SLINGSHOT ARGUMENTS AND THE INTENSIONALITY OF IDENTITY*

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ABSTRACT

It is argued that the slingshot argument does not soundly challenge the truth-maker correspondence theory of truth, by which at least some distinct true propositions are expected to have distinct truth-makers. Objections are presented to possible exact interpretations of the essential slingshot assumption, in which no fully acceptable reconstruction is discovered. A streamlined version of the slingshot is evaluated, in which explicit contradiction results, on the assumption that identity and nonidentity contexts are purely extensional relations, effectively establishing the intensionality of identity.

Keywords: Davidson, Donald; extension, extensionality; intension, intensionality; Leibnizian identity conditions; logic; Neale, Stephen; propositions; semantics; sentence tokens and types; slingshot argument(s); truth; truth-maker (theory)

1. Logical David Against a Truth-Maker Goliath

The slingshot argument purports to refute the standard correspondence theory of truth, in its requirement that there be distinct truth-makers for at least some distinct true propositions. The slingshot is supposed to accomplish this feat by proving from minimal logical means in a purely extensionalist environment that all true sentences must correspond to a single aggregative fact that serves as their massive collective truth-maker.¹

¹ This way of explaining the slingshot argument’s consequences for truth-maker theory was first articulated by Davidson, speaking of ‘The Great Fact’ [1967; 1990].
Originally suggested by Gottlob Frege, according to Alonzo Church, and later articulated by Kurt Gödel, slingshot arguments have been developed for different philosophical purposes by Donald Davidson. The general style of inference was designated *slingshot* by Jon Barwise and John Perry, in honor of the argument’s simplicity and use of minimal primitive resources in achieving exceptional conclusions. Slingshots have been more recently discussed, among others, by W.V.O. Quine, Dagfinn Follesdal, Stephen Neale, James O. Young, James Levine, Graham Oppy, Yaroslav Shramko, and Heinrich Wansing. The slingshot argument, like most interesting philosophical controversies and positions, in any of its forms, has both loyal adherents and ardent opponents.\(^2\)

### 2. Formalizing Slingshot Reasoning

There are several formulations of the slingshot argument. For convenience, we consider a common composite version based on four assumptions. The truth assertions of propositions \(p\) and \(q\) in assumptions (1) and (2) require no comment. Assumption (3) states that \(a\) is identical to the object identical to \(a\) and proposition \(p\) [is true]. What this further means is the subject later of more penetrating criticism. The slingshot is first drawn back and made ready to release:

\[
\begin{align*}
(1) & \quad p \\
(2) & \quad q \\
(3) & \quad a = \iota x \left[ x = a \land p \right]
\end{align*}
\]

If \(p = Fx\), closed by the definite descriptor, then step (3) can be instantiated as \(a = \iota x \left[ x = a \land Fa \right]\). The argument now proceeds by further supposing that:

\[
(4) \quad \text{The truth-maker of } p \neq \text{ the truth-maker of } q
\]

The truth-makers of \(p\) and \(q\) are posited as different facts. Both truth-making facts for propositions \(p\) and \(q\) happen to exist or obtain, such that the material equivalence \(p \leftrightarrow q\) is contingently true. From the presumed extensionality of (3), and the fact that where \(p\) and \(q\) by hypothesis are both true, it follows by truth table definition that \(p \leftrightarrow q\), it is deduced that, substituting \(q\) for \(p\) in the purely extensional definite description context in (3) implies:

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\(^2\) Church credits Frege’s [1892], as the origin of the slingshot argument, and it has become customary to mention him as the argument’s originator. See Church [1943]. Follesdal [1983], especially p. 92, points out some of the differences between Frege’s inspiration an Church’s application of a style of reasoning related to the family of slingshot arguments, which he also traces to Quine’s [1976]. Gödel [1944]. Barwise and Perry [1981]. See also Perry’s more recent essay [1996]. Neale [1995]. Also, Neale [2001]. Young [2002]. Levine [2006]. Oppy [1997]. Shramko and Wansing [2009].
(5) \( a = \tau[x = a \land q] \)
Transitivity of identity with (3) and (5) now delivers:

(6) \( \tau[x = a \land p] = \tau[x = a \land q] \)

The \( \tau[x = a](= a) \) component of (3), (5), and (6), however, is logically equivalent to \( a = a \), which, as a trivial logically necessary truth that follows from the universal reflexivity \( \forall x[x = x] \) condition on identity, drops out of consideration as among the truth-makers for \( \tau[x = a \land p] \) and \( \tau[x = a \land q] \), logically reducing (6) to:

(7) For any type distinct true sentences or propositions \( p, q \), the truth-maker of \( p \) = the truth-maker of \( q \)

Since (7) contradicts (4), the reductio is interpretable as showing that the truth-makers of any distinct true propositions are always identical rather than ever distinct.\(^3\) There are supposed to be surprising philosophical consequences resulting from slingshot arguments. However, what these are and whether and in what sense they might hold on the basis of the slingshot is sometimes even more hotly contested than whether in fact the slingshot works at all.\(^4\)

3. Stipulative Identity, License and Regulations

Formulated as in (1)-(7), it is hard on reflection to see how the slingshot argument could have ever been taken seriously. Several objections to the argument have been proposed, including Gödel’s suggestion that the inference does not go through when steps (3) and (5) are properly reformulated according to Russell’s analysis of definite descriptions.\(^5\)

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3 The comparable formalization in Neale, [1995], 789-790, regarded by many as definitive, similarly depends on an identity, derived by means of his specific formal principle of \( \tau \)-INTRO, in which the scope of the definite descriptor explicitly indicates that being the identical to object \( a \) is conjoined with a predication: [4] \( a = (\tau \tau)(x = a \land Fx) \).

4 Dissenting voices equally underwhelmed by the slingshot argument include especially Oppy [1997].

5 Gödel’s solution involving Russell’s theory of definite descriptions is a variation of the proposal offered here that merely appeals to Leibniz’s Law independently of Russell’s analysis. Russell’s translation of the definite description in (3) and (4) by itself does not solve the problem without further appeal to the indiscernibility of identicals, and the solution involving Leibniz’s principle does not require or logically depend on Russell’s analysis. Thus, the argument is avoided if steps (3) and (5), in our notation, are translated into Russell’s treatment of definite descriptions as follows:

(3R) \( a = \exists x[x = a \land p \land \forall y[y = a \iff x = y] \land p] \)

(5R) \( a = \exists x[x = a \land q \land \forall y[y = a \iff x = y] \land q] \)

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The slingshot depends essentially on a free exercise of stipulative identities. It applies relatively elementary logical principles together with modest conventional assumptions governing the identity relation in supposedly purely extensional logical contexts to define a universally generalizable identity statement, ostensibly concerning any propositional object \( a \) that is shown to require all facts or one all-inclusive cumulative fact as its do-all truth-maker. The difficulty in the argument is nevertheless apparent in a question that seems seldom if ever to have been asked, as to whether object \( a \) could possibly or intelligibly be defined as in (3), regardless of whether or not the truth-maker of \( p = (\text{or } \neq) \) the truth-maker of \( q \).

The slingshot is supposed to involve primitive means to achieve an impressive counter-semantic end. The slingshot certainly embodies something interesting. The question is, What? Whatever is going on in step (3) in the slingshot argument is by no means primitive or naïve, but a rather peculiar specially designed application of logical notation to proclaim an identity, and more especially the identity of a propositional object conjoined with any arbitrarily chosen true proposition. Even in Neale’s systematization of what he calls Gödel’s slingshot, built up constructively from the predicate assumption that \( Fa \), by means of definite descriptor syntax introduction and elimination, \( t\text{-INTRO} \) and \( t\text{-ELIM} \), a formula is produced that we do not actually need beyond \( Fa \). Neale does not merely introduce a definite descriptor from \( Fa \), as would plainly suffice, such as \( \iota x[Fx] \) and allow \( a = \iota x[Fx] \), or, say \( a = \iota x[\lambda y[Fy]x] \). Rather, \( t\text{-INTRO} \) is so designed that it takes any harmless predication directly into a form of the slingshot, by the principle that \( a = \iota x[x = a \land Fa] \). We do not need such formulations to be able to express constant and definite descriptor predications of properties to things. Nor are they firmly planted conceptually. Even if \( a \) has property \( F \), it does not follow that \( a \) is identical to the having of property \( F \), or even of being identical to \( a \), if objects possessing properties are in any sense distinct from the properties they possess. The slingshot argument is hard to take seriously largely because of its preposterous identity stipulations, as in \( t\text{-INTRO} \). Nevertheless, if \( t\text{-INTRO} \) can be built with what comes in the box, and what is not already included can be freely stipulated for logic to work with constructively, then logic and philosophy of logic must consider the consequences.

Stipulation is a sometimes indispensable instrument, but one that can also be too powerful for its own good. Sayso definition oversteps its bounds when it violates more firmly established logical principles, or trivializes what would otherwise be philosophically interesting implications. Bidding true identities into existence by means of logical syntax is not unrestricted, and should never be allowed to stray beyond the limits of logical consistency and such general significance requirements as noncircularity. In the slingshot argument, we cannot freely stipulate that
identities hold in flagrant violation of the predicative laws of logic, particular noncontradiction and excluded middle. We can only do so if we can independently make a strong brief against conventional identity principles, with disregard for the requirements of identicals prescribed by some form of Leibniz’s Law, and invoking more especially a version of the principle of the indiscernibility of identicals.

4. Supplementing Identity with Truth

Let us consider more scrupulously what happens in slingshot step (3). Although the argument’s inference principles themselves are minimal, there is nothing primitive or naïve about the identity statement on which the slingshot turns in this essential assumption. The slingshot itself might be a crude device, but the stone it casts in this sophisticated application is cleverly designed as something more like a logical smart missile.

Putatively, \( a \) is declared in the formula at (3) to be identical to \( \alpha \). Does this make sense? It looks to casual inspection as though in (3) \( \alpha \) is being identified with something other than itself, with \( a \) and the proposition \( p \), or perhaps the truth-maker of or state of affairs that \( p \). We describe the \( x \) such that \( x = a \) and \( p \) [is true], whereby object \( \alpha \) is said to be identical to \( a \) and or such that \( p \) is true. We may find ourselves at a loss to understand whatever this could mean, unless \( a = p \). That object \( a \) should turn out to be identical to proposition \( p \) is not necessarily problematic in and of itself. Propositions are also objects, and as such are included in the set of all objects of reference in thought and language in the logic’s referential domain. They can appear as predication relata in true or false identity statements. It must therefore be questioned whether we can freely add true propositions to an object’s identity conditions without thereby replacing reference to one object with reference to another distinct object. We would then be proceeding equivocally and hence fallaciously referring to two distinct intended objects before and after the replacement by the same propositional object name ‘\( a \)’ in the respective expressions. We can only proceed more charitably on the generous hypothesis that slingshot step (3), in order to be true, entails that \( a = p \).

The slingshot encounters a dilemma at just this point. The question is the legitimacy at step (3) of identifying object \( \alpha \) as the object identical to the conjunction of \( \alpha \) and proposition \( p \). Propositions, at least in the sense of sentence types in which the existence of a state of affairs is proposed, have identity conditions and are identical to some and not to other things, just like any other recognized category of entities. The only thinkable circumstance under which \( \alpha \) could be identified with both the thing that is identical to \( a \) and conjoined with the proposition or truth-maker of proposition \( p \), is presumably, as previously suggested, that in which \( a \)
itself is a proposition, and, in fact, in which \( a \) is identical to proposition \( p \). Truth functionally, only propositions can be conjoined together by the truth function \( \land \). If \( a \) is identical with proposition \( p \), then it could truly be extensionally identical to unlimited numbers of self-conjunctions, as we see when (3) is instantiated in a true application as: \( p = p \land p \). Else, if \( a \neq p \), then in (3) we might as well try to identify a pig with a pig and a cow. If \( a = p \), on the other hand, and if, as slingshot step (4) requires, the truth-maker of \( p \neq q \) the truth-maker of \( q \), if \( p \) and \( q \) have different truth-makers, then it must follow that \( p = q \), even if \( p \leftrightarrow q \). Although it is logically possible for \( p \) and \( q \) to have the same truth-maker even if they are two distinct propositions, \( p \neq q \), if \( p \) and \( q \) have different truth-makers, they evidently cannot be identical propositions, and we must conclude in this second dilemma horn that \( p \neq q \). From \( p \neq q \) and \( p \leftrightarrow q \), however, in an inference considered in greater detail below, it immediately follows by substitution of material equivalents \( p \leftrightarrow q \) in the presumed purely extensional nonidentity context \( p \neq q \), that \( p \neq p \) and \( q \neq q \).

This is an outright paradox, if we assume as standardly that identity is a minimally reflexive relation. The slingshot is not generally presented as a blatant logical antinomy, but as a logically consistent refutation of the correspondence theory of truth, contradicting the proposition *reductio ad absurdum* that distinct true propositions are generally made true by distinct truth-making state of affairs. The present objection proves that slingshot assumption (3) must finally be false, and hence that the argument does not soundly support the refutation of *reductio* target assumption (4) on the backs of otherwise true assumptions. The point is that the falsehood in the resulting contradiction in the proof can as well be accounted for as a consequence of the falsehood of assumption (3), and not exclusively because of the falsehood of assumption (4), that with (1)-(3), the truth-maker of \( p \neq q \) the truth-maker of \( q \). The *reductio* does not warrant rejecting the proposition that different propositions generally have different truth-makers, even if they share the same truth-value, because the contradiction can be attributed instead to the expansion in assumption or inference step of \( p \) or \( Fa \) in assumption (1), and with something like Neale’s \( \iota \)-INTRO supporting (3) in the background, so that it need not be taken as an assumption in addition to \( Fa \) (and \( Gb \)). If, contrariwise, slingshot step (3) is not meant to be true by stipulation, if it is simply an expression that could just as well be false, and if indeed it is demonstrably false, as we have just observed, then the slingshot argument in the forms we have considered does not represent sound reasoning undermining the correspondence theory of truth. The inference is not sufficiently powerful, given problems in the argument’s other assumptions, to collapse the truth-makers of every true proposition into a single massive internally indivisible truth-making state of affairs.
5. Extension to Neale’s Formulation

The problem is easily extended to recent popularly accepted slingshot formulations. Neale, to look more closely now at his reconstruction, in what might be considered the definitive statement of slingshot reasoning, assumes this starting-place, in his essay, ‘The Philosophical Significance of Gödel’s Slingshot’:

[1] \(Fa\)
[2] \(a \neq b\)
[3] \(Gb\)

Etc.\(^6\)

If \(a \neq b\), then \(Fa \neq Gb\). By truth tables, it follows as before, this time from [1] and [3], that \(Fa \leftrightarrow Gb\). Hence, the dire consequence once again obtains that \(Fa \neq Fa\) and \(Gb \neq Gb\). The problem is not that eventually Neale lands the argument in contradiction. As a reductio demonstration, that is its destination. The problem is that contradiction is reached too soon in the reconstruction.

Neale allows intersubstitution of material equivalents (not merely logical equivalents) in identity and therefore nonidentity contexts. If \(Fa\) and \(Gb\) are true, and if \(a \neq b\), then it follows first that \(Fa \leftrightarrow Gb\), and then, as indicated, with the same permissive rule of intersubstitution of material equivalents, we tumble into contradiction \(Fa \neq Fa\) and \(Gb \neq Gb\), even before the inference proceeds first to deduce from [1] the \(\iota\)-INTRO-derived expansion of \(Fa\), by which \(a = \iota x [x = a \land Fa]\). It is supposed to be exactly here that the identity statement exposes a proposition in the \(Fa\) conjunct of the identity statement, for replacement by any other material equivalent, thereby linking the truth-maker of \(Fa\) to that of any materially equivalent proposition \(Gb\). The objection is that Neale’s argument is embroiled in contradiction with the universal reflexivity of identity already in assumptions [1]-[3], prior to \(\iota\)-INTRO being brought into play.

Insofar as Neale accurately presents the logical structure of the slingshot, it appears that the argument embodies logically inconsistent assumptions, even independently of the slingshot’s main contradiction-driven reductio ad absurdum. As such, the slingshot cannot possibly represent, as it is sometimes portrayed, a sound refutation of that version of the correspondence theory of truth according to which distinct propositions are supposed to be made true by distinct truth-making facts or states of

\(^{6}\) Neale [1995], 788-789. Neale’s argument is more complex, but the present point depends only on these first three assumptions, indicated here by following the starting-place of his reconstruction of the slingshot with the marker ‘Etc.’. The interesting fact is not that Neale produces contradiction within an ultimate reductio inference structure, but that contradiction appears already in the first three assumptions before Neale has an opportunity to apply his principle \(\iota\)-INTRO of definite descriptor introduction.
affairs. The paradox suggests that there are even greater difficulties in the standard package of assumptions about propositions, their identity conditions, and especially about the extensionality of identity and nonidentity assertions.

The slingshot argument is interesting because it provides a litmus test for intensionalist-extensionalist leanings in logic and philosophical semantics. If you think that there is nothing logically wrong with the slingshot, and you are prepared to accept and build meaningful conclusions on it, then you may be somewhat of a radical extensionalist. If, on the contrary, despite your general open-mindedness, you think that slingshot inferences must be logically or materially faulty, and that in principle it should only be a technical question of discovering exactly where and how they go wrong, then at heart you are likely to be some kind of intensionalist in philosophical logic and semantics. Extensionalism is renowned for distinguishing propositional and fact identities in a rather coarse-grained way. There is nevertheless no reason even on radically extensionalist principles to suppose that the slingshot argument absurdly implies the collapse of all truth-making facts into a Parmenidean One, a single dense fact as the unified unarticulated ‘truth-maker’ for all true propositions. If the slingshot argument were sound, then the correspondence theory of truth would not be the only victim of its marksmanship, but the argument would evidently constitute the most decisive single-handed refutation of extensionalist semantics.

Another way to make the same criticism, if part only by force of repetition in different format, is to emphasize the fact that the slingshot is meant to follow a certain inferential progression. The difference this time is that we shall ignore the contradiction implicit already in assumptions (1)-(3). We take the opportunity to make explicit Neale’s use of introduction and elimination principles for the definite descriptor, which he presents in (vii) \( \tau \)-INTRO and (viii) \( \tau \)-ELIM rules:

\[
\begin{align*}
1. & \quad Fa \\
2. & \quad Gb \\
3. & \quad a \neq b \\
4. & \quad Fa \leftrightarrow Gb \\
5. & \quad a = \forall x [x = a \land Fa] \\
6. & \quad b = \forall x [x = b \land Gb] \\
7. & \quad [a = a = a \land Fa] \leftrightarrow Fa \\
8. & \quad [a = a = a \land Fa] \leftrightarrow Gb
\end{align*}
\]

(1),(2) elementary logic

(1 (vii) \( \tau \)-INTRO)

(2 (vii) \( \tau \)-INTRO)

(5 (viii) \( \tau \)-ELIM)

(4,7 elementary logic)

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\(^7\) Neale [1995], 789.
The obvious question is, Where is there to go after (8), when \(a = a = a\) drops out as a necessary consequence of the reflexivity of identity? What more of interest can be deduced, except that \(Fa \leftrightarrow Gb\), from (7) and (8), which is already explicit in (4)? Nor does the argument begin to suggest how any of these facts, undisputed in themselves, are supposed to qualify \(Gb\) as the truth-maker of ‘\(Fa\)’, or the reverse. We know that the slingshot inference is an artifact of a mistaken use of formalism, when we reformulate the essential assumption [5] as:

\[
[5']\ a = \nu[x = a \land Fa]
\]

From this we can validly instantiate:

\[
[5'']\ a = a = a \land Fa
\]

Which is to say, discounting necessary identities as contributing nothing to the proposition’s truth conditions, simply \(Fa\). If we assume that the left conjunct is \(a = a = a\), and that it drops out as guaranteed to be true by the universal reflexivity of identity, then we are right back where we started following Neale’s progression, with predication \(Fa\). We can perform this transformation of syntax endlessly, shifting back and forth by means of Neale’s \(\iota\)-INTRO and \(\iota\)-ELIM principles, without further substantive results. We shall merely change a form of expression from one mode to another and back again, without arriving at any moment in the process at any implications whatever, ontically comforting or otherwise, concerning the truth-makers of either \(Fa\) or \(Gb\).

Alternatively, among choices we have already explored in other ways and found grounds for rejecting, the inference might be parsed as \(a = Fa\). This, after desperate search, may appear to be the syntactically most sensible, perhaps the only possible, way to understand the construction. Unfortunately, it also disappoints. Since it is assumed to be true as part of the problem that \(Fa\), the putative identity statement so construed makes no sense, unless \(a\) is itself a predication, such that \(Fa \rightarrow FFa\). If \(Fa\), and \(a = Fa\), then it is already unavoidably true that \(FFa\). If it is not true or even intelligible that \(FFa\), then it is not the case that \(a = Fa\), and most logicians would sleep at night. The consequent is not standardly interpreted, let alone as true, and it appears to violate simple type theory restrictions against iterative predicative constructions of identical type. It can only be the case that \(FFa\), if \(a\) is the predication of property \(F\) to \(a\) itself, and if the predication of property \(F\) to object \(a\) has the property \(F\) of being a predication, although not a predication in such iteration directly to or of object \(a\). The question is not whether such nestings of predications \(F, FF\), and perhaps more, can be truly instantiated, but whether when they are instantiated they can truly be identical to object \(a\). The predications in question are anyway germane to metalogic and metasemantics, rather than to the truth-maker conditions of ordinary predications, such as ‘The flower is a red rose’. We cannot expect semantics to avail itself of the most questionable predicational structures.
for such elementary truth-value vehicles, merely for the sake of preserving a still more questionable slingshot objection to a correspondence truth-maker semantics that is otherwise minding its own business.

Thus, coming to terms with Neale’s $\iota$-INTRO, we are back to the riddle of the slingshot argument equivalence of trying to identify a pig with a pig plus a cow. How can any object $a$ be identified as both itself and its possession of one of its constitutive properties? It is enough that $a = a$. If $Fa$, then the definitely described object with that distinctive property of $a$ having property $F$ cannot be identical with $a$. The reason is the seasoned consideration that whereas objects have properties, they are not, in possessing them, identical to the properties they possess. I may have the property of being a horseman or of being a rational animal, but if I can no longer ride or reason, it will be I who have lost these properties, and therefore I possessing property $F$ cannot be identical with the having of property $F$, even if $F$ is the totality of all the essential properties I possess. The only relief for the application of Neale’s $\iota$-INTRO principle is if $a$ happens to be a predication, and $F$ is the property of being a predication (throughout, of a property to an object), so that $Fa$ means truly that $a$ is a predication, and $FFa$ truly means that it is a predication to say that $Fa$, that $a$ is a $F$-predication. It is also assumed without further ado that $a \neq b$ and that $Gb$. Surely, on reflection, none of these facts are enough individually or collectively to make random existent state of affairs $Gb$ a truth-maker of ‘$Fa$’.

Were that sufficient, we could manufacture the semantic absurdity of a ‘slingshot’ paradox for any truth-maker theory of truth we liked (and of truth-breakers for falsehoods), merely from the otherwise harmless assumptions that $Fa$ and $Fa \leftrightarrow Gb$. The truth-maker of ‘$Fa$’ is $E$, $Fa \leftrightarrow Gb$; therefore, the truth-maker of ‘$Gb$’ is $E$. That substitution has obviously gone awry. It is unlicensed on interchangeability salva veritate grounds, indicated syntactically by its need to transact the crucial substitution within an intensional quotation, rather than intersubstitution-supporting extensional, context. Truth-makers are associated in truth-maker semantics with mentioned ‘$Fa$’, not with used $Fa$ predications. Mentioning linguistic contexts, also for purposes of slingshot inferences, are intensional, referentially opaque, and impenetrable to intersubstitution of coreferential singular terms or materially equivalent propositions salva veritate.

The further implication is that all truth-maker correlation expressions are intensional. To say “The truth-maker of ‘$p$’ = $X$”, and that $p \leftrightarrow q$, therefore, “The truth-maker of ‘$q$’ = $X$”, is to attempt an intensional fallacy involving a substitution of materially equivalent propositions in an intensional context that is authorized exclusively for purely extensional contexts. To speak of a sentence’s or proposition’s truth-maker (or more generally of its truth conditions) is to say something more
intensionally fine-grained involving the most scrupulous application of Leibnizian identity requirements, than is available by such crude truth-functional devices as material equivalence. There appears no basis for suggesting that $Gb$ could be a truth-maker for $Fa$ or conversely, merely because they both share the truth-value True (T). That meager fact is nonetheless the sole justification for their intersubstitution in the slingshot argument. It appears that the slingshot inference is virtually played out of options by which to advance a plausible embarrassment for truth-maker semantics.

6. Slingshot Variation Defense

We might wonder whether the slingshot by proceeding in another way could not gain traction without engendering the logical contradiction that plagues the original version. Perhaps the assumptions in the customary slingshot are unnecessarily strong. We could accordingly try:

\[(4*) \beta = \nu x[x = a \land p]\]
\[(5*) \beta = \nu x[x = a \land q] \quad \text{ (as in (5))}\]
\[(6*) \nu x[x = a \land p] = \nu x[x = a \land q] \quad \text{ (as in (6))}\]

The revised argument does not support the inference to $p = q$ after step (6*), unfortunately, as it does when relying on the original (6). The fact that $\beta = a$ is not trivial implies that it does not cancel out of the extensionally identical definitely described objects in (6*) in the manner of the trivial truth-maker for $a = a$ in (6). We obtain instead in that case only $[\beta = a \land p] = [\beta = a \land q]$. From this logical nontriviality, we cannot immediately eliminate $\beta = a$, and we are prevented thereby from reducing the above identity extensionally to the equivalent of the proposition that the truth-maker of $p = \text{the truth-maker of } q$. Rather, the truth-maker of $\beta = a$ remains nontrivially on both sides of the main identity relation, and cannot drop out without excuse to leave the truth-maker of $p = \text{the truth-maker of } q$.

In this variation of the slingshot argument, we do not have the equivalent implicitly of parsing $[\beta = a] \land p$, but rather throughout of $\beta = [a \land p]$. The main difficulty with the latter parsing is that it implies that: (A) $a$, surprisingly, is a proposition that can be conjoined with other propositions like $p$, and that, even more astonishingly, (B) it follows that $a = [a \land a]$ (not to be confused with $a \leftrightarrow [a \land a]$). Whereas, there seems no reason why any friend or foe of extensionalism should care if the truth-maker of proposition $a$ is identical to the truth-maker of the logically equivalent conjunction, $a \land a$. Is that the slingshot’s best effort? If we are permitted to begin only with the nontrivial informative or significant true identity statement that $\beta = a$, then there is no deductively
valid passage to \( p = q \), or to the slingshot’s threatened conclusion that the truth-maker of random \( p \) = the truth-maker of random \( q \).

A dilemma that parallels the objection to the original slingshot is that either \( \beta = \alpha \), in which case we are back with the same problem as the original version of the slingshot. Or \( \beta \neq \alpha \), from which we cannot possibly detach the only relevant truth-maker as identical for the truth of both true propositions \( p \) and \( q \), in trying to advance beyond (6*). It is hard to imagine how such logically exhaustive and tightly mutually exclusive ‘realist’ choices at the base and conclusion of both dilemma horns could be nullified by any available parsings of essential slingshot assumption (4). The dilemma adds another burden to the classical slingshot argument that collectively begin to erode away all the argument’s typically presumed meaning, motivation and force. The viability of slingshot reasoning is not improving.

7. Streamlined Slingshot to Demonstrate the Intensionality of Identity

It appears logically possible for propositions \( p \) and \( q \) to have the same truth-maker even if they are distinct. If, on the contrary, \( p \) and \( q \) have different truth-makers, then they evidently cannot be identical propositions, and we must conclude in this second dilemma horn that in that case, \( p \neq q \). From \( p \neq q \) and \( p \leftrightarrow q \), however, it immediately follows by substitution of equivalents \( p \leftrightarrow q \) in the purely extensional nonidentity context \( p \neq q \), that \( p \neq p \) and \( q \neq q \). The argument, an even more compact reformulation of the classical slingshot, has this form:

Suppose that proposition \( p \) is true and proposition \( q \) is true, and that they are different propositions, \( p \neq q \).

Let \( p \) be the true proposition expressed by the sentence ‘Snow is white’, for example, and \( q \) the manifestly distinct true proposition expressed by the sentence ‘Grass is green’.

From \( p \land q \), assuming that both propositions are true, it follows by truth table definitions of the propositional connectives that materially \( p \leftrightarrow q \).

We are authorized to uniformly intersubstitute material or logical equivalents in any purely extensional context, \textit{salva veritate}.

Assume for the sake of argument, and, ultimately, as a hypothesis for \textit{reductio ad absurdum}, that identity, and hence also nonidentity expressions are purely extensional, or at least that the above nonidentity statement, \( p \neq q \), is purely extensional.

Under these assumptions, and from \( p \neq q \) and \( p \leftrightarrow q \) in particular, we can immediately derive \( p \neq p \) and \( q \neq q \), contradicting the conventional assumption of the reflexivity of identity.
The argument can also be more rigorously presented:

**Background assumptions:**

(E) \(=\) and \(\neq\) are purely extensional relations, in the sense that their corresponding expressive linguistic contexts in such identity and nonidentity statements as \(a = a\), \(a = \beta\) and \(a \neq \beta\) support the intersubstitution of coreferential terms and materially equivalent propositions *salva veritate*.  

(Extensionality of \(=\))

(R) \(\forall x [x = x]\)  

(Universal Reflexivity of \(=\))

**Modified slingshot counterexample to extensionality of \(=\) and \(\neq\):**

(1) \(p = \) Snow is white.  

(2) \(q = \) Grass is green.  

(3) \(p \neq q\)

(4) \(p \land q\)  

(Truth of (1) and (2))

(5) \(p \leftrightarrow q\)  

(From (4) in standard propositional logic)

(6) \(p \neq p \land q \neq q\)  

(((3),(5),(E))

Together with background assumption (R), we finally produce a *reductio* of background assumption (E):

(7) \(p = p\)  

(R)

(8) NOT-(E): \(=\) and \(\neq\) are not purely extensional relations, in the sense that their corresponding expressive linguistic contexts in such identity and nonidentity statements as \(a = a\), \(a = \beta\) and \(a \neq \beta\) do not support the intersubstitution of coreferential terms and materially equivalent propositions *salva veritate*.

(((E),(6),(7) *Reductio ad absurdum*)

If we assume as standardly that identity is a minimally reflexive relation, then we can mount a more comprehensive version of the slingshot argument that also covers but is not limited to definite description formulations. Shall we accept the conclusion that identity is an intensional rather than extensional relation? Or argue that the conclusion is an insufferable paradox, blatantly at odds with the intuitively obvious and logically indispensable truth that identity is extensional, and, indeed, that identity is an extensional relation *par excellence*?

The slingshot is not generally presented as a logical antinomy, but more conservatively as a logically consistent refutation of the correspondence theory of truth, contradicting the proposition that every distinct true proposition corresponds to a distinct truth-making state of affairs. The present objection proves that slingshot assumption (4) must finally be false, and hence that the argument does not soundly support an identification of the truth-makers of arbitrary distinct but jointly true and
hence materially equivalent propositions. If, contrariwise, slingshot step (4) is not meant to be true by stipulation, if it is simply an expression that could just as well be false, and if indeed it is demonstrably false, as we have just seen, then the slingshot argument so interpreted and applied does not represent sound reasoning undermining the correspondence theory of truth by conflating the truth-makers of every true proposition into an unarticulated single massive truth-making state of affairs.

The only obvious solutions to the paradoxical implications generated within the \textit{reductio} and based on its hypothesis are:

(i) Denying the assumption that \( p \neq q \), insisting that the proposition expressed by the sentence ‘Grass is green’ is after all identical to the proposition expressed by the sentence ‘Snow is white’.

(ii) Undoing the elementary propositional logic and classical truth table definitions of the truth functional connectives by which a conjunction logically implies a biconditional or logical equivalence.

(iii) Denying the assumption that identity and nonidentity contexts like \( p = q \) and \( p \neq q \) are considered to be purely extensional, and acknowledging instead that identities are always intensional expressions that do not support intersubstitution of coreferential terms or logically equivalent sentences \textit{salva veritate}.

Of these choices, and it is not clear that there are any others, (iii) is clearly preferable. It implies the independently supportable classification of identity and nonidentity contexts as intensional rather than purely extensional, given that Leibnizian property-based identity conditions are for that reason alone rightly categorized as intensional. An intensional semantics ontically prioritizes properties over objects and defines particular objects as having particular combinations of properties. Whereas an extensional semantics ontically prioritizes objects over their properties, and therefore acknowledges only existent objects, whose properties are given extensionally by virtue of inclusion in or exclusion from the extension of all existent objects with just those properties.

We can say in independent support of (iii) that in the relevant sense any foundationally property-based Leibnizian identity principles are by definition intensional. They stand in marked contrast to the existent object ontic prioritizing of an extensionalist semantics in which objects come before their properties, and the only theoretical burden is to offer set theoretical conditions for the truth-makers of propositions asserting that an object has or objects have a certain property, their inclusion in or exclusion from the extension of all those existent objects with exactly that property. The truth is that where identity conditions for objects are concerned, extensionalists generally accept Leibnizian property-based
identity principles. They typically do so, however, without realizing or in any case without acknowledging that in so doing they have contaminated their effort at maintaining a pure logical and semantic extensionalism with a healthy dose of intensionalism at the very logical core of their supposedly purely extensionalist semantics. They do not inquire as to what they are philosophically committing themselves to by adopting the Leibnizian identity of indiscernibles or indiscernibility of identicals as identity conditions for the existent objects on which any extensionalist semantics ultimately depends.

The solution in (i), besides being counterintuitive, trivializes all slingshot inferences. Slingshots have interest and force only if there are multiple distinct true propositions that a slingshot is able to prove surprisingly have the same truth-maker. If \( p = q \), if the proposition expressed by the sentence that ‘Snow is white’ is identical after all to the proposition expressed by the sentence that ‘Grass is green’, then there is nothing paradoxical about their having the same truth-maker.

Solution (ii) is drastic, and in a sense just as soundly breaks the back of extensionalism at another vertebra than accepting the proposal in (iii). The proposition that identity and nonidentity contexts are intensional is perfectly in keeping with the distinction between extensionality and intensionality, whereby what is extensional is ultimately logically and ontically committal (existent) object-prioritizing, and what is intensional is ultimately logically and ontically neutral property-prioritizing. Leibniz’s law in both its component conditions is by this distinction evidently intensional, identifying and individuating objects by virtue of their possession of a distinctive set of constitutive properties. The metasemantic proposition, “The truth-maker of \( 'p' = X' \) is intensional, partly and as a failsafe, because \( 'p' \) in single mentioning-citation quotes does not admit equivalents salva veritate, but also because the identity = relation and its expressive linguistic contexts is always intensional. The same can be said for the intensionality attributed to modal contexts by Quine in this formulation of his famous example:

\[
\begin{align*}
\text{(Q1) The number of planets} & = 9. \\
\text{(Q2) } & \Box (9 > 7) \\
\implies & \Box (\text{The number of planets} > 7)
\end{align*}
\]

Intensionality there may be somewhere in assumptions (Q1) or (Q2). It looks superficially, moreover, as though substitution failure salva veritate can only be laid at the door of assumption (Q2). The explicitly modal context into which the intersubstitution of ‘The number of planets’ for ‘9’ in (Q2) is attempted on the strength of the identity in (Q1), appears on the usual background assumptions how it is that we can proceed by syntactical substitution from true assumptions to a false conclusion. If the
identity context in (Q1) is intensional, however, as argued, then there is an alternative explanation for the substitution failure in (Q3), involving the intensionality or referential opacity of identity = contexts, rather than of modal □ contexts.

8. Philosophical Implications of Classical and Streamlined Slingshots

The further conclusion of this style of argument is to prove that identity is intensional rather than purely extensional. The following paradox is inspired by but appears logically far more toxic and at the same time more minimal than the slingshot argument.

Suppose that proposition \( p \) is true and proposition \( q \) is true, and that they are different propositions, \( p \neq q \). Perhaps \( p \) is the true proposition that Snow is white, and \( q \) is the presumably distinct true proposition that Grass is green. From \( p \land q \) it follows by truth table definitions of the propositional connectives that materially, \( p \leftrightarrow q \). We are authorized to uniformly intersubstitute logical or material equivalents in any purely extensional context, \( \textit{salva veritate} \). So, we assume for the sake of argument, and, finally, as a hypothesis for reductio, that identity and nonidentity expressions are purely extensional, or at least that the above nonidentity statement, \( p \neq q \), is purely extensional. Under these assumptions, from \( p \neq q \) and \( p \leftrightarrow q \), we can once again immediately derive the unwanted conclusions \( p \neq p \) and \( q \neq q \), contradicting universal reflexivity of identity assumptions.

The proposition that identity and nonidentity contexts are intensional is perfectly in keeping with the distinction between the extensional and intensional, whereby what is extensional is ultimately logically and ontically object-prioritizing, whereas what is intensional is ultimately logically and ontically property-prioritizing. The stripped-down version of the slingshot construed as a \( \textit{reductio} \) of the assumption that identity is extensional, leaves us with the comfortable conclusion that identity conditions formulated as variations of Leibniz’s Law effectively define objects by defining their identity conditions in terms of properties. To do so is necessarily to logically and ontically prioritize properties over objects, which is the hallmark of intensionalism. Identity, like reference, predication, and quantification, accordingly joins the list of intensional rather than purely extensional logical and semantic relations in a comprehensive philosophical logic.\(^8\)

\(^8\) A much condensed preliminary version of this essay was presented at the international philosophical conference, Crossing Borders, Austrian Society for Philosophy, Vienna, Austria, June 2-4, 2011. I am grateful to participants at the session for useful comments and criticisms, and to an anonymous journal referee who provided valuable suggestions leading to the essay’s improvement.
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