39)$$

$\tau\in R_{+0}$ and $a(\tau),\mathrm{b}(\tau),\mathrm{c}(\tau),\mathrm{d}(\tau):R_{+0}\to R$ are differentiable functions of class $C^1(R_{+0})$ such that $\mathrm{c}(\tau)+\mathrm{d}(\tau)=\frac{1}{C_1\cdot f_{\#}}\neq 0$ for all $\tau\in R_{+0}$ and for any $\tau,\mathrm{b}(\tau),\mathrm{d}(\tau)$ are not simultaneously zero. We have (40)

$$P(\lambda,\tau)=P_n(\lambda,\tau)=\lambda^2+a(\tau)\cdot\lambda+c(\lambda)=\lambda^2+\frac{1}{C_1\cdot R_1}\cdot\lambda$$

$$Q(\lambda,\tau)=Q_m(\lambda,\tau)=b(\tau)\cdot\lambda+d(\tau)=\frac{1}{C_1\cdot f_{\#}} \quad (41)$$

We assume that $P_n(\lambda,\tau)=P_n(\lambda)$ and $Q_m(\lambda,\tau)=Q_m(\lambda)$ cannot have common imaginary roots. That is, for any real number ω ;

$$p_n(\lambda=i\cdot\omega,\tau)+Q_m(\lambda=i\cdot\omega,\tau)\neq 0 \quad (42)$$

$$\frac{1}{C_1\cdot f_{\#}}-\omega^2+i\cdot\omega\cdot\frac{1}{C_1\cdot R_1}\neq 0; \quad (43)$$

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2=(c-\omega^2)^2+\omega^2\cdot a^2-(\omega^2\cdot b^2+d^2)$$

$$F(\omega,\tau)=\omega^4+\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_{\#})^2}\ ;\ \text{Hence} \quad (44)$$

$F(\omega,\tau)=0$ implies

$$\omega^4+\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_\#)^2}=0 \quad (45)$$

And its roots are given by

$$\omega_+^2=\frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)+\sqrt{\Delta}\}=\frac{1}{2}\cdot\{\sqrt{\Delta}-\frac{1}{(C_1\cdot R_1)^2}\} \quad (46)$$

$$\omega_-^2=\frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)-\sqrt{\Delta}\}=-\frac{1}{2}\cdot\{\sqrt{\Delta}+\frac{1}{(C_1\cdot R_1)^2}\} \quad (47)$$

$$\Delta=(b^2+2\cdot c-a^2)-4\cdot(c^2-d^2)=\frac{1}{C_1^2}\cdot[(\frac{2}{f_\#})^2-\frac{1}{R_1^2}] \quad (48)$$

Therefore the following holds true:

$$2\cdot\omega_{+/-}^2-(b^2+2\cdot c-a^2)=\pm\sqrt{\Delta} \quad (49)$$

$$2\cdot\omega_{+/-}^2+\frac{1}{(C_1\cdot R_1)^2}=\pm\sqrt{\Delta} \quad (50)$$

Furthermore $P_R(i\cdot\omega,\tau)=c(\tau)-\omega^2(\tau)=-\omega^2(\tau)$ (51)

$$P_I(i\cdot\omega,\tau)=\omega(\tau)\cdot a(\tau)=\omega(\tau)\cdot\frac{1}{C_1\cdot R_1} \quad (52)$$

$$Q_R(i\cdot\omega,\tau)=d(\tau)=\frac{1}{C_1\cdot f_\#} \quad (53)$$

$Q_I(i\cdot\omega,\tau)=\omega(\tau)\cdot b(\tau)=0$ hence

$$\sin\theta(\tau)=\frac{-P_R(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \quad (54)$$

$$\cos\theta(\tau)=-\frac{P_R(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \quad (55)$$

$$\sin\theta(\tau)=\frac{-(c-\omega^2)\cdot\omega\cdot b+\omega\cdot a\cdot d}{\omega^2\cdot b^2+d^2}=\omega\cdot\frac{f_\#}{R_1} \quad (56)$$

$$\cos\theta(\tau)=-\frac{(c-\omega^2)\cdot d+\omega^2\cdot a\cdot b}{\omega^2\cdot b^2+d^2}=\omega^2\cdot C_1\cdot f_\# \quad (57)$$

Which jointly with $\omega^4+\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_\#)^2}=0$ (58)

defines the maps $S_n(\tau)=\tau-\tau_n(\tau);\ \tau\in I,\ n\in\mathbb{N}_0$ (59)

which are continuous and differentiable in τ based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: a, b, c, d parameters are independent of delay parameter τ even if we use $a(\tau), b(\tau), c(\tau), d(\tau)$.

4. RFID Tag System Second Order

CHARACTERISTIC EQUATION $\tau_1=0$; $\tau_2=\tau$

The second case we analyze involves no delay in RFID Label voltage but does have a delay in voltage time derivative [5].

$$D(\lambda,\tau_1=0,\tau_2=\tau)=\lambda^2+\lambda\cdot\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau_2}+\frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\cdot\tau_2} \quad (60)$$

$$D(\lambda,\tau_1=0,\tau_2=\tau)=\lambda^2+(\lambda\cdot\frac{1}{C1\cdot R1}+\frac{1}{C1\cdot f_\#})\cdot e^{-\lambda\cdot\tau} \quad (61)$$

$$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)\cdot e^{-\lambda\tau} \quad (62)$$

The expression for $P_n(\lambda,\tau)$ is

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\cdot\lambda^k=P_0(\tau)+P_1(\tau)\cdot\lambda+P_2(\tau)\cdot\lambda^2=\lambda^2 \quad (63)$$

$$P_2(\tau)=1\ ;\ P_1(\tau)=0\ ;\ P_0(\tau)=0$$

The expression for $Q_m(\lambda,\tau)$ is

$$Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\cdot\lambda^k=\lambda\cdot\frac{1}{C1\cdot R_1}+\frac{1}{C1\cdot f_\#} \quad (64)$$

$$q_0(\tau)=\frac{1}{C1\cdot f_\#}\ ;\ q_1(\tau)=\frac{1}{C1\cdot R_1}\ ;\ q_2(\tau)=0$$

Our RFID system second order characteristic equation:

$$D(\lambda,\tau)=\lambda^2+a(\tau)\cdot\lambda+b(\tau)\cdot\lambda\cdot e^{-\lambda\cdot\tau}+c(\tau)+d(\tau)\cdot e^{-\lambda\cdot\tau} \quad (65)$$

Then

$$a(\tau)=0\ ;\ b(\tau)=\frac{1}{C_1\cdot R_1}\ ;\ c(\tau)=0\ ;\ d(\tau)=\frac{1}{C_1\cdot f_\#} \quad (66)$$

And like in our previous case analysis:

$$P(\lambda,\tau)=P_n(\lambda,\tau)=\lambda^2+a(\tau)\cdot\lambda+c(\lambda)=\lambda^2 \quad (67)$$

$$Q(\lambda,\tau)=Q_m(\lambda,\tau)=b(\tau)\cdot\lambda+d(\tau)=\lambda\cdot\frac{1}{C_1\cdot R_1}+\frac{1}{C_1\cdot f_\#} \quad (68)$$

we assume that $P_n(\lambda,\tau)=P_n(\lambda)$ and $Q_m(\lambda,\tau)=Q_m(\lambda)$ cannot have common imaginary roots. That is, for any real number ω ;

$$p_n(\lambda=i\cdot\omega,\tau)+Q_m(\lambda=i\cdot\omega,\tau)\neq 0 \quad (69)$$

$$\frac{1}{C_1\cdot f_\#}-\omega^2+i\cdot\omega\cdot\frac{1}{C_1\cdot R_1}\neq 0; \quad (70)$$

$$\begin{aligned}F(\omega,\tau)&=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2\\&=(c-\omega^2)^2+\omega^2\cdot a^2-(\omega^2\cdot b^2+d^2)\end{aligned} \quad (71)$$

$$F(\omega,\tau)=\omega^4-\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_\#)^2}; \quad (72)$$

Therefore $F(\omega,\tau)=0$ implies

$$\omega^4-\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_\#)^2}=0 \quad (73)$$

And its roots are given by

$$\omega_+^2=\frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)+\sqrt{\Delta}\}=\frac{1}{2}\cdot\{\sqrt{\Delta}+\frac{1}{(C_1\cdot R_1)^2}\} \quad (74)$$

$$\omega_-^2=\frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)-\sqrt{\Delta}\}=\frac{1}{2}\cdot\{-\sqrt{\Delta}+\frac{1}{(C_1\cdot R_1)^2}\} \quad (75)$$

$$\Delta=(b^2+2\cdot c-a^2)-4\cdot(c^2-d^2)=\frac{1}{C_1^2}\cdot[(\frac{2}{f_\#})^2+\frac{1}{R_1^2}] \quad (76)$$

Therefore the following holds:

$$2\cdot\omega_{+/-}^2-(b^2+2\cdot c-a^2)=\pm\sqrt{\Delta} \quad (77)$$

$$2\cdot\omega_{+/-}^2+\frac{1}{(C_1\cdot R_1)^2}=\pm\sqrt{\Delta} \quad (78)$$

Furthermore $P_R(i\cdot\omega,\tau)=c(\tau)-\omega^2(\tau)=-\omega^2(\tau)$ (79)

$$P_I(i\cdot\omega,\tau)=\omega(\tau)\cdot a(\tau)=0 \quad (80)$$

$$Q_R(i\cdot\omega,\tau)=d(\tau)=\frac{1}{C_1\cdot f_\#} \quad (81)$$

$Q_I(i\cdot\omega,\tau)=\omega(\tau)\cdot b(\tau)=\omega(\tau)\cdot\frac{1}{C_1\cdot R_1}$, hence:

$$\sin\theta(\tau)=\frac{-P_R(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \quad (82)$$

$$\cos\theta(\tau)=-\frac{P_R(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \quad (83)$$

$$\sin\theta(\tau)=\frac{-(c-\omega^2)\cdot\omega\cdot b+\omega\cdot a\cdot d}{\omega^2\cdot b^2+d^2}=\frac{\omega^3\cdot C_1\cdot R_1}{\omega^2+(\frac{R_1}{f_\#})^2} \quad (84)$$

$$\cos\theta(\tau)=-\frac{(c-\omega^2)\cdot d+\omega^2\cdot a\cdot b}{\omega^2\cdot b^2+d^2}=\frac{\omega^2\cdot C_1\cdot\frac{R_1}{f_\#}}{\omega^2+(\frac{R_1}{f_\#})^2} \quad (85)$$

Which, along with $\omega^4-\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2}-\frac{1}{(C_1\cdot f_\#)^2}=0$ (86)

Defines the maps $S_n(\tau)=\tau-\tau_n(\tau)$; $\tau\in\mathrm{I}$, $n\in\mathbb{N}_0$

Defines the maps $S_n(\tau)=\tau-\tau_n(\tau)$; $\tau\in\mathrm{I}$, $n\in\mathbb{N}_0$

which are continuous and differentiable in τ based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves the theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: a, b, c, d parameters are independent of the delay parameter τ even we use $a(\tau), b(\tau), c(\tau), d(\tau)$ [4] [5].

5. RFID Tag System Second Order

CHARACTERISTIC EQUATION $\tau_1=\tau$; $\tau_2=\tau$

The third case we analyze is when there is delay in both the RFID Label voltage and in the voltage time derivative [4] [5].

$$\begin{aligned}&D(\lambda,\tau_1=\tau,\tau_2=\tau)=\lambda^2\\&+\lambda\cdot\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau}+\frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\cdot\tau\cdot 2}\end{aligned} \quad (87)$$

$$\begin{aligned}&D(\lambda,\tau_1=\tau_2=\tau)\\&=\lambda^2+(\lambda\cdot\frac{1}{C1\cdot R1}+\frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\cdot\tau})\cdot e^{-\lambda\cdot\tau}\end{aligned} \quad (88)$$

$$D(\lambda,\tau) = P_n(\lambda,\tau) + Q_m(\lambda,\tau)\cdot e^{-\lambda\tau} \quad (89)$$

The expression for $P_n(\lambda,\tau)$ is

$$P_n(\lambda,\tau) = \sum_{k=0}^{n} P_k(\tau)\cdot\lambda^k = P_0(\tau) + P_1(\tau)\cdot\lambda + P_2(\tau)\cdot\lambda^2 = \lambda^2 \quad (90)$$
$$P_2(\tau)=1\ ;\ P_1(\tau)=0\ ;\ P_0(\tau)=0$$

The expression for $Q_m(\lambda,\tau)$ is

$$Q_m(\lambda,\tau) = \sum_{k=0}^{m} q_k(\tau)\cdot\lambda^k = \lambda\cdot\frac{1}{C1\cdot R_1} + \frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\tau} \quad (91)$$

Taylor expansion: $e^{-\lambda\tau} \approx 1 - \lambda\cdot\tau + \frac{\lambda^2\cdot\tau^2}{2}$ since we need n > m for [BK] analysis we choose $e^{-\lambda\tau} \approx 1 - \lambda\cdot\tau$ then we get $Q_m(\lambda,\tau) = \sum_{k=0}^{m} q_k(\tau)\cdot\lambda^k = \lambda\cdot\frac{1}{C1}(\frac{1}{R_1} - \frac{\tau}{f_\#}) + \frac{1}{C1\cdot f_\#}$

(92)

$$q_0(\tau,\lambda) = \frac{1}{C1\cdot f_\#}\ ;\ q_1(\tau) = \frac{1}{C1}\cdot(\frac{1}{R_1} - \frac{\tau}{f_\#})\ ;\ q_2(\tau) = 0 \quad (93)$$

Our RFID system second order characteristic equation:

$$D(\lambda,\tau) = \lambda^2 + a(\tau)\cdot\lambda + b(\tau)\cdot \lambda\cdot e^{-\lambda\cdot\tau} + c(\tau) + d(\tau)\cdot e^{-\lambda\cdot\tau} \quad (94)$$

Then

$$a(\tau)=0\ ;\ \mathrm{b}(\tau) = \frac{1}{C1}\cdot(\frac{1}{R_1} - \frac{\tau}{f_\#})\ ;\ \mathrm{c}(\tau)=0\ ;\ \mathrm{d}(\tau) = \frac{1}{C_1\cdot f_\#} \quad (95)$$

And much like our previous case analysis :

$$P(\lambda,\tau) = P_n(\lambda,\tau) = \lambda^2 \quad (96)$$

$$Q(\lambda,\tau) = Q_m(\lambda,\tau) = \lambda\cdot\frac{1}{C1}(\frac{1}{R_1} - \frac{\tau}{f_\#}) + \frac{1}{C1\cdot f_\#} \quad (97)$$

we assume that $P_n(\lambda,\tau) = P_n(\lambda)$ and $Q_m(\lambda,\tau)$ can't have common imaginary roots. That is for any real number ω ; $p_n(\lambda = i\cdot\omega,\tau) + Q_m(\lambda = i\cdot\omega,\tau) \neq 0$ (98)

$$-\omega^2 + i\cdot\omega\cdot\frac{1}{C1}(\frac{1}{R_1} - \frac{\tau}{f_\#}) + \frac{1}{C1\cdot f_\#} \neq 0; \quad (99)$$

$$F(\omega,\tau) = |P(i\cdot\omega,\tau)|^2 - |Q(i\cdot\omega,\tau)|^2\ ;\ P(i\cdot\omega,\tau) = -\omega^2 \quad (100)$$

$$P_R(i\cdot\omega,\tau) = -\omega^2\ ;\ P_I(i\cdot\omega,\tau) = 0 \quad (101)$$

$$Q(\lambda = i\cdot\omega,\tau) = i\cdot\omega\cdot\frac{1}{C1}(\frac{1}{R_1} - \frac{\tau}{f_\#}) + \frac{1}{C1\cdot f_\#} \quad (102)$$

$$Q_I(\lambda = i\cdot\omega,\tau) = \omega\cdot\frac{1}{C1}(\frac{1}{R_1} - \frac{\tau}{f_\#})$$
$$Q_R(\lambda = i\cdot\omega,\tau) = \frac{1}{C1\cdot f_\#} \quad (103)$$

$$|P(i\cdot\omega,\tau)|^2 = P_I^2 + P_R^2;\ \ |Q(i\cdot\omega,\tau)|^2 = Q_I^2 + Q_R^2 \quad (104)$$

$$|P(i\cdot\omega,\tau)|^2 = P_I^2 + P_R^2 = \omega^4 \quad (105)$$

$$|Q(i\cdot\omega,\tau)|^2 = \omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2 + \frac{1}{(C1\cdot f_\#)^2} \quad (106)$$

$$F(\omega,\tau) = \omega^4 - \omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2 - \frac{1}{(C1\cdot f_\#)^2} \quad (107)$$

Hence $F(\omega,\tau) = 0$ implies that:

$$\omega^4 - \omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2 - \frac{1}{(C1\cdot f_\#)^2} = 0 \quad (108)$$

$$F_\omega = 4\cdot\omega^3 - 2\cdot\omega\cdot\frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2$$
$$= 2\cdot\omega\cdot[2\cdot\omega^2 - \frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2] \quad (109)$$

$$F_\tau = \frac{2\cdot\omega^2}{C1^2\cdot f_\#}\cdot(\frac{1}{R_1} - \frac{\tau}{f_\#}) \quad (110)$$

$$P_{I\omega} = 0\ ;\ P_{R\omega} = -2\cdot\omega\ ;\ Q_{I\omega} = \frac{1}{C_1}\cdot[\frac{1}{R_1} - \frac{\tau}{f_\#}]$$
$$Q_{R\omega} = 0\ ;\ P_{I\tau} = 0\ ;\ P_{R\tau} = 0 \quad (111)$$

$$Q_{I\tau} = -\frac{\omega}{C_1\cdot f_\#}\ ;\ Q_{R\tau} = 0 \quad (112)$$

The expressions for U and V can be derived easily [BK] :

$$U = (P_R\cdot P_{I\omega} - P_I\cdot P_{R\omega}) - (Q_R\cdot Q_{I\omega} - Q_I\cdot Q_{R\omega}) \quad (113)$$

$$V = (P_R\cdot P_{Ix} - P_I\cdot P_{Rx}) - (Q_R\cdot Q_{Ix} - Q_I\cdot Q_{Rx}) \quad (114)$$

$$V = \frac{\omega}{C_1^2 \cdot f_\#^2} \;;\; U = \frac{1}{C_1^2 \cdot f_\#} \cdot [\frac{\tau}{f_\#} - \frac{1}{R_1}] \tag{115}$$

$\omega_\tau = -\frac{F_\tau}{F_\omega}$ and we get the expression:

$$\omega_\tau = -\frac{\frac{\omega}{C1^2 \cdot f_\#} \cdot (\frac{1}{R_1} - \frac{\tau}{f_\#})}{2 \cdot \omega^2 - \frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2} \tag{116}$$

Defines the maps $S_n(\tau) = \tau - \tau_n(\tau)\;;\; \tau \in \mathrm{I}\,,\, \mathrm{n} \in \mathbb{N}_0$

Defines the maps $S_n(\tau) = \tau - \tau_n(\tau)\;;\; \tau \in \mathrm{I}\,,\, \mathrm{n} \in \mathbb{N}_0$

which are continuous and differentiable in τ based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: Taylor approximation for $e^{-\lambda\tau} \approx 1 - \lambda \cdot \tau$ gives us good stability analysis approximation for restricted delay time intervals only.

6. RFID Tag System Stability Analysis under Delayed Variables In Time

Our RFID homogeneous system for v_1, v_2 leads to a characteristic equation for the eigenvalue λ having the form

$$P(\lambda) + Q(\lambda) \cdot e^{-\lambda \cdot \tau} = 0 \;;\; \text{first case } \tau_1 = \tau \;;\; \tau_2 = 0$$

$$D(\lambda, \tau_1 = \tau, \tau_2 = 0) = \lambda^2 + \lambda \cdot \frac{1}{C1 \cdot R1} + \frac{1}{C1 \cdot f_\#} \cdot e^{-\lambda \cdot \tau_1} \tag{117}$$

We use different parameter terminology in this case:

$$k \to j \;;\; p_k(\tau) \to a_j \;;\; q_k(\tau) \to c_j \;;\; n = 2 \;;\; m = 0$$

Additionally $P_n(\lambda, \tau) \to P(\lambda)\;;\; Q_m(\lambda, \tau) \to Q(\lambda)$

then $P(\lambda) = \sum_{j=0}^{2} a_j \cdot \lambda^j$ and $Q(\lambda) = \sum_{j=0}^{0} c_j \cdot \lambda^j$. (118)

$$P(\lambda) = \lambda^2 + \lambda \cdot \frac{1}{C1 \cdot R1}\;;\; Q(\lambda) = \frac{1}{C1 \cdot f_\#} \tag{119}$$

$n, m \in \mathbb{N}_0$, $n > m$ and $a_j, c_j : \mathrm{R}_{+0} \to R$ are continuous and differentiable function of τ such that $a_0 + c_0 \neq 0$. In the following $\_\_\_$, "$-$" denotes complex and conjugate. $P(\lambda), Q(\lambda)$

Are analytic functions in λ and differentiable in τ.

The coefficients:

$\{a_j(C_1, R_1), c_j(C_1, \text{antenna parametrs})\} \in \mathbb{R}$ depend on RFID C_1, R_1 values and antenna parameters but not on τ.

$$a_0 = 0,\; a_1 = \frac{1}{C_1 \cdot R_1},\; a_2 = 1,\; a_3 = 0\;;\; c_0 = \frac{1}{C_1 \cdot f_\#},\; c_1 = c_2 = 0$$

Unless absolutely necessary, the designation of the variation arguments $(R_1, C_1, \text{antenna parametrs})$ will subsequently be omitted from P, Q, a_j, c_j. The coefficients a_j, c_j are continuous, and differentiable functions of their arguments, and direct substitution shows that $a_0 + c_0 \neq 0\;;\; \frac{1}{C_1 \cdot f_\#} \neq 0$

$\forall\; C_1$, antenna parameters $\in \mathbb{R}_+$ i.e $\lambda = 0$ is not a root of the characteristic equation. Furthermore $P(\lambda)$, $Q(\lambda)$ are analytic functions of λ for which the following requirements of the analysis (see Kuang, 1993, section 3.4) can also be verified in the present case [4] [5].

(a) If $\lambda = i \cdot \omega$, $\omega \in \mathbb{R}$ then $P(i \cdot \omega) + Q(i \cdot \omega) \neq 0$, i.e. P and Q have no common imaginary roots. This condition was verified numerically in the entire $(R_1, C_1, \text{antenna parametrs})$ domain of interest.

(b) $|Q(\lambda) / P(\lambda)|$ is bounded for $|\lambda| \to \infty$, $\mathrm{Re}\,\lambda \geq 0$. No roots bifurcation from ∞.

Indeed, in the limit

$$|Q(\lambda) / P(\lambda)| = \left| \frac{1}{C_1 \cdot f_\# \cdot (\lambda^2 + \lambda \cdot \frac{1}{C_1 \cdot R_1})} \right| \tag{120}$$

(c) $F(\omega) = |P(i \cdot \omega)|^2 - |Q(i \cdot \omega)|^2$ (121)

$$F(\omega, \tau) = \omega^4 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_\#)^2} \tag{122}$$

Has at most a finite number of zeros. Indeed, this is a bi-cubic polynomial in ω (second degree in ω^2).

(d) Each positive root $\omega(C_1, R_1, \text{antenna parametrs})$

of $F(\omega)=0$ is continuous and differentiable with respect to C_1, R_1, antenna parametrs. This condition can only be assessed numerically.

In addition, since the coefficients in P and Q are real, we have

$\overline{P(-i\cdot\omega)} = P(i\cdot\omega)$, and $\overline{Q(-i\cdot\omega)} = Q(i\cdot\omega)$ thus $\lambda = i\cdot\omega,\ \omega > 0$ may be on eigenvalue of characteristic equation. The analysis consists in identifying the roots of characteristic equation situated on the imaginary axis of the complex λ – plane, where by increasing the parameters

C_1, R_1, antenna parametrs and delay τ, Reλ may, at the crossing ,change its sign from (-) to (+), i.e. from a stable focus

$E^{(0)}(V_1^{(0)}, V_2^{(0)}) = (0,0)$ to an unstable one, or vice versa.

This feature may be further assessed by examining the sign of the partial derivatives with respect to C_1, R_1 and antenna parameters.

$$\wedge^{-1}(C_1) = \left(\frac{\partial \operatorname{Re}\lambda}{\partial C_1}\right)_{\lambda=i\cdot\omega},\ \mathrm{R}_1, \text{antenna parametrs} = const$$

$$\wedge^{-1}(\mathrm{R}_1) = \left(\frac{\partial \operatorname{Re}\lambda}{\partial \mathrm{R}_1}\right)_{\lambda=i\cdot\omega},\ C_1, \text{antenna parametrs} = const$$

$$\wedge^{-1}(f_\#) = \left(\frac{\partial \operatorname{Re}\lambda}{\partial f_\#}\right)_{\lambda=i\cdot\omega},\ C_1, \mathrm{R}_1 = const;$$

$$\wedge^{-1}(\tau) = \left(\frac{\partial \operatorname{Re}\lambda}{\partial \tau}\right)_{\lambda=i\cdot\omega},\ C_1, \mathrm{R}_1, \text{antenna parameters} = const;\ \text{where } \omega \in \mathbb{R}_+. \quad (123)$$

For the first case $\tau_1 = \tau$; $\tau_2 = 0$ we get the following results

$$P_R(i\cdot\omega) = -a_2\cdot\omega^2 + a_0 = -\omega^2 \quad (124)$$

$$P_I(i\cdot\omega) = -a_3\cdot\omega^3 + a_1\cdot\omega = \omega\cdot\frac{1}{C_1\cdot R_1} \quad (125)$$

$$Q_R(i\cdot\omega) = -c_2\cdot\omega^2 + c_0 = \frac{1}{C_1\cdot f_\#} \quad (126)$$

$Q_I(i\cdot\omega) = c_1\cdot\omega = 0$; $F(\omega)=0$ yield to

$$\omega = \pm\sqrt{\frac{1}{2\cdot(C_1\cdot R_1)^2} \pm \frac{1}{2}\cdot\sqrt{\frac{1}{(C_1\cdot R_1)^4} + 4\cdot\frac{1}{[C_1\cdot f_\#]^2}}} \quad (127)$$

$\frac{1}{(C_1\cdot R_1)^4} + 4\cdot\frac{1}{[C_1\cdot f_\#]^2} > 0$ always and additional for

$$\omega \in R;\ \frac{1}{2\cdot(C_1\cdot R_1)^2} \pm \frac{1}{2}\cdot\sqrt{\frac{1}{(C_1\cdot R_1)^4} + 4\cdot\frac{1}{[C_1\cdot f_\#]^2}} > 0 \quad (128)$$

There are two options: first, it is always true that

$$\frac{1}{2\cdot(C_1\cdot R_1)^2} + \frac{1}{2}\cdot\sqrt{\frac{1}{(C_1\cdot R_1)^4} + 4\cdot\frac{1}{[C_1\cdot f_\#]^2}} > 0 \quad (129)$$

Second $\frac{1}{2\cdot(C_1\cdot R_1)^2} - \frac{1}{2}\cdot\sqrt{\frac{1}{(C_1\cdot R_1)^4} + 4\cdot\frac{1}{[C_1\cdot f_\#]^2}} > 0$ (130)

Not exist and always negative for any RFID TAG overall parameters values.

When writing $P(\lambda) = P_R(\lambda) + i\cdot P_I(\lambda)$ and $Q(\lambda) = Q_R(\lambda) + i\cdot Q_I(\lambda)$, and inserting $\lambda = i\cdot\omega$

into RFID characteristic equation , ω must satisfy the following :

$$\sin\omega\cdot\tau = g(\omega) = \frac{-P_R(i\cdot\omega)\cdot Q_I(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_R(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (131)$$

$$\cos\omega\cdot\tau = h(\omega) = -\frac{P_R(i\cdot\omega)\cdot Q_R(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_I(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (132)$$

Where $|Q(i\cdot\omega)|^2 \neq 0$ in light of requirement (a) above, and $(g,h) \in R$. Furthermore, it follows that in the $\sin\omega\cdot\tau$ and $\cos\omega\cdot\tau$ equations, by squaring and adding the sides, ω must be a positive root of $F(\omega) = |P(i\cdot\omega)|^2 - |Q(i\cdot\omega)|^2 = 0$.

Note that $F(\omega)$ is independent of τ. It is important to notice that if $\tau \notin I$ (assume that $I \subseteq R_{+0}$ is the set where $\omega(\tau)$ is a positive root of $F(\omega)$ and for $\tau \notin I$, $\omega(\tau)$ is not define. Then for all τ in I $\omega(\tau)$ it satisfies that $F(\omega,\tau) = 0$)

Then there are no positive $\omega(\tau)$ solutions for $F(\omega,\tau) = 0$, and we cannot have stability switches. For any $\tau \in I$ where $\omega(\tau)$ is a positive solution of

$F(\omega,\tau)=0$, we can define the angle $\theta(\tau)\in[0,2\cdot\pi]$ as the solution of

$$\sin\theta(\tau)=\frac{-P_R(i\cdot\omega)\cdot Q_I(i\cdot\omega)+P_I(i\cdot\omega)\cdot Q_R(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (133)$$

$$\cos\theta(\tau)=-\frac{P_R(i\cdot\omega)\cdot Q_R(i\cdot\omega)+P_I(i\cdot\omega)\cdot Q_I(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (134)$$

And the relation between the argument $\theta(\tau)$ and $\omega(\tau)\cdot\tau$ for $\tau\in I$ must be $\omega(\tau)\cdot\tau=\theta(\tau)+n\cdot 2\cdot\pi \;\; \forall \;\; n\in\mathbb{N}_0$. Hence we can define the maps $\tau_n : I\to R_{+0}$ given by

$\tau_n(\tau)=\frac{\theta(\tau)+n\cdot 2\cdot\pi}{\omega(\tau)}; n\in\mathbb{N}_0, \tau\in I$. Let us introduce the functions $I\to R$; $S_n(\tau)=\tau-\tau_n(\tau),\;\tau\in I,\;n\in\mathbb{N}_0$ which are continuous and differentiable in τ. In the following, the subscripts $\lambda,\;\omega, R_1,\;C_1$ and RFID TAG antenna parameters $(\text{w,g,B0,A0,A}_{AVG}, B_{AVG}, etc.,)$ indicate the corresponding partial derivatives. Let us first concentrate on $\wedge(x)$, remember in $\lambda(R_1, C_1, \text{w,g,B0,A0,A}_{AVG}, B_{AVG}, etc.,)$

And $\omega(R_1, C_1, \text{w,g,B0,A0,A}_{AVG}, B_{AVG}, etc.,)$, and keeping all parameters except one (x) and τ. The derivation closely follows that in reference [BK]. Differentiating RFID characteristic equation $P(\lambda)+Q(\lambda)\cdot e^{-\lambda\cdot\tau}=0$ with respect to specific parameter (x), and inverting the derivative for convenience, one calculates:

<u>Remark</u>: $x=R_1, C_1, \text{w,g,B0,A0,A}_{AVG}, B_{AVG}, etc.,$

$$(\frac{\partial\lambda}{\partial x})^{-1} = \frac{-P_\lambda(\lambda,x)\cdot Q(\lambda,x)+Q_\lambda(\lambda,x)\cdot P(\lambda,x)-\tau\cdot P(\lambda,x)\cdot Q(\lambda,x)}{P_x(\lambda,x)\cdot Q(\lambda,x)-Q_x(\lambda,x)\cdot P(\lambda,x)} \quad (135)$$

Where $P_\lambda=\frac{\partial P}{\partial\lambda}$,.... etc., substituting $\lambda=i\cdot\omega$, and

bearing i $\overline{P(-i\cdot\omega)}=P(i\cdot\omega)$, $\overline{Q(-i\cdot\omega)}=Q(i\cdot\omega)$ Then $i\cdot P_\lambda(i\cdot\omega)=P_\omega(i\cdot\omega)$ and $i\cdot Q_\lambda(i\cdot\omega)=Q_\omega(i\cdot\omega)$ and that on the surface $|P(i\cdot\omega)|^2=|Q(i\cdot\omega)|^2$, one obtains

$$(\frac{\partial\lambda}{\partial x})^{-1}|_{\lambda=i\cdot\omega} = (\frac{i\cdot P_\omega(i\cdot\omega,x)\cdot\overline{P(i\cdot\omega,x)}+i\cdot Q_\lambda(i\cdot\omega,x)\cdot\overline{Q(\lambda,x)}-\tau\cdot|P(i\cdot\omega,x)|^2}{P_x(i\cdot\omega,x)\cdot\overline{P(i\cdot\omega,x)}-Q_x(i\cdot\omega,x)\cdot\overline{Q(i\cdot\omega,x)}}) \quad (136)$$

Upon separating this into real and imaginary parts, with

$$P=P_R+i\cdot P_I \;;\; Q=Q_R+i\cdot Q_I\;;\; P_\omega=P_{R\omega}+i\cdot P_{I\omega} \quad (137)$$

$$Q_\omega=Q_{R\omega}+i\cdot Q_{I\omega};\, P_x=P_{Rx}+i\cdot P_{Ix};\, Q_x=Q_{Rx}+i\cdot Q_{Ix} \quad (138)$$

$P^2=P_R^2+P_I^2$. When (x) can be any RFID TAG parameters R1, C1, and time delay τ etc,. For convenience, we have dropped the arguments $(i\cdot\omega,x)$, and

$$F_\omega=2\cdot[(P_{R\omega}\cdot P_R+P_{I\omega}\cdot P_I)-(Q_{R\omega}\cdot Q_R+Q_{I\omega}\cdot Q_I)] \quad (139)$$

$$F_x=2\cdot[(P_{Rx}\cdot P_R+P_{Ix}\cdot P_I)-(Q_{Rx}\cdot Q_R+Q_{Ix}\cdot Q_I)] \quad (140)$$

$\omega_x=-F_x/F_\omega$. We define U and V:

$$U=(P_R\cdot P_{I\omega}-P_I\cdot P_{R\omega})-(Q_R\cdot Q_{I\omega}-Q_I\cdot Q_{R\omega}) \quad (141)$$

$$V=(P_R\cdot P_{Ix}-P_I\cdot P_{Rx})-(Q_R\cdot Q_{Ix}-Q_I\cdot Q_{Rx}) \quad (142)$$

We choose our specific parameter as time delay x = τ.

$$U=\frac{\omega^2}{C_1\cdot R_1};\, P^2=\omega^4+\omega^2\cdot\frac{1}{(C_1\cdot R_1)^2};F_\tau=0 \quad (143)$$

$$P_{I\tau}=0\;;\;P_{R\tau}=0\;;\;Q_{I\tau}=0\;;\;Q_{R\tau}=0\;\Rightarrow V=0$$

$$\frac{\partial F}{\partial\omega}=F_\omega=2\cdot[2\cdot\omega^3+\omega\cdot\frac{1}{(C1\cdot R1)^2}];\;F(\omega,\tau)=0 \quad (144)$$

Differentiating with respect to τ, we get

$$F_\omega\cdot\frac{\partial\omega}{\partial\tau}+F_\tau=0\;;\;\tau\in I\Rightarrow\frac{\partial\omega}{\partial\tau}=-\frac{F_\tau}{F_\omega} \quad (145)$$

$$\wedge^{-1}(\tau)=(\frac{\partial\,\mathrm{Re}\,\lambda}{\partial\tau})_{\lambda=i\cdot\omega} \quad (146)$$

$$\wedge^{-1}(\tau)=\mathrm{Re}\{\frac{-2\cdot[U+\tau\cdot|P|^2]+i\cdot F_\omega}{F_\tau+i\cdot 2\cdot[V+\omega\cdot|P|^2]}\}=\frac{2\cdot\omega^2+\frac{1}{(C1\cdot R1)^2}}{\omega^4+\omega^2\cdot\frac{1}{(C1\cdot R1)^2}} \quad (147)$$

$$sign\{\wedge^{-1}(\tau)\} = sign\{(\frac{\partial \operatorname{Re}\lambda}{\partial \tau})_{\lambda=i\cdot\omega}\} \quad (148)$$

$$sign\{\wedge^{-1}(\tau)\} = sign\{F_\omega\}\cdot sign\{\tau\cdot\frac{\partial\omega}{\partial\tau}+\omega+\frac{U\cdot\frac{\partial\omega}{\partial\tau}+V}{|P|^2}\} \quad (149)$$

$\frac{\partial\omega}{\partial\tau} = \omega_\tau = -\frac{F_\tau}{F_\omega}$; $F_\tau = 0 \Rightarrow \frac{\partial\omega}{\partial\tau} = 0$ then we get

$$sign\{\wedge^{-1}(\tau)\} = sign\{2\cdot\omega\cdot[2\cdot\omega^2+\frac{1}{(C1\cdot R1)^2}]\}\cdot sign\{\omega\} \quad (150)$$

Result : $\wedge^{-1}(\tau) > 0$ for all $\omega, R1, C1$ values. The sign of $\wedge^{-1}(\tau)$ is independent of τ values so in the first case $\tau_1 = \tau$; $\tau_2 = 0$ there is no stability switch for different values of τ .

We now inspect the third interesting case wherein $\tau_1 = \tau$; $\tau_2 = \tau$ when there are delays both in RFID Label voltage and voltage time derivative [4] [5].

$$\begin{aligned} &D(\lambda, \tau_1 = \tau, \tau_2 = \tau) \\ &= \lambda^2 + \lambda\cdot\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau} + \frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\cdot\tau\cdot 2} \end{aligned} \quad (151)$$

Taylor expansion: $e^{-\lambda\tau} \approx 1 - \lambda\cdot\tau + \frac{\lambda^2\cdot\tau^2}{2}$ since we need n > m [BK] analysis we choose $e^{-\lambda\tau} \approx 1 - \lambda\cdot\tau$ and then we get our RFID system second order characteristic equation:

$$\begin{aligned} D(\lambda,\tau) &= \lambda^2 + a(\tau)\cdot\lambda + b(\tau)\cdot\lambda\cdot e^{-\lambda\cdot\tau} \\ &\quad + c(\tau) + d(\tau)\cdot e^{-\lambda\cdot\tau} \end{aligned} \quad (152)$$

$a(\tau) = 0$; $b(\tau) = \frac{1}{C1}\cdot(\frac{1}{R_1} - \frac{\tau}{f_\#})$; $c(\tau) = 0$; $d(\tau) = \frac{1}{C_1\cdot f_\#}$

$$\begin{aligned} F(\omega,\tau) &= |P(i\cdot\omega,\tau)|^2 - |Q(i\cdot\omega,\tau)|^2 \\ &= (c-\omega^2)^2 + \omega^2\cdot a^2 - (\omega^2\cdot b^2 + d^2) \end{aligned} \quad (153)$$

$$F(\omega,\tau) = \omega^4 - \omega^2\cdot\frac{1}{(C_1\cdot R_1)^2} - \frac{1}{(C_1\cdot f_\#)^2} \quad (154)$$

Hence $F(\omega,\tau) = 0$ implies

$$\omega^4 - \omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1} - \frac{\tau}{f_\#})^2 - \frac{1}{(C1\cdot f_\#)^2} = 0 \quad (155)$$

And its roots are given by

$$\begin{aligned} \omega_+^2 &= \frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)+\sqrt{\Delta}\} \\ &= \frac{1}{2}\cdot\{\sqrt{\Delta} + \frac{1}{C1^2}(\frac{1}{R_1}-\frac{\tau}{f_\#})^2\} \end{aligned} \quad (156)$$

$$\begin{aligned} \omega_-^2 &= \frac{1}{2}\cdot\{(b^2+2\cdot c-a^2)-\sqrt{\Delta}\} \\ &= \frac{1}{2}\cdot\{-\sqrt{\Delta} + \frac{1}{C1^2}(\frac{1}{R_1}-\frac{\tau}{f_\#})^2\} \end{aligned} \quad (157)$$

$$\begin{aligned} \Delta &= (b^2+2\cdot c-a^2) - 4\cdot(c^2-d^2) \\ &= \frac{1}{C1^2}\cdot(\frac{1}{R_1}-\frac{\tau}{f_\#})^2 + \frac{4}{(C1\cdot f_\#)^2} \end{aligned} \quad (158)$$

Therefore the following holds:

$$2\cdot\omega_{+/-}^2 - (b^2+2\cdot c-a^2) = \pm\sqrt{\Delta} \quad (159)$$

$$\begin{aligned} &\sin\theta(\tau) = \\ &\frac{-P_R(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau) + P_I(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \end{aligned} \quad (160)$$

$$\begin{aligned} &\cos\theta(\tau) = \\ &-\frac{P_R(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau) + P_I(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \end{aligned} \quad (161)$$

$$\begin{aligned} \sin\theta(\tau) &= \frac{-(c-\omega^2)\cdot\omega\cdot b + \omega\cdot a\cdot d}{\omega^2\cdot b^2 + d^2} \\ &= \frac{\omega^3\cdot\frac{1}{C1}\cdot(\frac{1}{R_1}-\frac{\tau}{f_\#})}{\omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1}-\frac{\tau}{f_\#})^2 + \frac{1}{(C1\cdot f_\#)^2}} \end{aligned} \quad (162)$$

$$\begin{aligned} \cos\theta(\tau) &= -\frac{(c-\omega^2)\cdot d + \omega^2\cdot a\cdot b}{\omega^2\cdot b^2 + d^2} \\ &= \frac{\omega^2\cdot\frac{1}{C1\cdot f_\#}}{\omega^2\cdot\frac{1}{C1^2}(\frac{1}{R_1}-\frac{\tau}{f_\#})^2 + \frac{1}{(C1\cdot f_\#)^2}} \end{aligned} \quad (163)$$

For our stability switching analysis we choose typical RFID parameter values:

$C_1 = 23pF$; $R_1 = 100k\Omega = 10^5$; $L_{calc} = f_\# = 2.65mH$

Then $\frac{1}{C_1^2} = 1.89 \cdot 10^{21}$; $\frac{1}{C_1^2 \cdot f_\#^2} = 2.69 \cdot 10^{26}$

We find those ω, τ values which fulfill $F(\omega, \tau) = 0$. We ignore negative, complex, and imaginary values of ω for specific τ values. The following table gives the list. $\tau \in [0.001..10]$ and can be expressed using a straight line ($\omega = \tau \bullet 1.64 \bullet 10^{13}$)

τ	ω
0.001	$1.64 \bullet 10^{10}$
0.01	$1.64 \bullet 10^{11}$
0.05	$8.2 \bullet 10^{11}$
0.1	$1.64 \bullet 10^{12}$
0.2	$3.28 \bullet 10^{12}$
1	$1.64 \bullet 10^{13}$
5	$8.2 \bullet 10^{13}$
10	$1.64 \bullet 10^{14}$

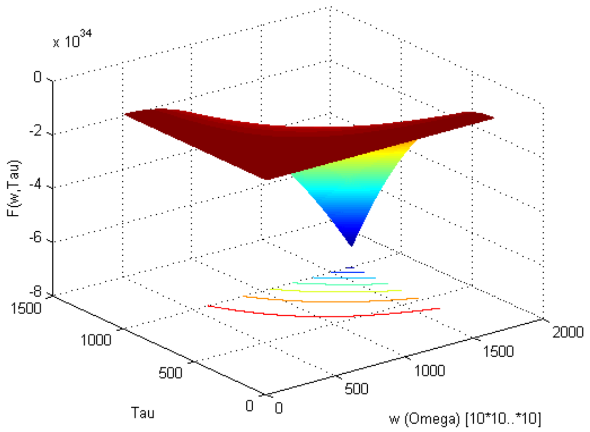

Figure 3. RFID TAG $F(\omega,\tau)$ function for $\tau_1 = \tau_2 = \tau$

Remark: In the above figure ω variable is 10^{10} units.

Matlab : [w,t]=meshgrid(1:1:1640,0:0.01:10); f=w.*w.*w.*w-w.*w.*1.89*10^21.*(10^-5-(t./(2.65*10^-3))).^2 -2.69*10^26; meshc(f); % $\omega \to w, \tau \to t$

We plot the stability switch diagram based on different delay values of our RFID TAG system.

$$\wedge^{-1}(\tau) = \left(\frac{\partial \operatorname{Re} \lambda}{\partial \tau}\right)_{\lambda = i \cdot \omega} = \operatorname{Re}\left\{\frac{-2 \cdot [U + \tau \cdot |P|^2] + i \cdot F_\omega}{F_\tau + i \cdot 2 \cdot [V + \omega \cdot |P|^2]}\right\} \quad (164)$$

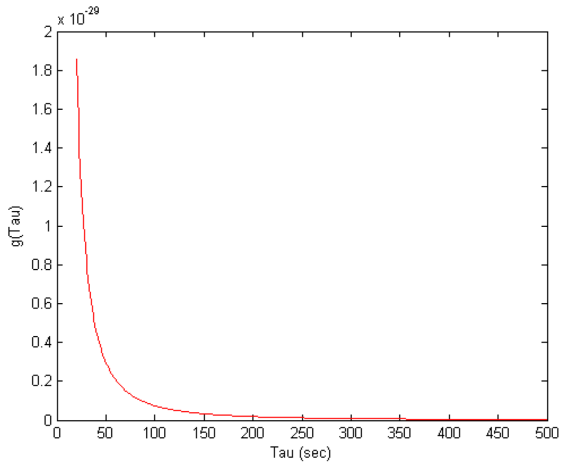

Figure 4. RFID TAG stability switch diagram based on different delay values of our RFID TAG system.

$$\wedge^{-1}(\tau) = \left(\frac{\partial \operatorname{Re} \lambda}{\partial \tau}\right)_{\lambda = i \cdot \omega} = \frac{2 \cdot \{F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2)\}}{F_\tau^2 + 4 \cdot (V + \omega \cdot P^2)^2} \quad (165)$$

$$g(Tau) = \wedge^{-1}(\tau) = \left(\frac{\partial \operatorname{Re} \lambda}{\partial \tau}\right)_{\lambda = i \cdot \omega} \quad (166)$$

The stability switch occur only on those delay values (τ) that fit the equation: $\tau = \frac{\theta_+(\tau)}{\omega_+(\tau)}$ and $\theta_+(\tau)$ is the solution of

$$\sin \theta(\tau) = \frac{\omega^3 \cdot \frac{1}{C1} \cdot \left(\frac{1}{R_1} - \frac{\tau}{f_\#}\right)}{\omega^2 \cdot \frac{1}{C1^2} \left(\frac{1}{R_1} - \frac{\tau}{f_\#}\right)^2 + \frac{1}{(C1 \cdot f_\#)^2}} \quad (167)$$

$$\cos \theta(\tau) = \frac{\omega^2 \cdot \frac{1}{C1 \cdot f_\#}}{\omega^2 \cdot \frac{1}{C1^2} \left(\frac{1}{R_1} - \frac{\tau}{f_\#}\right)^2 + \frac{1}{(C1 \cdot f_\#)^2}} \quad (168)$$

When $\omega = \omega_+(\tau)$ if only ω_+ is feasible. Additionally when all RFID TAG parameters are known, the stability switch due to various time delay values τ is described in the following expression:

$$sign\{\wedge^{-1}(\tau)\} = sign\{F_\omega(\omega(\tau), \tau)\} \cdot sign\{\tau \cdot \omega_\tau(\omega(\tau)) + \omega(\tau) + \frac{U(\omega(\tau)) \cdot \omega_\tau(\omega(\tau)) + V(\omega(\tau))}{|P(\omega(\tau))|^2}\}$$

(169)

Remark: we know $F(\omega,\tau)=0$ implies it roots $\omega_i(\tau)$ and finding those delay values τ for which ω_i is feasible. There are τ values for which ω_i is complex or imaginary number, leaving us unable to analyze stability [4] [5].

7. Conclusion

A RFID TAG environment is characterize by electromagnetic interferences which can influence RFID TAGs stability in time. There are two main RFID TAGs variables which are affected by electromagnetic interferences-- voltage developed on the RFID Label and the voltage time derivative. Each RFID Label variable under electromagnetic interference is characterized by respective time delay. The two time delays are not the same but can be categorized to some subcases due to interference behaviors.

The first case is when there is RFID Label voltage time delay but no voltage derivative time delay. The second case is when there is no RFID Label voltage time delay but there is a voltage derivative time delay. The third case is when both RFID Label voltage time delay and voltage derivative time delay exist. For simplicity we consider the two delays in the third case to be the same (the difference exists but it is negligible in our analysis). In each case we derive the related characteristic equation. The characteristic equation is dependent on the RFID Label's overall parameters and interference time delays. Following mathematical manipulation and [BK] theorems and definitions, we derived the expression which gives us a clear understanding of the RFID Label stability map. The stability map gives all possible options for stability segments where each segment belongs to different time delay values segments. RFID Label stability analysis can be influenced by TAG overall parameter values. We do not discuss this analysis in the current article.

Appendix A: Lemma 1.1

Assume that $\omega(\tau)$ is a positive and real root of $F(\omega,\tau)=0$

Defined for $\tau \in I$, which is continuous and differentiable. Assume further that if $\lambda = i\cdot\omega$, $\omega \in R$, then $P_n(i\cdot\omega,\tau)+Q_n(i\cdot\omega,\tau)\neq 0,\ \tau \in \mathrm{R}$ hold true. Then the functions $S_n(\tau)$, $n \in N_0$, are continuous and differentiable on I.

Appendix B : Theorem 1.2

Assume that $\omega(\tau)$ is a positive real root of $F(\omega,\tau)=0$ defined for $\tau \in I,\ I \subseteq R_{+0}$, and at some $\tau^* \in I$, $S_n(\tau^*)=0$

For some $n \in N_0$ then a pair of simple conjugate pure imaginary roots $\lambda_+(\tau^*)=i\cdot\omega(\tau^*),\ \lambda_-(\tau^*)=-i\cdot\omega(\tau^*)$ of $D(\lambda,\tau)=0$ exist at $\tau=\tau^*$ which crosses the imaginary axis from left to right if $\delta(\tau^*)>0$ and crosses the imaginary axis from right to left if $\delta(\tau^*)<0$ where

$$\delta(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda=i\omega(\tau^*)}\} = sign\{F_\omega(\omega(\tau^*),\tau^*)\}\cdot sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (170)$$

The theorem becomes

$$sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda=i\omega\pm}\} = sign\{\pm\Delta^{1/2}\}\cdot sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (171)$$

Appendix C: Theorem 1.3

The characteristic equation: $\tau_1=\tau,\tau_2=0$; $\tau_1=0,\tau_2=\tau$

$$D(\lambda,\tau)=\lambda^2+a(\tau)\cdot\lambda+b(\tau)\cdot\lambda\cdot e^{-\lambda\cdot\tau}+c(\tau)+d(\tau)\cdot e^{-\lambda\cdot\tau} \quad (172)$$

$$D(\lambda,\tau_1,\tau_2)=\lambda^2+\lambda\cdot\frac{1}{C1\cdot R1}\cdot e^{-\lambda\cdot\tau_2}+\frac{1}{C1\cdot f_\#}\cdot e^{-\lambda\cdot(\tau_1+\tau_2)} \quad (173)$$

has a pair of simple and conjugate pure imaginary roots $\lambda=\pm\omega(\tau^*)$, $\omega(\tau^*)$ real at $\tau^* \in I$ if $S_n(\tau^*)=\tau^*-\tau_n(\tau^*)=0$ for some $n \in N_0$. If $\omega(\tau^*)=\omega_+(\tau^*)$. This pair of simple conjugate pure imaginary roots crosses the imaginary axis from left to right if $\delta_+(\tau^*)>0$ and crosses the imaginary axis from right to left if $\delta_+(\tau^*)<0$ where

$$\delta_+(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda=i\omega_+(\tau^*)}\} = sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (174)$$

If $\omega(\tau^*)=\omega_-(\tau^*)$, this pair of simple conjugate pure imaginary roots cross the imaginary axis from left to right if $\delta_-(\tau^*)>0$ and crosses the imaginary axis from right to left

If $\delta_-(\tau^*)<0$ where

$$\delta_-(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda=i\omega_-(\tau^*)}\} = -sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (175)$$

If $\omega_+(\tau^*) = \omega_-(\tau^*) = \omega(\tau^*)$ then $\Delta(\tau^*) = 0$ and $sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda=i\omega(\tau^*)}\} = 0$, the same is true when $S_n^{'}(\tau^*)=0$

The following result can be useful in identifying values of τ where there have been stability switches.

Appendix D: Theorem 1.4

Assume that for all $\tau \in I$, $\omega(\tau)$ is defined as a solution of $F(\omega,\tau) = 0$ then

$$\delta_\pm(\tau) = sign\{\pm\Delta^{1/2}(\tau)\}\cdot signD_\pm(\tau)$$

$$D_\pm(\tau) = \omega_\pm^2\cdot[(\omega_\pm^2\cdot b^2 + d^2) + a^{'}\cdot(c-\omega_\pm^2) + b\cdot d^{'} - b^{'}\cdot d - a\cdot c^{'}]$$
$$+\omega_\pm\cdot\omega_\pm^{'}\cdot[\tau\cdot(\omega_\pm^2\cdot b^2 + d^2) - b\cdot d + a\cdot(c-\omega_\pm^2) + 2\cdot\omega_\pm^2\cdot a] \quad (176)$$

$$a^{'} = \frac{da(\tau)}{d\tau};\ \mathrm{b}^{'} = \frac{db(\tau)}{d\tau};\ \mathrm{c}^{'} = \frac{dc(\tau)}{d\tau};\ \mathrm{d}^{'} = \frac{dd(\tau)}{d\tau} \quad (177)$$

8. References

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